

Hadronic Top quark polarimetry with particleNet

“playing tag with the down quark at the $t\bar{t}$ playground”

Jules Vandenbroeck

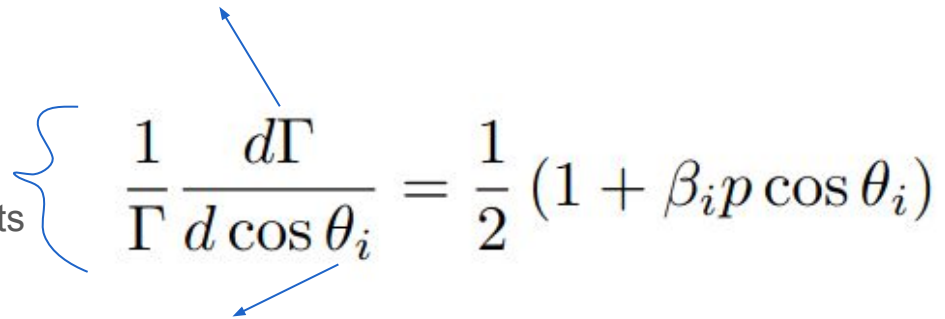
Motivation

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_i} = \frac{1}{2} (1 + \beta_i p \cos \theta_i)$$

Motivation

partial decay width of top quark

angular distribution
of the decay products



The diagram shows the equation $\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_i} = \frac{1}{2} (1 + \beta_i p \cos \theta_i)$. A blue arrow points from the text 'partial decay width of top quark' to the $d\Gamma$ in the numerator. Another blue arrow points from the text 'angle of the top quark decay product with the top spin axis in the top quark rest frame' to the θ_i in the denominator. A blue curly bracket is placed to the left of the fraction, with the text 'angular distribution of the decay products' next to it.

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_i} = \frac{1}{2} (1 + \beta_i p \cos \theta_i)$$

angle of the top quark decay
product with the top spin axis in
the top quark rest frame

Motivation

angular distribution of the decay products

partial decay width of top quark

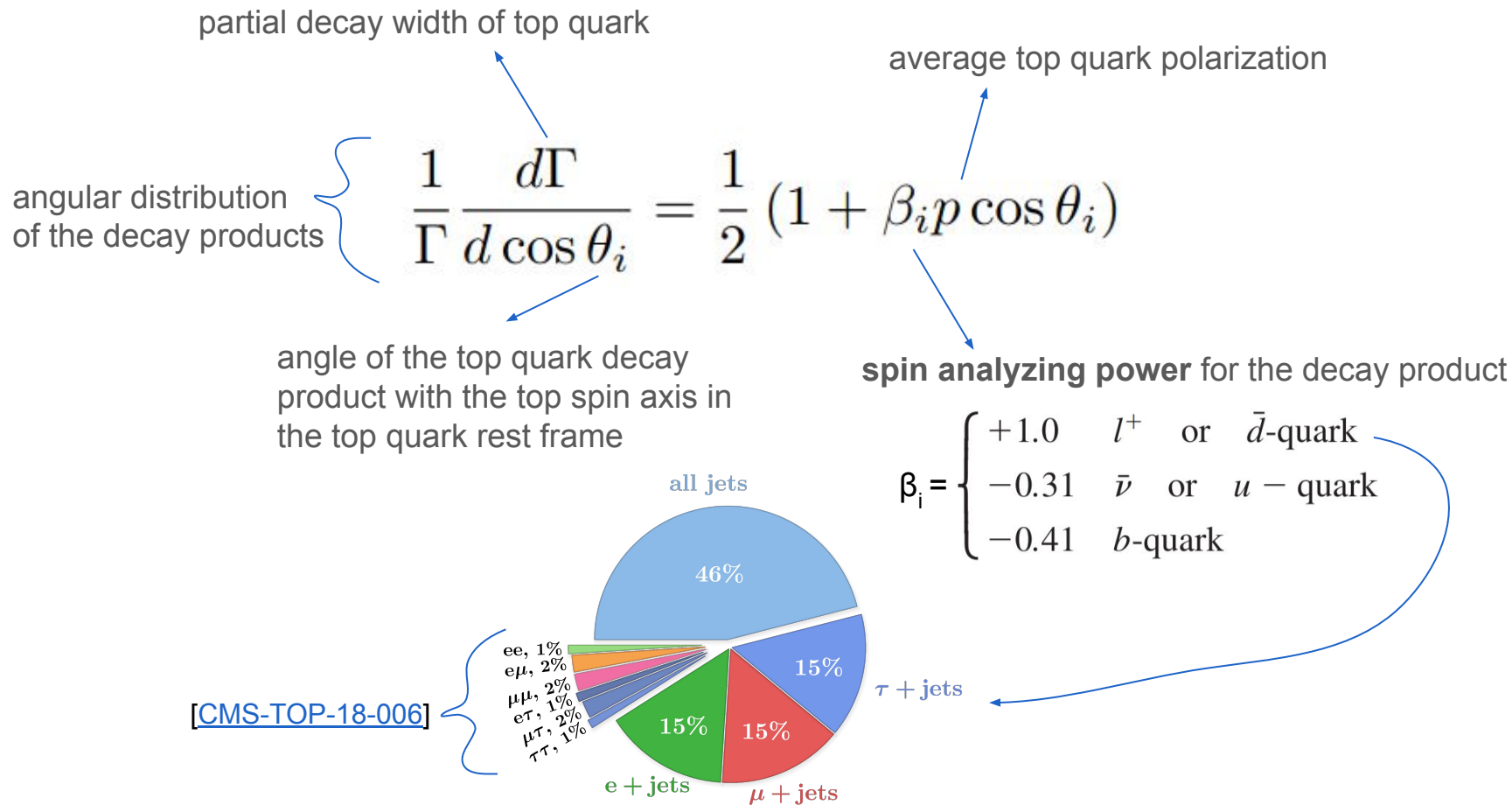
average top quark polarization

angle of the top quark decay product with the top spin axis in the top quark rest frame

spin analyzing power for the decay product

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_i} = \frac{1}{2} (1 + \beta_i p \cos\theta_i)$$
$$\beta_i = \begin{cases} +1.0 & l^+ \text{ or } \bar{d}\text{-quark} \\ -0.31 & \bar{\nu} \text{ or } u\text{-quark} \\ -0.41 & b\text{-quark} \end{cases}$$

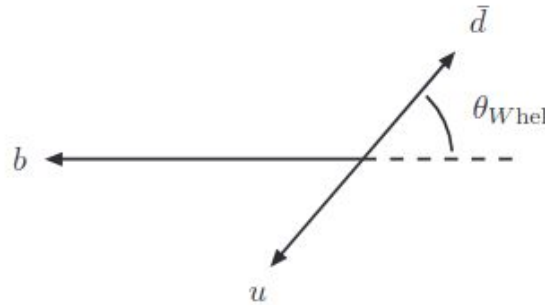
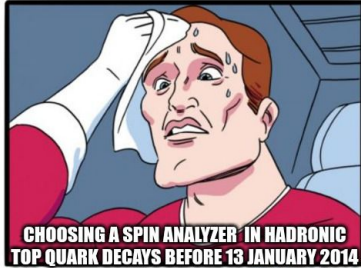
Motivation



History of down tagging

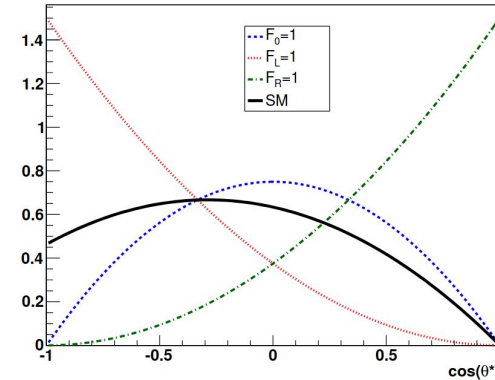
we want to maximize spin analyzing power!

“W-bosons in top decay are produced on average with negative helicity, favoring the down-type quark to be emitted closer to the b-quark”



$$f_0 \simeq \frac{m_t^2}{m_t^2 + 2m_W^2} \simeq 0.70$$

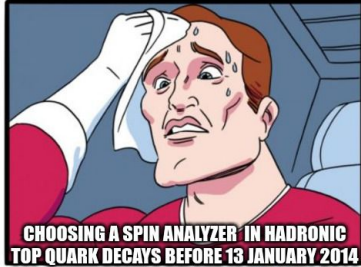
$$f_L \simeq \frac{2m_W^2}{m_t^2 + 2m_W^2} \simeq 0.30,$$



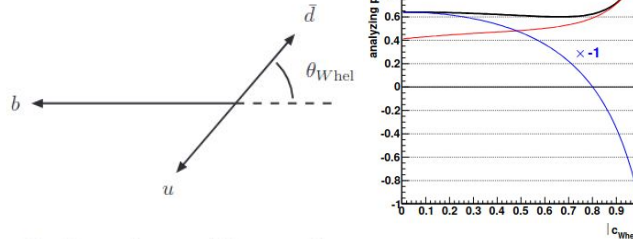
1994

History of down tagging ○ ○ ○

*we want to maximize spin
analyzing power!*



Better Hadronic Top Quark Polarimetry



$$\vec{q}_{\text{opt}}(|c_{W^{\text{hel}}}|) \equiv p(d \rightarrow q_{\text{soft}})\hat{q}_{\text{soft}} + p(d \rightarrow q_{\text{hard}})\hat{q}_{\text{hard}}$$

$$\kappa_{\text{opt}}(|c_{W^{\text{hel}}}|) = |\vec{q}_{\text{opt}}(|c_{W^{\text{hel}}}|)|$$

$$\simeq \frac{\sqrt{s_{W^{\text{hel}}}^4 m_t^2 (m_t^2 - 2m_W^2) + (1 + c_{W^{\text{hel}}}^2)^2 m_W^4}}{s_{W^{\text{hel}}}^2 m_t^2 + (1 + c_{W^{\text{hel}}}^2) m_W^2},$$

Spin Analyzer	Power
optimal hadronic	0.64

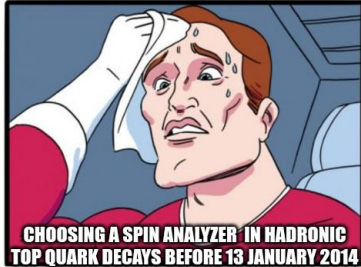
1994

2014

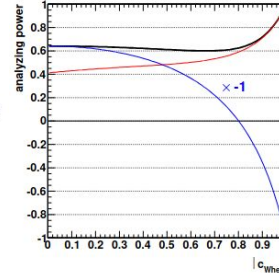
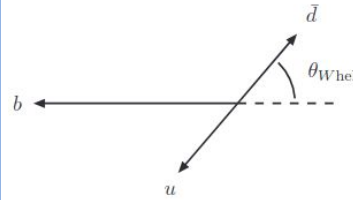
²Methods for measuring the charge of the progenitor quark do exist, but are not very statistically powerful for separating charge +1/3 from +2/3 (see [43]). It might nonetheless be interesting to explore what further gains could be achieved by folding in this information.

History of down tagging ○ ○ ○

*we want to maximize spin
analyzing power!*



Better Hadronic Top Quark Polarimetry

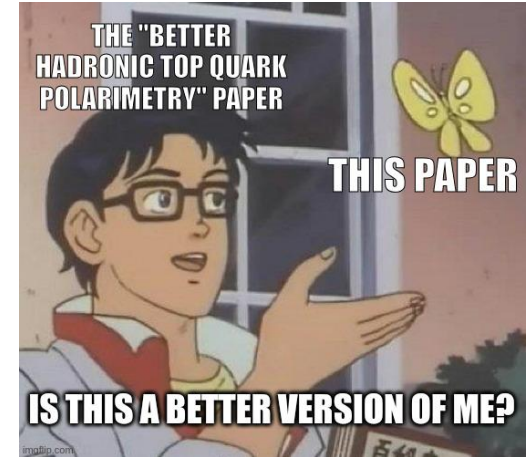


$$\vec{q}_{\text{opt}}(|c_{W\text{hel}}|) \equiv p(d \rightarrow q_{\text{soft}})\hat{q}_{\text{soft}} + p(d \rightarrow q_{\text{hard}})\hat{q}_{\text{hard}}$$

$$\kappa_{\text{opt}}(|c_{W\text{hel}}|) = |\vec{q}_{\text{opt}}(|c_{W\text{hel}}|)|$$

$$\simeq \frac{\sqrt{s_{W\text{hel}}^4 m_t^2 (m_t^2 - 2m_W^2) + (1 + c_{W\text{hel}}^2)^2 m_W^4}}{s_{W\text{hel}}^2 m_t^2 + (1 + c_{W\text{hel}}^2) m_W^2}$$

Spin Analyzer	Power
optimal hadronic	0.64



1994

2014

NOW

²Methods for measuring the charge of the progenitor quark do exist, but are not very statistically powerful for separating charge +1/3 from +2/3. It might nonetheless be interesting to explore what further gains could be achieved by folding in this information.

The maths...

including non-kinematic observables $\{\mathcal{O}\}$ other than helicity

$$\vec{q}_{\text{opt}} = p(d \rightarrow q_{\text{hard}} | c_W, \{\mathcal{O}\}) \hat{q}_{\text{hard}} + p(d \rightarrow q_{\text{soft}} | c_W, \{\mathcal{O}\}) \hat{q}_{\text{soft}} .$$

...

$$\langle |\vec{q}_{\text{opt}}|^2 \rangle_{\mathcal{O}} = 1 - 2p(d \rightarrow q_{\text{hard}} | c_W) p(d \rightarrow q_{\text{soft}} | c_W) (1 - \hat{q}_{\text{hard}} \cdot \hat{q}_{\text{soft}})$$

$$\times \left[p(d \rightarrow q_{\text{hard}} | c_W) \int d\mathcal{L}_{\mathcal{O}} \frac{\mathcal{L}_{\mathcal{O}} p(\mathcal{L}_{\mathcal{O}} | d \rightarrow q_{\text{hard}})}{(p(d \rightarrow q_{\text{hard}} | c_W) + \mathcal{L}_{\mathcal{O}} p(d \rightarrow q_{\text{soft}} | c_W))^2} + p(d \rightarrow q_{\text{soft}} | c_W) \int d\mathcal{L}_{\mathcal{O}} \frac{\mathcal{L}_{\mathcal{O}}^2 p(\mathcal{L}_{\mathcal{O}} | d \rightarrow q_{\text{hard}})}{(p(d \rightarrow q_{\text{hard}} | c_W) + \mathcal{L}_{\mathcal{O}} p(d \rightarrow q_{\text{soft}} | c_W))^2} \right] .$$

...

$$\langle |\vec{q}_{\text{opt}}|^2 \rangle_{\mathcal{O}} = 1 \quad (15)$$

$$\begin{aligned} & - 2p(d \rightarrow q_{\text{hard}} | c_W) p(d \rightarrow q_{\text{soft}} | c_W) (1 - \hat{q}_{\text{hard}} \cdot \hat{q}_{\text{soft}}) \\ & + 2\sigma^2 p(d \rightarrow q_{\text{hard}} | c_W)^2 p(d \rightarrow q_{\text{soft}} | c_W)^2 (1 - \hat{q}_{\text{hard}} \cdot \hat{q}_{\text{soft}}) \\ & + \dots \end{aligned}$$

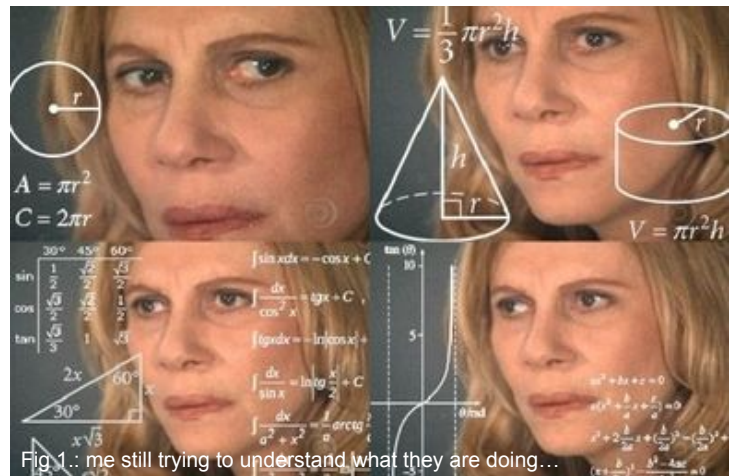


Fig.1.: me still trying to understand what they are doing...

The maths...

including non-kinematic observables $\{O\}$ other than helicity

$$\vec{q}_{\text{opt}} = p(d \rightarrow q_{\text{hard}} | c_W, \{O\}) \hat{q}_{\text{hard}} + p(d \rightarrow q_{\text{soft}} | c_W, \{O\}) \hat{q}_{\text{soft}} .$$

...

$$\langle |\vec{q}_{\text{opt}}|^2 \rangle_O = 1 - 2p(d \rightarrow q_{\text{hard}} | c_W) p(d \rightarrow q_{\text{soft}} | c_W) (1 - \hat{q}_{\text{hard}} \cdot \hat{q}_{\text{soft}})$$

$$\times \left[p(d \rightarrow q_{\text{hard}} | c_W) \int d\mathcal{L}_O \frac{\mathcal{L}_O p(\mathcal{L}_O | d \rightarrow q_{\text{hard}})}{(p(d \rightarrow q_{\text{hard}} | c_W) + \mathcal{L}_O p(d \rightarrow q_{\text{soft}} | c_W))^2} + p(d \rightarrow q_{\text{soft}} | c_W) \int d\mathcal{L}_O \frac{\mathcal{L}_O^2 p(\mathcal{L}_O | d \rightarrow q_{\text{hard}})}{(p(d \rightarrow q_{\text{hard}} | c_W) + \mathcal{L}_O p(d \rightarrow q_{\text{soft}} | c_W))^2} \right] .$$

...

$$\langle |\vec{q}_{\text{opt}}|^2 \rangle_O = 1 \quad (15)$$

$$- 2p(d \rightarrow q_{\text{hard}} | c_W) p(d \rightarrow q_{\text{soft}} | c_W) (1 - \hat{q}_{\text{hard}} \cdot \hat{q}_{\text{soft}})$$

$$+ 2\sigma^2 p(d \rightarrow q_{\text{hard}} | c_W)^2 p(d \rightarrow q_{\text{soft}} | c_W)^2 (1 - \hat{q}_{\text{hard}} \cdot \hat{q}_{\text{soft}})$$

$$+ \dots$$

depends on the non-kinematic observables $\{O\}$

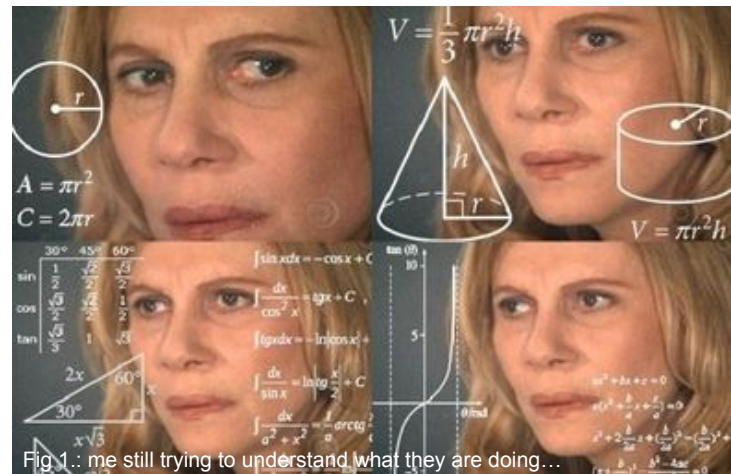


Fig.1.: me still trying to understand what they are doing...

Term is strictly positive

The maths...

including non-kinematic observables $\{O\}$ other than helicity

$$\sqrt{\langle |\vec{q}_{\text{opt}}|^2 \rangle} \approx 0.643 + 0.163\sigma^2 + \dots$$

σ^2 is the variance of $p(\mathcal{L}_O | d \rightarrow q_{\text{hard}})$

$$\mathcal{L}_O \equiv \frac{p(\{O\} | d \rightarrow q_{\text{soft}})}{p(\{O\} | d \rightarrow q_{\text{hard}})}.$$



Analysis

Samples:

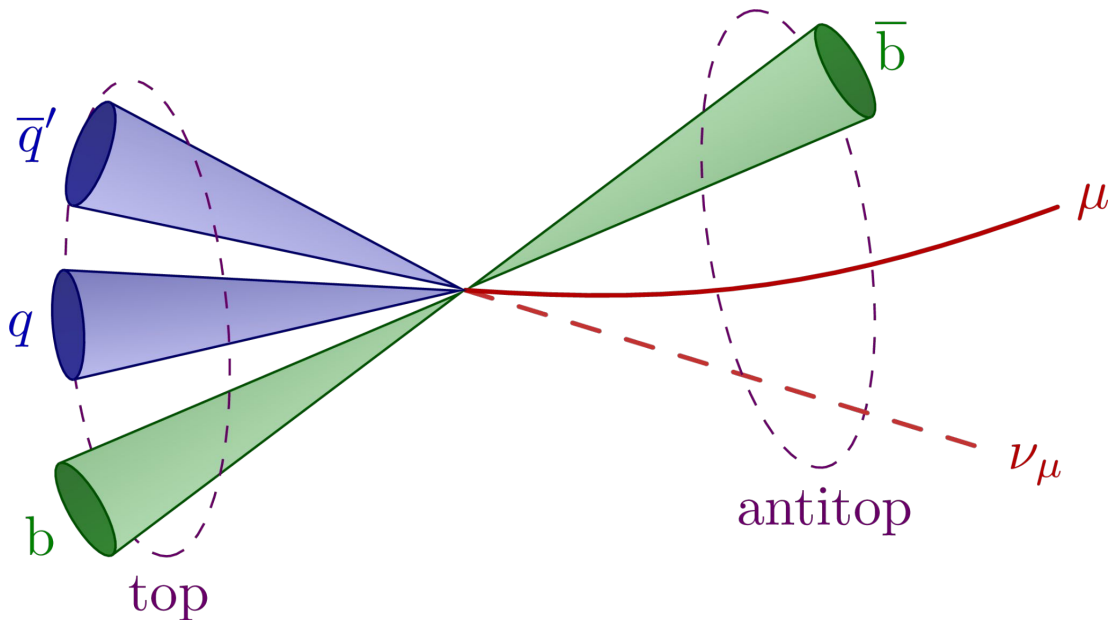
- semileptonic $t\bar{t}$ samples at 14 TeV (left-, right-, and unpolarized top quarks)
- gen-level top $p_T > 200$ GeV

Objects:

- all particles ($|\eta| < 3$ and $p_T > 1$ GeV)
- CA fatjet ($R=1.5$) with $p_T > 250$ GeV
- declustering the fatjet
→ ≥ 3 subjects ($m_{\text{subject}} < 30$ GeV)

Selection:

- hardest 4 subjects
- top quark reconstruction with 3rd or 4th jet, $m(j_1, j_2, j_{3/4}) \in [165, 190]$ GeV
- parton matching



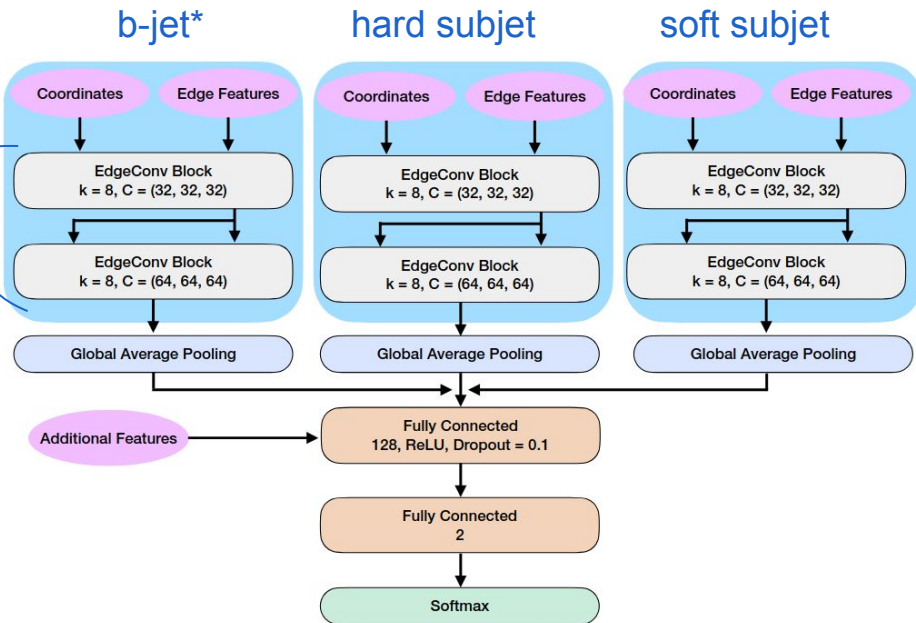
Analysis

Fancy Graph Neural Network

- modified the Particle Net architecture
- trained on unpolarized samples

constituents of the subject

modified
Particle Net



*In no way they state how they tag the b-jet... (I assume gen-level information and one should keep in mind the b-tagging efficiency when applying on reco level?)

Analysis

constituents of the subjet

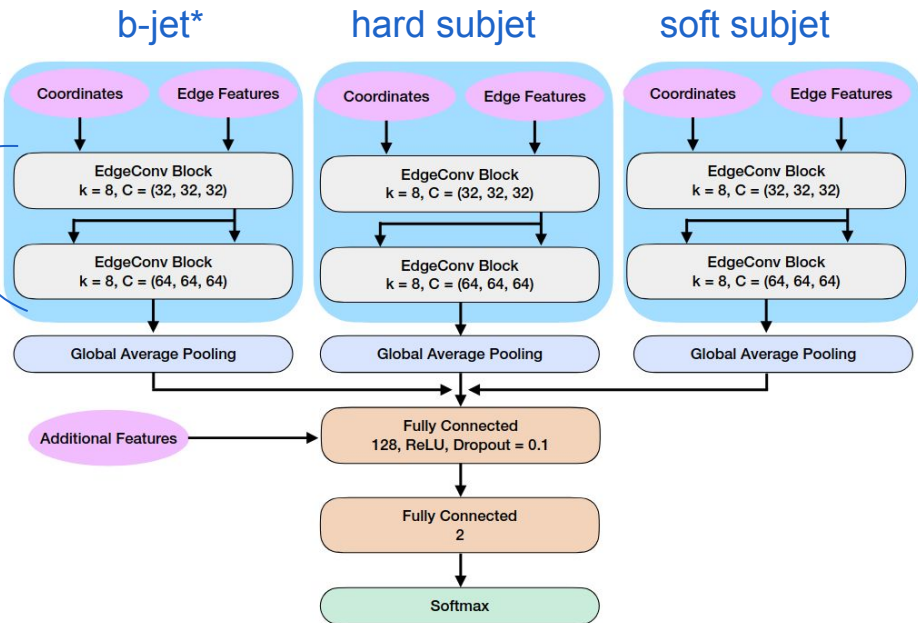
Fancy Graph Neural Network

- modified the Particle Net architecture
- trained on unpolarized samples

modified
Particle Net

Variable	Definition
$\Delta\eta_t$	difference in pseudorapidity between the particle and the top jet axis
$\Delta\phi_t$	difference in azimuthal angle between the particle and the top jet axis
$\Delta\eta_j$	difference in pseudorapidity between the particle and the subjet axis
$\Delta\phi_j$	difference in azimuthal angle between the particle and the subjet axis
$\log p_T$	logarithm of the particle's p_T
$\log E$	logarithm of the particle's Energy
q	electric charge of the particle
isElectron	if the particle is an electron
isMuon	if the particle is a muon
isPhoton	if the particle is a photon
isChargedHadron	if the particle is a charged hadron
isNeutralHadron	if the particle is a neutral hadron

TABLE I: Input features in GNN: the positions of points in the graph (top) and the attributes of each particle (bottom).



*In no way they state how they tag the b-jet... (I assume gen-level information and one should keep in mind the b-tagging efficiency when applying on reco level?)

Analysis

constituents of the subjet

Fancy Graph Neural Network

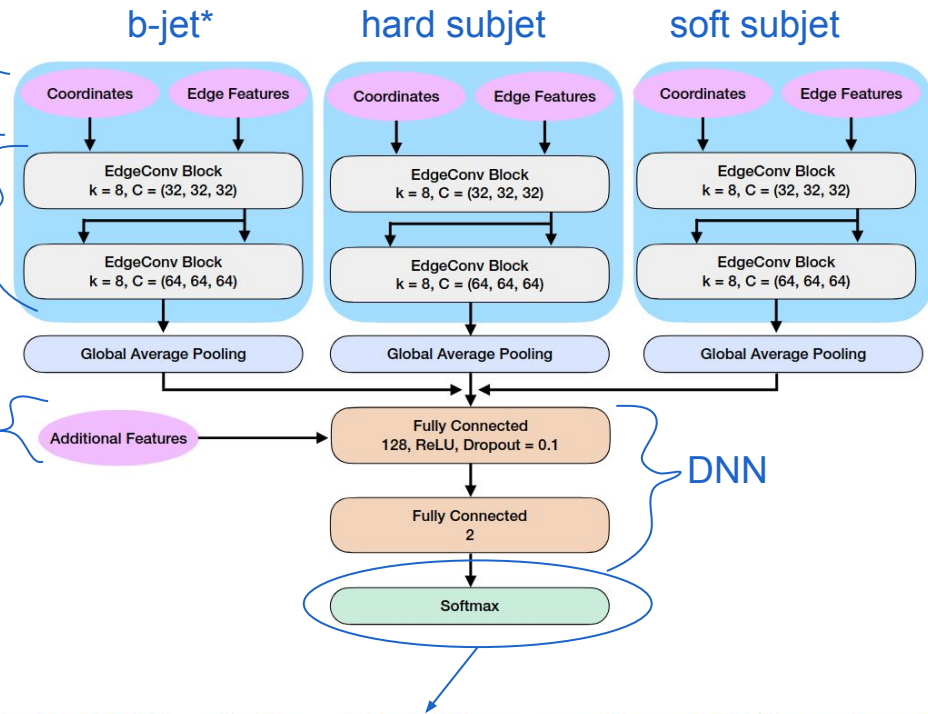
- modified the Particle Net architecture
- trained on unpolarized samples

Variable	Definition
$\Delta\eta_t$	difference in pseudorapidity between the particle and the top jet axis
$\Delta\phi_t$	difference in azimuthal angle between the particle and the top jet axis
$\Delta\eta_j$	difference in pseudorapidity between the particle and the subjet axis
$\Delta\phi_j$	difference in azimuthal angle between the particle and the subjet axis
$\log p_T$	logarithm of the particle's p_T
$\log E$	logarithm of the particle's Energy
q	electric charge of the particle
isElectron	if the particle is an electron
isMuon	if the particle is a muon
isPhoton	if the particle is a photon
isChargedHadron	if the particle is a charged hadron
isNeutralHadron	if the particle is a neutral hadron

TABLE I: Input features in GNN: the positions of points in the graph (top) and the attributes of each particle (bottom).

modified Particle Net

W helicity



the probability of the soft $p(d \rightarrow q_{\text{soft}}|c_W, \{\mathcal{O}\})$ or hard subjet $p(d \rightarrow q_{\text{hard}}|c_W, \{\mathcal{O}\})$ being the down-type jet. In

*In no way they state how they tag the b-jet... (I assume gen-level information and one should keep in mind the b-tagging efficiency when applying on reco level?)

Analysis

Fancy Graph Neural Network

- modified the Particle Net architecture
- trained on unpolarized samples

constituents of the subjet

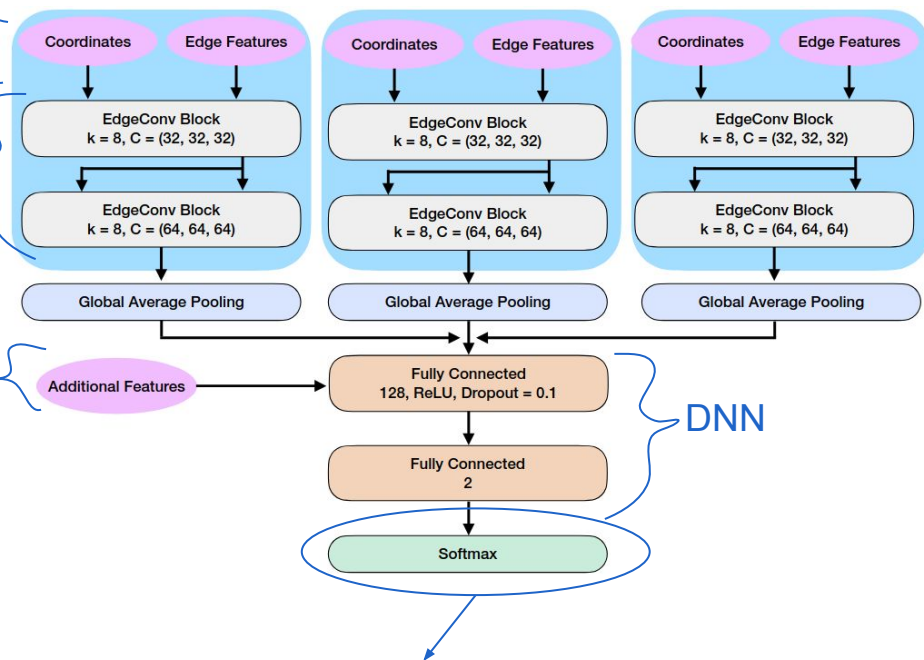
modified
Particle Net

W helicity

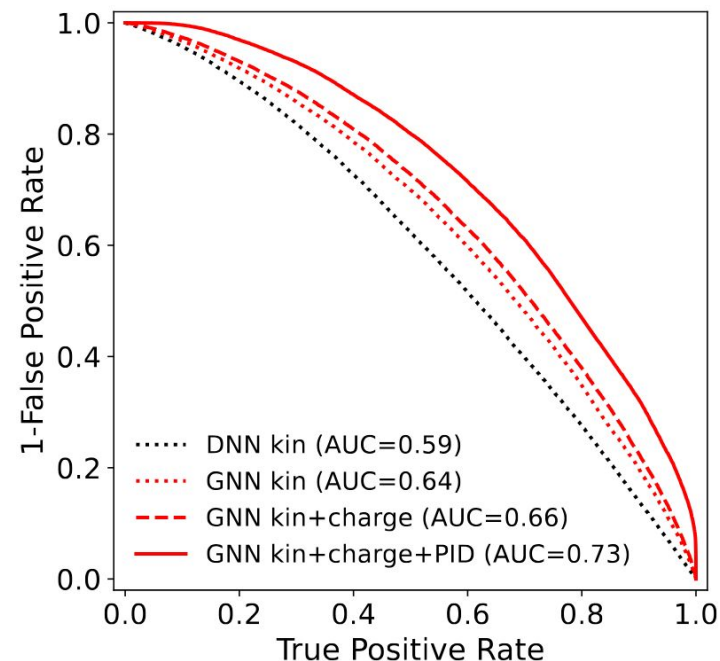
b-jet*

hard subjet

soft subjet



the probability of the soft $p(d \rightarrow q_{\text{soft}} | c_W, \{\mathcal{O}\})$ or hard subjet $p(d \rightarrow q_{\text{hard}} | c_W, \{\mathcal{O}\})$ being the down-type jet. In



*In no way they state how they tag the b-jet... (I assume gen-level information and one should keep in mind the b-tagging efficiency when applying on reco level?)

Analysis

constituents of the su

b-jet*

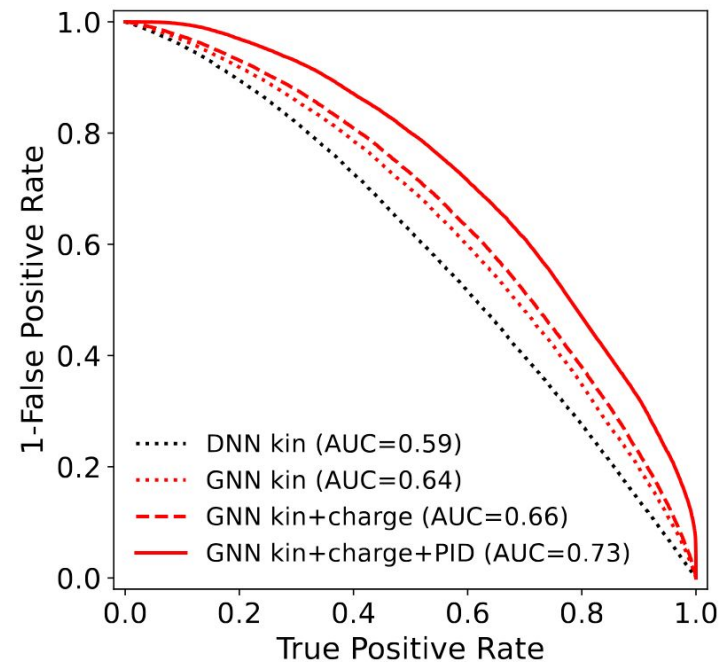
hard subjet

soft subjet

Fancy Graph Neural Network

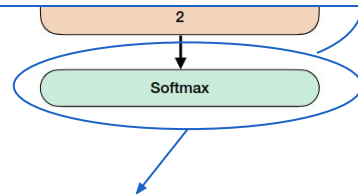
- modified the Particle Net architecture
- trained on unpolarized samples

mod
Par



	β_{opt}^{tL}	β_{opt}^{tR}
DNN _{Eff=100%}	0.622	0.625
GNN _{Eff=100%}	0.678	0.685
GNN _{Eff=50%}	0.751	0.758
GNN _{Eff=20%}	0.863	0.869

TABLE II: Spin analyzing power for different methods and efficiencies.



the probability of the soft $p(d \rightarrow q_{\text{soft}}|c_W, \{\mathcal{O}\})$ or hard subjet $p(d \rightarrow q_{\text{hard}}|c_W, \{\mathcal{O}\})$ being the down-type jet. In

$$\vec{q}_{\text{opt}} = p(d \rightarrow q_{\text{hard}}|c_W, \{\mathcal{O}\}) \hat{q}_{\text{hard}} + p(d \rightarrow q_{\text{soft}}|c_W, \{\mathcal{O}\}) \hat{q}_{\text{soft}} .$$

Conclusion

down tagging in top decays

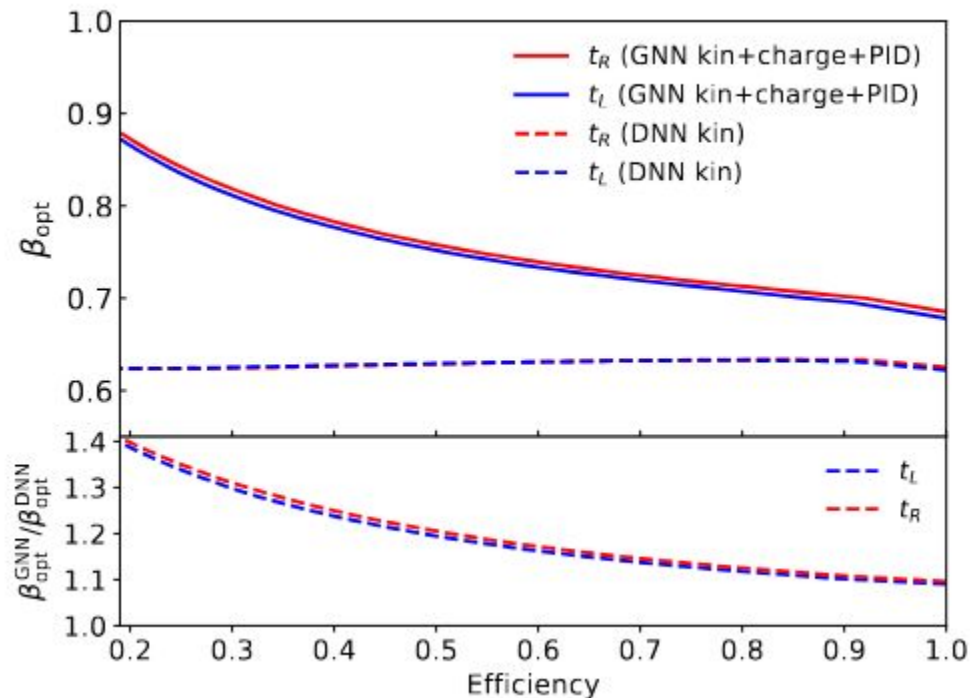
- GNN for tagging
- jet substructure kinematics and PID most important for down tagging

top quark polarization

- tagged down-type jet can be used as a proxy for top quark spin in $t\bar{t}$ semileptonic
- boost the top quark precision physics studies

Future Outlook?

- incorporate charm tagging since half of the hadronic top decays to charm quarks.
- actually be used in a measurement



Hadronic Top quark polarimetry with particleNet

“playing tag with the down quark at the $t\bar{t}$ playground”

Jules Vandenbroeck

Appendix

W 's polarization causes $c_{W\text{hel}}$ to be distributed as

$$\rho(c_{W\text{hel}}) \equiv \frac{3}{8}f_R(1+c_{W\text{hel}})^2 + \frac{3}{4}f_0(1-c_{W\text{hel}}^2) + \frac{3}{8}f_L(1-c_{W\text{hel}})^2, \quad (1)$$

3

where f_R , f_0 , and f_L are respectively the fractions of right-handed helicity, zero helicity, and left-handed helicity W bosons in top-frame. In the $V - A$ electroweak theory, f_R is nearly zero, and

$$\begin{aligned} f_0 &\simeq \frac{m_t^2}{m_t^2 + 2m_W^2} \simeq 0.70 \\ f_L &\simeq \frac{2m_W^2}{m_t^2 + 2m_W^2} \simeq 0.30, \end{aligned} \quad (2)$$

in the approximation $m_b = 0$ and taking $m_t = 172$ GeV. By the approximate CP-invariance of the decay, anti-tops have a nearly identical distribution.

Appendix

For the DNN implementation, we input the four-momenta as (p_T, η, ϕ, E) of the b -jet, harder jet, and softer jet in that order. Additionally, the helicity angle in the top rest frame is included as an input feature, resulting in a total of 13 input features. The binary label given to each event indicates whether the harder jet corresponds to the down-type jet. The network architecture consists of three hidden layers, each with 32 dimensions with RELU activation function. The output layer is one-

Table I. After the Edge Convolution blocks, each graph is pooled and flattened in the same manner, then concatenated into a single linear input of total dimension of 192. The helicity angle is included as a supplementary feature, by concatenating it with the linear layer following the edge convolutions. The combined inputs are then fed into a fully connected linear layer with 128 neurons before the output layer. The network architecture is depicted in Fig. 1. The subjects' input order and labeling are consistent with the DNN case. Both networks are optimized using the Adam optimizer.

W 's polarization causes $c_{W\text{hel}}$ to be distributed as

$$\rho(c_{W\text{hel}}) \equiv \frac{3}{8}f_R(1+c_{W\text{hel}})^2 + \frac{3}{4}f_0(1-c_{W\text{hel}}^2) + \frac{3}{8}f_L(1-c_{W\text{hel}})^2, \quad (1)$$

3

where f_R , f_0 , and f_L are respectively the fractions of right-handed helicity, zero helicity, and left-handed helicity W bosons in top-frame. In the $V - A$ electroweak theory, f_R is nearly zero, and

$$\begin{aligned} f_0 &\simeq \frac{m_t^2}{m_t^2 + 2m_W^2} \simeq 0.70 \\ f_L &\simeq \frac{2m_W^2}{m_t^2 + 2m_W^2} \simeq 0.30, \end{aligned} \quad (2)$$

in the approximation $m_b = 0$ and taking $m_t = 172$ GeV. By the approximate CP-invariance of the decay, anti-tops have a nearly identical distribution.