Extreme mass-ratio inspirals with a scalar environment

Robrecht Keijzer

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Master Thesis Presentation - March

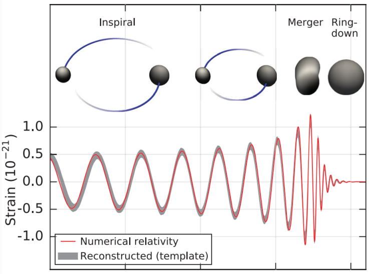
Int. promotor: prof. Archisman Ghosh (UGent)
Ext. promotor: prof. Thomas Hertog (KULeuven)
Supervisor: Simon Maenaut (KULeuven)

Introduction

	Mass Ratio:
GW191219_163120	~ 26
Advanced LIGO, ET, CE	~ 100
LISA	$\gtrsim 10^4$

→ Extreme Mass Ratio Inspiral: EMRI

→ Difficult for LVK waveform modelling



Binary merger [1]

Solution: Black Hole Perturbation Theory

Geodesics

Three constants of motion:

- Energy E
- Angular momentum L
- Carter's constant Q

$\rightarrow \boxed{\dot{E} = \dot{L} = \dot{Q} = 0}$

Small secondary mass: EMRIs

Gravitational Self Force!

$$\boxed{\begin{aligned}\frac{dE}{dt} &= -\dot{E}_{\mathrm{GW}} \\ \frac{dL}{dt} &= -\dot{L}_{\mathrm{GW}}\end{aligned}}$$

Expansion in mass ratio:

$$q = \mu/M < 10^{-4}$$

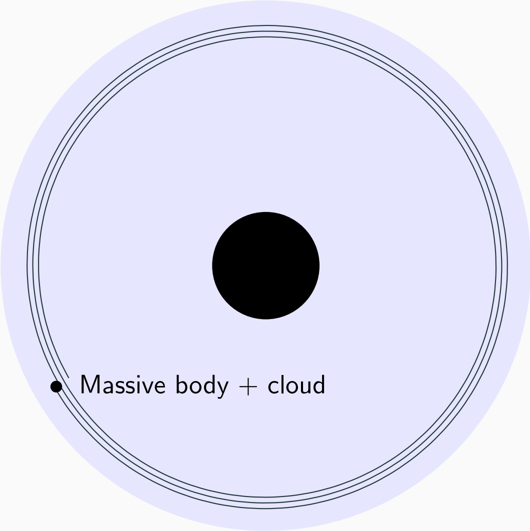
Two parts:

1. **Flux calculation**
   $\rightarrow$ focus of this thesis
2. Time evolution

EMRIs with scalar environments

$m_{\text{scalar}} \simeq 10^{-21} - 10^{-11}\text{eV}$

Recently: fully relativistic framework (2023) [2]



Ultralight Scalar Fields

1. Production mechanism around BH: Superradiance (1970)

2. Fuzzy dark matter, ultralight axions, (light) QCD axion

3. Consistent with ΛCDM $\rightarrow$ Cuspy Halos?

   "[...] ultra-light scalars [...], among the most promising alternatives to WIMPs" (2022)[3]

4. Detectable with LISA, by only gravitational interactions!

   Recently: constrain mass within 0.5% (2024)[4]

Approach I [2]

Expansion at order $\mathcal{O}(\epsilon^n, q^m)$:

$$
\begin{aligned}
g_{\mu\nu}^{\text{exact}} &= g_{\mu\nu}^{\text{BG}} + q h_{\mu\nu}^{(0,1)} + \epsilon h_{\mu\nu}^{(1,0)} + \epsilon q h_{\mu\nu}^{(1,1)} + \dots \\
\Phi^{\text{exact}} &= \epsilon \phi^{(1,0)} + q \phi^{(0,1)} + \epsilon q \phi^{(1,1)} + \dots
\end{aligned}
$$

Solve Einstein equations:

$$
\begin{aligned}
G_{\mu\nu}(g_{\mu\nu}^{\text{exact}}) &= 8\pi (T_{\mu\nu}^{\Phi} + T_{\mu\nu}^{p}) \\
\Box^{\text{exact}} \Phi^{\text{exact}} &= \mu^2 \Phi^{\text{exact}}
\end{aligned}
$$

Eventual goal:

$$
\boxed{
\begin{aligned}
\frac{dE}{dt} &= -\dot{E}_{\text{GW}}(h^{(0,1)}) - \dot{E}_{\text{scalar}}(\phi^{(1,1)}) \\
\frac{dL}{dt} &= -\dot{L}_{\text{GW}}(h^{(0,1)}) - \dot{L}_{\text{scalar}}(\phi^{(1,1)})
\end{aligned}
}
$$

New thing: **scalar fluxes**!

Approach II [2]

For now: **circular orbits, non-rotating black hole**

At order $\mathcal{O}(\epsilon^0, q^1)$:

$$\delta G_{\mu\nu}[h^{(0,1)}] = 8\pi T^{p}_{\mu\nu}[g^{\mathrm{BG}}] \tag{1}$$

At order $\mathcal{O}(\epsilon^1, q^0)$:

$$(\Box^{\mathrm{BG}} - \mu^2)\phi^{(1,0)} = 0 \tag{2}$$

At order $\mathcal{O}(\epsilon^1, q^1)$:

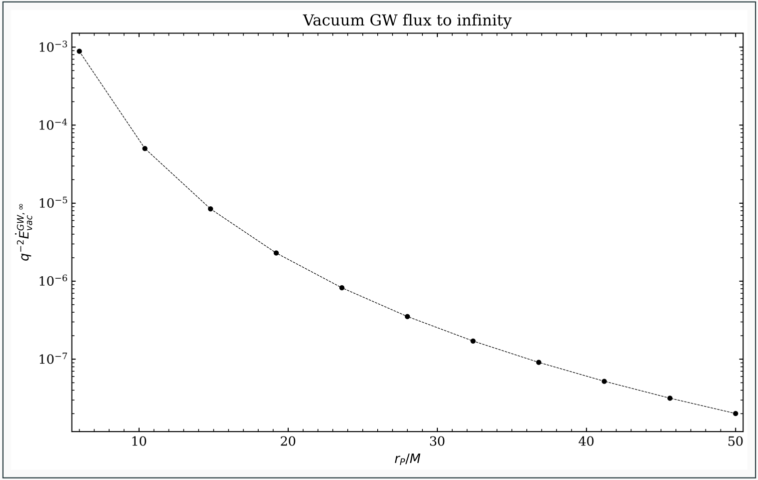
$$(\Box^{\mathrm{BG}} - \mu^2)\phi^{(1,1)} = S^{\phi}[h^{(0,1)}, \phi^{(1,0)}] \tag{3}$$

Results I: GW fluxes

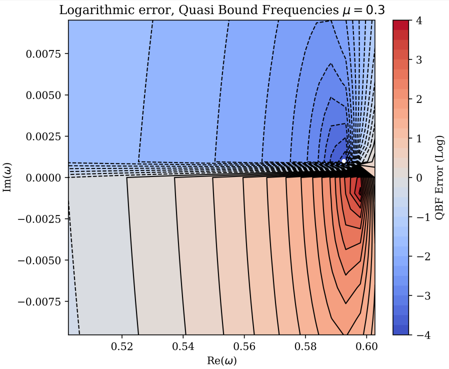
l	m	$\dot{E}_\infty$ (my result)	$\dot{E}_\infty$ (ref. [5])	rel. diff.
2	1	8.1661e-07	8.1633e-07	0.04%
	2	1.7064e-04	1.7063e-04	0.006%
3	1	2.1747e-09	2.1731e-09	0.08%
	2	2.5203e-07	2.5199e-07	0.02%
	3	2.5473e-05	2.5471e-05	0.008%
4	1	8.4058e-13	8.3956e-13	0.2%
	2	2.5098e-09	2.5091e-09	0.03%
	3	5.7757e-08	5.7751e-08	0.02%
	4	4.7258e-06	4.7256e-06	0.005%
5	1	1.2617e-15	1.2594e-15	0.2%
	2	2.7908e-12	2.7896e-12	0.05%
	3	1.0935e-09	1.0933e-09	0.02%
	4	1.2325e-08	1.2324e-08	0.009%
	5	9.4567e-07	9.4563e-07	0.005%

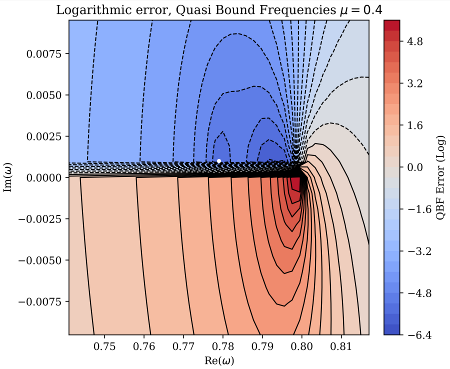
Distance $r_{\text{secondary}} = 7.9456M$

Results II: GW fluxes

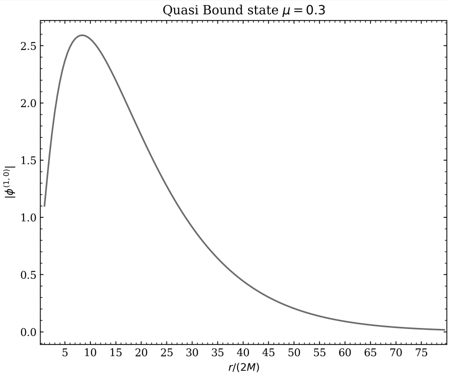


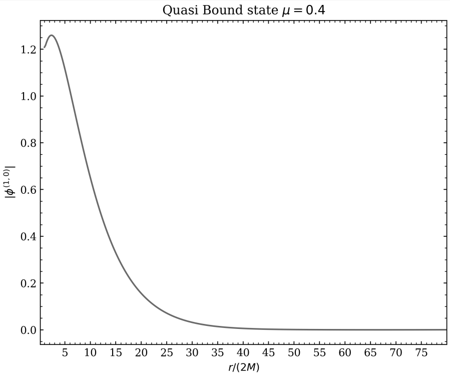
Results III: Scalar cloud background





Results III: Scalar cloud background





Prospect

Initial goal already published (january 2025): rotating black hole

Depending on success reproducing [2]:

- Analyse accuracy of [2]
  $\rightarrow$ Extend to wider mass ranges and distances?

- Include small eccentricity, with hope of maintaining convergence

$\rightarrow$ Calculations programmed with more general systems in mind

Scalar perturbations: rotating black hole

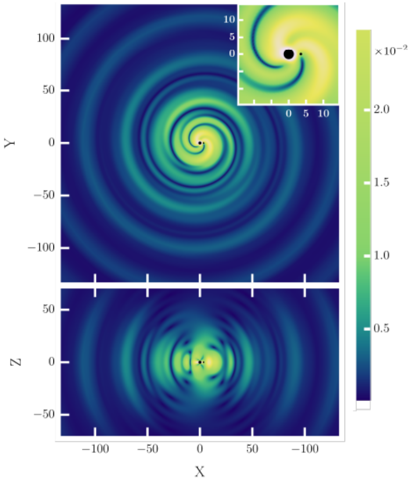


FIG. 1. We show the absolute value of the perturbed scalar field $|\phi^{(1,1)}|$ for $\ell \geq 2$, taking $\alpha = 0.3$, $a = 0.88M$ and $r_{\rm p} = 3.5M$. In the top panel, we show an equatorial slice of the field solution, in which the $\hat{Z}$–axis is aligned with the BH spin. In the bottom panel, we show an azimuthal slice of the field, where the secondary moves "into the plane."

Questions?

Bibliography

[1] Abbott et al., Physical Review Letters **116**, 10.1103/physrevlett.116.061102 (2016).

[2] R. Brito and S. Shah, Physical Review D **108**, 10.1103/physrevd.108.084019 (2023).

[3] M. Mina, D. F. Mota, and H. A. Winther, Astronomy amp; Astrophysics **662**, A29 (2022).

[4] H. Khalvati, A. Santini, F. Duque, L. Speri, J. Gair, H. Yang, and R. Brito, **Impact of relativistic waveforms in lisa's science objectives with extreme-mass-ratio inspirals,** 2024, arXiv:2410.17310 [gr-qc].

[5] K. Martel, Physical Review D **69**, 10.1103/physrevd.69.044025 (2004).

[6] H. Lazare, J. Flitter, and E. D. Kovetz, **Constraints on the fuzzy dark matter mass window from high-redshift observables,** 2025, arXiv:2407.19549 [astro-ph.CO].

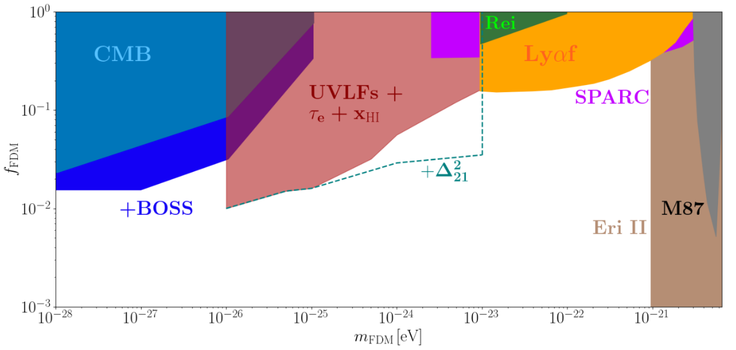


Figure 2: FDM mass window [6]