Phase locking of spiral waves in a rotating electrical field

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1. Introduction

2. Phase-locking of spiral waves (2D)

3. Phase-locking of scroll waves (3D)

Outline

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2. Phase-locking of spiral waves (2D)
3. Phase-locking of scroll waves (3D)

Excitable and oscillatory media

- Excitable medium: small stimulus invokes activation cycle
  - Forest fire
  - Mexican wave
  - Epidemics
  - Cardiac tissue
- Oscillatory medium: self-sustained activity
  - Cardiac pacemaker cells
  - Biological signaling
  - Belousov-Zhabotinsky (BZ) chemical reaction

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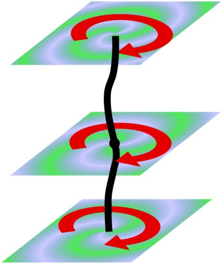
- Oscillatory medium:
  self-sustained activity
  - Cardiac pacemaker cells
  - Biological signaling
  - Belousov-Zhabotinsky (BZ) chemical reaction
- Both types can be modeled by a reaction-diffusion equation:

$$\partial_t \mathbf{u} = D_0 \mathbf{P} \Delta \mathbf{u} + \mathbf{F}(\mathbf{u})$$

Spiral wave formation

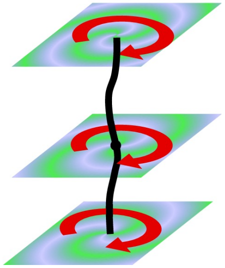
- Can be initiated by well-timed pulse
- Activation pattern:
  - 2D: spiral wave
  - 3D: scroll wave

Scroll wave filaments



- **Filament**:= scroll's rotation axis

Scroll wave filaments



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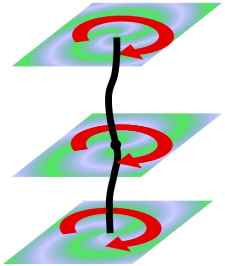
- Filaments are lines of phase singularity, similar to
  - Eye of a hurricane/tornado
  - Hydrodynamical vortex
  - Cosmic strings

Scroll wave filaments



- **Filament**:= scroll's rotation axis
- Filaments are lines of phase singularity, similar to
  - Eye of a hurricane/tornado
  - Hydrodynamical vortex
  - Cosmic strings
- Filaments offer a way to describe and control complexity

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Electroforetic drift in chemical media

- In a chemical system where different reactants diffuse, imposing an external electrical field $\vec{E}$ results in additional convection:

$$\dot{\mathbf{u}} = \mathbf{P}\Delta\mathbf{u} + \mathbf{F}(\mathbf{u}) - \mathbf{M}\vec{E}\cdot\vec{\nabla}\mathbf{u}. \qquad (1)$$

- In the limit where $||\vec{E}||$ is small, this is a special case of

$$\dot{\mathbf{u}} = \mathbf{P}\Delta\mathbf{u} + \mathbf{F}(\mathbf{u}) - \epsilon\mathbf{h}. \qquad (2)$$

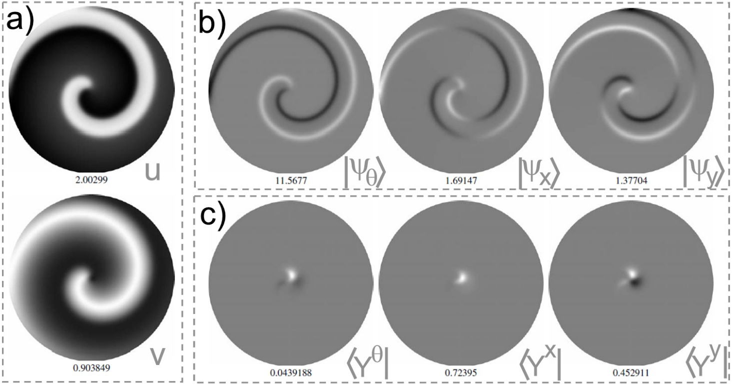
in which case the spiral wave drift can be found perturbatively using the spiral wave's response functions $\mathbf{W}^x, \mathbf{W}^y$, $\mathbf{W}^0$:

$$\begin{aligned} \dot{X} &= \langle \mathbf{W}^x \mid \mathbf{h} \rangle + \mathcal{O}(\epsilon^2) \\ \dot{Y} &= \langle \mathbf{W}^y \mid \mathbf{h} \rangle + \mathcal{O}(\epsilon^2) \\ \dot{\Phi} &= \omega_0 + \langle \mathbf{W}^0 \mid \mathbf{h} \rangle + \mathcal{O}(\epsilon^2) \end{aligned} \qquad (3)$$

where $\langle \mathbf{f} \mid \mathbf{g} \rangle = \iint_{\mathbb{R}^2} \mathbf{f}^H \mathbf{g} dS$.

Localization of spiral response functions

- The response functions can be computed explicitly by numerically solving an adjoint problem (see e.g. *Biktasheva et al. PRE 2009*).



- Localization of their response functions enables spiral waves to behave as a hybrid between a particle and wave (*Biktasheva et al. PRE 2003*)

Electroforetic drift in a **constant** field

- In a constant $\vec{E}$-field, we get in the laboratory frame:

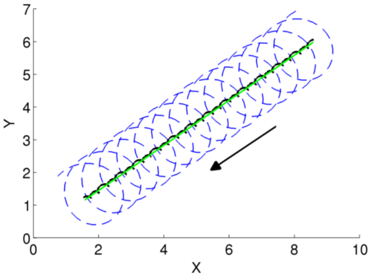
$$\dot{X}^a = M^a{}_b(\Phi) E^b, \tag{4}$$

$$M^a{}_b(\Phi) = R^a{}_A(\Phi) M^A{}_B R^B{}_b(\Phi) \tag{5}$$

$$M^A{}_B = \langle \mathbf{W}^A \mid \mathbf{M} \mid \partial_B \mathbf{u}_0 \rangle \tag{6}$$

After averaging over one period, we get

$$\dot{\vec{X}} = \frac{M^x{}_x + M^y{}_y}{2} \vec{E} + \frac{M^y{}_x - M^x{}_y}{2} \vec{T} \times \vec{E}, \qquad \dot{\Phi} = \omega_0. \tag{7}$$



Phase-locking in a **rotating** field

- If we impose an external $\vec{E}$-field rotating at frequency ω_f, we find Adler's equation for phase-locking:

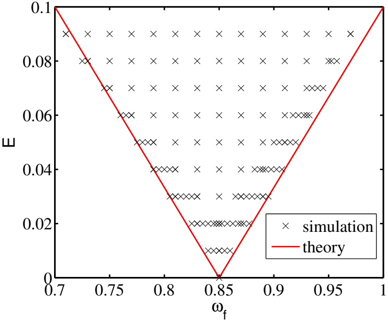
$$\begin{aligned}\dot{\Phi} &= \omega_0 - \omega_f + {M^0}_A E^A, \\ &= -\Delta\omega + ME\cos(\Phi - \alpha), \qquad M = \sqrt{({M^0}_x)^2 + ({M^0}_y)^2} \qquad (8)\end{aligned}$$

- If $|\Delta\omega| < ME$, the 1D dynamical system has a stable equilibrium (phase-locked spiral wave) in

$$\Phi = \alpha + \arccos\left(\frac{\Delta\omega}{M}\right) \qquad (9)$$

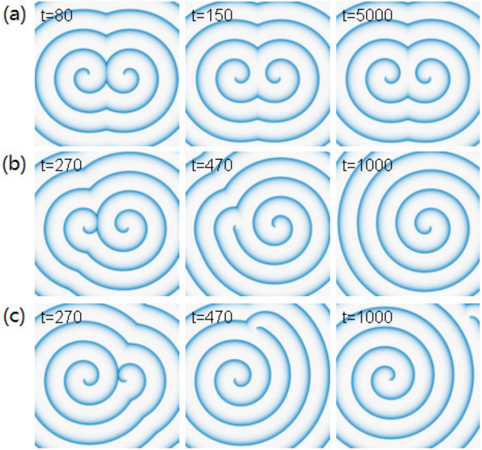
Li, Dierckx et al.

J Chem Phys 2013



Spiral chirality selection

- If one spiral rotates faster than a neighboring one, it will push the slower one away
  - $|\omega_f| > |\omega_0|$: only phase-locked spirals survive
  - $|\omega_f| < |\omega_0|$: only non phase-locked spirals survive



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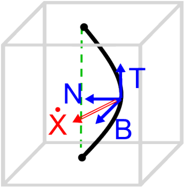
Filament dynamics



- Motion of a filament $X^i(\sigma, t)$ depends on its curvature $k(\sigma, t)$ *(Keener 1988, Biktashev 1994)*

$$\begin{aligned} \dot{\vec{X}} &= \gamma_1\, \partial_\sigma^2 \vec{X} + \gamma_2\, \partial_\sigma \vec{X} \times \partial_\sigma^2 \vec{X} \\ &= \gamma_1\, k\vec{N} + \gamma_2\, k\vec{B} \end{aligned} \tag{10}$$

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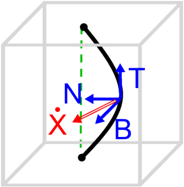

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- γ_1:= 'filament tension'
  - $\gamma_1 > 0 \;\Rightarrow\;$ filament length $\downarrow \Rightarrow$ stable
  - $\gamma_1 < 0 \;\Rightarrow\;$ filament length $\uparrow \Rightarrow$ unstable

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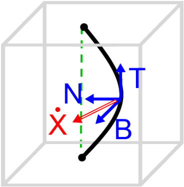

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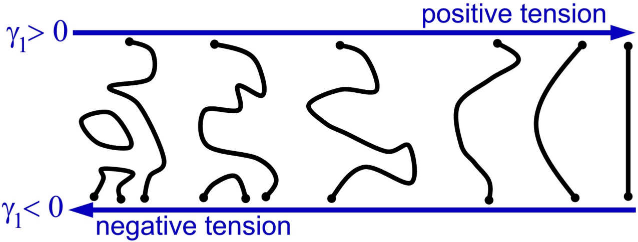


Filament tension of a phase-locked scroll wave

- When $\vec{E} = \vec{0}$, filament tension is given by

$$\gamma_1 = \frac{P^x{}_x + P^y{}_y}{2} = \frac{1}{2}\langle \mathbf{W}^A \mid \mathbf{P} \mid \partial_A \mathbf{u}_0 \rangle \tag{11}$$

- When $\vec{E} \neq \vec{0}$, there are shifts:

$$\omega_0 \to \omega_f, \qquad \mathbf{u}_0 \to \mathbf{U}_0, \qquad \mathbf{W}^A \to \boldsymbol{\mathcal{W}}^A, \qquad \gamma_1 \to \Gamma_1 \tag{12}$$

- If $|\Delta\omega| < ME$, the relative orientation of the phase-locked spiral with respect to the $\vec{E}$ is given by

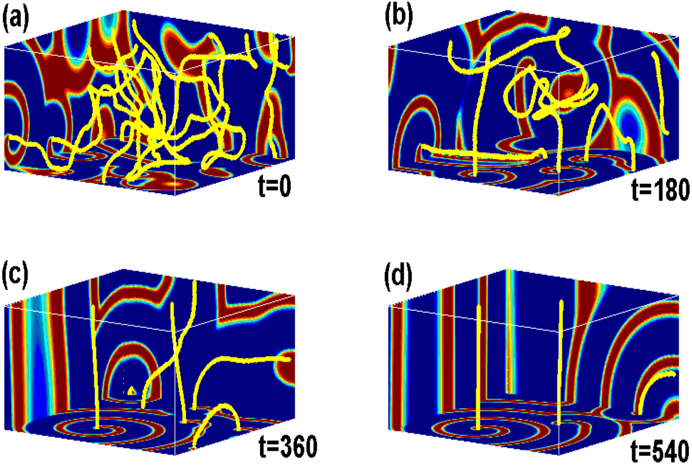
$$\Delta\omega = \vec{M} \cdot \vec{E}, \tag{13}$$

with resulting filament tension

$$\Gamma_1 = \gamma_1 + \vec{a} \cdot \vec{E}. \tag{14}$$

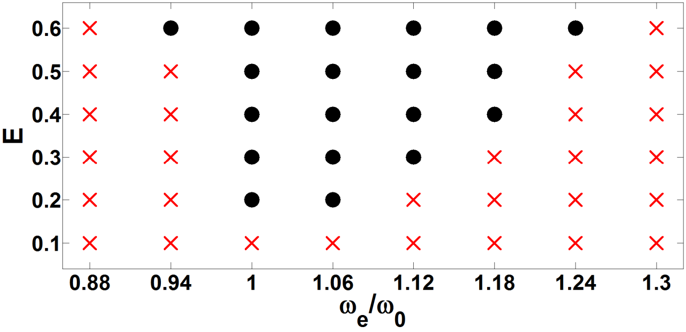
Re-ordering of scroll wave turbulence

- Numerical example of restoring order in a system with negative filament tension (Barkley kinetics):



Conditions for re-ordering turbulence

1. Phase-locking should be possible: $|\Delta\omega| < ME$
2. Positive filament tension of the phase-locked scroll: $\Gamma_1 > 0$
3. External field rotates faster than the phase-locked scroll: $|\omega_f| > |\omega_0|$



Conclusions

1. Applying a rotating field enables to phase-lock a spiral wave
2. By forcing faster or smaller rotation of the spiral wave, spirals of only one chirality survive
3. This procedure can be applied in chemical media to convert 3D turbulence to a state with straight scroll waves (which can be eliminated by a DC field)

References

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Thank you for your attention! More questions?