BPS meeting 2016

How non-perturbative effects can resurge from perturbation theory

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Based on arXiv:1604.07851
"Resurgence in η-deformed Principal Chiral Models";
SD, D. Dorigoni, D. Thompson

Perturbation theory in QFT and QM

- Generally, exact results are inaccessible
- Instead (Taylor) series expansion in some small parameter g

$$\mathbb{O}(g) \sim a_0 + a_1 g + a_2 g^2 + \cdots$$
$$\sim \sum_{n=1}^{+\infty} a_n \, g^n$$

- Go beyond this simple structure, to *trans-series* using *resurgence*

$$\mathbb{O}(g) \overset{?}{\sim} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} a_{n,k} g^n [\exp(-s/g)]^k$$

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$$\sim \sum_{n=1}^{+\infty} a_n\, g^n \;?$$

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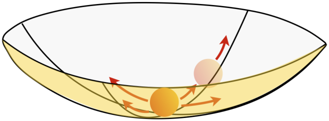
Perturbation theory in QFT and QM

Perturbation theory works BUT ...

1. ... in general this series is divergent/ ambiguous as $a_n \sim n!$

$$\mathbb{O}(g) \stackrel{?}{=} \sum_{n=1}^{+\infty} a_n\, g^n = \infty$$

2. ... does not take into account non-perturbative effects

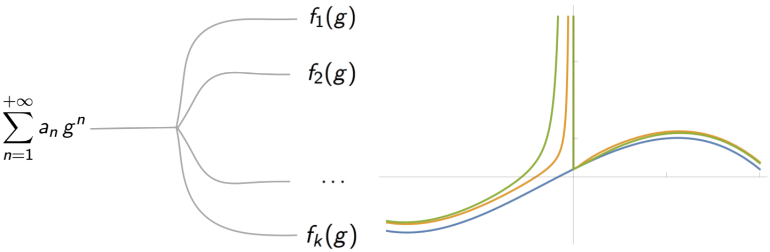


Problem 1. and 2. are actually not independent ... !

Perturbation theory in QFT and QM

In QFT/QM, can we make sense of $\sum_{n=1}^{+\infty} a_n g^n = ?$

- Not convergent but asymptotic series
- { Asympt. series } $\longleftrightarrow$ { several functions }



- Do not capture function with a vanishing Taylor expansion

$$f(g) = e^{-1/g} \sim 0 \text{ around } g \sim 0$$

Resurgence for Borel summable functions

Resurgence program: redefine what you mean by "the sum"

$$\mathbb{O}(g) \stackrel{?}{=} \sum_{n=1}^{\infty} a_n g^n \,, \text{ with } a_n \sim n!$$

1. Regularize by a *Borel transform*

$$B[\mathbb{O}](t) \equiv \sum_{n=1}^{\infty} \frac{a_n}{(n-1)!} t^{n-1}$$

   is a series with a non-zero radius of convergence

2. *Define* the sum $\tilde{\mathbb{O}}(g)$ as the *Borel resummation* of $B[\mathbb{O}]$

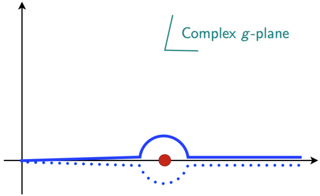
$$\widetilde{\mathbb{O}}(g) \equiv \frac{1}{g} \int_0^{\infty} \mathrm{d}t \, e^{-t/g} B[\mathbb{O}](t)$$

Resurgence for non-Borel summable functions

What if $B[\mathbb{O}]$ has singularities along the positive real axis?

$$\widetilde{\mathbb{O}}(g) \equiv \frac{1}{g}\int_0^\infty \mathrm{d}t\, e^{-t/g} B[\mathbb{O}](t)$$

- Instead dodge the singularity ... ambiguous



- Get imaginary NP contribution/ambiguity (= *instantons*)

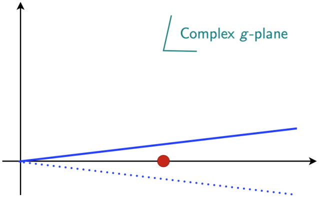
$$\left[\widetilde{\mathbb{O}}_{\mathrm{up}}(\lambda) - \widetilde{\mathbb{O}}_{\mathrm{down}}(\lambda)\right] \sim i\, e^{-A/\lambda}\,.$$

Resurgence for non-Borel summable functions

What if $B[\mathbb{O}]$ has singularities along the positive real axis?

$$\widetilde{\mathbb{O}}(g) \equiv \frac{1}{g}\int_0^{e^{i\theta}\infty} \mathrm{d}t\, e^{-t/g} B[\mathbb{O}](t)$$

- Instead dodge the singularity .... ambiguous choice



- Get imaginary NP contribution/ambiguity ($=$ *instantons*)

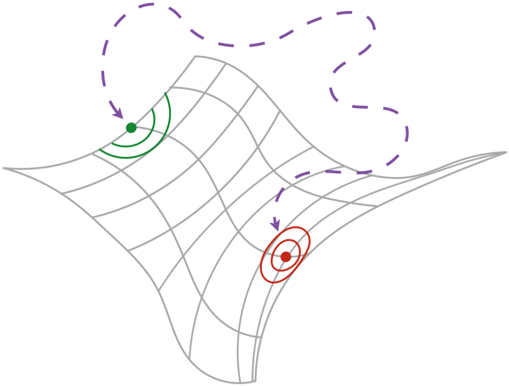
$$\left[\widetilde{\mathbb{O}}_{\text{up}}(\lambda) - \widetilde{\mathbb{O}}_{\text{down}}(\lambda)\right] \sim i\, e^{-A/\lambda} .$$

The magics of resurgence

The asymptotic nature of
perturbative expansion observables ⇒ ambiguities

in the mean time...

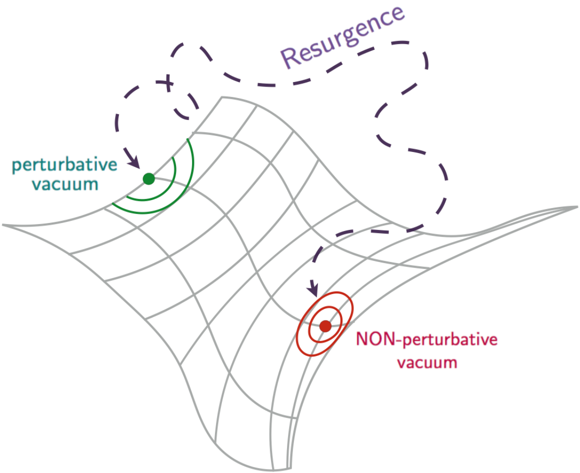
Resummation leads to non-perturbative effects ⇒ ambiguities



The magics of resurgence

Resurgence theory

(ambiguities + ambiguities = well-defined quantity)



The η-deformed PCM and it's resurgent structure

$$S_\eta = \frac{1}{2\pi t}\int d^2\sigma \ \mathrm{tr}\ \left[\partial_+ g g^{-1}\frac{1}{1+\eta\mathcal{R}}\partial_- g g^{-1}\right] .$$

η-deformed Principal Chiral Model

=

deformed integrable model in 2d

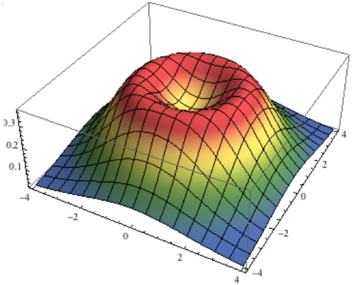
- Integrable = 'a lot of' symmetries → 'solve exactly'
- Deformed = investigate integrability
- Why? String model (?), AdS/CFT correspondence, Yang-Baxter equations, proxy to QCD_2,...

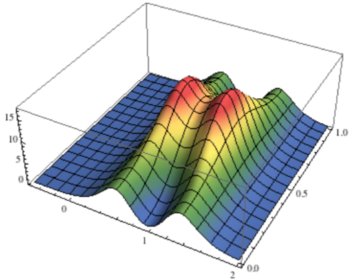
Can we understand it's resurgent structure?

Resurgence and the PCM

- But puzzle: no instantons in PCM... $\to$ well-defined observables?
- Resurgence predicts existence of NP objects
- Intro *Unitons* and their fractionalization: *Fractons*
- Investigated by reducing to a 1D effective theory (= QM)

$$\mathcal{H} = \frac{g^2}{4L} p_\theta^2 + \frac{L\pi^2}{4g^2} \sin^2(\theta) \left(1 + \eta^2 \sin^2(\theta)\right) .$$





Conclusions

- Perturbative expansion of observables are factorially divergent
- Resurgence intertwines the P and NP sector while curing ambiguities
- We investigated the resurgent structure of the integrable η-deformed PCM
- By reducing to QM we identified the expected non-perturbative objects from resurgence theory

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Thank you for your attention!