From BEC polaron to BCS polaron

G. Lombardi, J. Tempere

Universiteit Antwerpen, Theory of Quantum and Complex systems

Theory of Quantum Systems & Complex Systems University of Antwerp

Fröhlich Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \sum_{\boldsymbol{q}} \hbar\omega_{\boldsymbol{q}} \hat{\alpha}^\dagger_{\boldsymbol{q}} \hat{\alpha}_{\boldsymbol{q}} + \sum_{\boldsymbol{q}} V_{\boldsymbol{q}}\, e^{-i\boldsymbol{q}\cdot\hat{\boldsymbol{r}}} \left(\hat{\alpha}_{\boldsymbol{q}} + \hat{\alpha}^\dagger_{-\boldsymbol{q}}\right)$$

Fröhlich solid state polaron

Quasiparticle consisting of an electron and the polarization cloud that it drags along while moving in a polar crystal

- $m \to m_e$ electron mass.
- $\hbar\omega_{\boldsymbol{q}} \to \hbar\omega_{LO}$ dispersion relation for longitudinal-optical phonons.
- $V_{\boldsymbol{q}}$ interaction amplitude for the electron to emit/absorb LO phonons.

BEC polaron

Quasiparticle deriving from the interaction of an impurity with the Bogoliubov excitations of a Bose-Einstein condensate.

- $m \to m_I$ mass of the impurity.
- $\hbar\omega_{\boldsymbol{q}}$ dispersion for the Bogoliubov excitations of the Bose-Einstein condensate.
- $V_{\boldsymbol{q}}$ interaction amplitude for the impurity to emit/absorb a Bogoliubov excitation.

"BCS polaron"

Quasiparticle deriving from the interaction of an impurity with the collective excitations of a Fermi superfluid.

- $m \to m_I$ mass of the impurity.
- $\hbar\omega_{\boldsymbol{q}}$ dispersion for the collective excitations of the Fermi superfluid.
- $V_{\boldsymbol{q}}$ interaction amplitude for the impurity to emit/absorb an excitation.

Effective field theory for Fermi superfluids

Ultracold fermions interacting via a *s-wave* contact potential governed by the interaction parameter $(k_F a_{FF})^{-1}$. *(a_{FF} fermion-fermion scattering length)*

$(k_F a_{FF})^{-1} < 0 \to$ BCS $\quad (k_F a_{FF})^{-1} = 0 \to$ unitarity $\quad (k_F a_{FF})^{-1} > 0 \to$ BEC

$\longrightarrow$ EFT describes the system in terms of the pairing field Φ

$\longrightarrow$ Basic assumption: Φ varies slowly in time and space

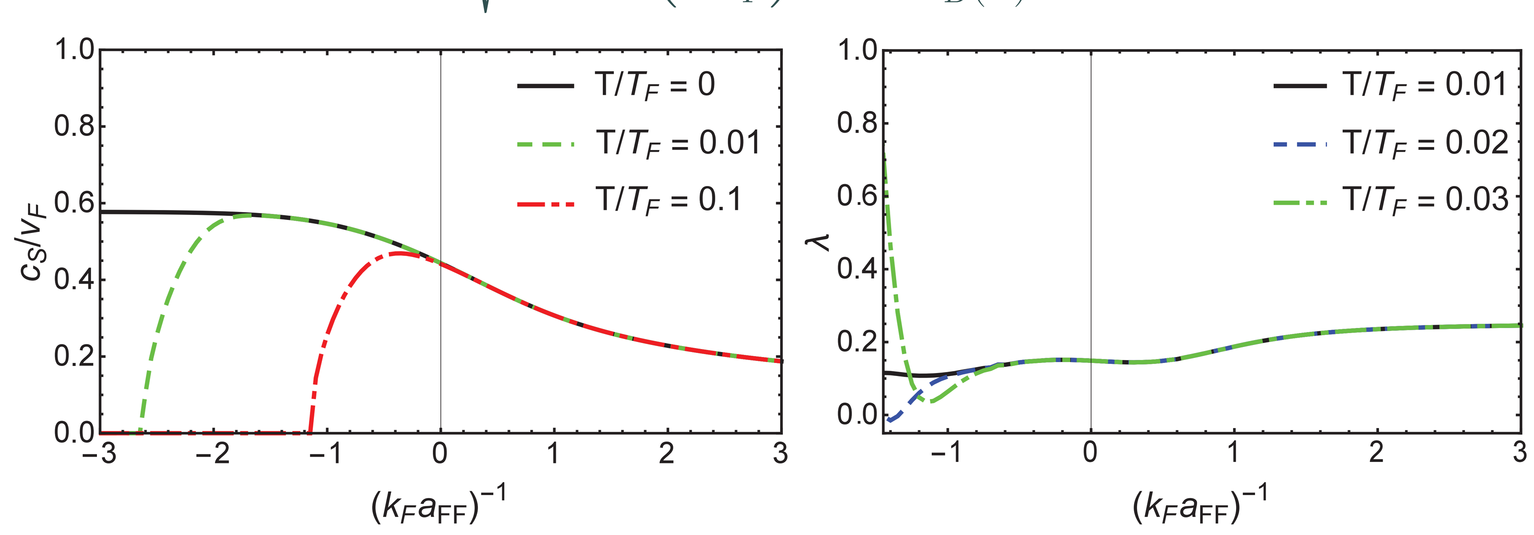
$\longrightarrow$ Complete EFT action $\quad [w = |\Phi|^2]$

$$\mathcal{S}_{EFT} = \int d^4x \left[\Omega_s(w) + \frac{D(w)}{2}\left(\frac{\partial\bar{\Phi}}{\partial\tau}\Phi - \bar{\Phi}\frac{\partial\Phi}{\partial\tau}\right) + Q(w)\frac{\partial\bar{\Phi}}{\partial\tau}\frac{\partial\Phi}{\partial\tau} + \frac{R(w)}{2w}\left(\frac{\partial|\Phi|^2}{\partial\tau}\right)^2 + \frac{C(w)}{2m_F}|\nabla_{\boldsymbol{r}}\Phi|^2 - \frac{E(w)}{2m_F}\left(\nabla_{\boldsymbol{r}}|\Phi|^2\right)^2\right]$$

(we have analytic expressions for all the coefficients [1]*)*

Collective excitations

- the pair field is rewritten as the sum of mean-field and fluctuation contributions as
$$\Phi(\boldsymbol{r},\tau) = \Delta + \varphi(\boldsymbol{r},\tau)\,.$$
- an expansion up to second order leads to the *quadratic fluctuation action*
$$S^{quad}_{EFT} = \frac{1}{2}\sum_{\boldsymbol{q},n}\left(\bar{\varphi}_{\boldsymbol{q},n}\ \ \varphi_{-\boldsymbol{q},-n}\right) \times \mathbb{M}(\boldsymbol{q}, i\Omega_n) \times \begin{pmatrix}\varphi_{\boldsymbol{q},n}\\ \bar{\varphi}_{-\boldsymbol{q},-n}\end{pmatrix}.$$
- the spectrum of collective excitations is determined by the solution of
$$\det\left(\mathbb{M}(\boldsymbol{q},\omega)\right) = 0 \qquad \text{after the transformation } i\Omega_n \to \omega$$
- up to second order in q the dispersion for the collective excitations is given by
$$\hbar\omega_{\boldsymbol{q}} = \hbar q\sqrt{c_S^2 + \lambda\left(\frac{\hbar q}{2m_F}\right)^2} = \frac{\hbar^2}{2m_B(\lambda)} q\sqrt{q^2 + 2/\xi^2}\,.$$



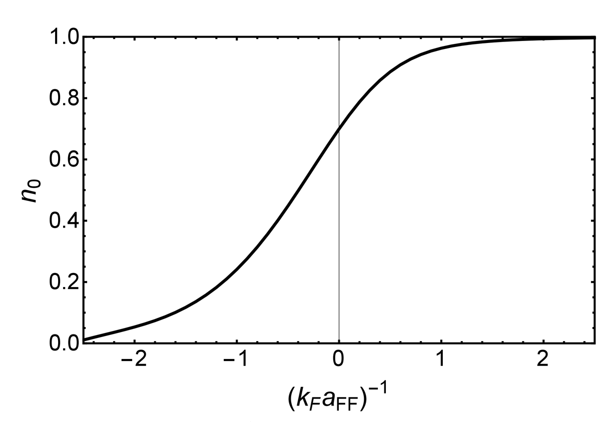
$m_B(\lambda) \equiv m_F/\sqrt{\lambda}$ can be interpreted as the interaction-dependent mass of the fermion pairs. In the BEC limit $\lambda \to 1/4$ and $m_B(\lambda) \to 2m_F$.

Weak coupling regime: $T = 0$ perturbation theory

The Hamiltonian for the system becomes then

$$\hat{H} = \frac{\hat{p}^2}{2m_I} + \sum_{\boldsymbol{q}}\hbar\omega_{\boldsymbol{q}}\hat{\alpha}^\dagger_{\boldsymbol{q}}\hat{\alpha}_{\boldsymbol{q}} + g_{IB}\sqrt{N_0}\sum_{\boldsymbol{q}}\sqrt{\frac{\epsilon_{\boldsymbol{q}}}{\hbar\omega_{\boldsymbol{q}}}}e^{-i\boldsymbol{q}\cdot\hat{\boldsymbol{r}}}\left(\hat{\alpha}_{\boldsymbol{q}} + \hat{\alpha}^\dagger_{-\boldsymbol{q}}\right) \equiv \hat{H}_0 + \hat{V}$$

- $g_{IB} \to$ impurity-fermion pair ($\sim$ boson) coupling constant (function of the *impurity-boson scattering length* a_{IB}).
- $N_0 \to$ number of particles in the condensate.
- $\epsilon_{\boldsymbol{q}} \equiv \frac{\hbar q^2}{2m_B(\lambda)} \to$ dispersion for the free bosons



We start from an unperturbed eigenstate of $\hat{H}_0$ of the form $|\psi_{\boldsymbol{k}}\rangle|\emptyset\rangle$ consisting of a free impurity described by a plane-wave eigenfunction $\psi_{\boldsymbol{q}} = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}/\sqrt{V}$ and the vacuum state for the phonons $|\emptyset\rangle$ and calculate the energy up to second order corrections.

$$E^{(0)}_{\boldsymbol{k}} = \langle\emptyset|\langle\psi_{\boldsymbol{k}}|\,\hat{H}_0\,|\psi_{\boldsymbol{k}}\rangle|\emptyset\rangle = \frac{\hbar^2k^2}{2m_I} \quad \Big| \quad \Delta E^{(1)}_{\boldsymbol{k}} = 0 \quad \Big| \quad \Delta E^{(2)}_{\boldsymbol{k}} = \sum_{|exc\rangle\neq|\psi_{\boldsymbol{k}}|\emptyset\rangle}\frac{\left|\langle exc|\,\hat{V}\,|\psi_{\boldsymbol{k}}\rangle|\emptyset\rangle\right|^2}{E^{(0)}_{\boldsymbol{k}} - E^{(0)}_{exc}}$$

Results

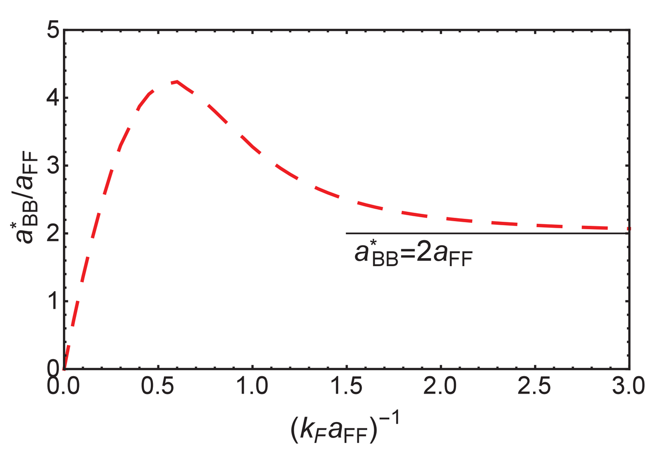
Polaronic coupling constant

The energy up to second order in perturbation can be written in terms of the *polaronic coupling constant* α that is defined as

$$\alpha \equiv \frac{a_{IB}}{a^*_{BB}\xi}$$

where $a^*_{BB} = 1/16\pi n_0 v_S^2/\epsilon_0^2$. In analogy with the BEC polaron case (where $\alpha \equiv \frac{a_{IB}}{a_{BB}\xi}$), a^*_{BB} is thus expected to give a measure of the scattering length between the fermion pairs.

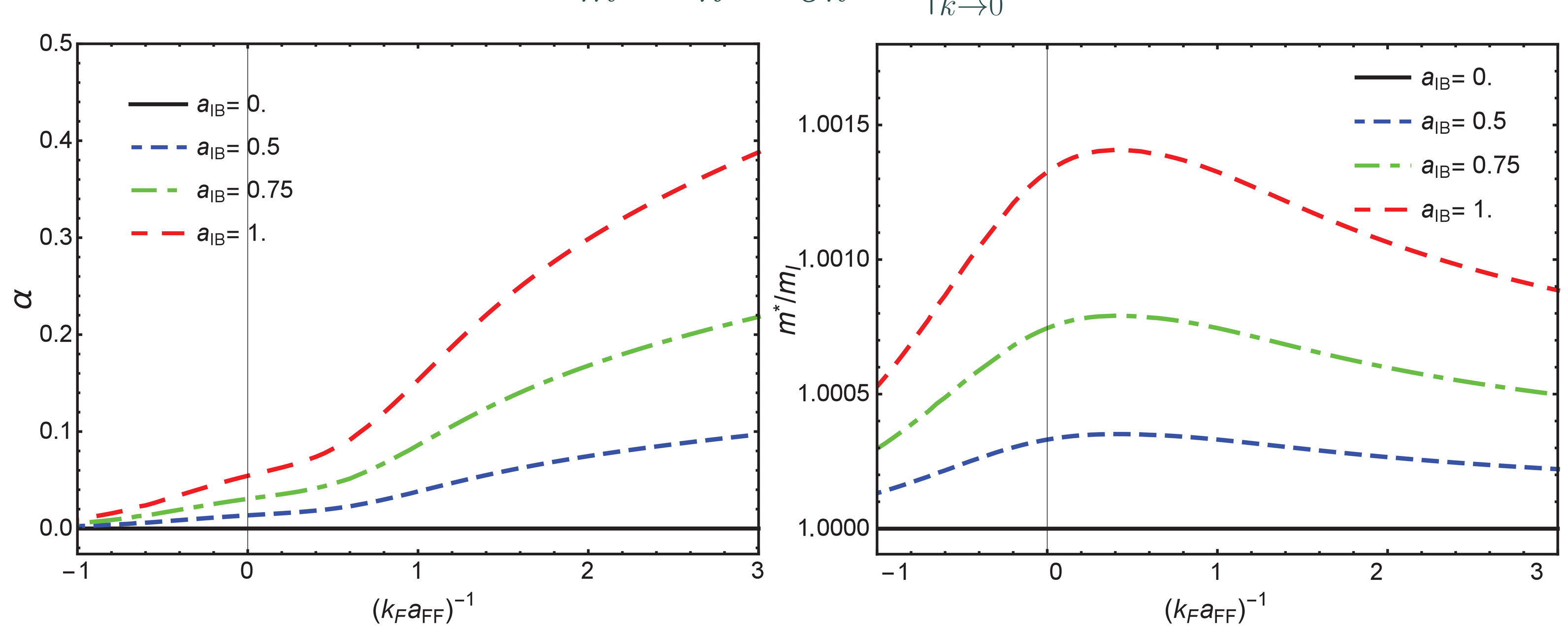
$\Rightarrow$ in the BEC limit the mean-field prediction $a^*_{BB} = 2a_{FF}$ is correctly obtained.



Effective mass

The *effective mass* is calculated by using the definition

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{\partial^2\left(E^{(2)}_{\boldsymbol{k}}\right)}{\partial k^2}\Bigg|_{k\to 0}$$



- The polaronic coupling constant increases monotonically when going from the BCS towards the BEC regime.
- A maximum is found for the ratio m^*/m_I for small positive values of the interaction parameter $(k_F a_{FF})^{-1}$.
- For a fixed value of $(k_F a_{FF})^{-1}$ both α and m^* increase with a_{IB}.

Conclusions

- The problem of an impurity interacting with the collective excitations of a Fermi superfluid was mapped on the Fröhlich Hamiltonian.
- The dispersion relations for the collective excitations of a fermionic superfluid were calculated in the framework of an effective field theory [1] suitable to describe superfluid Fermi gases in a wide range of the $\{T, (k_F a_{FF})^{-1}\}$−space.
- The behavior of the polaronic coupling constant and of the effective mass was analysed across the BEC-BCS crossover for different regimes of the interaction between the impurity and the fermion pairs. Interesting features are observed in the near BEC regime.

References

[1] S. N. Klimin, J. Tempere, G. Lombardi, J. T. Devreese, *Finite temperature effective field theory and two-band superfluidity in Fermi gases*, Eur. Phys. J. B **88**, 122 (2015), arXiv:1309.1421 [cond-mat.quant-gas].

[2] G.Lombardi, J. Tempere, *Polaronic effects of an impurity in a Fermi superfluid away from the BEC limit*, arXiv:1604.00776 [cond-mat.quant-gas] (2016).

Acknowledgements

This research was supported by the Flemish Research Foundation (FWO-Vl), project No. G.0115.12N, No. G.0119.12N, No. G.0122.12N, No. G.0429.15N, No. G0G6616N, by the Scientific Research Network of the Research Foundation-Flanders, WO.033.09N, and by the Research Fund of the University of Antwerp.

Contact: giovanni.lombardi@uantwerpen.be