Read-Green resonances in a topological superconductor coupled to a bath

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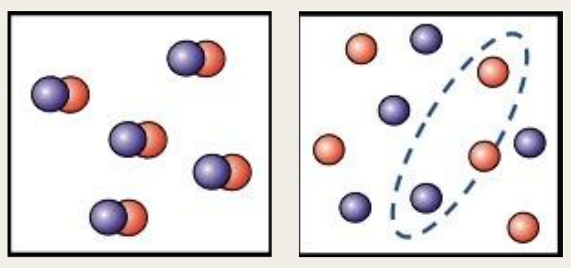
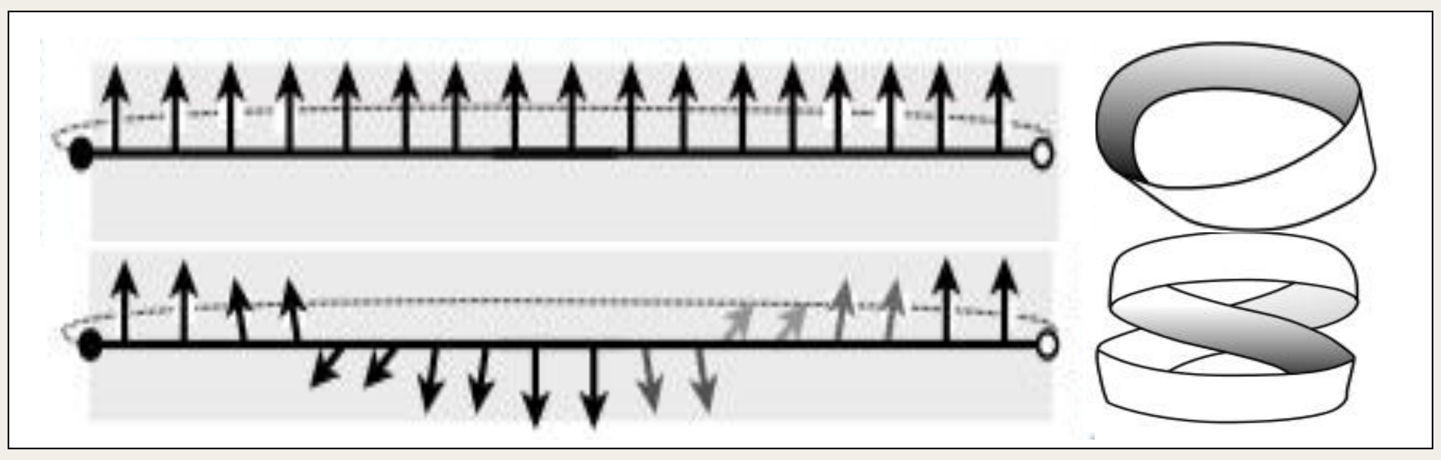
1. What is topological superconductivity?



- Superconductivity: Electrons at the Fermi level form **Cooper pairs**
- Topological: Phases have a **nontrivial** topological structure

- Example: 2D chiral *p*-wave superconductor

$$H_{p+ip} = \sum_{\mathbf{k}} \frac{|\mathbf{k}|^2}{2m} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} - \frac{G}{4m} \sum_{\mathbf{k}\neq\pm\mathbf{k}'} (k_x + ik_y)(k'_x - ik'_y) c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}} c_{-\mathbf{k}'} c_{\mathbf{k}'}$$

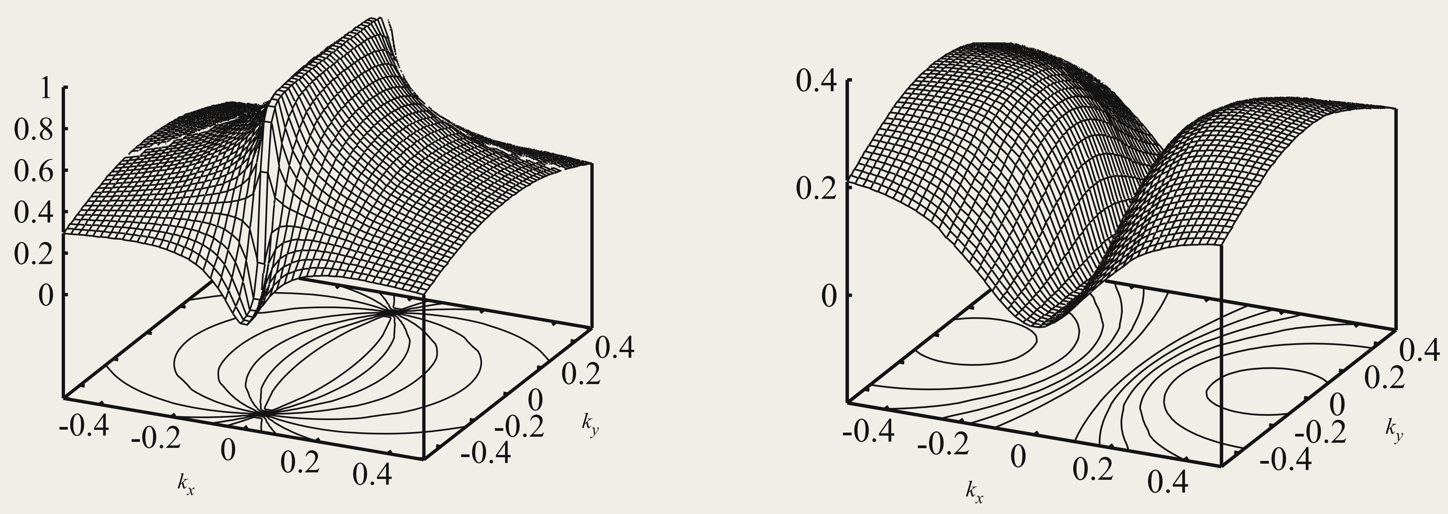


2. The mean-field approach...

Approximate the ground state by a coherent mean-field **BCS** state

$$|BCS\rangle = e^{\sum_{\mathbf{k}} \theta_k c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}}} |\theta\rangle$$

- Variationally optimize (strongly correlated) wave function
- Breaks particle-number symmetry: introduce **chemical potential**
- BUT fluctuations can be neglected in thermodynamic limit
  - $\mu > 0$: Topologically nontrivial phase
  - $\mu = 0$: Phase transition - Quasiparticle spectrum becomes gapless
  - $\mu < 0$: Topologically trivial phase



$\mu > 0$ $\qquad$ $\mu < 0$

3.... or the Bethe ansatz approach

The Hamiltonian is **Richardson-Gaudin integrable** with eigenstates exactly given by a **Bethe ansatz** wavefunction

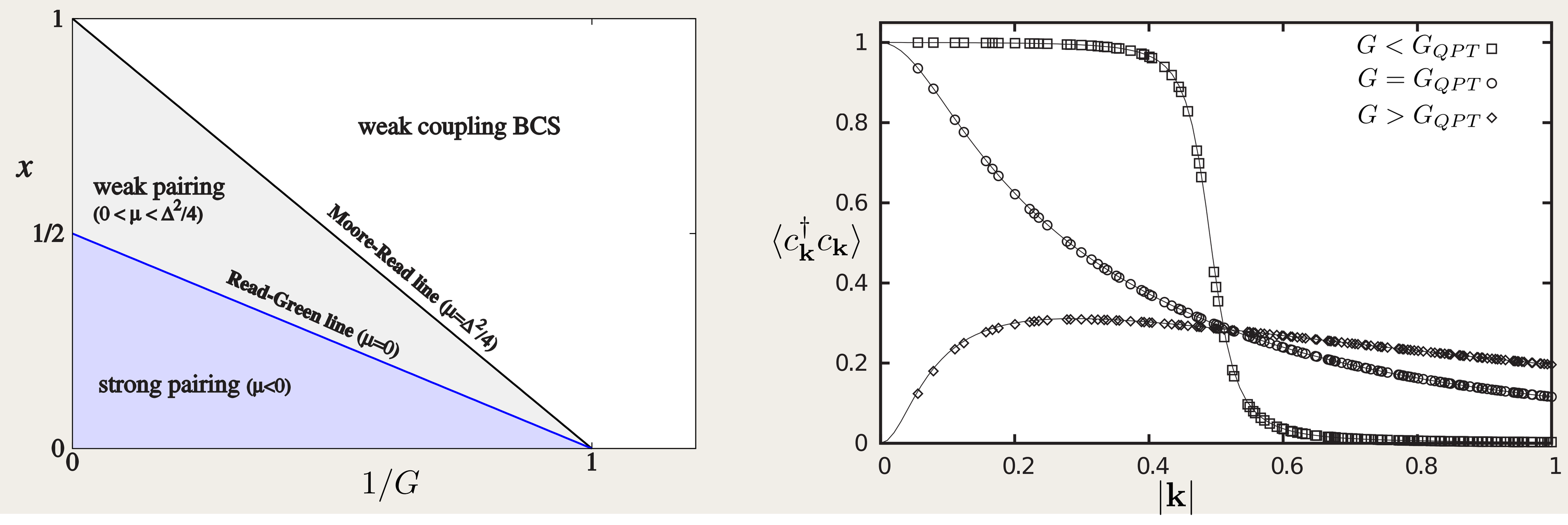
$$|RG\rangle = \prod_{\alpha=1}^{N} \left(\sum_{\mathbf{k}} \frac{k_x - ik_y}{|\mathbf{k}|^2 - v^2_\alpha} c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}} \right) |\theta\rangle$$

~ product of Cooper pairs

where the parameters in the wavefunction behave as **pair energies** and are coupled through the RG equations

$$(1+G) - G\sum_{|\mathbf{k}|} \frac{|\mathbf{k}|^2}{|\mathbf{k}|^2 - v^2_\alpha} + 2G \sum_{\beta\neq\alpha} \frac{v^2_\beta}{v^2_\beta - v^2_\alpha} = 0$$

Read-Green points mark phase transition ($v^2_\alpha = 0$)

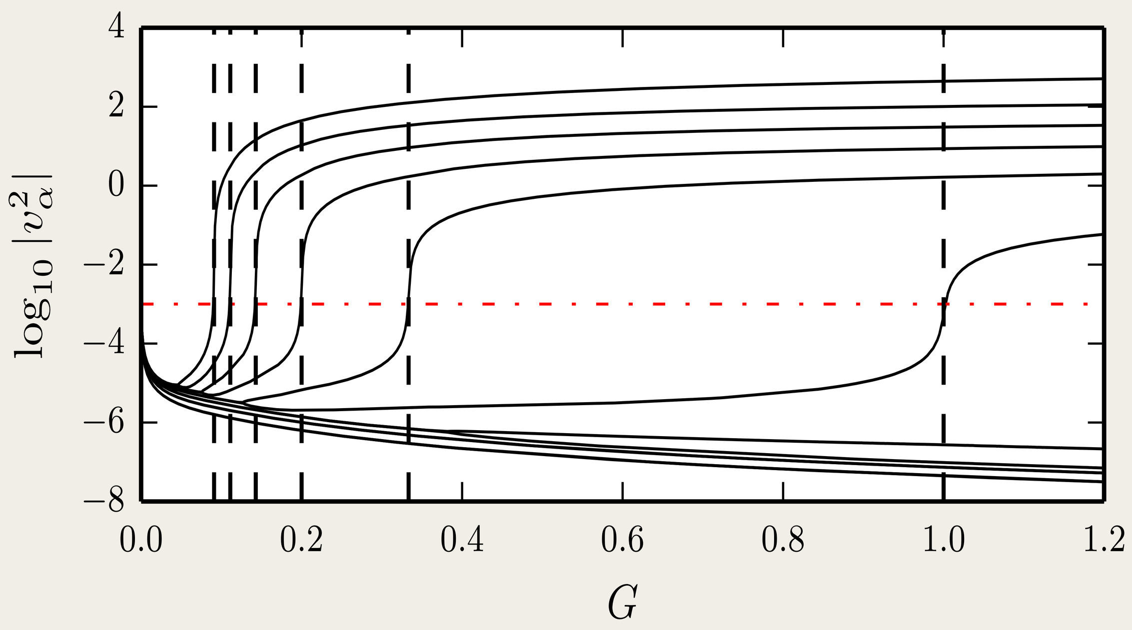


4. Introducing a system-environment interaction

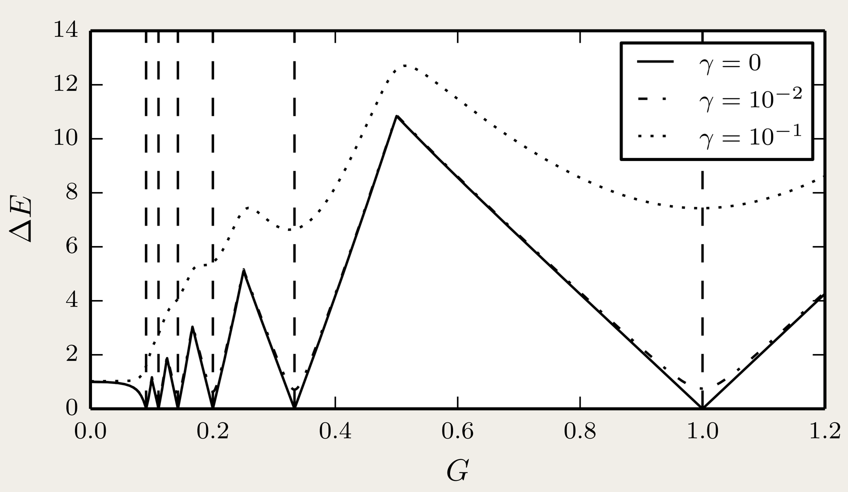
- Bath/environment allows for **exchange of Cooper pairs** = Breaks particle-number symmetry
- Interpolates between mean-field theory for finite systems and exact theory
- Does not break integrability = Can be exactly solved for large systems!

$$H = H_{p+ip} + \frac{\gamma}{\sqrt{2m}} \sum_{\mathbf{k}} \left[(k_x + ik_y) c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}} + \text{h.c.} \right]$$ ⟶ Does not change pairing symmetry

$$|RG\rangle = \prod_{\alpha=1}^{N} \left(\frac{\gamma}{v^2_\alpha} + G \sum_{\mathbf{k}} \frac{k_x - ik_y}{|\mathbf{k}|^2 - v^2_\alpha} c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}} \right) |\theta\rangle$$ ⟶ No definite particle number

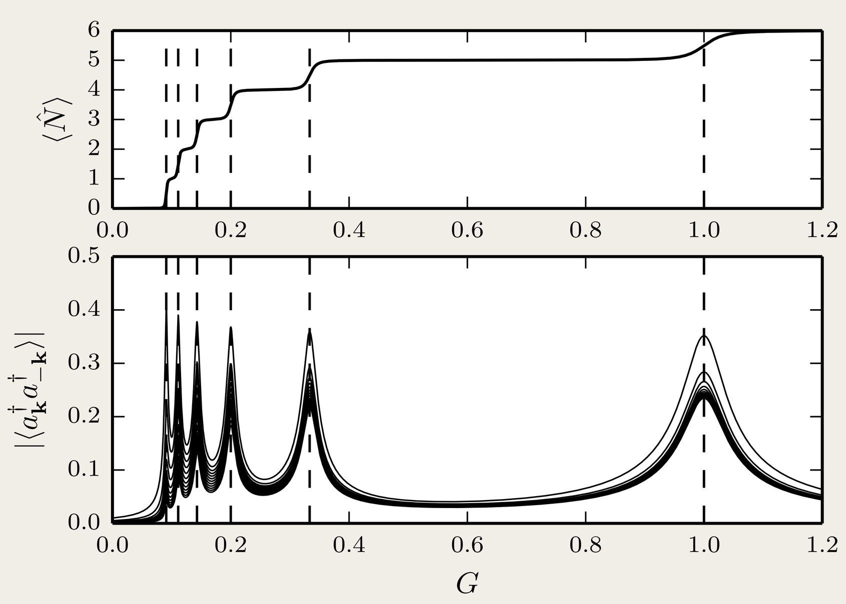


5. What happens to the phase transition?



Excitation gap

- System gapless at Read-Green points
- Density fluctuations allow for series of gaps
- Gaps open up due to system-bath coupling

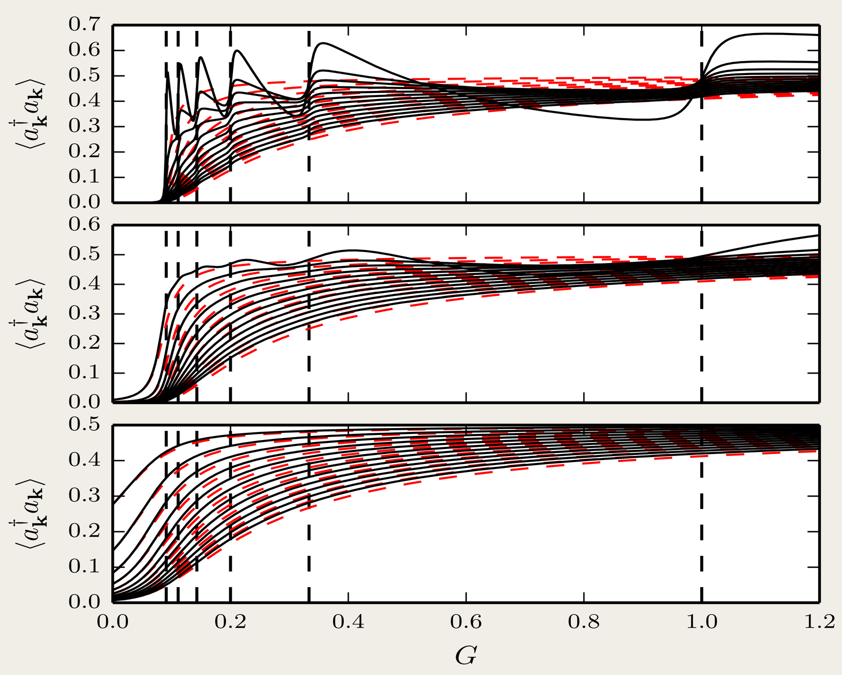


Occupation numbers

- Instead of phase transition the particle number increases
- Resonances (linked with zero-energy excitations)
- System remains in topologically non-trivial phase

- Read-Green resonances

5. Compare with mean-field theory



- Small system-bath coupling ($|\gamma| \ll$) Resonances at Read-Green points undetected by mean-field theory
- Resonances spread out with increasing $|\gamma|$
- Large system-bath coupling ($|\gamma| \gg$) mean-field theory becomes exact (bath coupling = mean-field interaction)

6. Conclusions

Due to the existence of zero-energy excitations, the system-bath coupling introduces avoided crossings at the phase transition, accompanied by **Read-Green resonances** in the low-energy and -momentum states.

References

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