Rotating Fermi Gases within the Finite Temperature Effective Field Theory

S. N. Klimin[1], J. Tempere[1, 2], N. Verhelst[1] and V. M. Milošević[3]

[1] TQC, Universiteit Antwerpen, Universiteitsplein 1, B-2610 Antwerpen, Belgium
[2] Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA
[3] Departement Fysica, Universiteit Antwerpen, Groenenborgerlaan 171, B-2020 Antwerpen, Belgium

Universiteit Antwerpen

Theory of Quantum and Complex systems

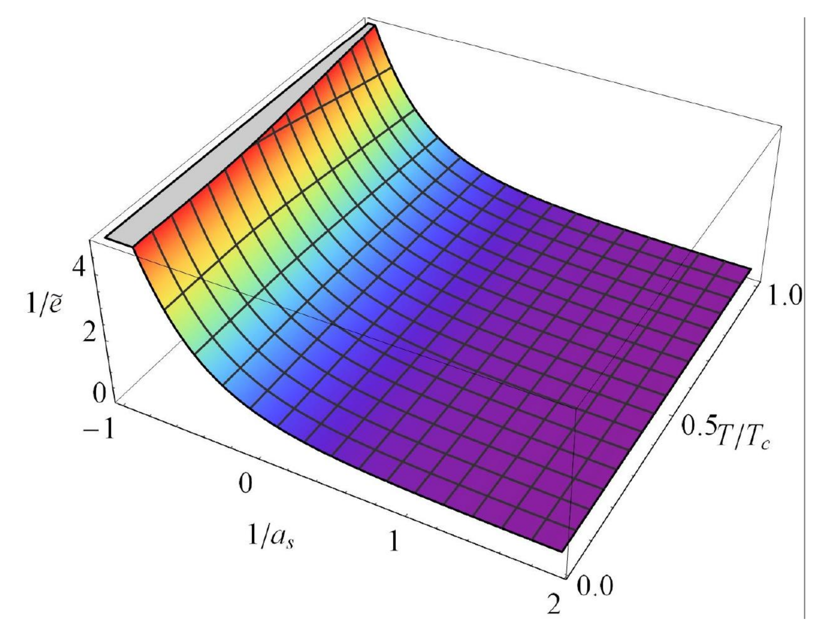
Abstract

In the present work, superfluid Fermi gases are described in terms of an effective field theory (EFT) [1] for a macroscopic wave function representing the field of condensed pairs, analogous to the Ginzburg – Landau theory for superconductors. We have established how rotation modifies this effective field theory [2], by deriving it on the basis of the *microscopic* Hamiltonian of Fermi gas in the rotating frame of reference. The rotation leads to the appearance of an effective vector potential, and the coupling strength of this vector potential to the macroscopic wave function depends on the interaction strength between the fermions, due to a *renormalization* of the pair effective mass in the effective field theory. The mass renormalization is in agreement with results of functional renormalization group theory. We use the macroscopic wave function description to study vortices and the critical rotation frequencies to form them. The derived phase diagrams for vortex states are in good agreement with results of the Bogoliubov – De Gennes theory and with experimental data.

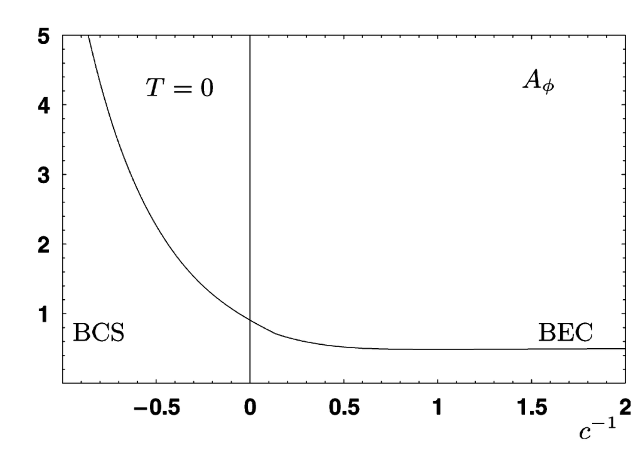
[1] S. N. Klimin, J. Tempere, G. Lombardi, and J. T. Devreese, Eur. Phys. Journal B **88**, 122 (2015); arXiv:1309.1421.
[2] S. N. Klimin, J. Tempere, N. Verhelst, and M. V. Milošević, arXiv:1512.00214 (*to be published*).

Renormalization of the pair mass

Finite temperature EFT [2]



Functional renormalization group theory at T = 0 [3]



EFT predicts the renormalization of the pair effective mass in line with the functional renormalization group theory.

[3] S. Diehl and C. Wetterich, Nucl. Phys. B **770**, 206 (2007).

Incorporation of rotation in EFT

We start from the path integral partition function

$$\mathcal{Z} \propto \int \mathcal{D}\left[\bar{\psi}, \psi\right] e^{-S}$$

with the microscopic action of interacting fermions

$$S = \int_0^\beta d\tau \int d\mathbf{r} \left[\sum_{\sigma=\uparrow,\downarrow} \bar{\psi}_\sigma \left(\frac{\partial}{\partial \tau} + H - \mu_\sigma(\mathbf{r}) \right) \psi_\sigma + g \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow \right]$$

The single-particle Hamiltonian in the rotating frame of reference

$$H = -\frac{\left(\nabla - i\mathbf{A}(\mathbf{r})\right)^2}{2m} + \frac{m\left(\omega_\perp^2 - \omega^2\right)}{2}\left(x^2 + y^2\right) + \frac{m\omega_z^2}{2} z^2 \qquad \mathbf{A}(\mathbf{r}) = m\omega\left[\mathbf{e}_z \times \mathbf{r}\right]$$

The Hubbard-Stratonovich transformation, the integration out fermion fields, and the gradient expansion lead to the effective field action

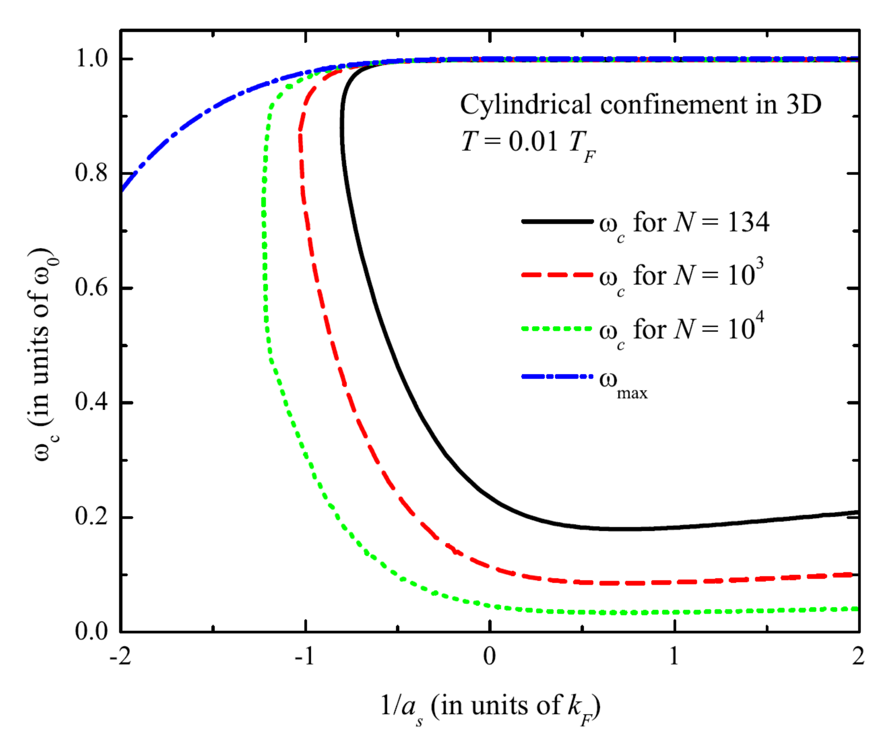
$$S_{eff} = \int_0^\beta d\tau \int d\mathbf{r} \left\{ \left[\Omega_s(w) + \frac{D}{2}\left(\bar{\Psi}\frac{\partial \Psi}{\partial \tau} - \frac{\partial \bar{\Psi}}{\partial \tau}\Psi \right) \right. \right.$$
$$\left. + Q \frac{\partial \bar{\Psi}}{\partial \tau}\frac{\partial \Psi}{\partial \tau} - \frac{R}{2w}\left(\frac{\partial w}{\partial \tau}\right)^2 + C\left(\nabla_\mathbf{r}\bar{\Psi}\cdot\nabla_\mathbf{r}\Psi\right) - E\left(\nabla_\mathbf{r} w\right)^2 \right.$$
$$\left. + iD\mathbf{A}\cdot\left(\bar{\Psi}\nabla_\mathbf{r}\Psi - \Psi\nabla_\mathbf{r}\bar{\Psi}\right) \right] \Big\} \qquad w = |\Psi|^2$$

This term appears due to rotation, which breaks the inversion symmetry.

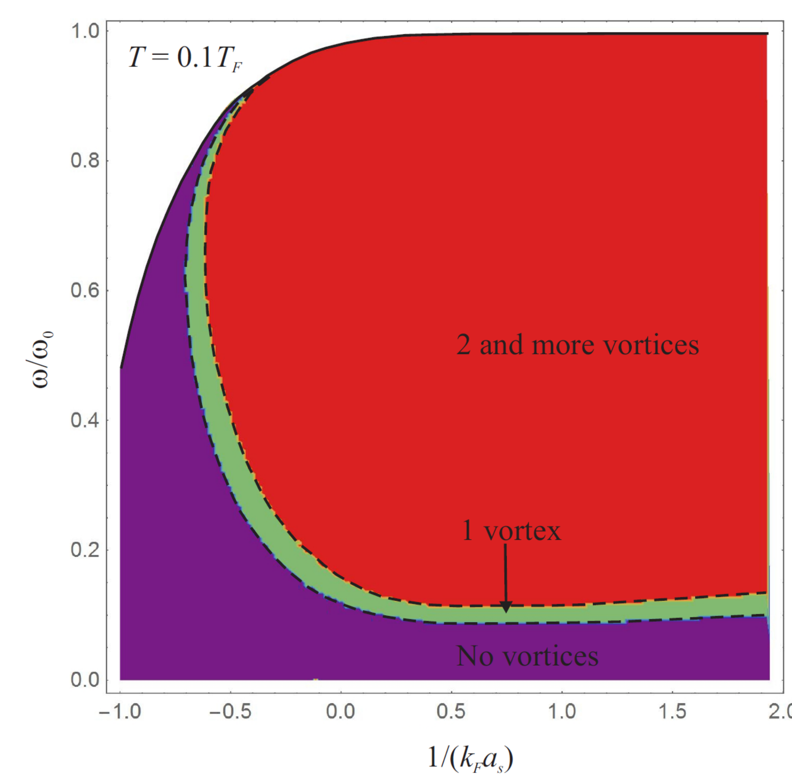
The coefficients are the same as in [1].

Area of vortex stability

Area of stability for vortices at different numbers of particles per unit length in a cylindrically symmetric confinement potential [2]



Phase diagram for one- and two-vortex states in the variables $(\omega/\omega_0, 1/(k_F a_s))$ [2]

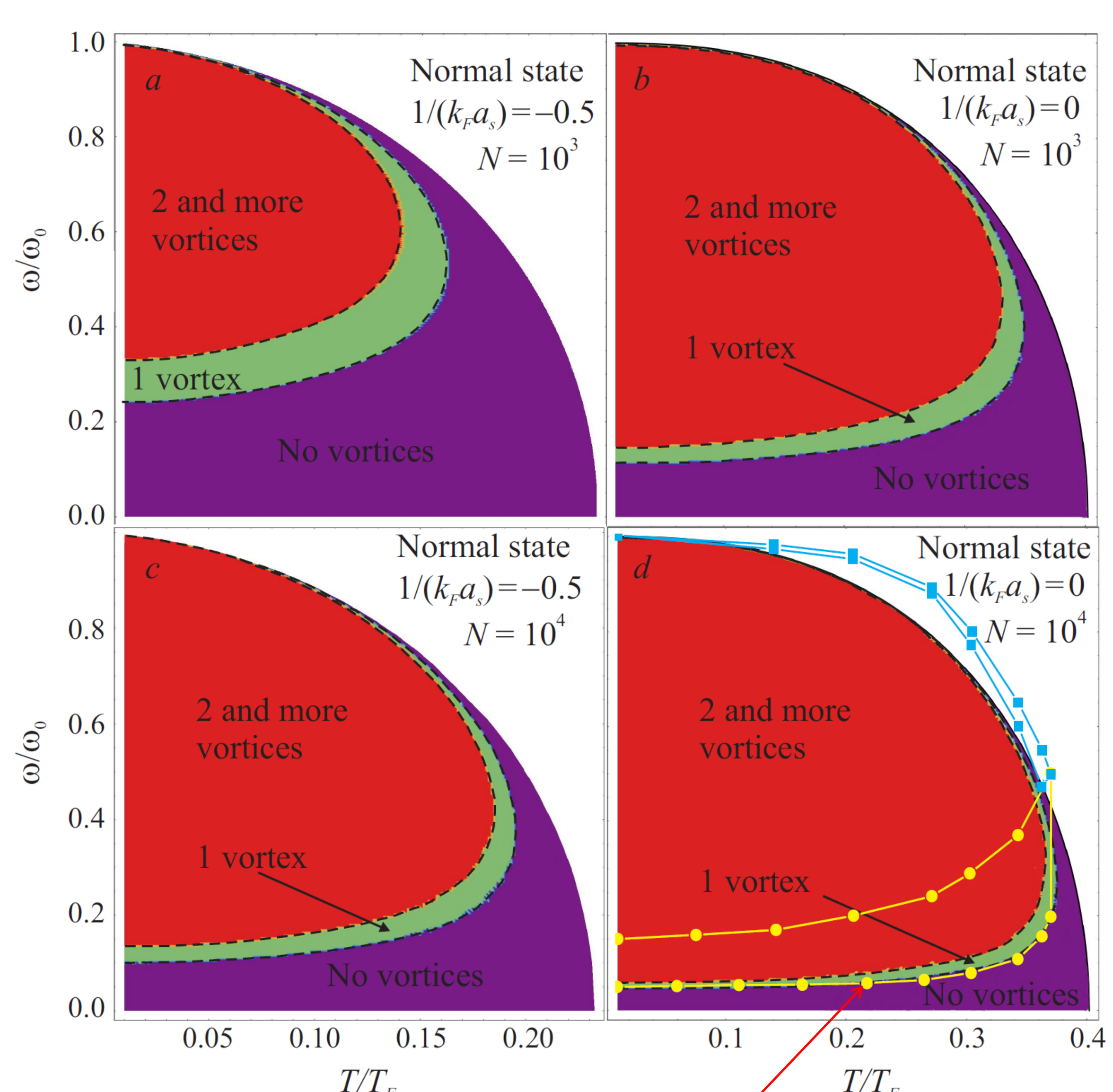


There can be *two* critical rotation frequencies for the vortex formation.
At low rotation frequencies, vortices are not stable. When increasing ω, vortices can become stable starting from a lower critical rotation frequency $\omega_{c,1}$.
There may exist also an upper critical rotation frequency $\omega_{c,2}$ such that the vortex state turns back to the superfluid state for $\omega_{c,1} < \omega < \omega_{c,2}$, since the radius of the superfluid state falls down with an increasing rotation frequency.
The obtained phase diagrams are in line with the BdG calculation [4].

[4] H. J. Warringa, Phys. Rev. A **86**, 043615 (2012).

Phase diagrams: temperature dependence

Phase diagrams for one- and two-vortex states in the variables $(\omega/\omega_0, T/T_F)$ [2]



Comparison with BdG, Ref. [5]

In the phase diagrams, the transition lines between the regimes with different numbers of vortices bend over leading to *reentrant behavior* of the critical rotation frequencies as a function of temperature. This reentrant dependence has a clear physical sense.

On one hand, at higher temperatures, the radius of the superfluid phase decreases. On the other hand, the healing length increases when the temperature rises towards T_c.

When the healing length is sufficiently large, the existence of stable vortices becomes energetically non-favorable with respect to the superfluid state.

The obtained phase diagrams exhibit a clear similarity to those obtained in Refs. [4, 5].

The critical rotation frequency is in a good agreement with the results of the coarse grained BdG method [5].

[5] S. Simonucci, P. Pieri, and G. C. Strinati, Nature Physics **11**, 941 (2015).

Conclusions

- The finite temperature effective field theory for fermionic superfluids is extended to the case of rotating Fermi gases.
- A non-trivial physical result is the renormalization of the effective mass for the fermion pair. It is in agreement with the prediction of the functional renormalization group theory.
- The vortex phase diagrams exhibit close similarity with the results of the coarse grained BdG method.
- The lowest critical frequencies calculated in both EFT and BdG approaches lie close to each other despite the fact that our calculation lies within the picture where the effective mass of "dressed" pairs is renormalized, while in the BdG treatment, masses of pairs are non-renormalized.
- This coincidence is remarkable and promising to throw a bridge between these two paradigms.

Ref: S. N. Klimin, J. Tempere, N. Verhelst, and M. V. Milošević, arXiv:1512.00214.

We acknowledge support from by the Flemish Research Foundation (FWO-Vl), project nrs. G.0115.12N, G.0119.12N, G.0122.12N, G.0429.15N, by the Scientific Research Network of the Research Foundation-Flanders, WO.033.09N, and by the Research Fund of the University of Antwerp.