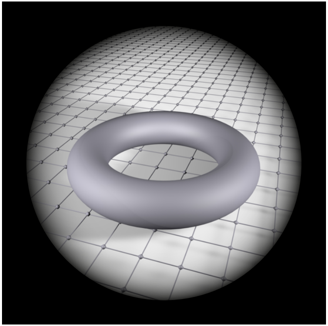
Measuring Chern numbers in Atomic Gases: 2D and 4D Quantum Hall Physics in the Lab

Nathan Goldman



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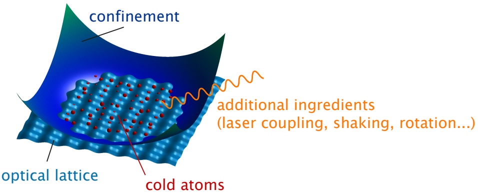
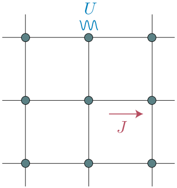
Building band structures for ultracold atoms in optical lattices

- Building a Hamiltonian

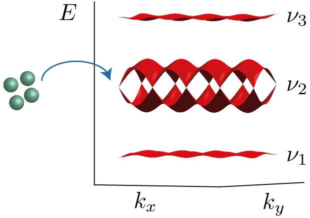


$$\hat{H}_{\text{model}} = -J \sum_{\text{link}} \hat{a}_j^\dagger \hat{a}_{j+1} + U \sum_{\text{sites}} \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j + \text{ingredients} \; \ldots$$



Reviews: Dalibard et al. **Rev. Mod. Phys. '11** and Goldman et al. **Rep. Prog. Phys. '14**

- Loading atoms into the corresponding band structure

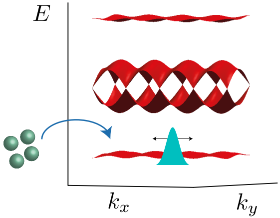


How will the atomic cloud behave,

if the bands are topological?

Loading atoms into the bands: measuring geometry vs topology

- Wave-packet preparation : **local in k-space**



Probe:

- The local dispersion of the band (i.e. band velocity)
- The local **geometry** of the band,

  as captured by the **Berry curvature**

$$\Omega_n^{xy} = i\left[\left\langle \partial_{k_x} u_n \middle| \partial_{k_y} u_n \right\rangle - \left\langle \partial_{k_y} u_n \middle| \partial_{k_x} u_n \right\rangle\right]$$

Experiments: Jotzu et al. (Nature '14), Duca et al. (Science '15), Fläschner et al. (arXiv:1509.05763)

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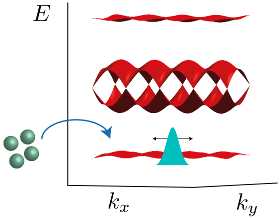

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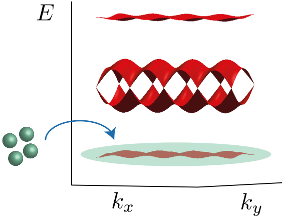
- Filling the band **uniformly in k-space**



Probe:

- The global **topology** of the band

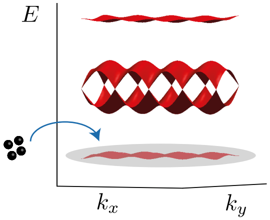
  Example: the **first Chern number** of the band

$$\frac{1}{2\pi}\int_{\mathbb{T}^2} \Omega_n = \nu \in \mathbb{Z}$$

Experiments: Aidelsburger et al. (Nat. Phys. '15), Wu et al. (arXiv:1511.08170)

Hall conductivity and center-of-mass observables

- **Solid-state physics**: filling a band with electrons



electric field

$$+\boldsymbol{E} = E_y \mathbf{1}_y$$

Transport equation for the current density

$$j^x = \frac{e^2}{h} E_y \nu$$

(TKNN formula)

⟶ quantized Hall conductivity (IQHE)

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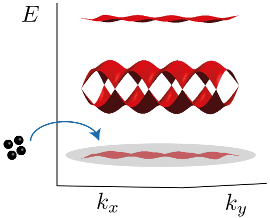

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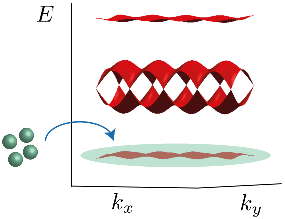

linear gradient

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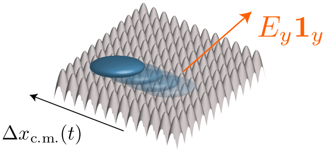
Center-of-mass imaged in-situ

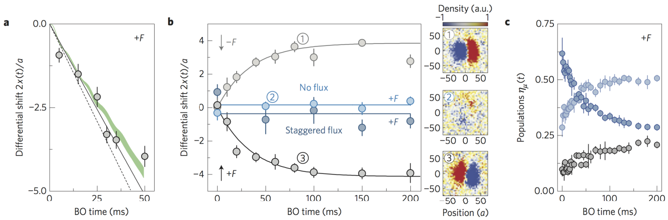


$$\boldsymbol{v}_{\text{c.m.}} = \frac{\boldsymbol{v}_{\text{tot}}}{N_{\text{tot}}} = \frac{\boldsymbol{j}(\nu)}{n}$$

LETTERS
PUBLISHED ONLINE: 8 DECEMBER 2014 | DOI: 10.1038/NPHYS3171
nature physics

Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms

M. Aidelsburger[1,2*], M. Lohse[1,2], C. Schweizer[1,2], M. Atala[1,2], J. T. Barreiro[1,2†], S. Nascimbène[3], N. R. Cooper[4], I. Bloch[1,2] and N. Goldman[3,5]



By fitting this equation to the experimental data $x(t)$, with the Chern number being the only fit parameter, we obtain an experimental value for the Chern number of the lowest band

$$\nu_{\mathrm{exp}} = 0.99(5)$$

Hall conductivity and center-of-mass observables

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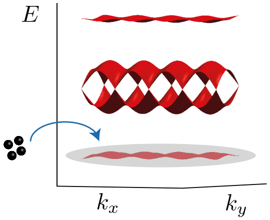

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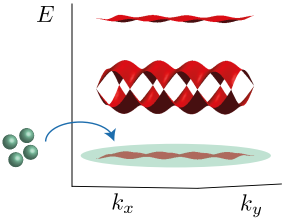

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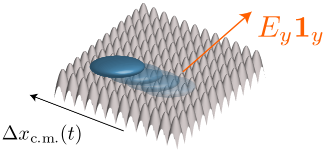

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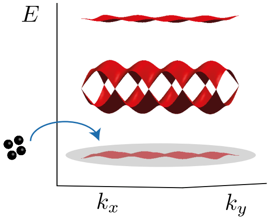

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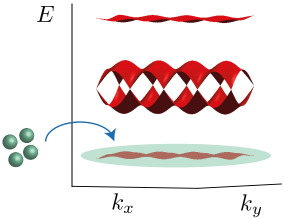

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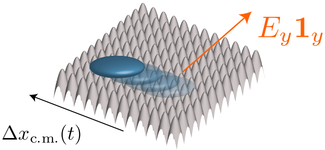
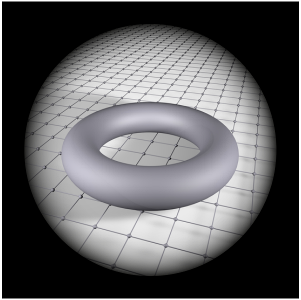


$$\boxed{\boldsymbol{v}_{\text{c.m.}} = \frac{\boldsymbol{v}_{\text{tot}}}{N_{\text{tot}}} = \frac{\boldsymbol{j}(\nu)}{n(\nu)}}$$

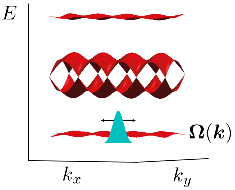
On the measurement of Chern numbers through center-of-mass responses



H. M. Price, O. Zilberberg, T. Ozawa, I. Carusotto, N. G., **arXiv:1602.01696**

Semiclassics and the modified density of states

- Let us consider a wave-packet prepared in a band $\mathcal{E}(\boldsymbol{k})$ with curvature $\boldsymbol{\Omega}(\boldsymbol{k})$



$+\boldsymbol{E}$: electric field/linear gradient

$+\boldsymbol{B}$: weak (perturbing) magnetic field

- The semiclassical equations of motion are given by [$\hbar = e = 1$]

$$\dot{\boldsymbol{r}} = \frac{\partial \mathcal{E}(\boldsymbol{k})}{\partial \boldsymbol{k}} - \dot{\boldsymbol{k}} \times \boldsymbol{\Omega}(\boldsymbol{k}),$$
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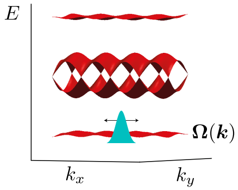

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- For a **completely filled band**, the particle density is given by

$$n = \frac{1}{V} \sum_{\boldsymbol{k}} = \int_{\mathbb{T}^2} D(\boldsymbol{k}) \mathrm{d}^2 k, \qquad D(\boldsymbol{k}) \text{ : phase-space density of states}$$

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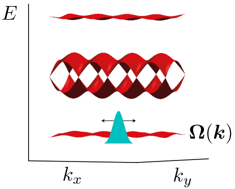

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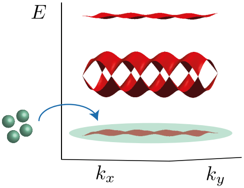
- Care is required in the presence of **both** $\boldsymbol{\Omega}$ and $\boldsymbol{B}$ [see Xiao et al. PRL '05, Bliokh PLA '06]

$$n = \int_{\mathbb{T}^2} D(\boldsymbol{k}) \mathrm{d}^2 k = \frac{1}{(2\pi)^2} \int_{\mathbb{T}^2} (1 + \boldsymbol{B}\cdot\boldsymbol{\Omega}) \mathrm{d}^2 k$$

[the phase-space volume element $\Delta \boldsymbol{R} \Delta \boldsymbol{K}$ is constant for **canonical** variables !]

Center-of-mass responses

- Summary of the configuration :



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[Streda]

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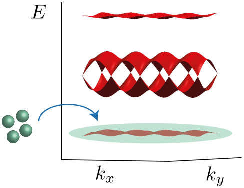

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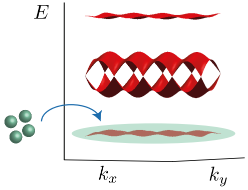

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- The **transverse center-of-mass velocity** is given by

$$v^x_{\text{c.m.}} = \frac{j^x}{n} = \left(\frac{E_y}{\frac{A_{\text{MBZ}}}{2\pi} + B\nu}\right)\nu \approx \frac{2\pi}{A_{\text{MBZ}}}E_y\nu - \left(\frac{2\pi}{A_{\text{MBZ}}}\right)^2 E_y B\,(\nu)^2 \quad \left[\frac{|2\pi B\nu|}{A_{\text{MBZ}}} \ll 1\right]$$

- There are **two types of quantized responses** :

linear response ($\sim E_y$) + non-linear response ($\sim E_y B$)

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- Simple check (homework !) : square lattice with flux per plaquette $\Phi_{\text{tot}} = 1/5$

Picture A : $\Phi = 1/5$ and $\Phi_{\text{pert}} = 0 \longrightarrow v^x_{\text{c.m.}}(\text{A})$

Picture B : $\Phi = 1/4$ and $\Phi_{\text{pert}} = -1/20 \longrightarrow v^x_{\text{c.m.}}(\text{B}) = v^x_{\text{c.m.}}(\text{A})$

This simple equivalence is possible thanks to the **modified density of states** !

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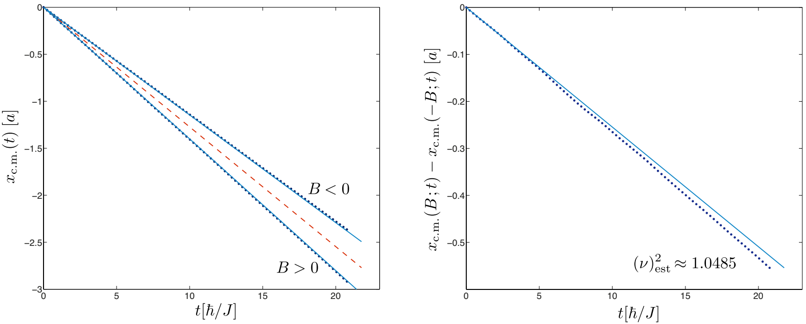
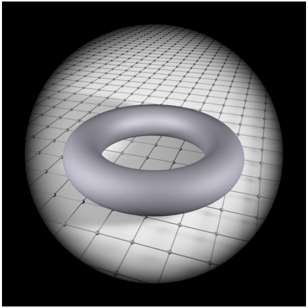
- Full numerical simulations for $\Phi = 1/4$ and $\Phi_{\text{pert}} = \pm 10\% \times \Phi$



4D Physics with Cold Atoms



H. M. Price, O. Zilberberg, T. Ozawa, I. Carusotto, N. G., **PRL 115, 195303 (2015)**

Brief warm-up : Topological structures in 4D

- The curvature Ω of a given band (defined over the FBZ) is a **two-form**

$$\Omega = \frac{1}{2}\Omega^{\mu\nu}\mathrm{d}k_{\mu} \wedge \mathrm{d}k_{\nu} \quad \neq 0 \text{ for } \dim(\text{FBZ}) \geq 2 \qquad \text{(no curvature in 1D)}$$

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$$\Omega^2 = \Omega \wedge \Omega = \frac{1}{4}\Omega^{\mu\nu}\Omega^{\gamma\delta}\,\mathrm{d}k_\mu \wedge \mathrm{d}k_\nu \wedge \mathrm{d}k_\gamma \wedge \mathrm{d}k_\delta \quad \neq 0 \text{ for } \dim(\text{FBZ}) \geq 4$$

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- **Topology in 2D** : the first Chern number

$$\nu_1 = \frac{1}{2\pi}\int_{\text{FBZ}} \mathrm{Tr}\,\Omega \qquad \dim(\text{FBZ}) = 2$$

- The first Chern number is associated with the 2D quantum Hall effect ($\text{FBZ} = \mathbb{T}^2$)

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- **Topology in 4D** : the second Chern number

$$\nu_2 = \frac{1}{8\pi^2}\int_{\text{FBZ}} \mathrm{Tr}\,\Omega^2 \qquad \dim(\text{FBZ}) = 4$$

- The second Chern number is associated with the 4D QH effect ($\text{FBZ} = \mathbb{T}^4$)
  see Zhang and Hu Science 2001 and Avron et al. PRL 1988 about 4D systems with TRS

4D quantum Hall effect from semiclassics

- We consider a wave-packet in a 4D Bloch band $\mathcal{E}(\boldsymbol{k})$ with Berry curvature $\Omega^{\mu\nu}(\boldsymbol{k})$

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- We consider a wave-packet in a 4D Bloch band $\mathcal{E}(\boldsymbol{k})$ with Berry curvature $\Omega^{\mu\nu}(\boldsymbol{k})$
- We write the semiclassical equations of motion in $D = 4$ spatial dimensions

$$\dot{r}^{\mu}(\boldsymbol{k}) = \frac{\partial \mathcal{E}(\boldsymbol{k})}{\partial k_{\mu}} - \dot{k}_{\nu}\Omega^{\mu\nu}(\boldsymbol{k}); \qquad \mu, \nu = x, y, z, w \tag{1}$$

$$\dot{k}_{\mu} = -E_{\mu} - \dot{r}^{\nu} B_{\mu\nu}; \qquad B_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

see Xiao et al. RMP ’10 ; Gao,Yang,Niu PRB ’15

4D quantum Hall effect from semiclassics

- We consider a wave-packet in a 4D Bloch band $\mathcal{E}(\boldsymbol{k})$ with Berry curvature $\Omega^{\mu\nu}(\boldsymbol{k})$
- We write the semiclassical equations of motion in $D = 4$ spatial dimensions

$$\dot{r}^{\mu}(\boldsymbol{k}) = \frac{\partial \mathcal{E}(\boldsymbol{k})}{\partial k_{\mu}} - \dot{k}_{\nu}\Omega^{\mu\nu}(\boldsymbol{k}); \qquad \mu, \nu = x, y, z, w \tag{1}$$

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- Let us combine these Eqs. (1) repeatedly :

$$\begin{aligned}
\dot{r}^{\mu}(\boldsymbol{k}) &= \frac{\partial \mathcal{E}(\boldsymbol{k})}{\partial k_{\mu}} + E_{\nu}\Omega^{\mu\nu}(\boldsymbol{k}) + \dot{r}^{\gamma} B_{\nu\gamma}\Omega^{\mu\nu}(\boldsymbol{k}) \\
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$\rightarrow$ Combining $\boldsymbol{E}$ and $\boldsymbol{B}$ produces a term $\sim \Omega^2$

- Topological response in the **current density of a filled band** ?

Topological responses in 4D

- The velocity in a state $\boldsymbol{k}$ is

$$\dot{r}^{\mu}(\boldsymbol{k}) \approx \frac{\partial \mathcal{E}(\boldsymbol{k})}{\partial k_{\mu}} + E_{\nu}\Omega^{\mu\nu}(\boldsymbol{k}) + \frac{\partial \mathcal{E}(\boldsymbol{k})}{\partial k_{\gamma}} B_{\nu\gamma}\Omega^{\mu\nu}(\boldsymbol{k}) + \Omega^{\gamma\delta}(\boldsymbol{k})\Omega^{\mu\nu}(\boldsymbol{k})E_{\delta}B_{\nu\gamma} + \dots$$

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- For a TRS system, we recover the topological-field-theory prediction

$$j^{\mu} = \frac{\nu_2}{4\pi^2}\varepsilon^{\mu\alpha\beta\nu}E_{\nu}B_{\alpha\beta} \qquad \text{[Ref : Qi, Hughes, Zhang PRB '08]}$$

Introducing a 4D framework

- We want to investigate the transport equation **using ultracold atoms**

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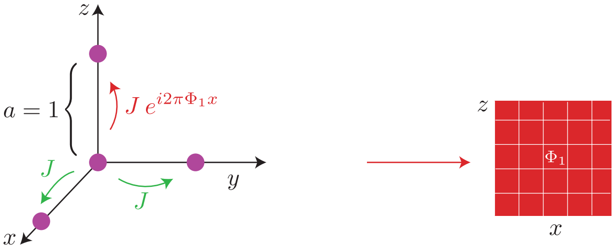

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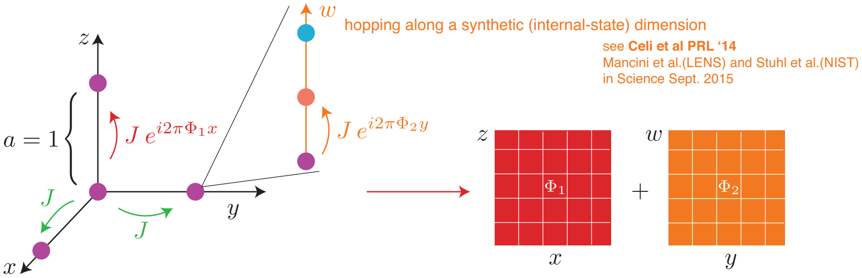

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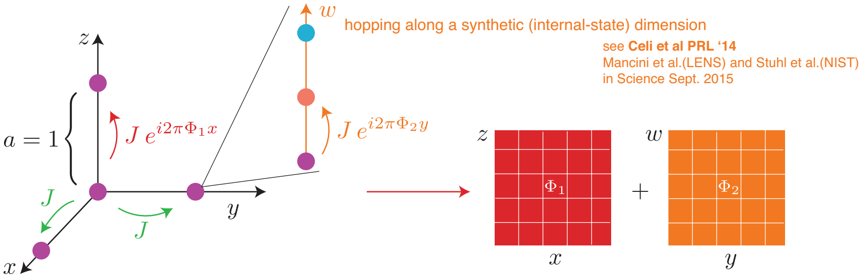
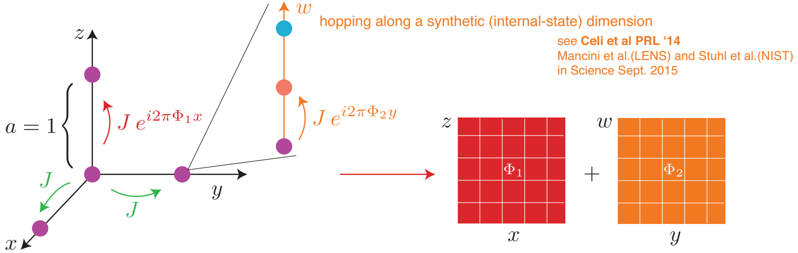
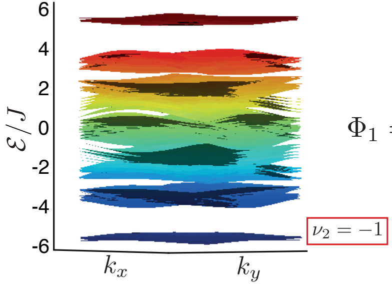


- Physical realization with cold atoms in a 3D optical lattice : Easy !
  - A superlattice along z + resonant $x-z$-dependent time-modulation
  - Raman transitions between internal states with recoil momentum along y



- **Topological band structure** :

The energy spectrum displays a low-energy **flat** topological band with $\boldsymbol{\nu_2 = -1}$



$$\Phi_1 = \Phi_2 = \frac{1}{4}$$

Probing the transport equations

- Let us come back to our transport equation, with $\Omega^{zx}, \Omega^{yw} \neq 0$

$$j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{\mathbb{T}^4} \Omega^{\mu\nu} \mathsf{d}^4 k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta}, \quad \nu_2 = \frac{1}{4\pi^2} \int_{\mathbb{T}^4} \Omega^{zx} \Omega^{yw} \mathsf{d}^4 k$$

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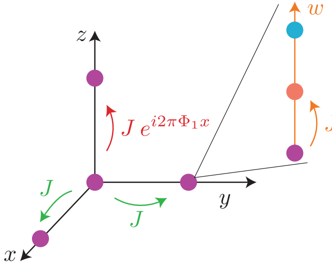
- We now choose an electric field $\boldsymbol{E} = E_y \mathbf{1}_y$ and a magnetic field $B_{\alpha\beta} = B_{zw}$



hopping along a synthetic (internal-state) dimension

see Celi et al PRL '14

(simply tune the Raman lasers!)

where $\tilde{\Phi} = B_{zw}/2\pi \approx 1/10$

- The transport equations yield two non-trivial contributions :

$$j^w = E_y \frac{1}{(2\pi)^4} \int_{\mathbb{T}^4} \Omega^{wy} \mathrm{d}^4 k \text{ : linear response along } w \text{ } (\sim \text{2D QH effect})$$

$$j^x = \frac{\nu_2}{4\pi^2} E_y B_{zw} \text{ : non-linear response along } x \text{ } (\sim \text{4D QH effect})$$

- Predictions have been validated through numerical simulations [$\nu_2^{\text{exp}} \approx -1.07$]

The center-of-mass drifts in a 4D topological system

- Reminder : the center-of-mass velocity is related to the current density via

$$\boldsymbol{v}_{\text{c.m.}} = \boldsymbol{j}/n$$

- The current density is

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- The particle density is [using the modified density of states !]

$$n = \frac{1}{(2\pi)^4} \int_{\mathbb{T}^4} \left[1 + \frac{1}{2} B_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{64} \left(\varepsilon^{\alpha\beta\gamma\delta} B_{\alpha\beta} B_{\gamma\delta} \right) \left(\varepsilon_{\mu\nu\lambda\rho} \Omega^{\mu\nu} \Omega^{\lambda\rho} \right) \right] \mathrm{d}^4 k$$

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We find situations where $j^{\mu} = j^{\mu}(\nu_2)$ but $v^{\mu}_{\text{c.m.}} \neq v^{\mu}_{\text{c.m.}}(\nu_2)$!

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- Here we show results for $B_{zw} \neq 0$ and $\Omega^{zw} = 0$: simple case !

The center-of-mass drift : Numerical simulations

- The predicted center-of-mass drift along x (2nd-Chern-number response) :

$$v^x_{\text{c.m.}} = j^x V_{\text{cell}} = j^x \left(4a \times 4a \times a \times a\right), \quad \text{for } \Phi_1 = \Phi_2 = 1/4$$

$$= \left(\frac{\nu_2}{4\pi^2} E_y \times B_{zw}\right) \times 16a^4 \approx 2a/T_B, \qquad T_B = 2\pi/aE_y \approx 50\text{ms}$$

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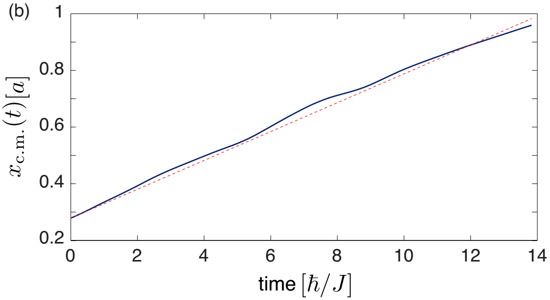
- We have calculated the COM trajectory for $E_y = 0.2J/a$ and $B_{zw}/2\pi = -1/10$



- From these simulations : $\nu_2 \approx -0.98$

The 4D responses are **of the same order**
as the effects reported in the 2D measurement [*Aidelsburger et al '15*] !

Conclusions

- **Center-of-mass observables** have a much richer dependence on topological invariants than previously discussed
- The **particle density** is related to topological invariants (Streda)
- **Ultracold gases** are suitable platforms to probe these intriguing topological responses (e.g. EM effects in COM observables)

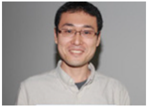
- **4D quantum Hall physics** (e.g. 2nd Chern numbers) is accessible using ultracold gases extended by **synthetic dimensions**
- Not discussed : 4D QH physics can also be explored with **photonics** (see Ozawa, Price, Goldman, Zilberberg, Carusotto, PRA **93**, 0438270 2016)

- Cold atoms offer a platform to study **interaction effects in 4D** topological bands

The 4D team

- Hannah M. Price, Tomoki Ozawa and Iacopo Carusotto, **BEC Center (Trento)**
- Oded Zilberberg, **ETH (Zurich)**

Hannah



Tomoki

Iacopo

Oded

- **On the measurement of Chern numbers through center-of-mass responses**
  H. M. Price, O. Zilberberg, T. Ozawa, I. Carusotto, N. G., **arXiv:1602.01696**

- **Four-Dimensional Quantum Hall Effect with Ultracold Atoms**
  H. M. Price, O. Zilberberg, T. Ozawa, I. Carusotto, N. G., **PRL 115, 195303 (2015)**

- **Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms**
  M. Aidelsburger, M. Lohse, C. Schweizer, M. Atala, J. T. Barreiro,
  S. Nascimbène, N. R. Cooper, I. Bloch and N. G., **Nature Physics 11, 162 (2015)**