Power laws in the dynamic hysteresis of quantum nonlinear photonic resonator

W. Casteels, F. Storme, A. Le Boité, C. Ciuti

wim.casteels@univ-paris-diderot.fr

Laboratoire Matériaux et Phénomènes Quantiques
Université Paris Diderot-Paris-7

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Outline

- Introduction
- optical bistability: semiclassical approach
- optical bistability at the quantum level
- Dynamical hysteresis
- Conclusions & perspectives

Introduction: Kerr model

$\hbar = 1$

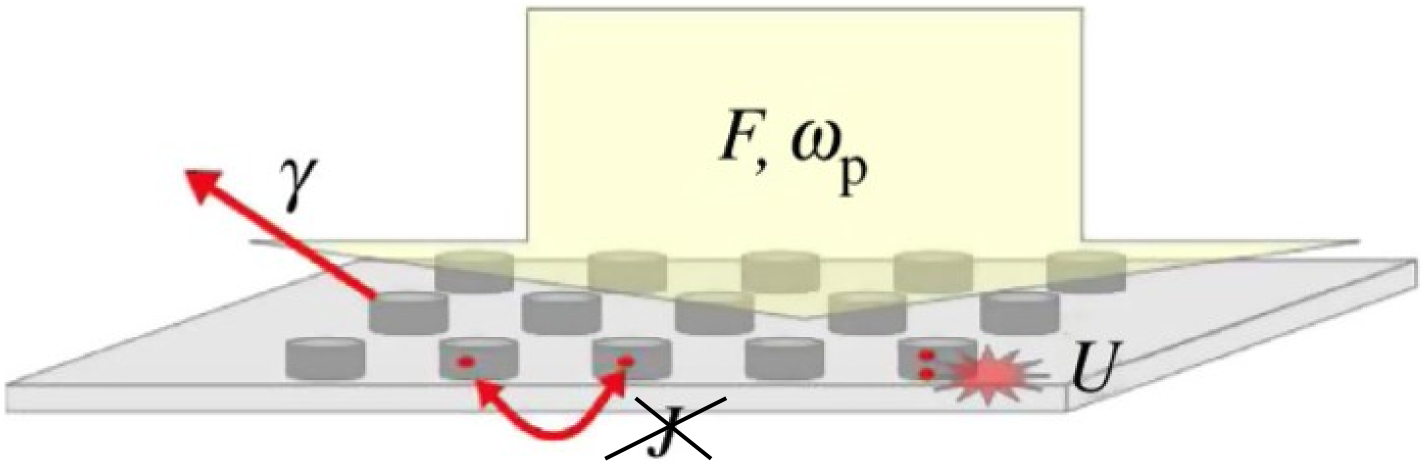
System Hamiltonian: $\hat{H}_S = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{U}{2} \hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a}$

Coherent drive term: $\hat{H}_P = F(t)\, e^{-i\omega_p t} \hat{a}^\dagger + F^*(t)\, e^{i\omega_p t} \hat{a}$

Total Hamiltonian: $\hat{H} = \hat{H}_S + \hat{H}_P$

Losses described by Lindblad master equation for density operator:

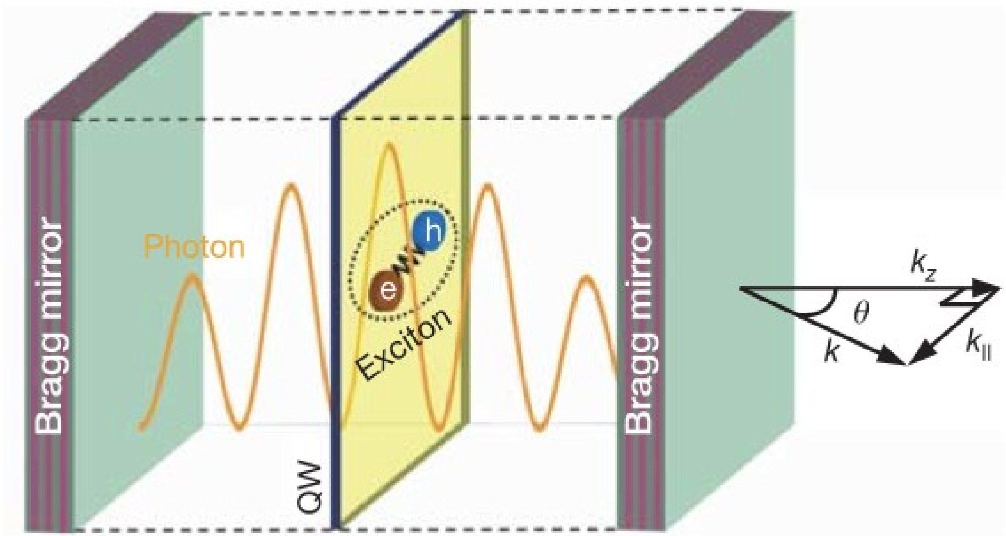
$$\partial_t \hat{\rho} = -i \left[\hat{H}, \hat{\rho}\right] + \frac{\gamma}{2} \left(2\hat{a}^\dagger \hat{\rho}\hat{a} - \hat{a}^\dagger \hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger \hat{a}\right)$$



Review article: I. Carusotto and C. Ciuti – Rev. Mod. Phys. **85**, 299 (2013)

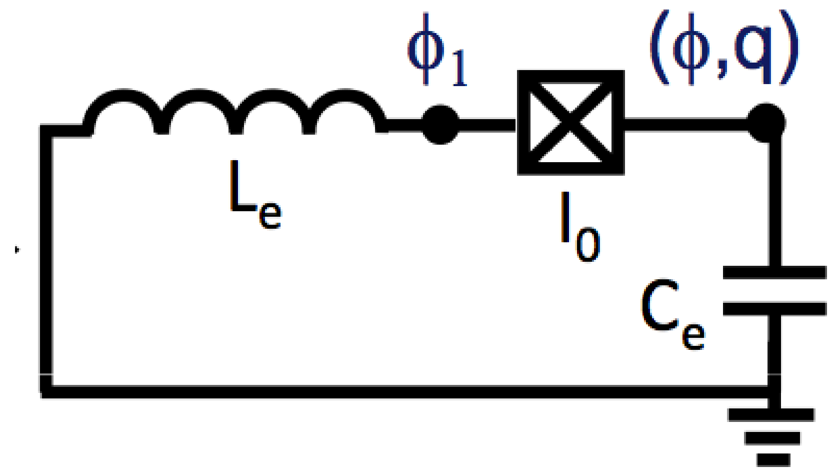
Experimental implementations

microcavity polaritons:



J. Kasprzak et. al. – Nature **443**, 409 (2006)
Daniele Bajoni et. al. – Phys. Rev. Lett. **100**, 047401 (2008)

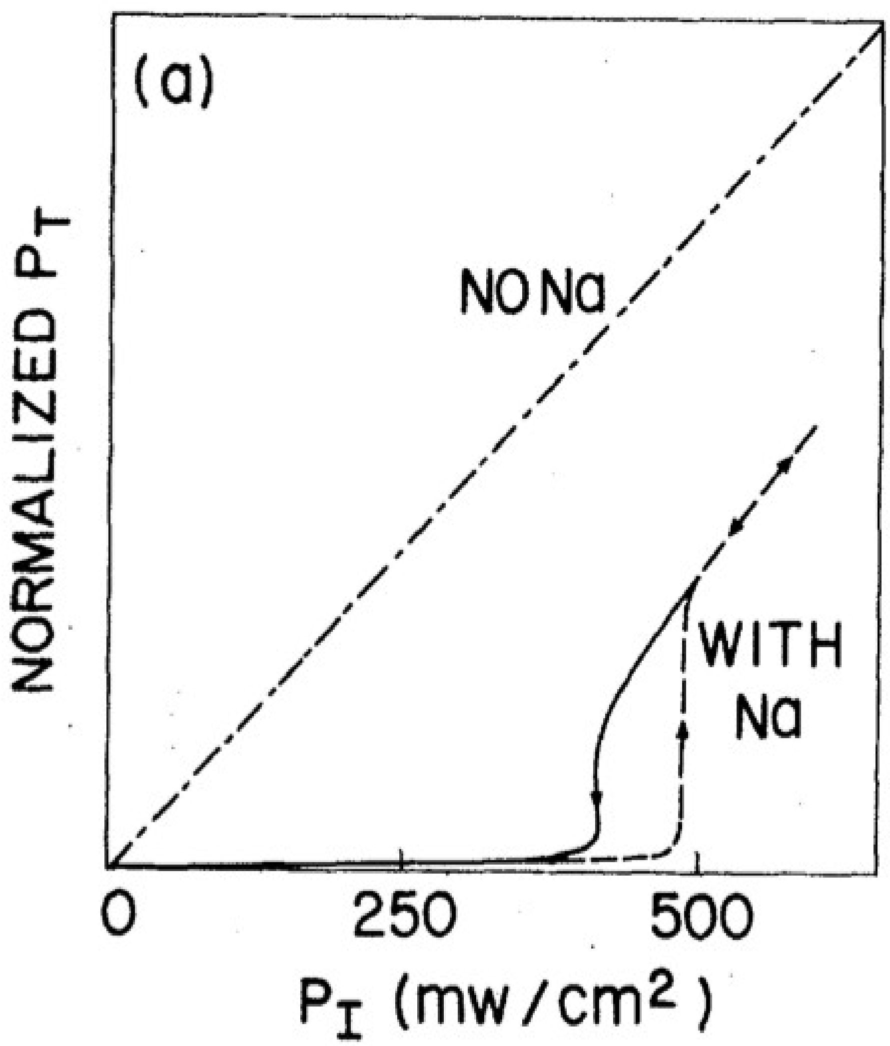
Superconducting circuit:



F. R. Ong et. al. – Phys. Rev. Lett. **106**, 167002 (2011)

Optical bistability

Experimental signature:



H. M. Gibbs, S. L. McCall, and T. N. C. Venkatesan
Phys. Rev. Lett. **36**, 1135 (1976)

Optical bistability: semiclassical approach

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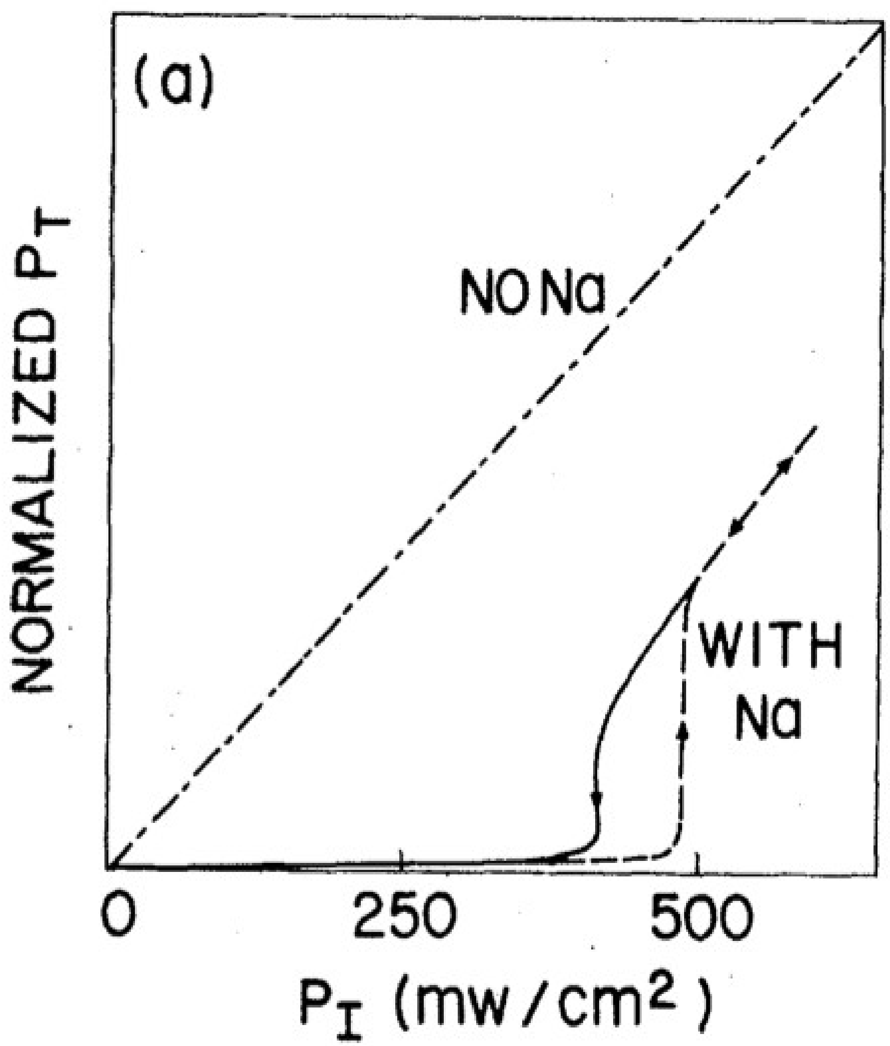

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Semiclassical analysis:

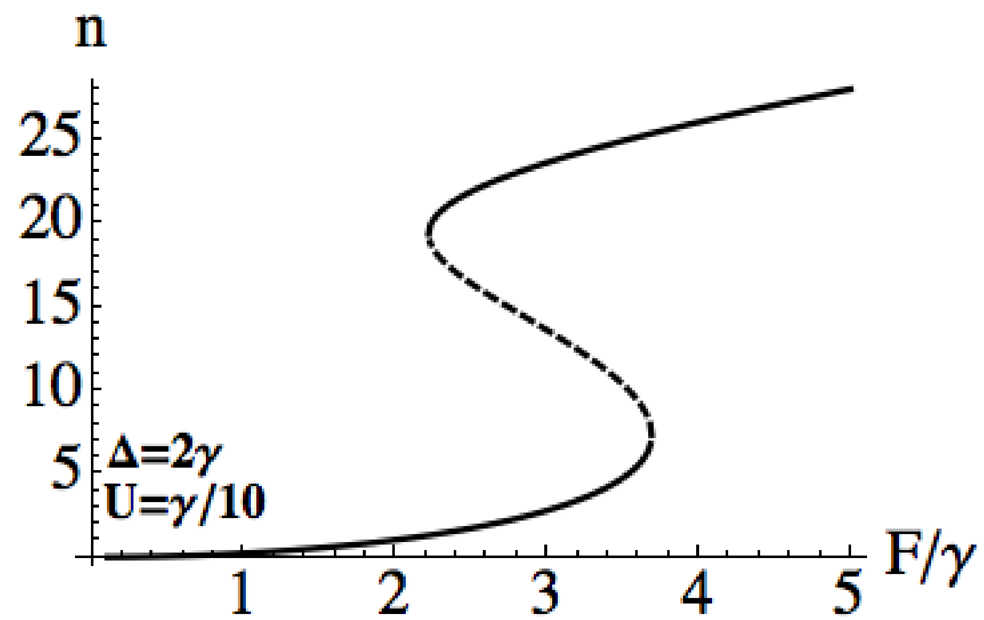
$$\hat{a} \rightarrow \alpha$$

$$i\partial_t \alpha = \left(\omega_c - i\frac{\gamma}{2} + U\left|\alpha\right|^2\right)\alpha + F(t)\, e^{-i\omega_p t}$$

Steady-state:

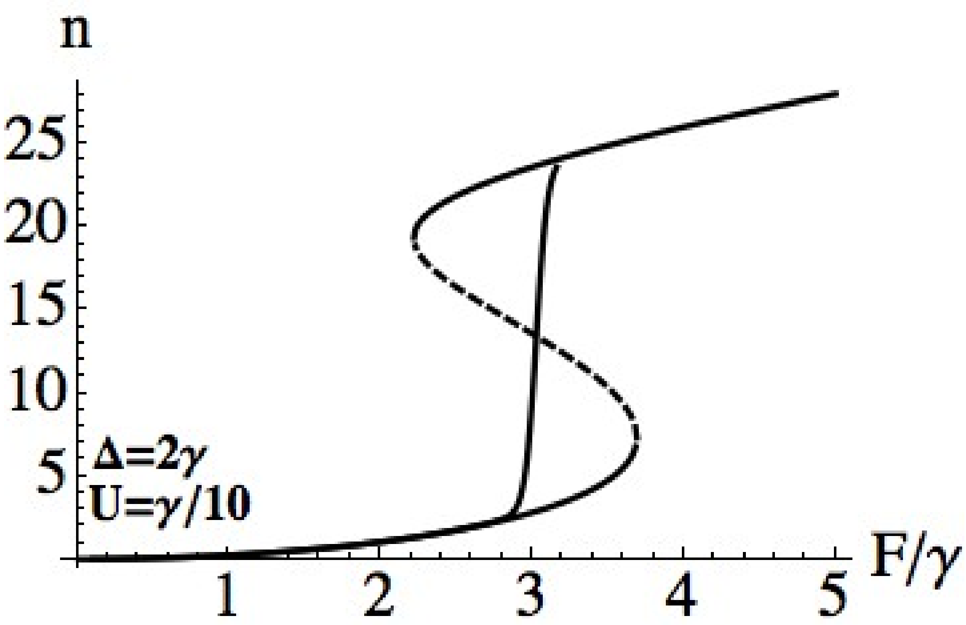
$$\left|\alpha\right|^2 = n_{SS} = \frac{\left|F\right|^2}{\left(-\Delta + U n_{SS}\right)^2 + \gamma^2/4}$$

Laser-cavity detuning: $\Delta = \omega_p - \omega_c$



optical bistability at the quantum level

The quantum solution is unique:



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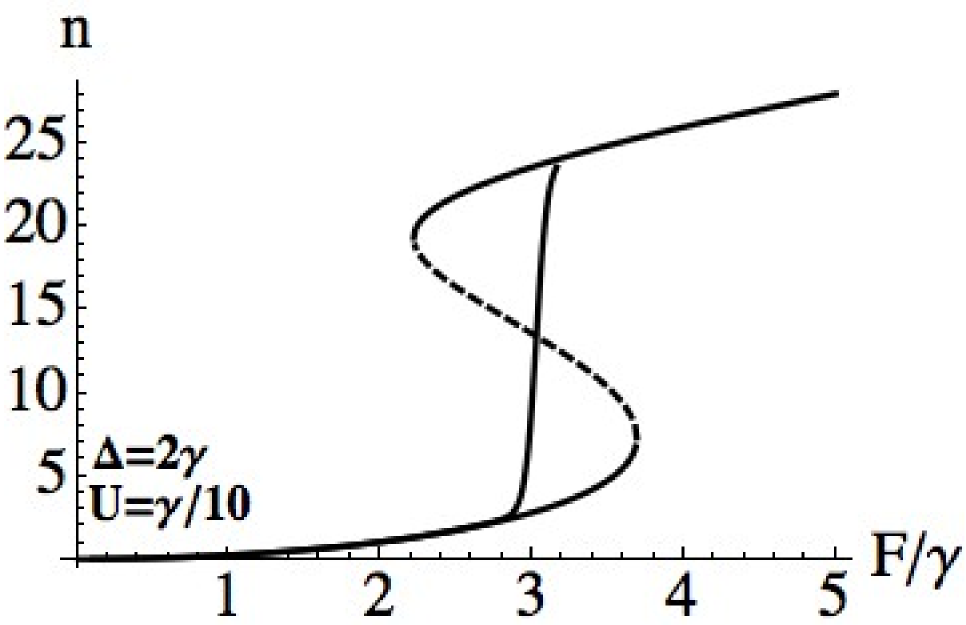

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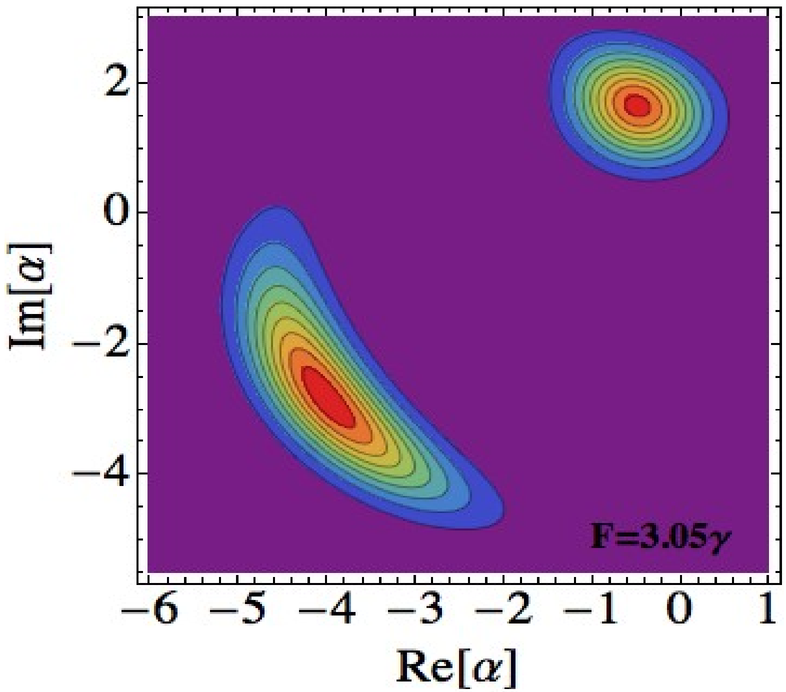
Bimodal Wigner distribution:



K. Vogel and H. Risken – Phys. Rev. A **39**, 4675 (1989)

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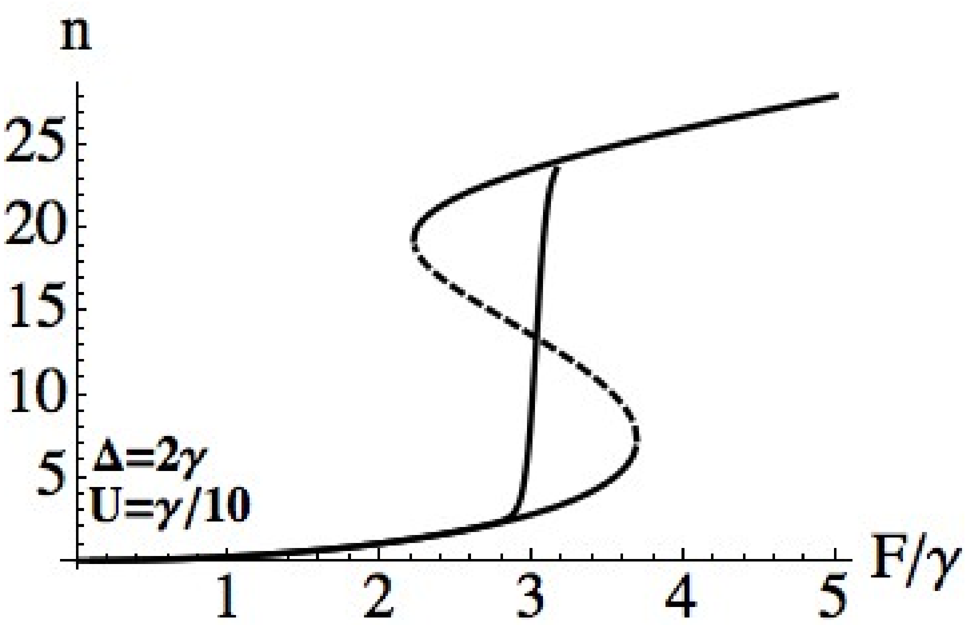

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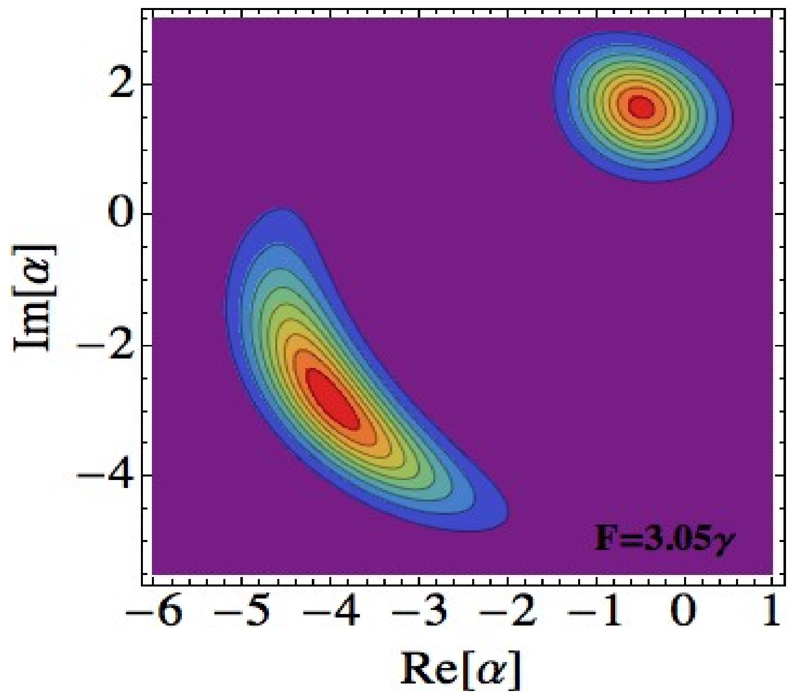

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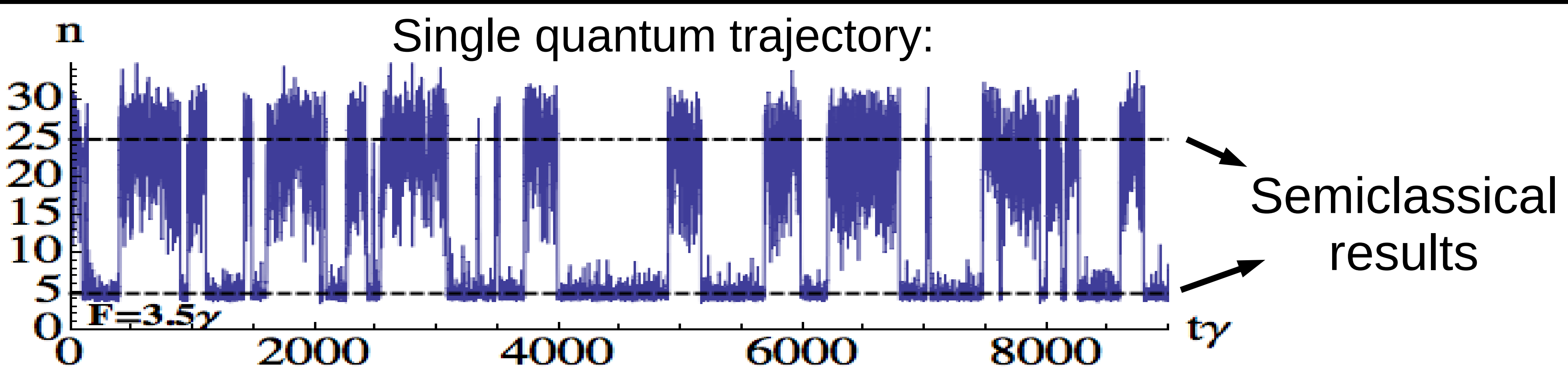
Single quantum trajectory:



J. Kerckhoff, M. A. Armen, and H. Mabuchi, Opt. Express 19, 24468 (2011).

Dynamical hysteresis

So far: analysis of the steady-state properties

Q: what about time dependence of the experimental sweep?

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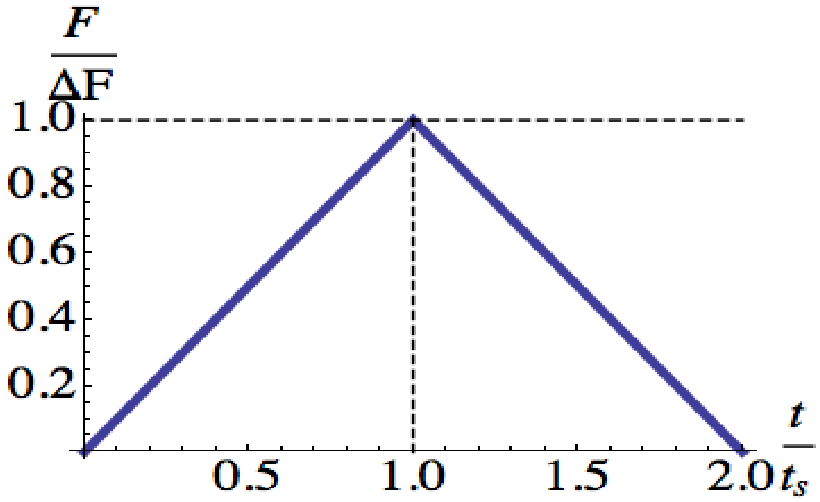
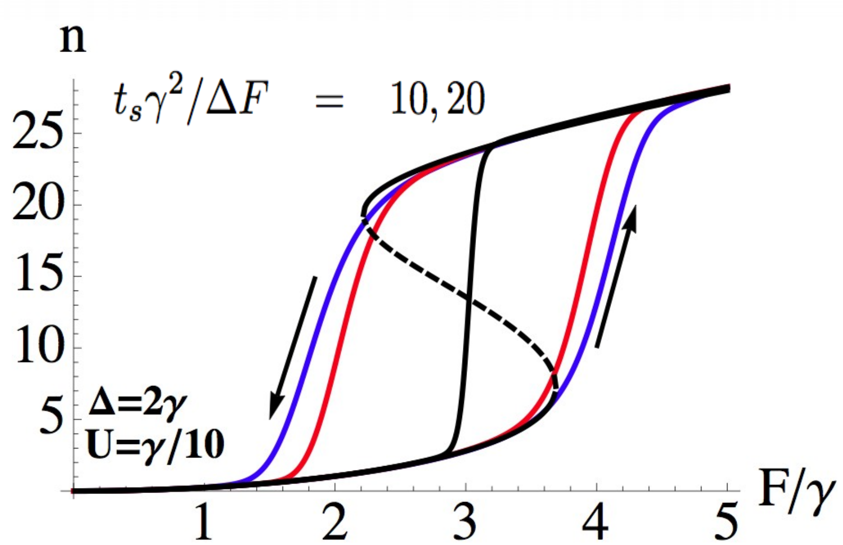
Q: what about time dependence of the experimental sweep?

Triangular time dependence:



→ dynamical hysteresis:



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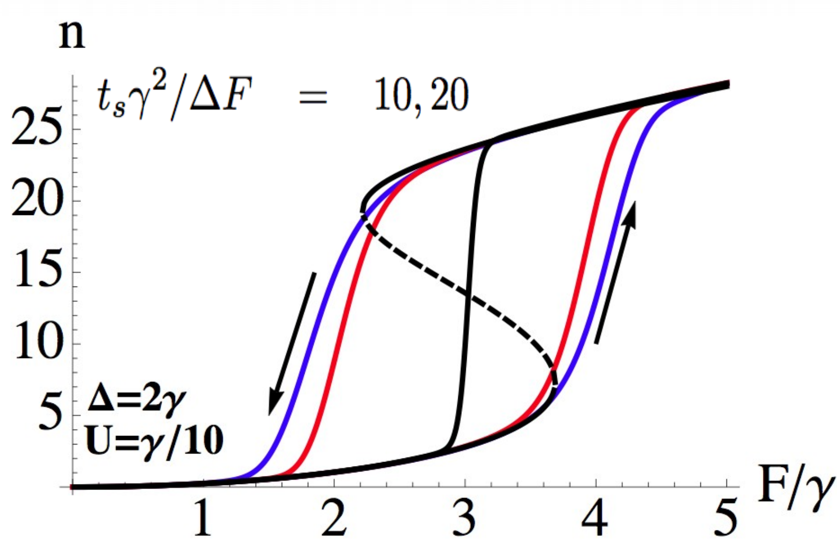
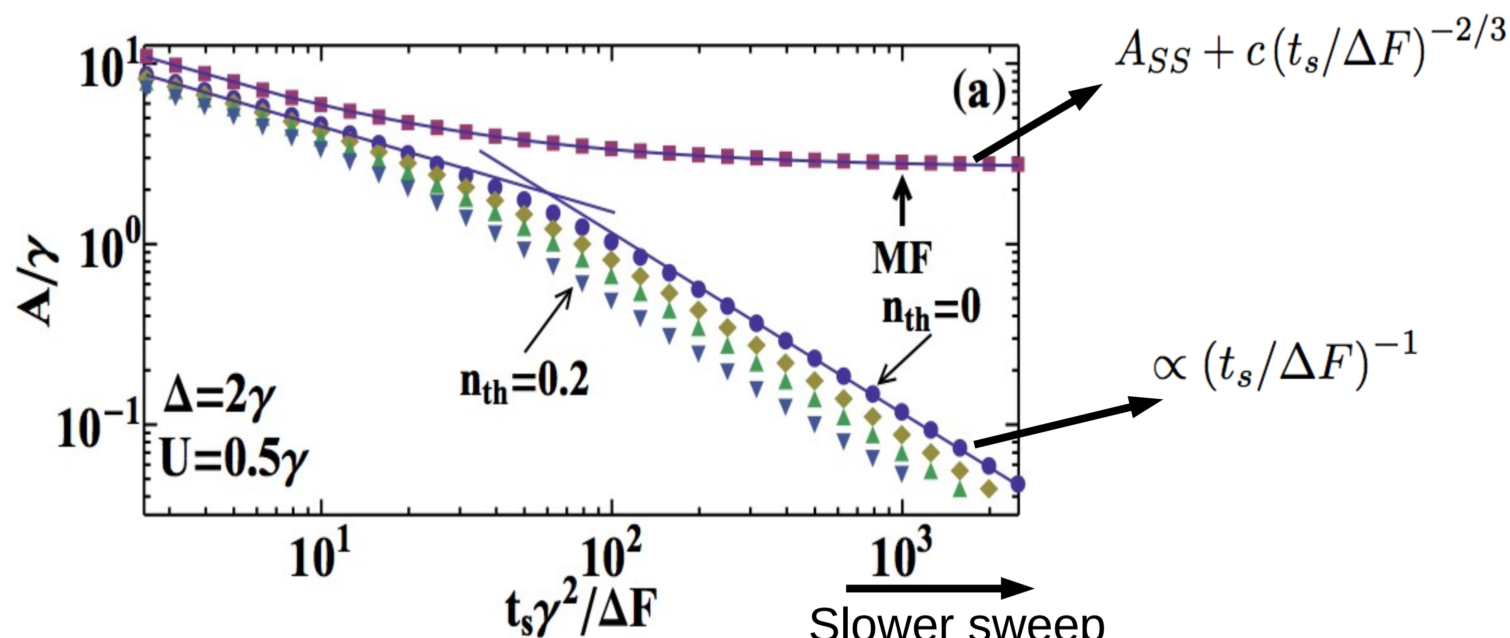


Power law behavior of the hysteresis area A:

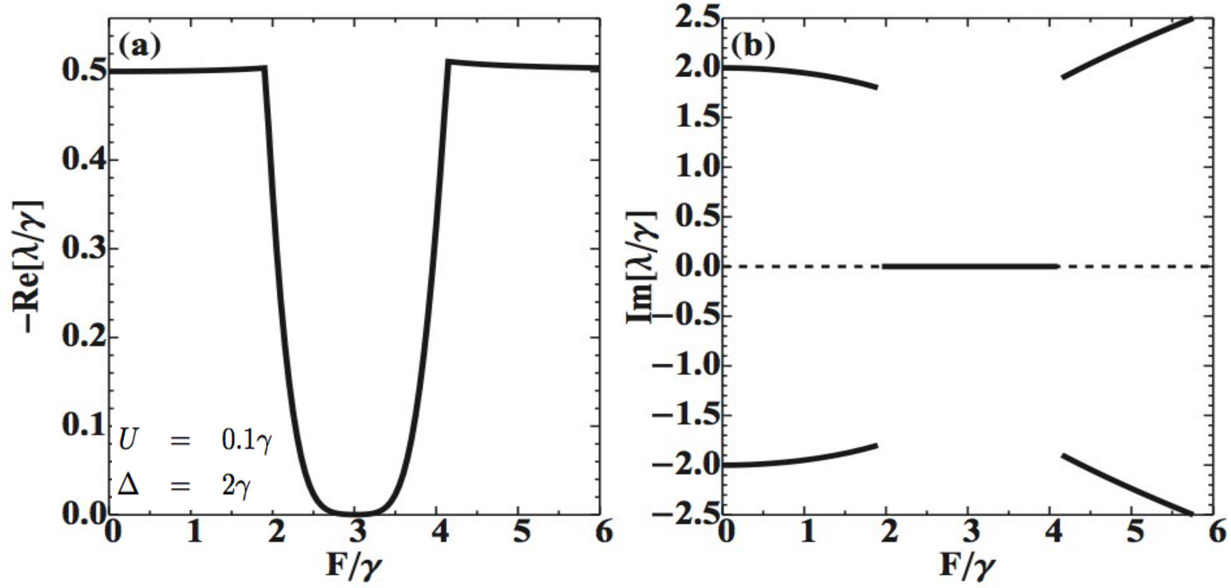


Dynamical hysteresis

$$\partial_t \hat{\rho} \quad = \quad -i\left[\hat{H},\hat{\rho}\right] + \frac{\gamma}{2}\left(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right)$$

Liouvillian gap:

$$\begin{aligned} \partial_t \hat{\rho} &= \hat{L}\hat{\rho} \\ \hat{L}\hat{\rho}_\lambda &= \lambda\hat{\rho}_\lambda \\ \hat{\rho}_0 &= \hat{\rho}_{SS} \\ \lambda \quad \text{with} & \quad \max\left\{Re\left[\lambda\right]\right\} \end{aligned}$$



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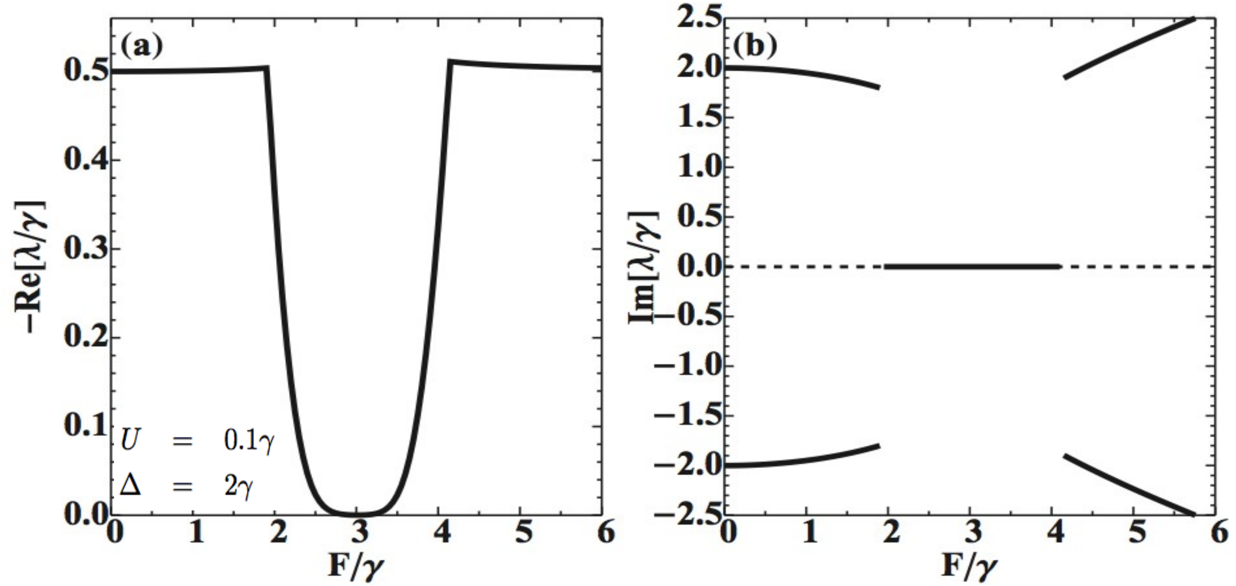


Characteristic timescales

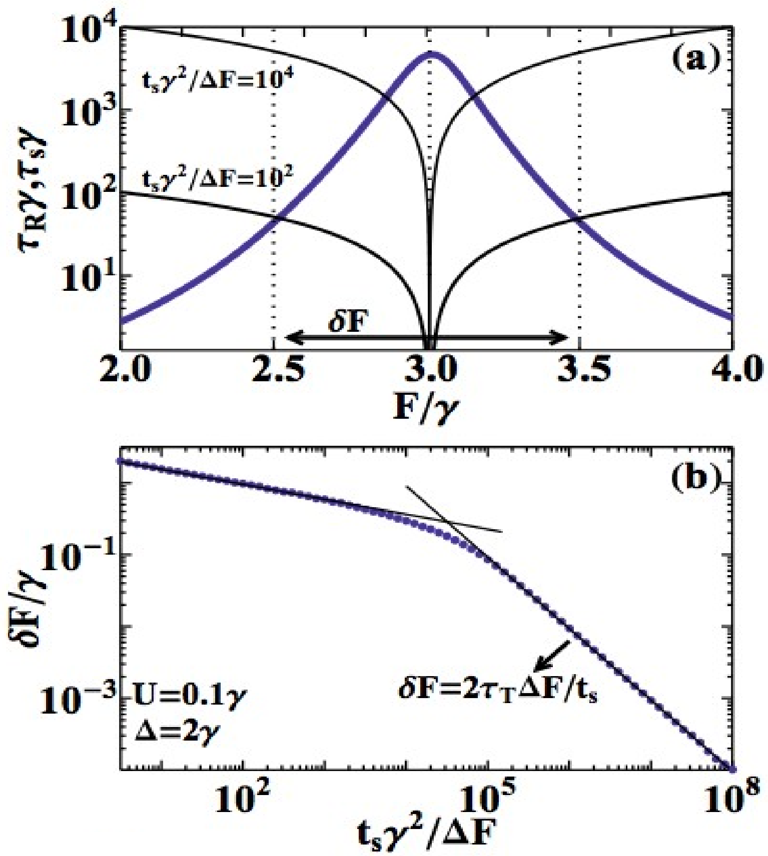
reaction time: $\tau_R = -1/Re[\lambda]$

distance from transition: $\epsilon(t) = F_c - F(t)$

transition rate: $\left|\frac{\dot{\epsilon}(t)}{\epsilon(t)}\right| = \frac{\Delta F}{t_s}\frac{1}{|F_c - F(t)|} = \frac{1}{\tau_s}$

Sweep timescale

Dynamical hysteresis



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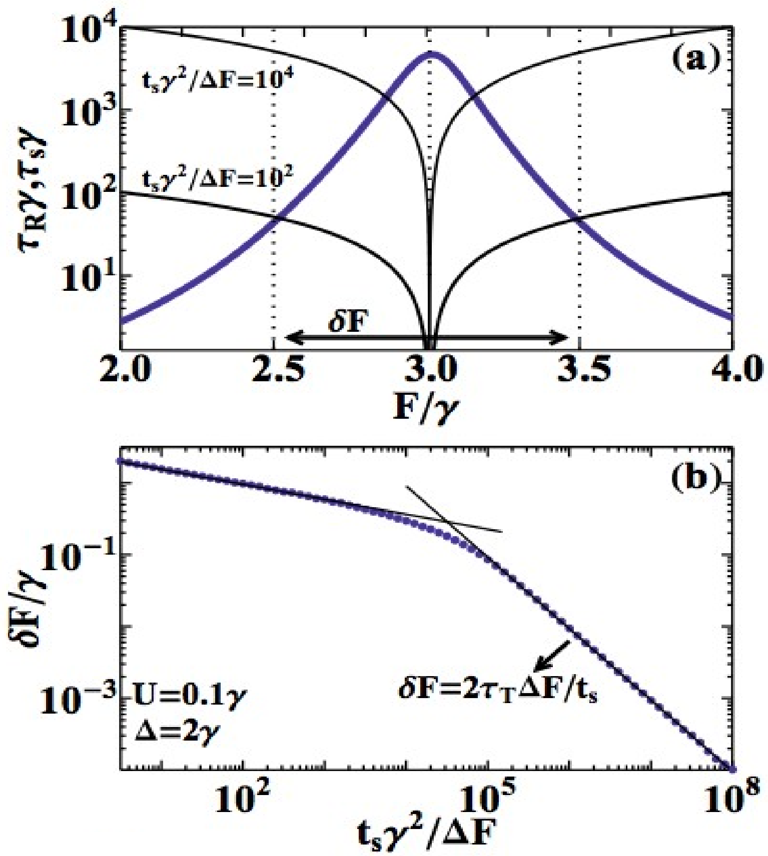

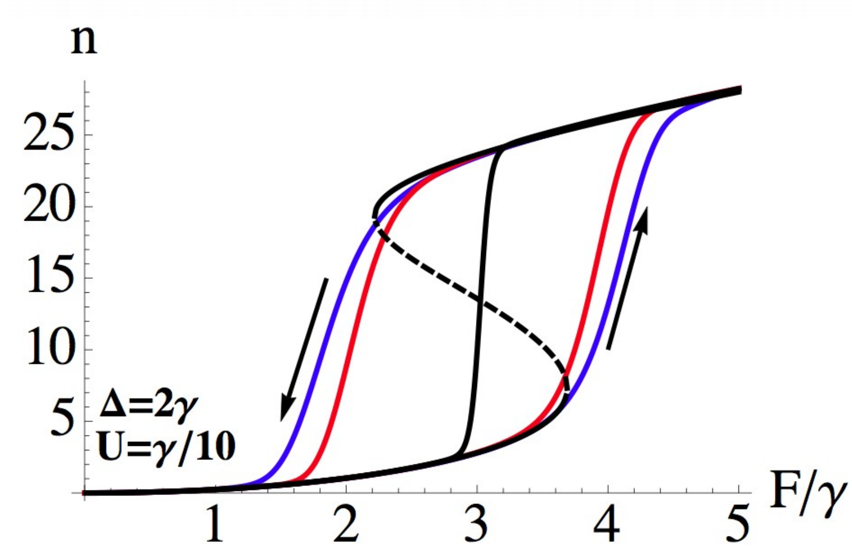

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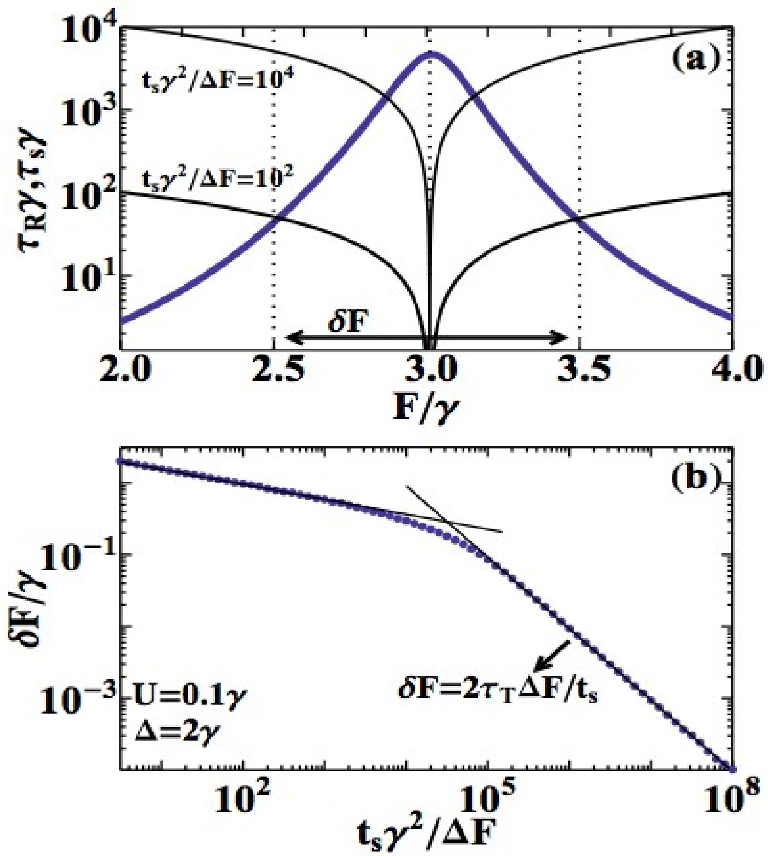

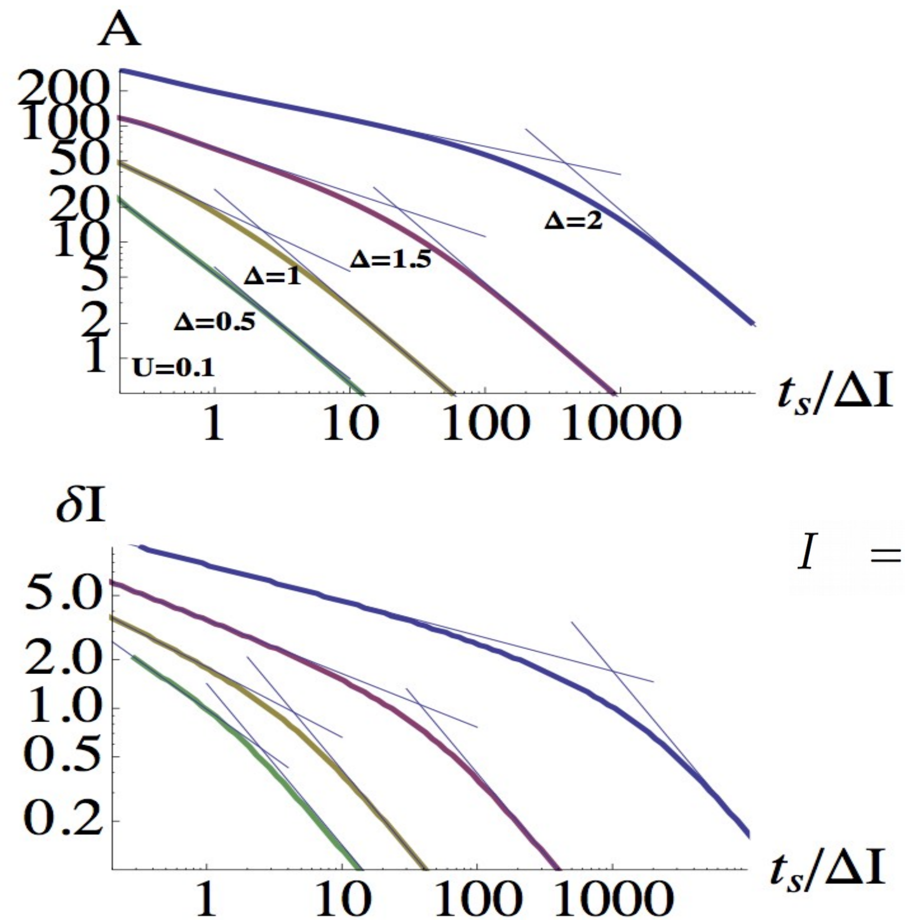

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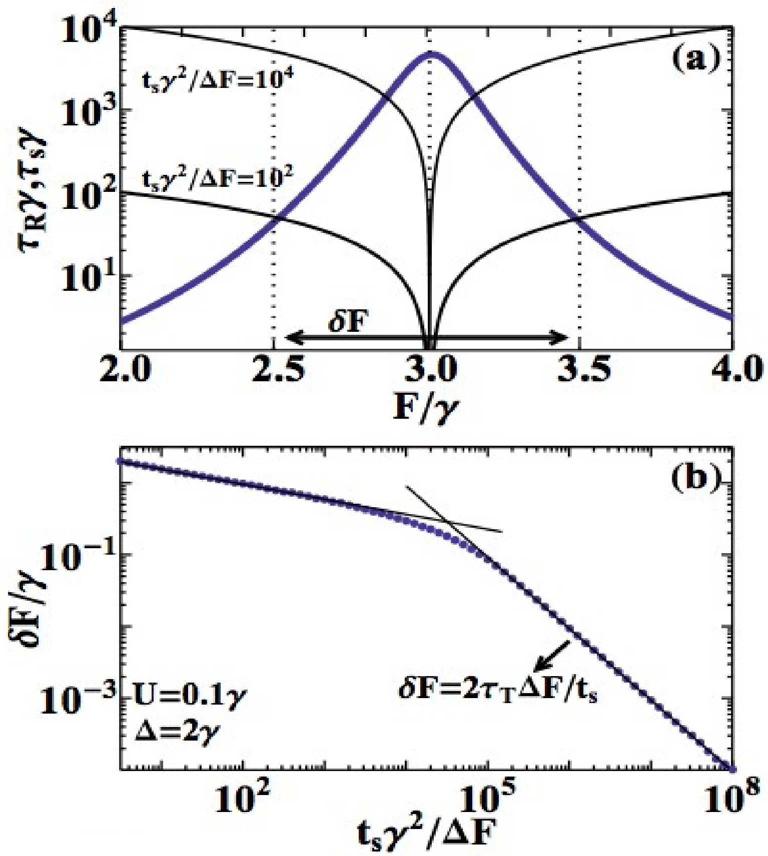

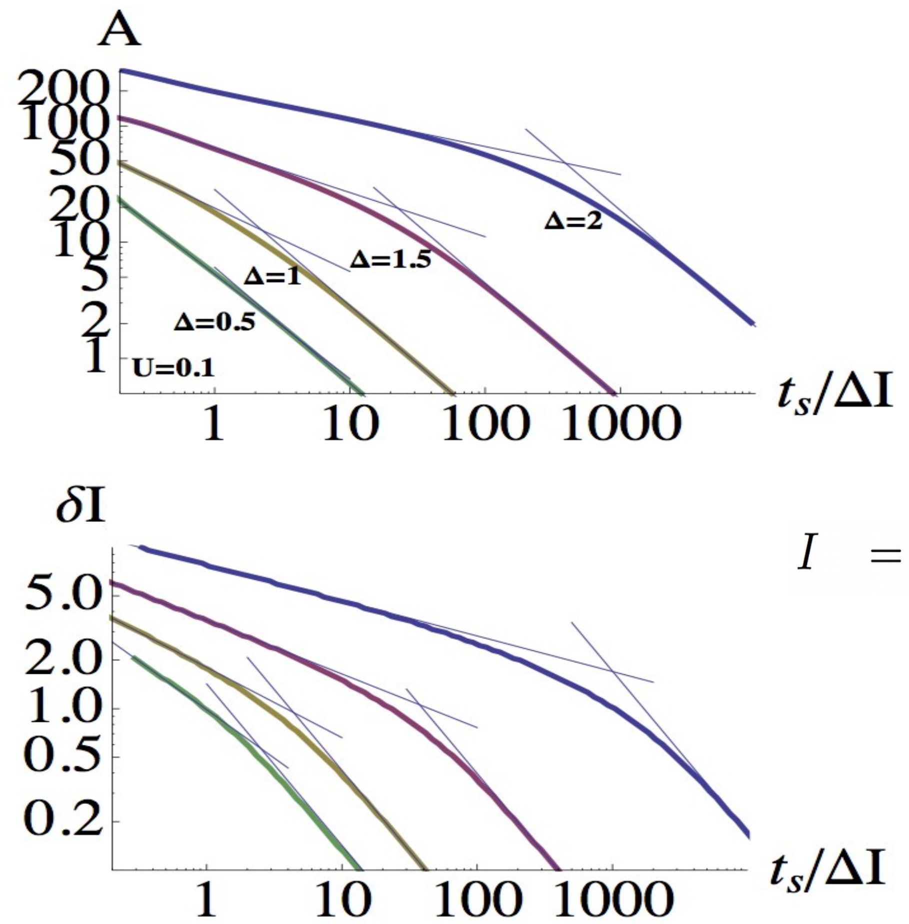

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→ Closely related to Kibble- Zurek mechanism

Sweep timescale

Conclusions & perspectives

- We revealed the dynamical character of the optical bistabilty hysteresis cycle which is qualitatively different from the semiclassical static hysteresis.
- We showed that the behavior of the Liouvillian gap allows to qualitatively understand the double power law behavior of the hysteresis area.

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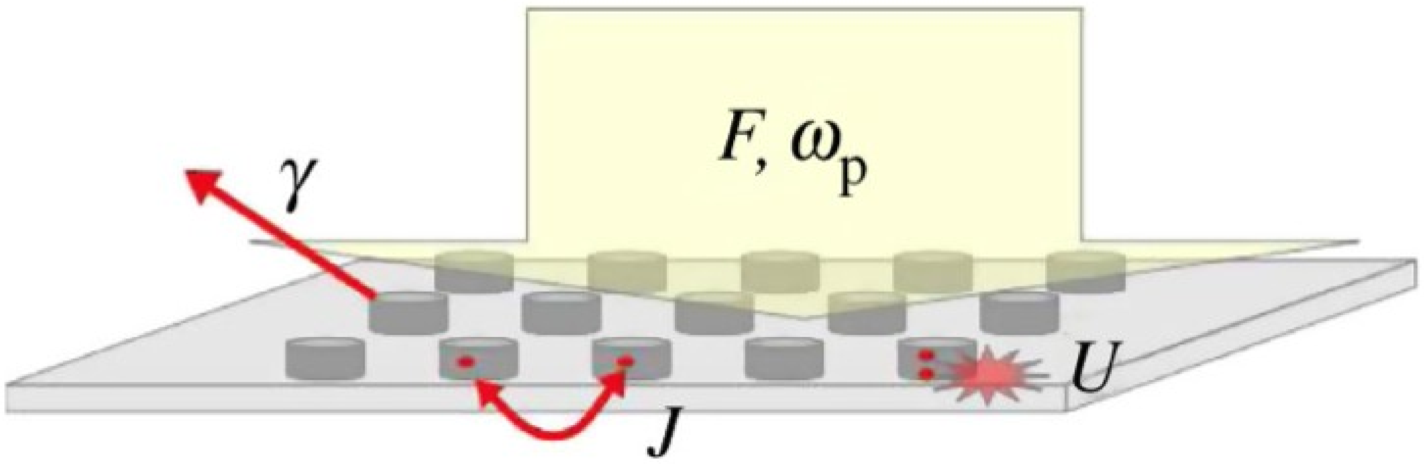
What about coupled arrays of photonic resonators?



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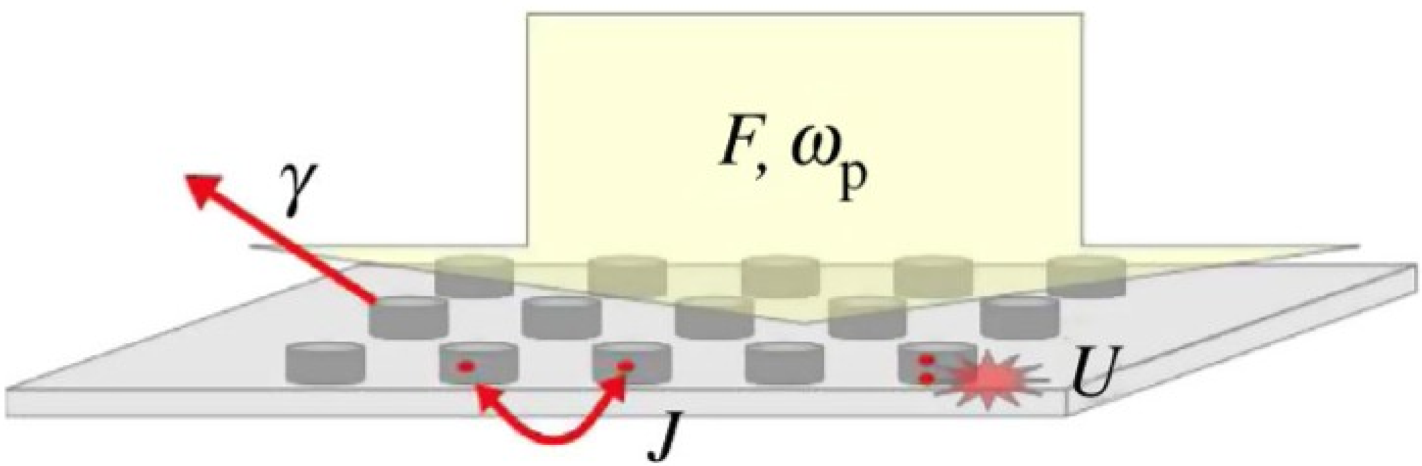

Thanks for your attention!

Ref.: W. Casteels, F. Storme, A. Le Boité, and C. Ciuti – Phys. Rev. A **93**, 033824 (2016)