

Machine Learning for HEP

Lecture III – Generative models for the LHC



Dall-E



UNIVERSITÀ
DEGLI STUDI
DI MILANO

BND Graduate School – Blankenberge 2024
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Plan of attack

Wednesday

1. Introduction to Machine Learning

- Basic concepts of machine learning

Thursday

3. Generative Models for the LHC

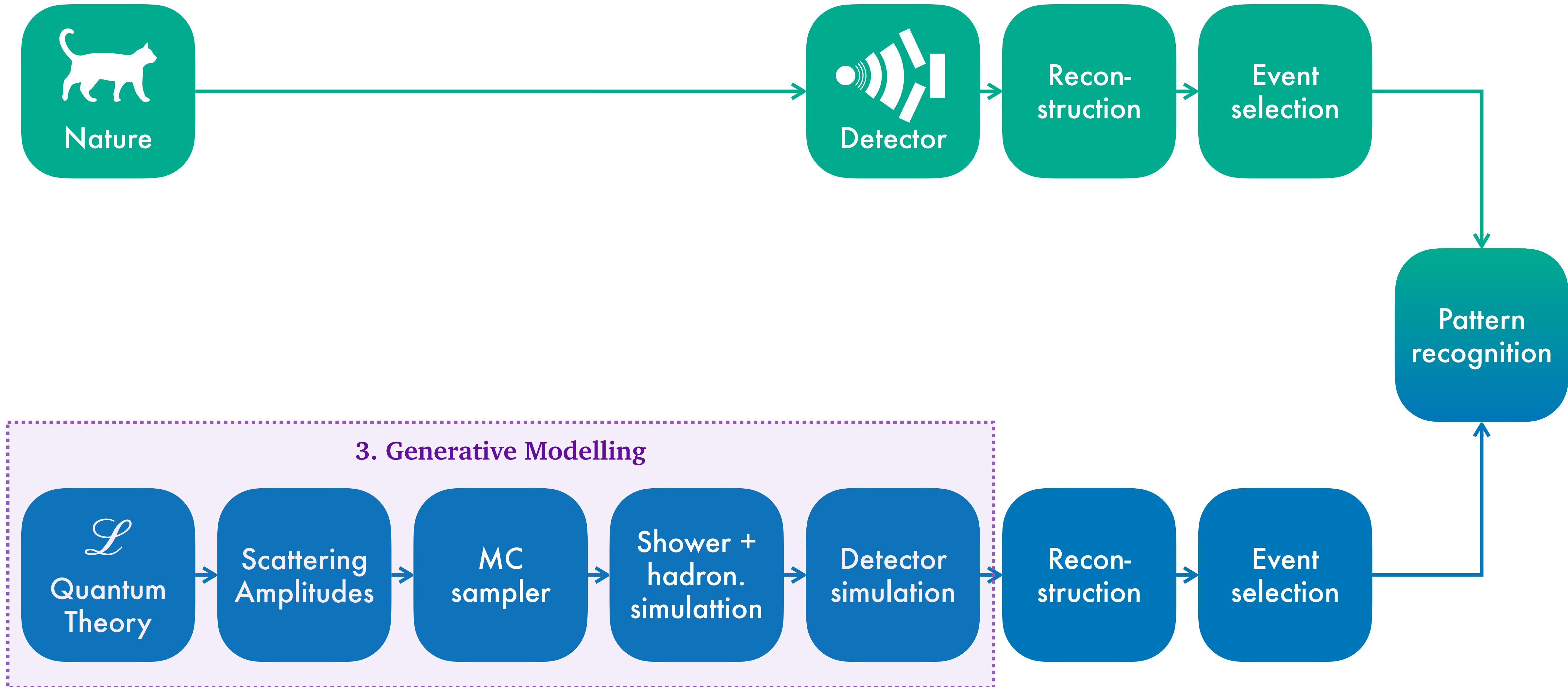
- GANs, normalizing flows & diffusion models

4. Anomaly Detection

- Autoencoders, CWoLa,...

LHC analysis + ML

3



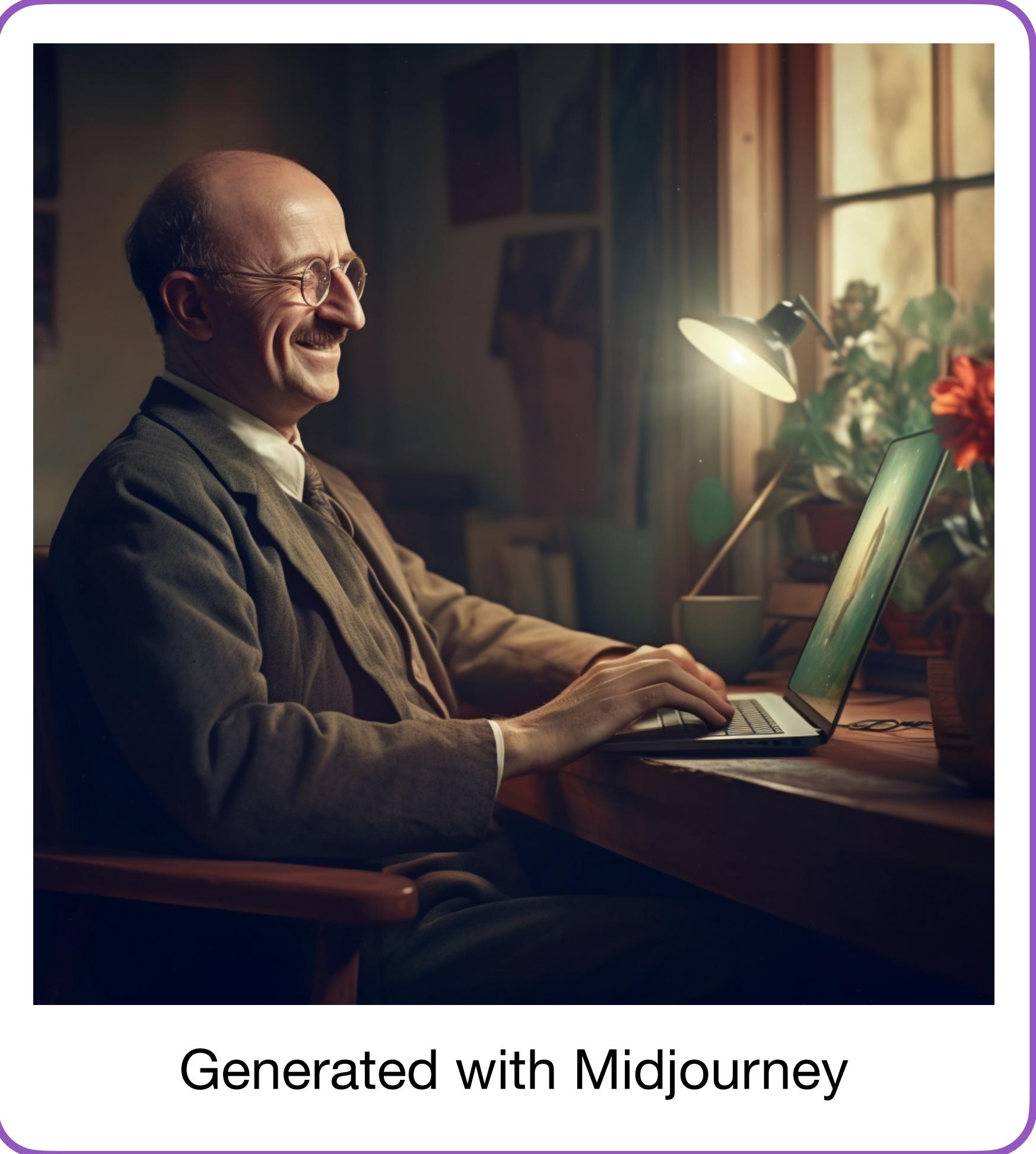
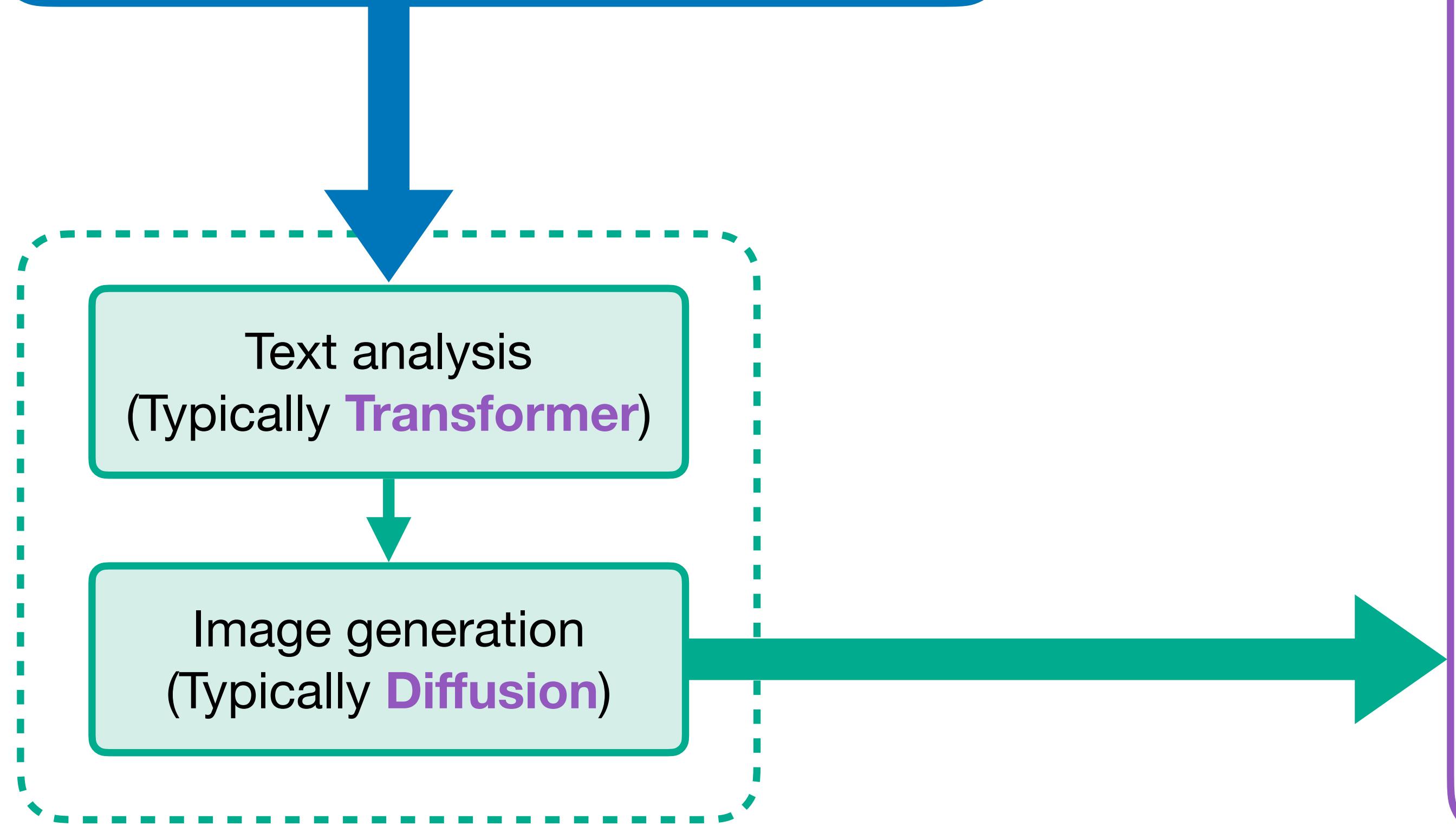
Lecture III

Generative Models for the LHC

Current Hype: Text-to-Image Models

Text prompt

Realistic photography of Max Planck sitting at his laptop being super happy while coding



Generative Models

GAN



GAN Art (2018)
→ sold for \$432,500

Diffusion Models



State-of-the-art
image generation

Transformer



State-of-the-art
text generation

What is a Generative Model?

We have: $p_{\text{truth}} \equiv p_{\text{data}}(x)$



We want to generate new samples

$x \sim p_{\omega}(x) \simeq p_{\text{data}}(x)$

The distribution p_{truth} is usually given as:

- **explicit** as function (e.g. $d\sigma \propto$ differential cross-section)
- **implicit** via a set of training data $\{x\} \sim p_{\text{data}}(x)$

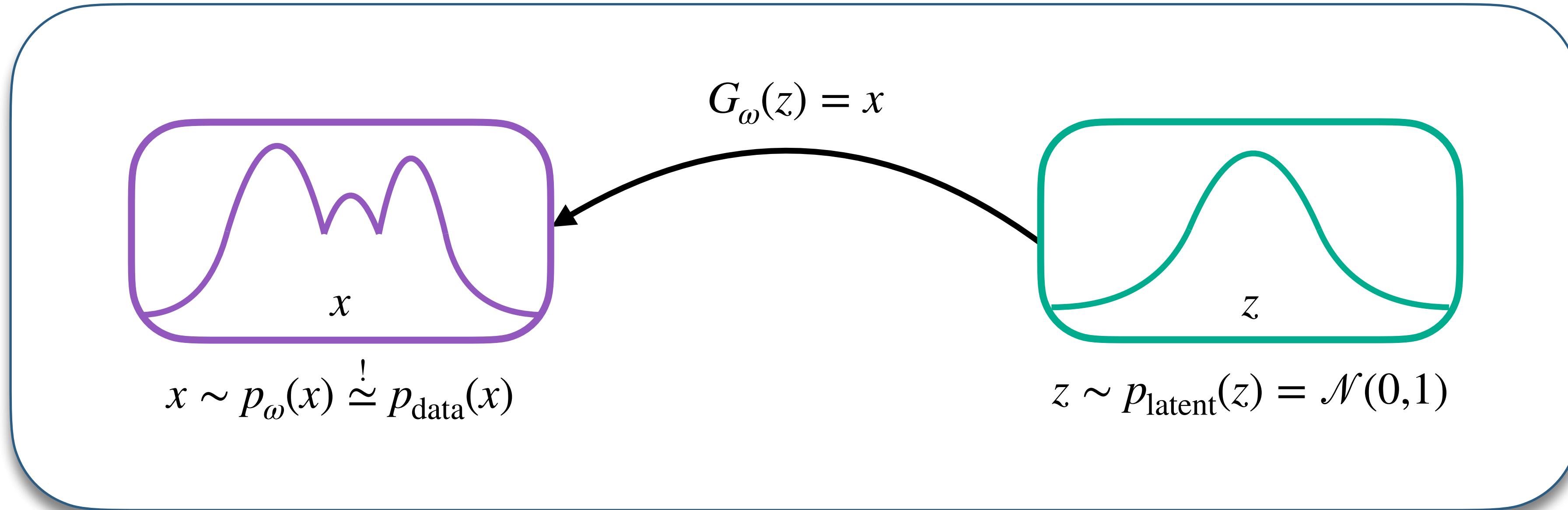
In **particle physics**:

- Event generation
- Calorimeter simulation
- Unfolding
- MEM (transfer function)

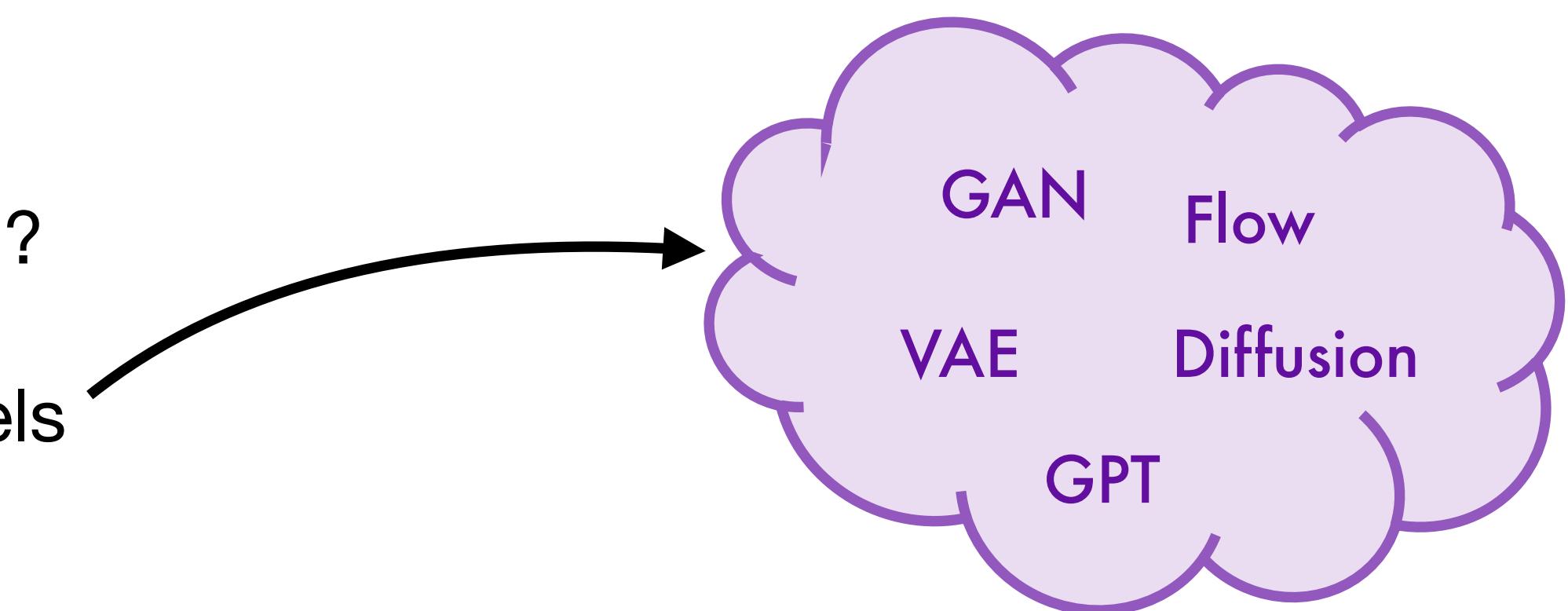
→ this is a stochastic (random) process (RNG)

→ needs “random” input

What is a Generative Model?

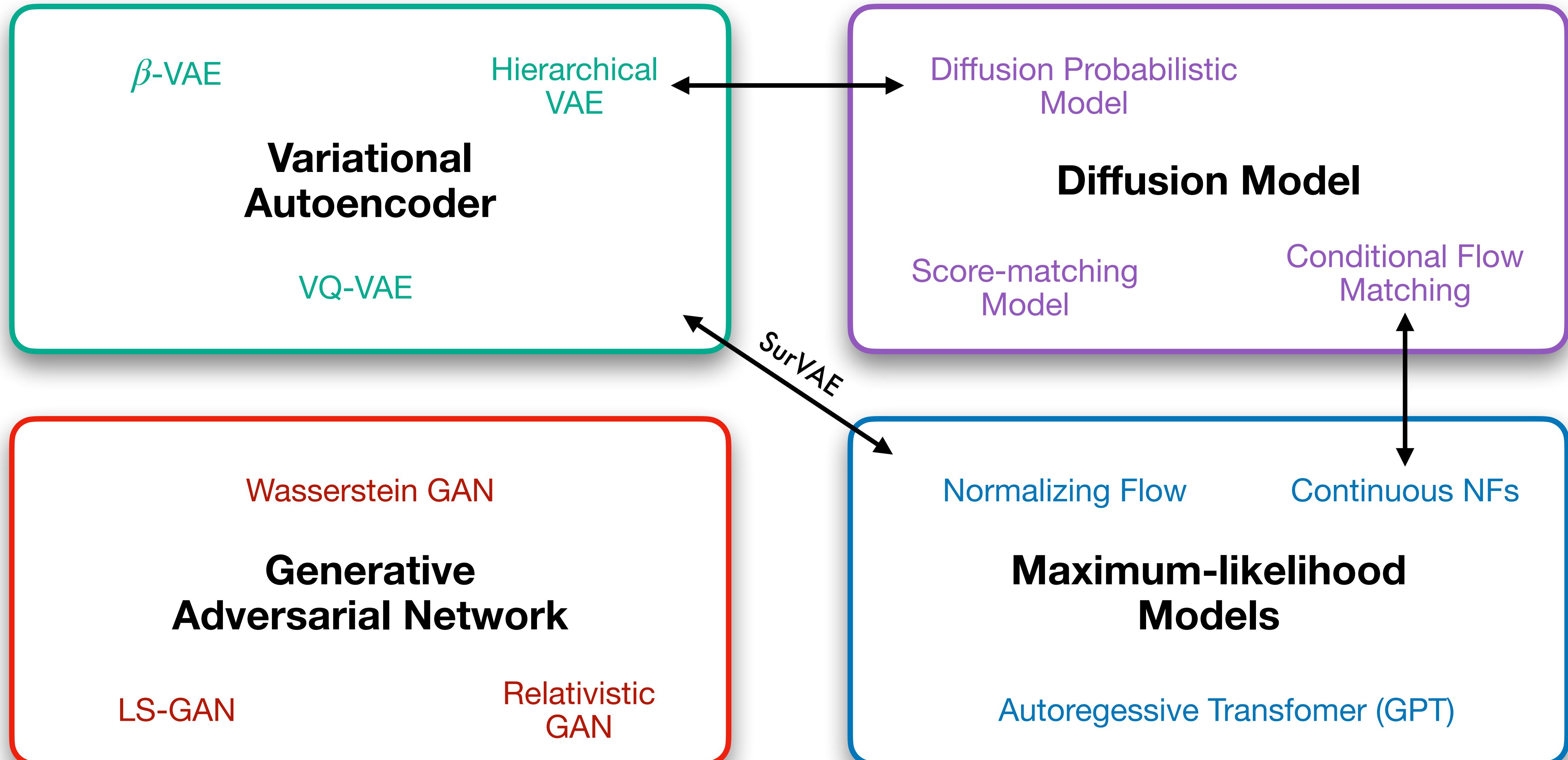


- How to **construct** and **train** $G_\omega(z)$?
- **Multiple types** of generative models

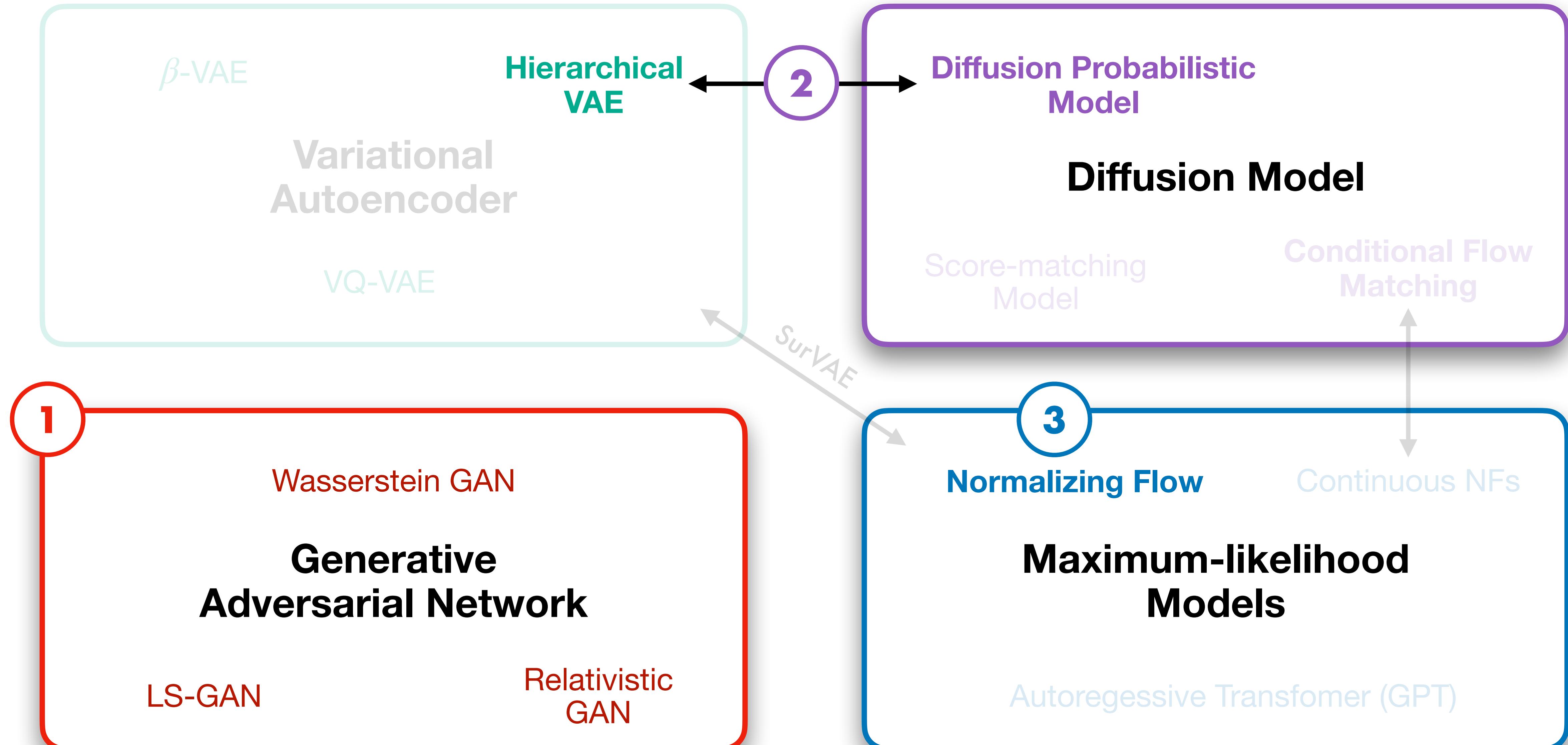


Types of deep generative models

Deep generative models



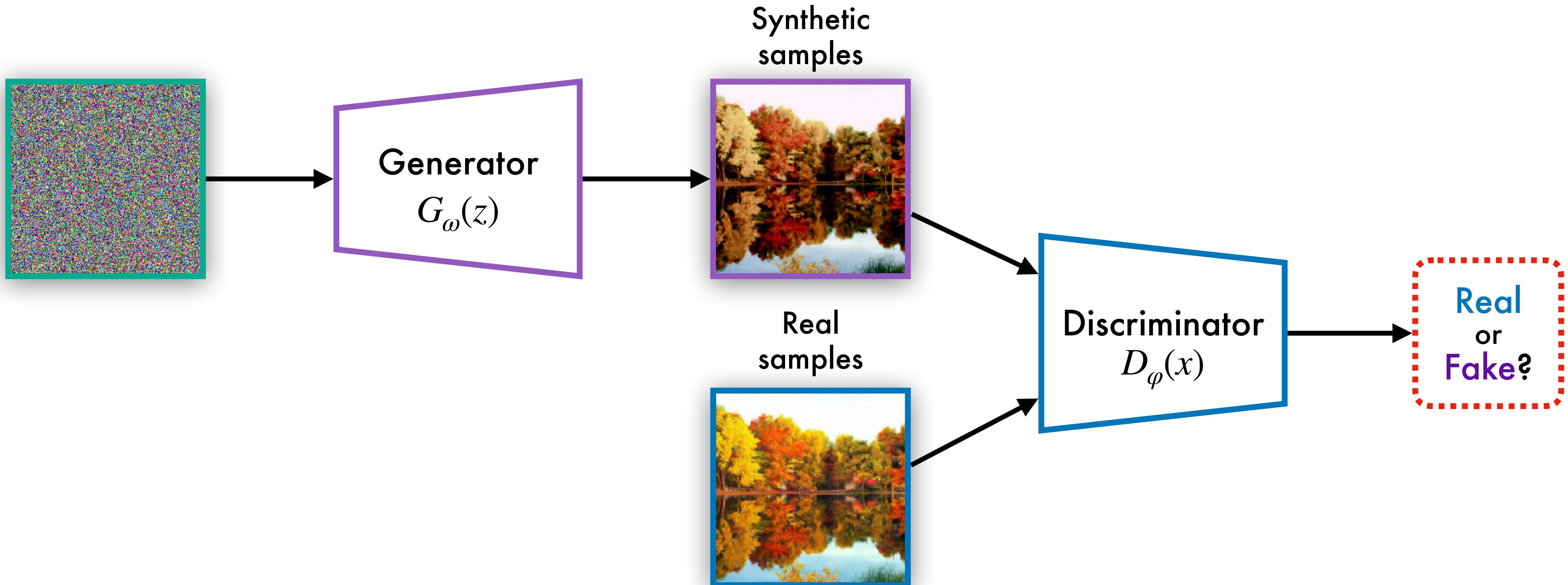
Deep generative models



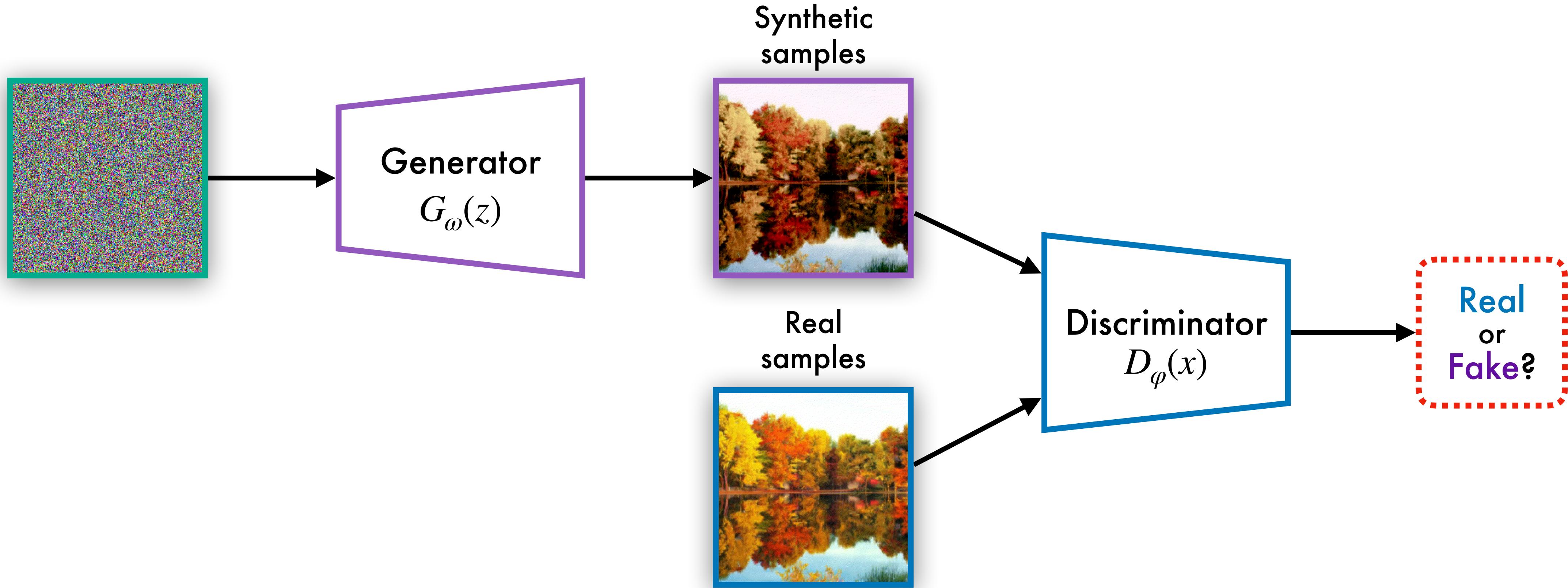
Part I

Generative Adversarial Networks

Generative adversarial network



Generative adversarial network



$$\begin{aligned}\mathcal{L}_D &= - \left\langle \log D_\varphi(x) \right\rangle_{x \sim p_{\text{data}}} - \left\langle \log(1 - D_\varphi(x)) \right\rangle_{x \sim p_{\hat{\omega}}} \\ &= - \left\langle \log D_\varphi(x) \right\rangle_{x \sim p_{\text{data}}} - \left\langle \log(1 - D_\varphi(G_{\hat{\omega}}(z))) \right\rangle_{z \sim p_{\text{latent}}}\end{aligned}$$

Iterative
Training

$$\begin{aligned}\mathcal{L}_G &= \left\langle \log(1 - D_{\hat{\varphi}}(x)) \right\rangle_{x \sim p_\omega} \\ &= \left\langle \log(1 - D_{\hat{\varphi}}(G_\omega(z))) \right\rangle_{z \sim p_{\text{latent}}}\end{aligned}$$

Generative adversarial network

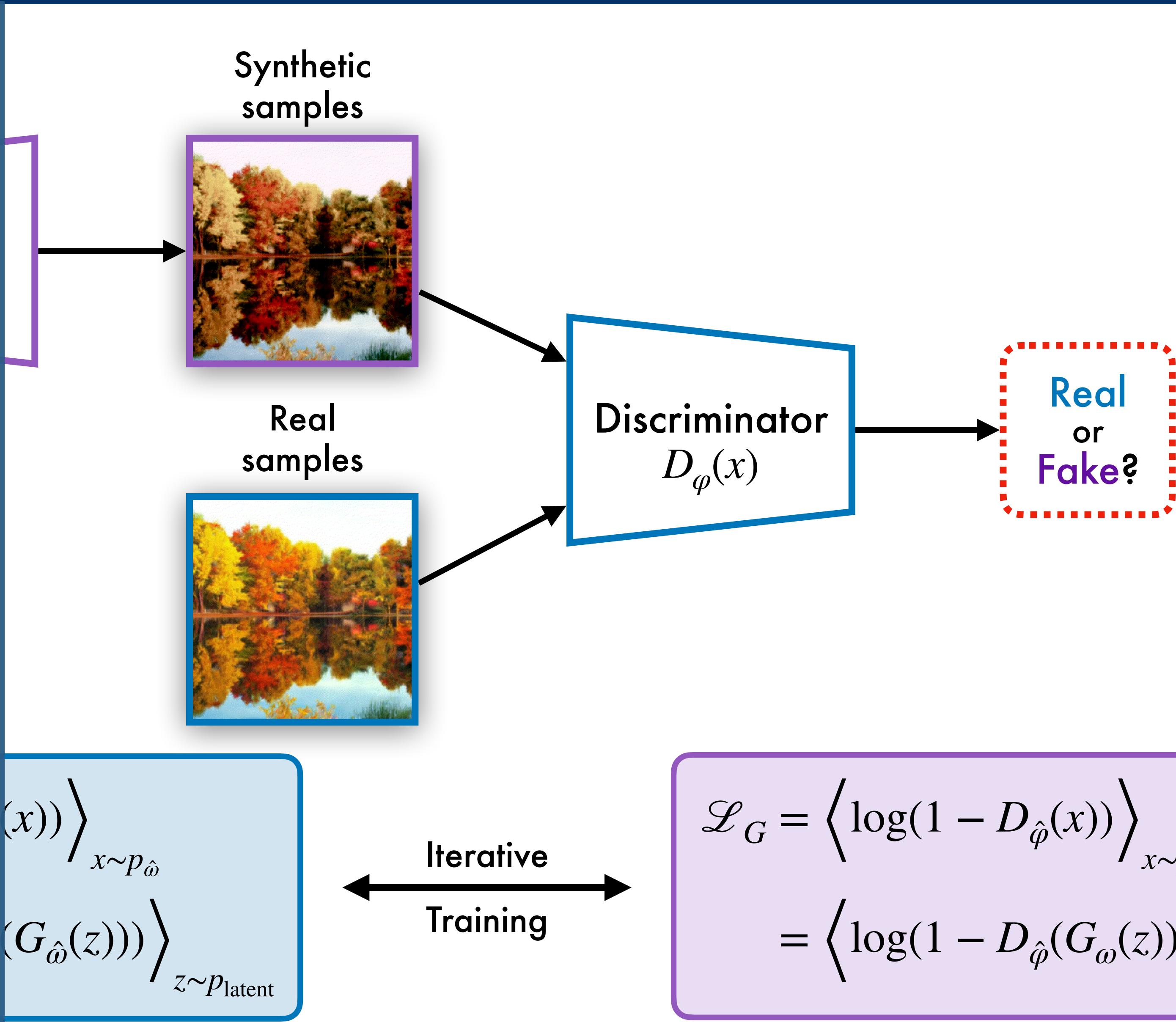
Problems with GANs

- ⊖ Min-max training unstable
→ vanishing gradients
- ⊖ Metric for success?
→ loss only shows competition!
- ⊖ Mode collapse

Ways to improve

- ⊕ Wasserstein GAN [1701.07875, 1704.00028]
- ⊕⊕ Gradient penalty [1705.09367, 1801.04406]
- ⊕⊕ Spectral normalization [1802.05957]

$$\mathcal{L}_D = \left\langle \log D_\phi(x) \right\rangle_{x \sim p_{\text{data}}} - \left\langle \log(1 - D_\phi(x)) \right\rangle_{x \sim p_{\hat{\omega}}(G_{\hat{\omega}}(z))} - \left\langle \log(1 - D_\phi(G_{\hat{\omega}}(z))) \right\rangle_{z \sim p_{\text{latent}}}$$



$$\begin{aligned}\mathcal{L}_G &= \left\langle \log(1 - D_{\hat{\phi}}(x)) \right\rangle_{x \sim p_{\omega}} \\ &= \left\langle \log(1 - D_{\hat{\phi}}(G_{\omega}(z))) \right\rangle_{z \sim p_{\text{latent}}}\end{aligned}$$

Part II

Diffusion Models

Midjourney

Dall-E

Recently...

Stable Diffusion



Midjourney AI

Diffusion Models



Diffusion Models

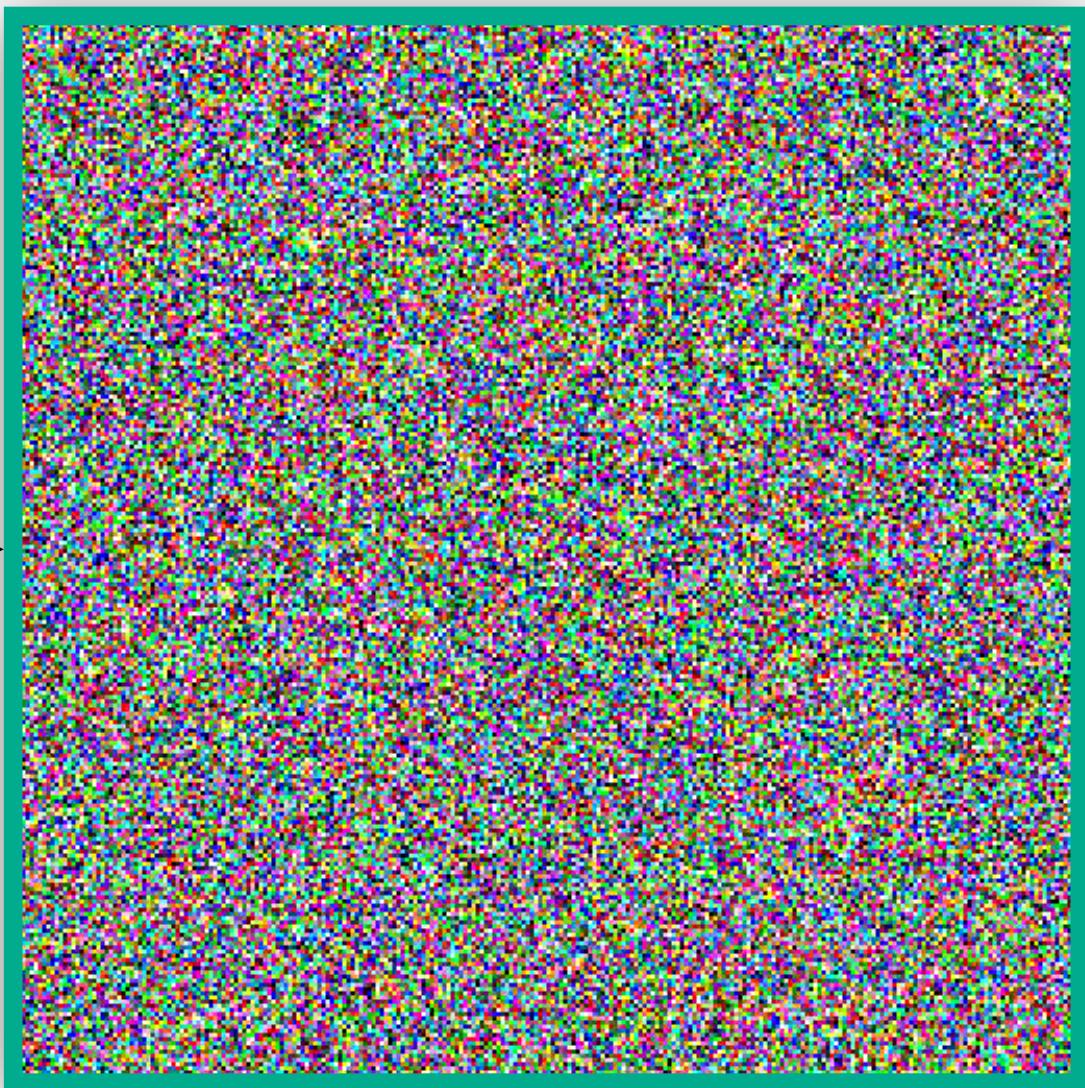
Define mapping as **time-dependent** diffusion process



$$x \sim p_{\omega}(x) \stackrel{!}{\simeq} p_{\text{data}}(x)$$

Parametrization of time

- discrete: $t = 0, 1, \dots, T$
- continuous: $t \in [0, 1]$



$$z \sim p_{\text{latent}}(z) = \mathcal{N}(0, 1)$$

- Gradually **add noise** to data samples to transform them to gaussians
- ← Gradually **remove noise** from gaussians to obtain data samples

VAEs Revisited

Bayes' Theorem

$$p(z|x) = \frac{p(x|z)p(z)}{p(x)}$$

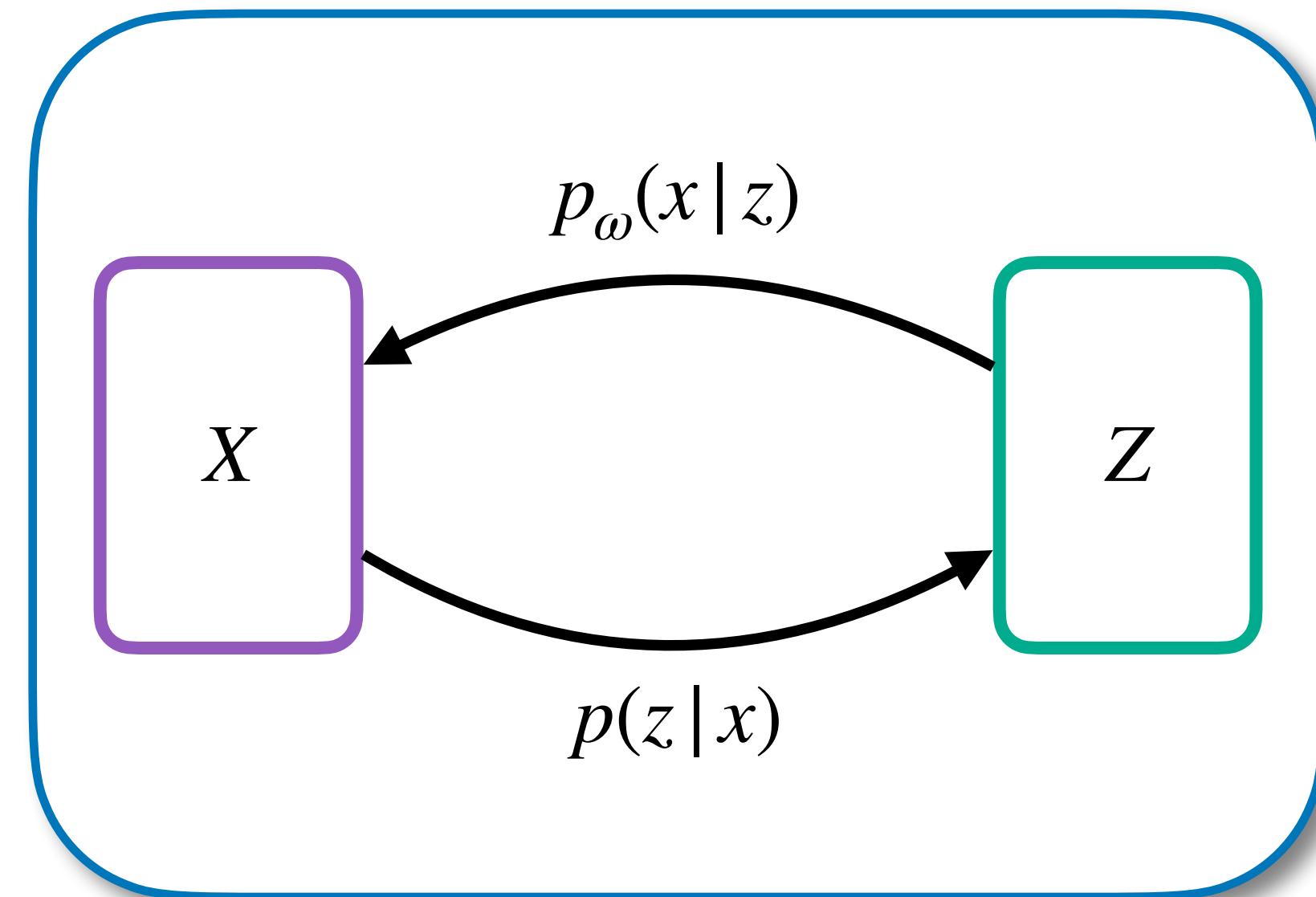
Prior Evidence

Problem: evidence intractable

$$p(x) = \int dz p(x|z)p(z)$$

Variational inference

Encoder: $q_\phi(z|x) = \mathcal{N}(z; \mu_\phi(x), \sigma_\phi^2(x))$



Likelihood: $p_\omega(x|z)$ ← Decoder

Posterior: $p(z|x)$ ← Want!

Match!

Evidence Lower Bound – ELBO

Minimize KL

$$\text{KL}(q_\phi(z|x), p(z|x)) = \int dz q_\phi(z|x) \log \left(\frac{q_\phi(z|x)}{p(z|x)} \right)$$



Maximize ELBO



ELBO \neq Elmo

Evidence Lower Bound – ELBO

Minimize KL

$$\text{KL}(q_\phi(z|x), p(z|x)) = \int dz q_\phi(z|x) \log \left(\frac{q_\phi(z|x)}{p(z|x)} \right)$$

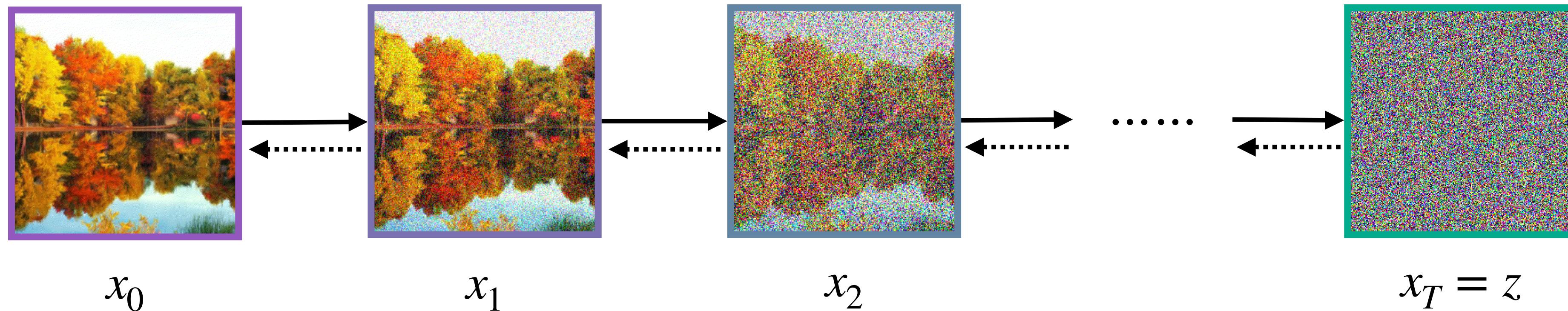
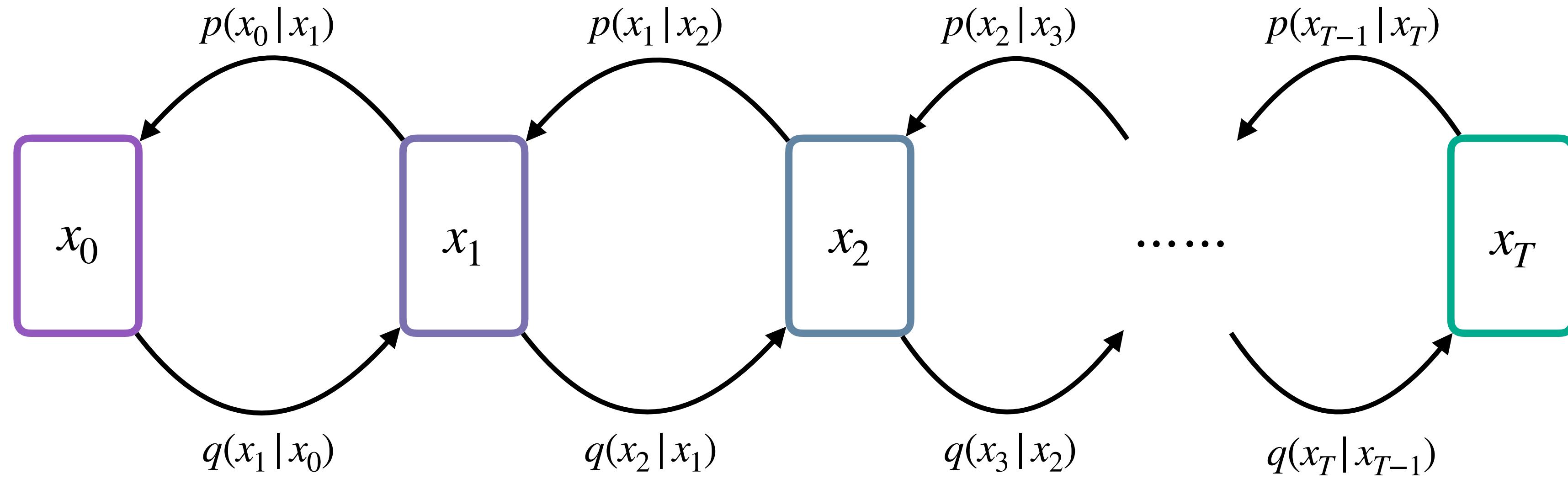
Maximize ELBO

$$\begin{aligned} \log p(x) &= \int dz q_\phi(z|x) \log p(x) \\ &= \int dz q_\phi(z|x) \log \frac{p_\omega(x|z)p(z)}{p(z|x)} \quad \text{Bayes' theorem} \\ &= \int dz q_\phi(z|x) \left[\log p_\omega(x|z) - \log \frac{q_\phi(z|x)}{p(z)} + \log \frac{q_\phi(z|x)}{p(z|x)} \right] \\ &= \underbrace{\langle \log p_\omega(x|z) \rangle_{q_\phi(z|x)}}_{\text{ELBO}} - \text{KL}(q_\phi(z|x), p(z)) + \underbrace{\text{KL}(q_\phi(z|x), p(z|x))}_{\geq 0} \end{aligned}$$

SurVAE [2007.02731]

$$\log p(x) = \int dz q_\phi(z|x) \left[\log p(z) + \log \frac{p_\omega(x|z)}{q_\phi(z|x)} + \log \frac{q_\phi(z|x)}{p(z|x)} \right]$$

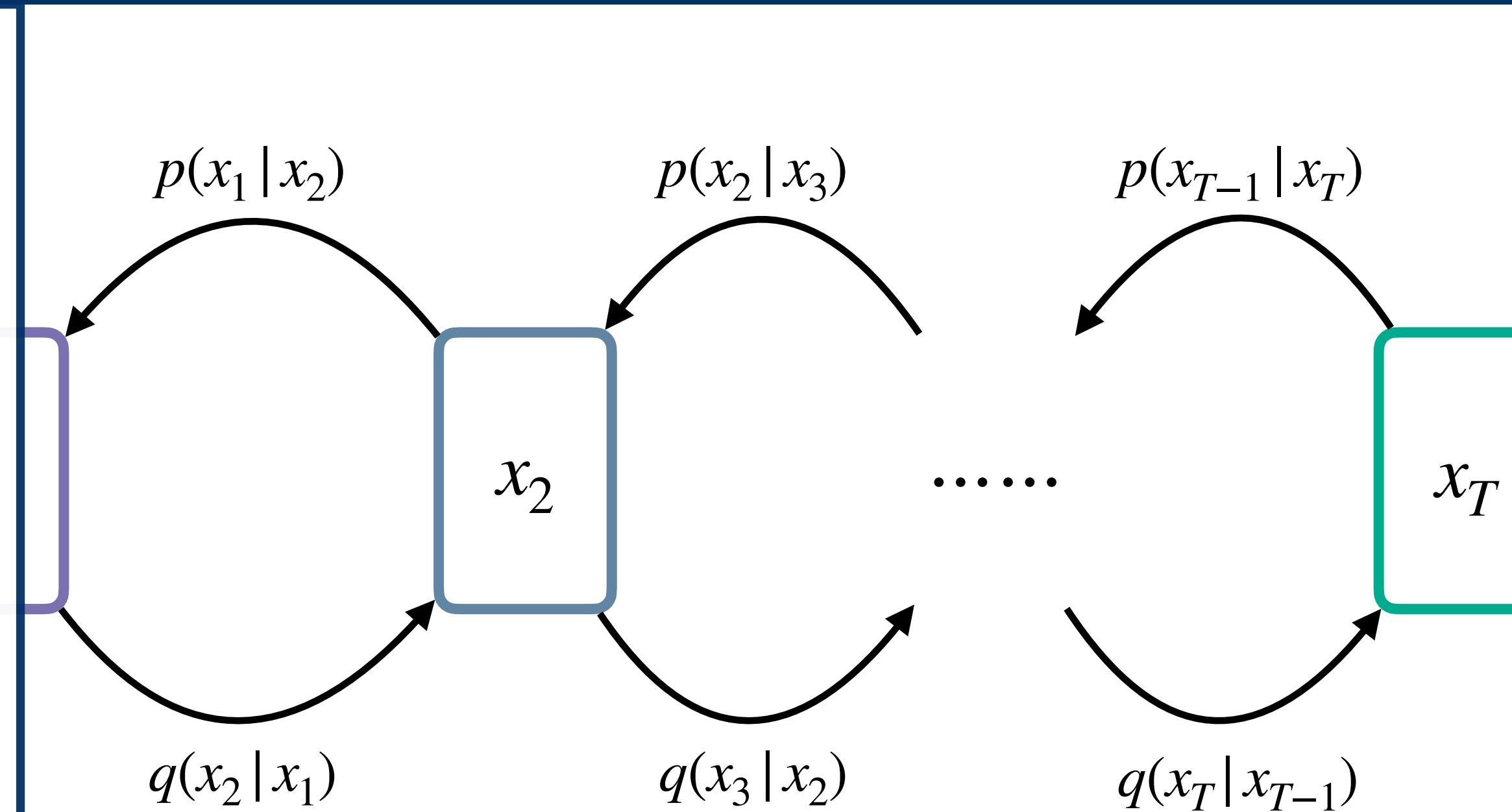
Denoising Diffusion Model (DDPM)



Denoising Diffusion Model (DDPM)

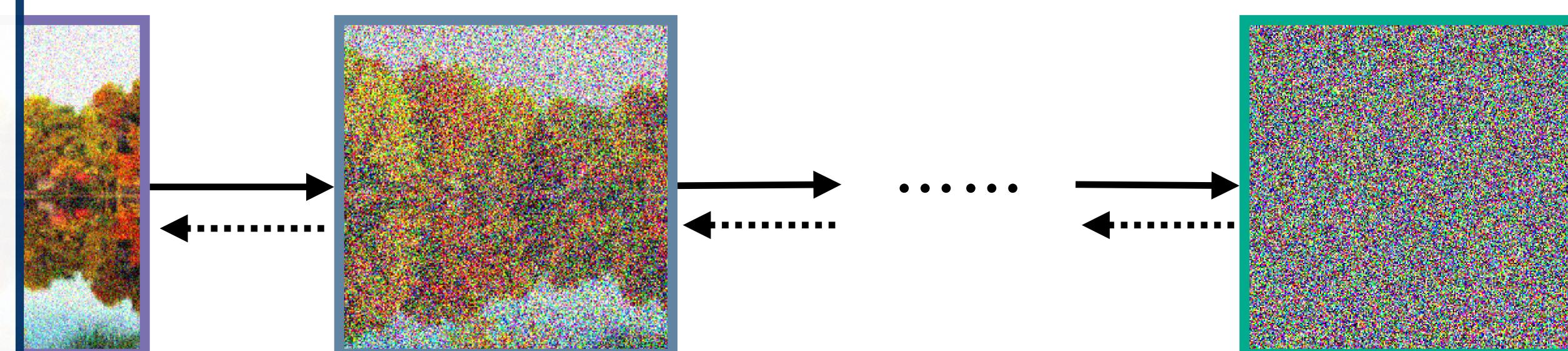
Hierachical VAEs = DDPMs

- explicit **diffusion process**: $q(x_t | x_{t-1})$
- no **bottleneck**: $\dim x_0 = \dim x_i$
- find **reverse process**: $p_\omega(x_t | x_{t+1})$



Key facts - Diffusion Model

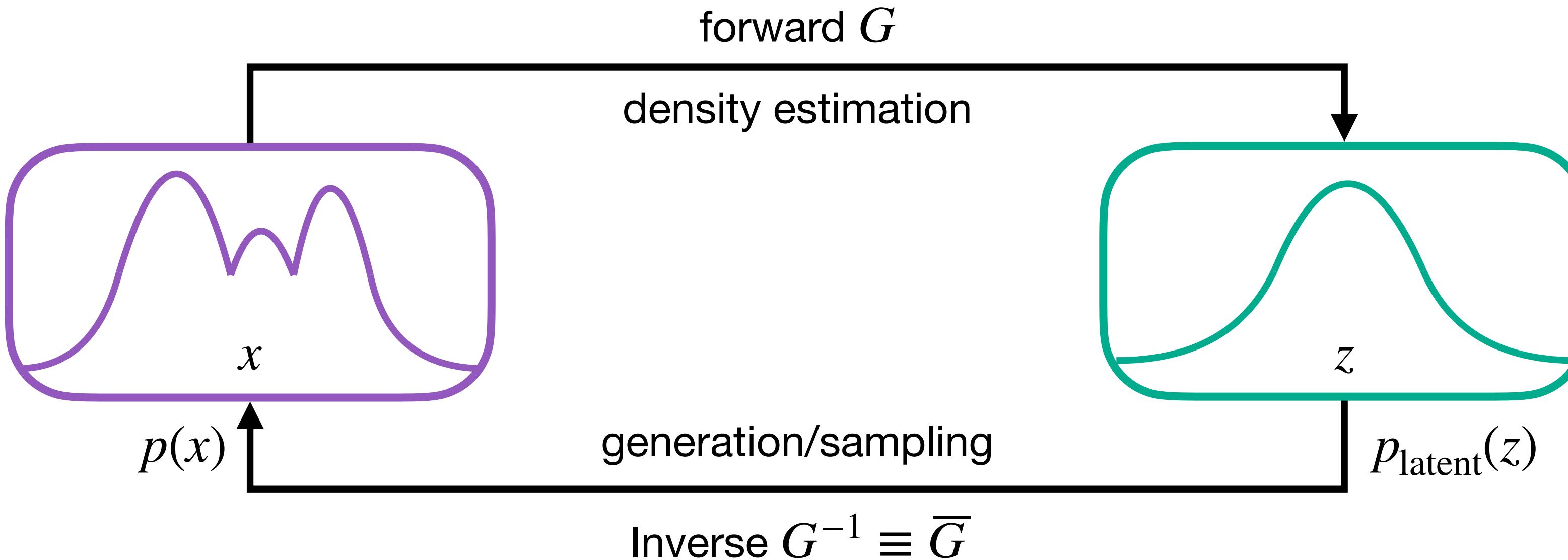
- ⊕ State-of-the-art in precision
- ⊕ Fast and stable training
- ⊖ Slow evaluation



Part III

Normalizing Flows

Normalizing flow – Basics

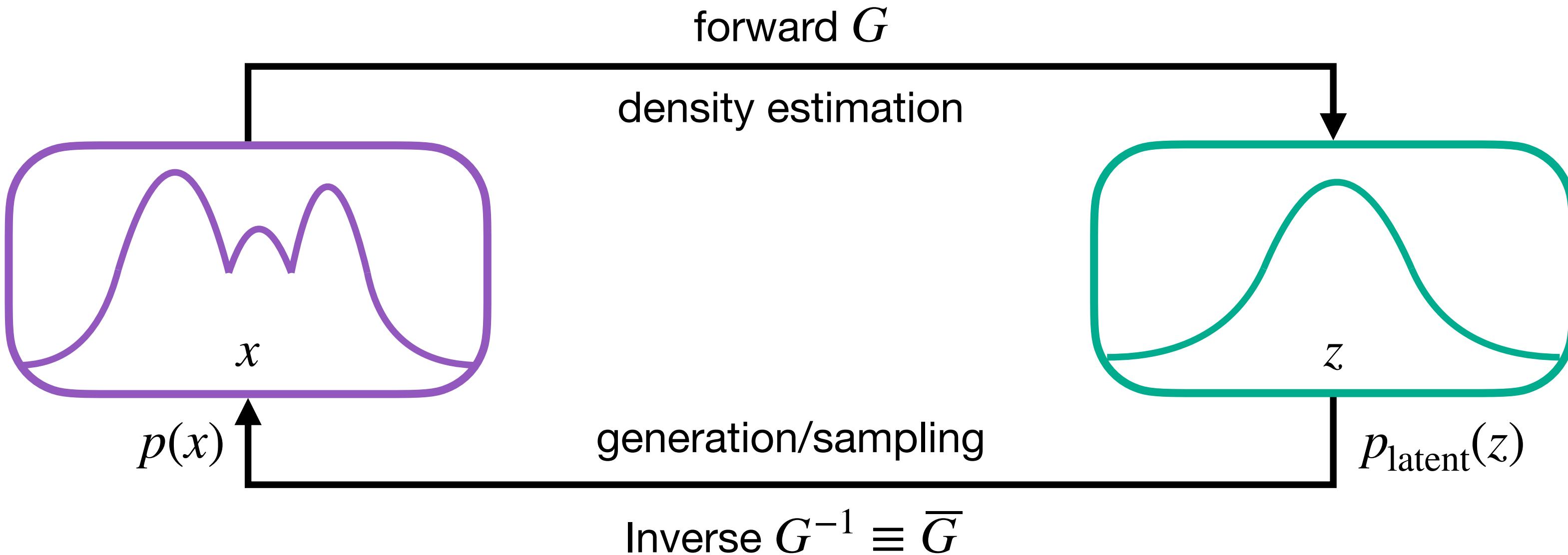


Conservation of probability: $p(x) dx = p_{\text{latent}}(z) dz$ with $z = G_\omega(x) \quad x = \bar{G}_\omega(z)$

Change-of-variables formula:

$$p_\omega(x) = p_{\text{latent}}(z = G_\omega(x)) \cdot \left| \frac{\partial G_\omega(x)}{\partial x} \right|$$

Normalizing flow – Basics



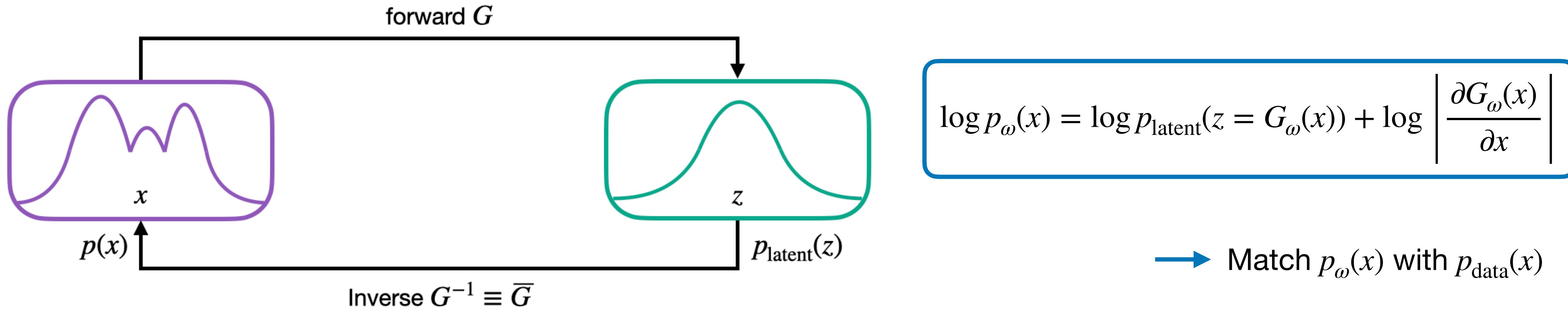
Conservation of probability: $p(x) dx = p_{\text{latent}}(z) dz$ with $z = G_\omega(x) \quad x = \bar{G}_\omega(z)$

Change-of-variables formula:

$$\log p_\omega(x) = \log p_{\text{latent}}(z = G_\omega(x)) + \log \left| \frac{\partial G_\omega(x)}{\partial x} \right|$$

How to train it?

Normalizing flow – Training



Kullback-Leibler divergence:

$$\begin{aligned} \text{KL}(p_{\text{data}}(x) \mid p_\omega(x)) &= \int dx p_{\text{data}}(x) \log \frac{p_{\text{data}}(x)}{p_\omega(x)} \\ &= - \int dx p_{\text{data}}(x) \log p_\omega(x) + \boxed{\int dx p_{\text{data}}(x) \log p_{\text{data}}(x)} \end{aligned}$$

No ω dependence

Negative log-likelihood loss:

$$\mathcal{L}_{\text{NLL}} = - \int dx p_{\text{data}}(x) \log p_\omega(x) = \langle -\log p_\omega(x) \rangle_{x \sim p_{\text{data}}}$$

Tractable Jacobian?

$$\log p_\omega(x) = \log p_{\text{latent}}(z = G_\omega(x)) + \log \left| \frac{\partial G_\omega(x)}{\partial x} \right| \rightarrow \text{Requires tractable Jacobian!}$$

In general: $g_\omega(x) = \left| \frac{\partial G_\omega(x)}{\partial x} \right|$ is $d \times d$ matrix \rightarrow Scales with $\mathcal{O}(d^3)$ 😞

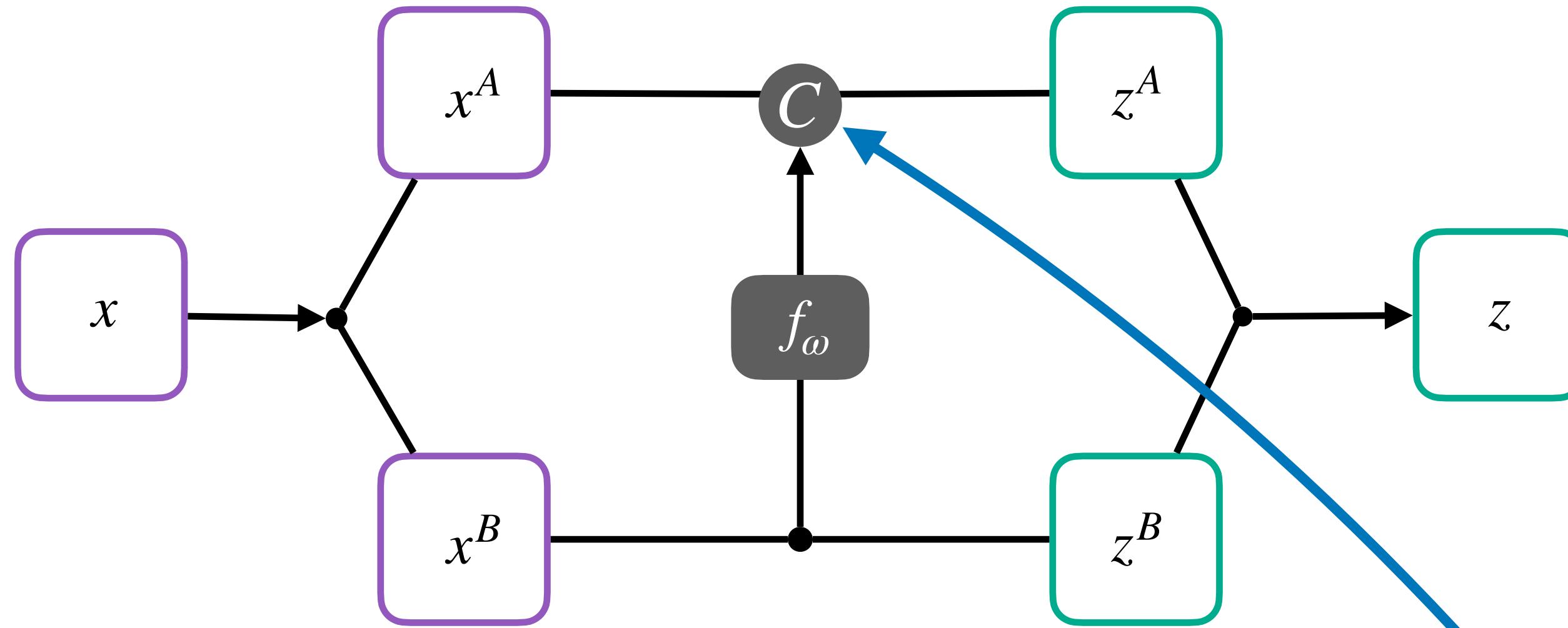
Solution: **Autoregressive transformations**

$$z = \begin{pmatrix} z_1 \\ \vdots \\ z_d \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ \vdots \\ x_d \end{pmatrix}$$

$$\begin{aligned} z_1 &\equiv z_1(x_1) \\ z_2 &\equiv z_2(x_1, x_2) \\ &\vdots \\ z_d &\equiv z_d(x_1, x_2, \dots, x_d) \end{aligned}$$

$$\rightarrow J_{ij}(x) = \begin{pmatrix} \frac{\partial z_1}{\partial x_1} & \frac{\partial z_2}{\partial x_1} & \cdots & \frac{\partial z_d}{\partial x_1} \\ 0 & \frac{\partial z_2}{\partial x_2} & \cdots & \frac{\partial z_d}{\partial x_1} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \frac{\partial z_d}{\partial x_d} \end{pmatrix} \rightarrow \det J = \prod_i J_{ii} \sim \mathcal{O}(d) \text{ 😊}$$

Coupling block



Forward pass:

$$\begin{aligned} z^A &= C(x^A; f_\omega(x^B)) \\ z^B &= x^B \end{aligned}$$

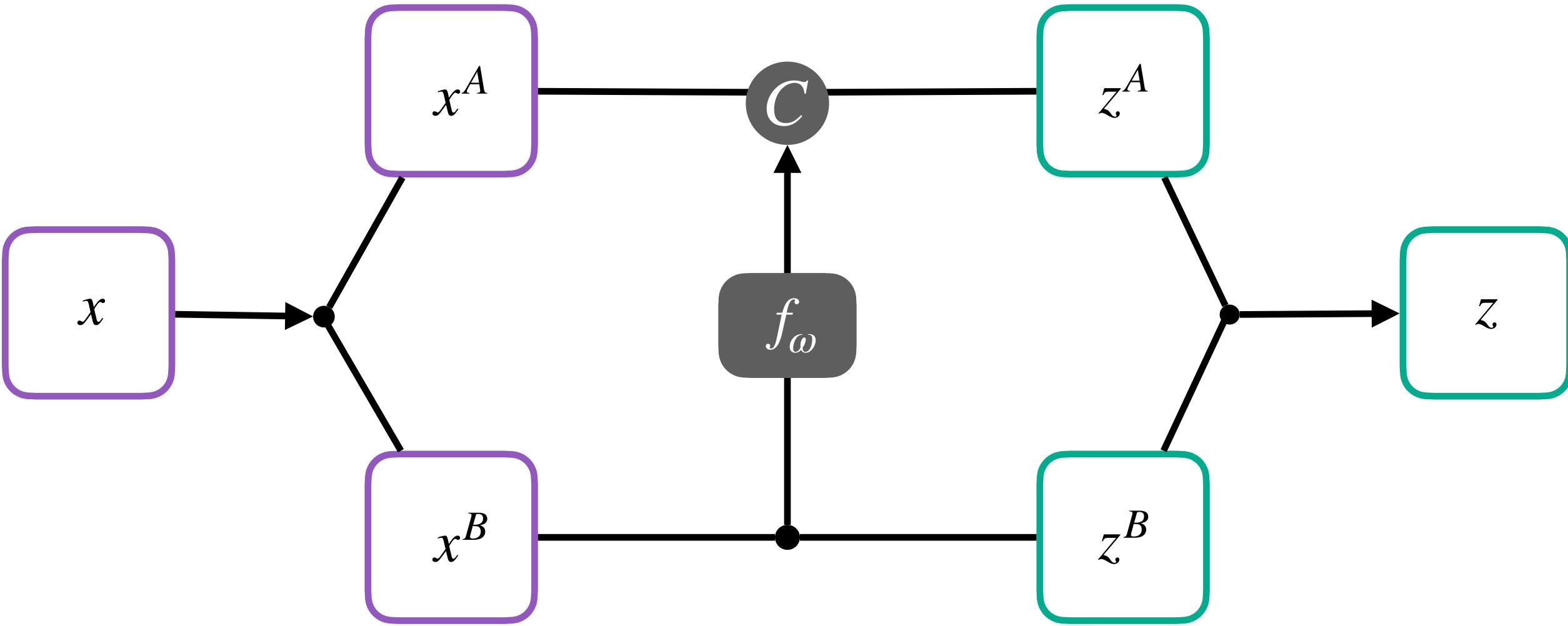
$$J_{ij}(x) = \begin{pmatrix} \frac{\partial C}{\partial x^A} & \frac{\partial C}{\partial f_\omega} \frac{\partial f_\omega}{\partial x^B} \\ 0 & I_m \end{pmatrix}$$

Inverse pass:

$$\begin{aligned} x^A &= C^{-1}(z^A; f_\omega(z^B)) \\ x^B &= z^B \end{aligned}$$

What is the function C ?

Coupling block



Forward pass:

$$\begin{aligned} z^A &= C(x^A; f_\omega(x^B)) \\ z^B &= x^B \end{aligned}$$

Inverse pass:

$$\begin{aligned} x^A &= C^{-1}(z^A; f_\omega(z^B)) \\ x^B &= z^B \end{aligned}$$

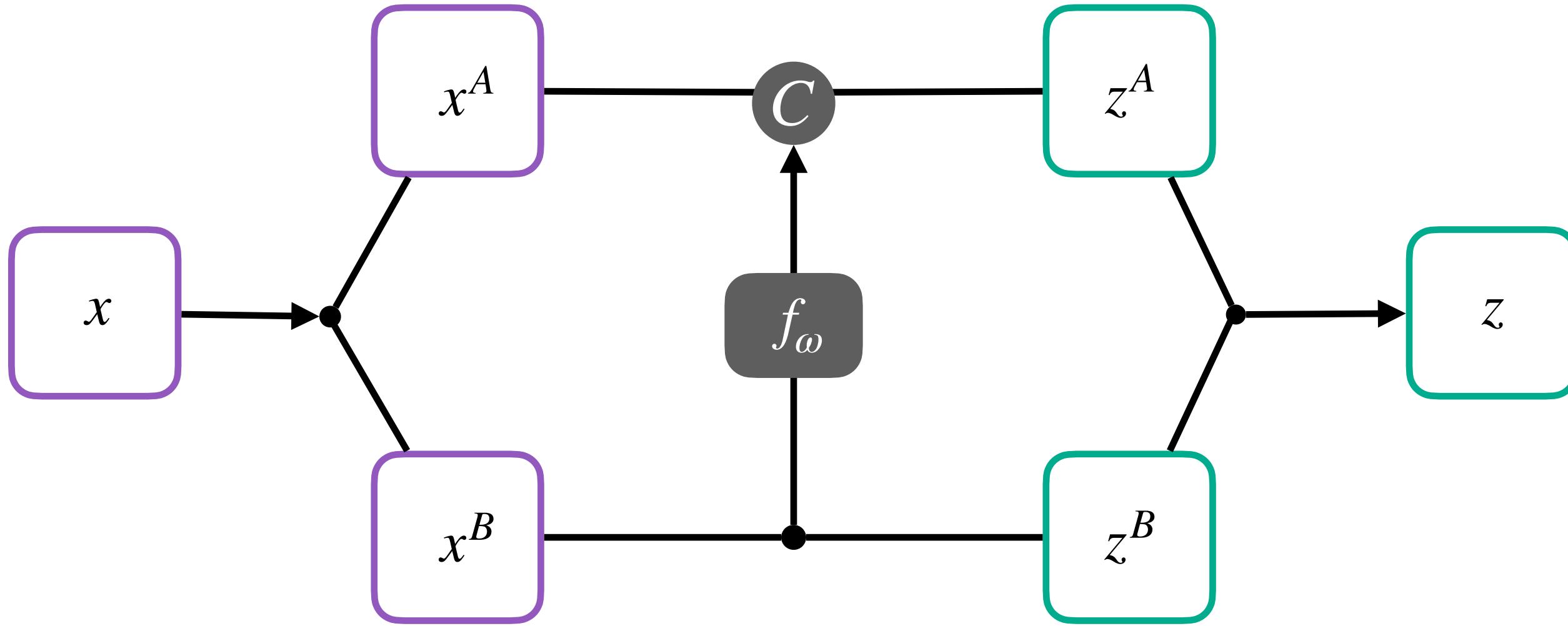
$$J_{ij}(x) = \begin{pmatrix} \frac{\partial C}{\partial x^A} & \frac{\partial C}{\partial f_\omega} \frac{\partial f_\omega}{\partial x^B} \\ 0 & I_m \end{pmatrix}$$

Affine
[1605.08803]

$$C^A = \alpha_\omega(x^B) \cdot x^A + \mu_\omega(x^B)$$

parametrized by NN

Coupling block



Forward pass:

$$\begin{aligned} z^A &= C(x^A; f_\omega(x^B)) \\ z^B &= x^B \end{aligned}$$

Inverse pass:

$$\begin{aligned} x^A &= C^{-1}(z^A; f_\omega(z^B)) \\ x^B &= z^B \end{aligned}$$

$$J_{ij}(x) = \begin{pmatrix} \frac{\partial C}{\partial x^A} & \frac{\partial C}{\partial f_\omega} \frac{\partial f_\omega}{\partial x^B} \\ 0 & I_m \end{pmatrix}$$

Affine
[1605.08803]

Quadratic
[1808.03856]

Rational quadratic
[1906.04032]

$$C^A = \alpha_\omega(x^B) \cdot x^A + \mu_\omega(x^B)$$

$$C = a_\omega x^2 + b_\omega x + c_\omega$$

$$C = \frac{a_\omega x^2 + b_\omega x + c_\omega}{d_\omega x^2 + e_\omega x + f_\omega}$$

Coupling block

Key facts - Normalizing Flows

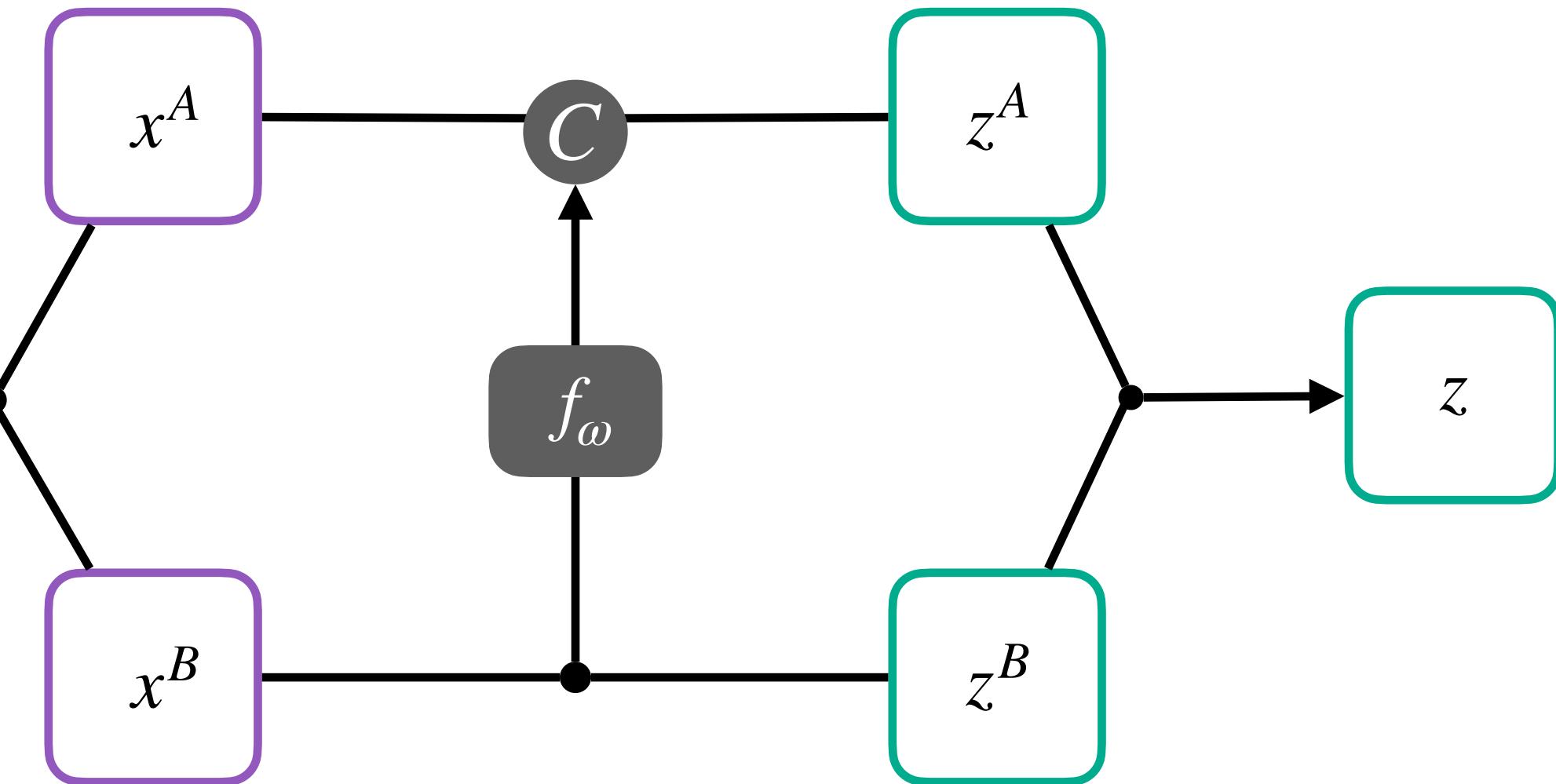
- ⊕ Fast training and evaluation
- ⊕ Tractable and fast likelihoods
- ⊖ Reduced flexibility and expressivity

Application
dependent

Forward pass

Key facts - Diffusion Model

- ⊕ State-of-the-art in precision
- ⊕ Fast and stable training
- ⊖ Slow evaluation



$$J_{ij}(x) = \begin{pmatrix} \frac{\partial C}{\partial x^A} & \frac{\partial C}{\partial f_\omega} \frac{\partial f_\omega}{\partial x^B} \\ 0 & I_m \end{pmatrix}$$

Affine
[1605.08803]

Quadratic
[1808.03856]

Rational
quadratic
[1906.04032]

$$C^A = \alpha_\omega(x^B) \cdot x^A + \mu_\omega(x^B)$$

$$C = a_\omega x^2 + b_\omega x + c_\omega$$

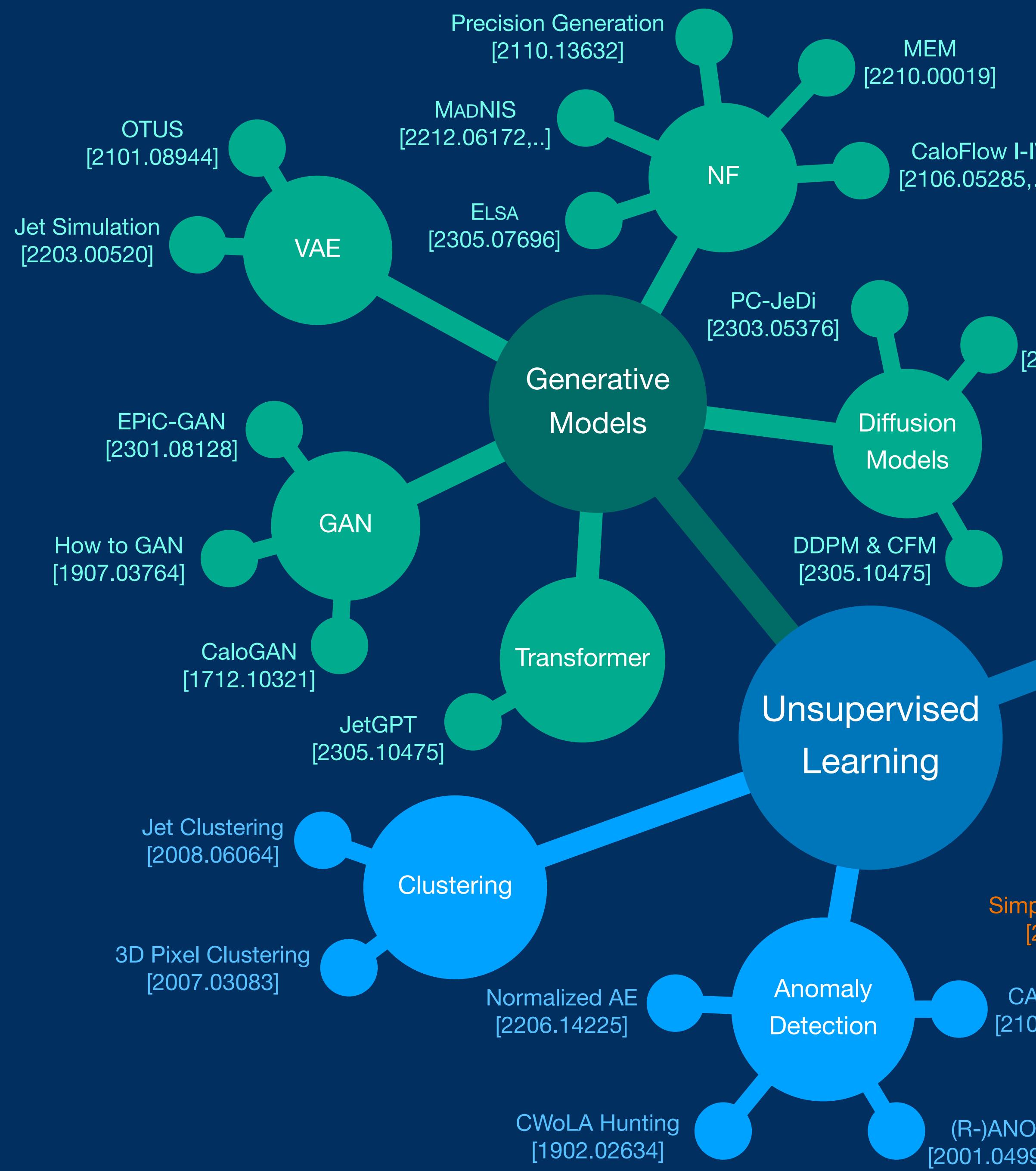
$$C = \frac{a_\omega x^2 + b_\omega x + c_\omega}{d_\omega x^2 + e_\omega x + f_\omega}$$



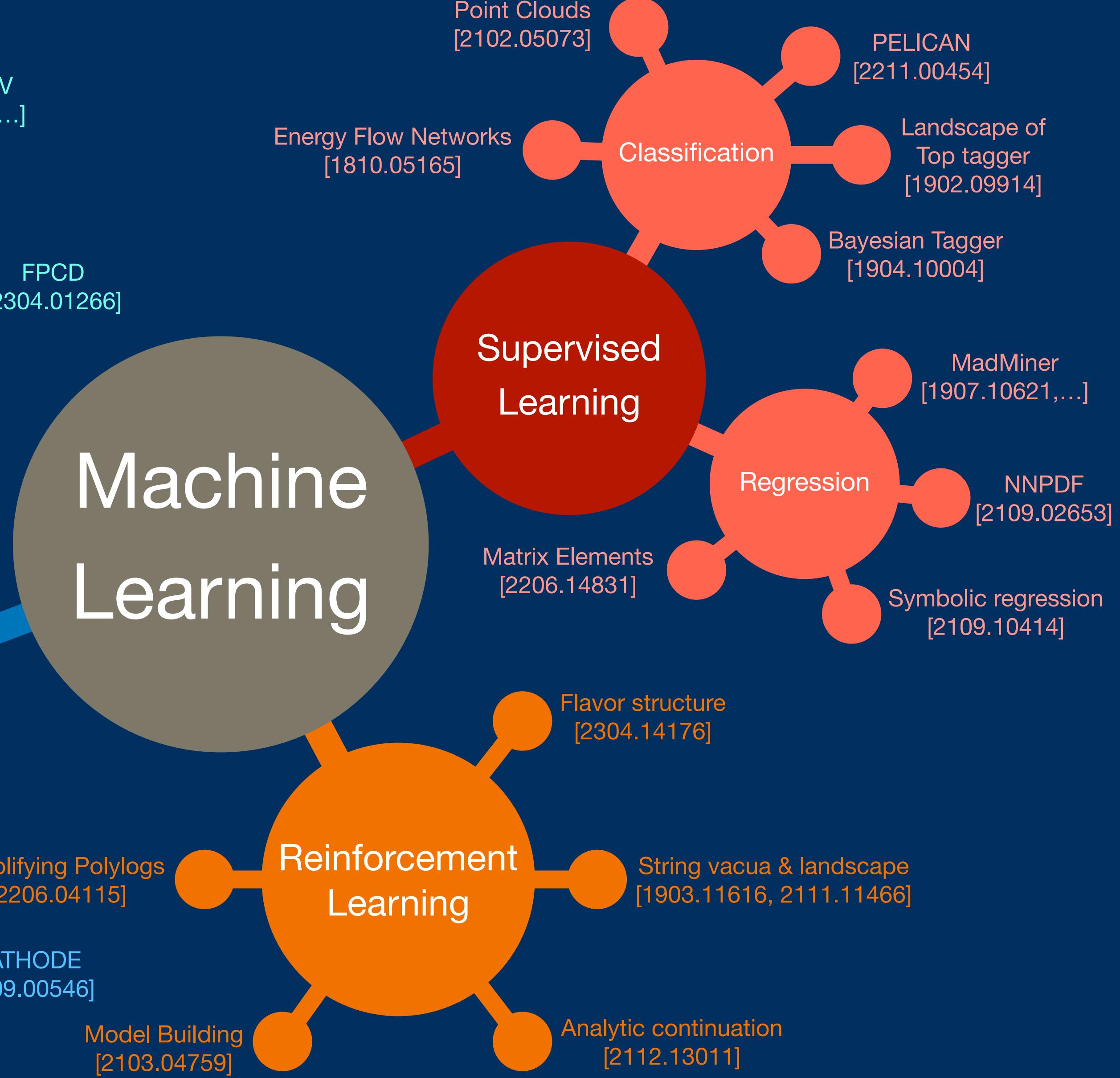
Questions?

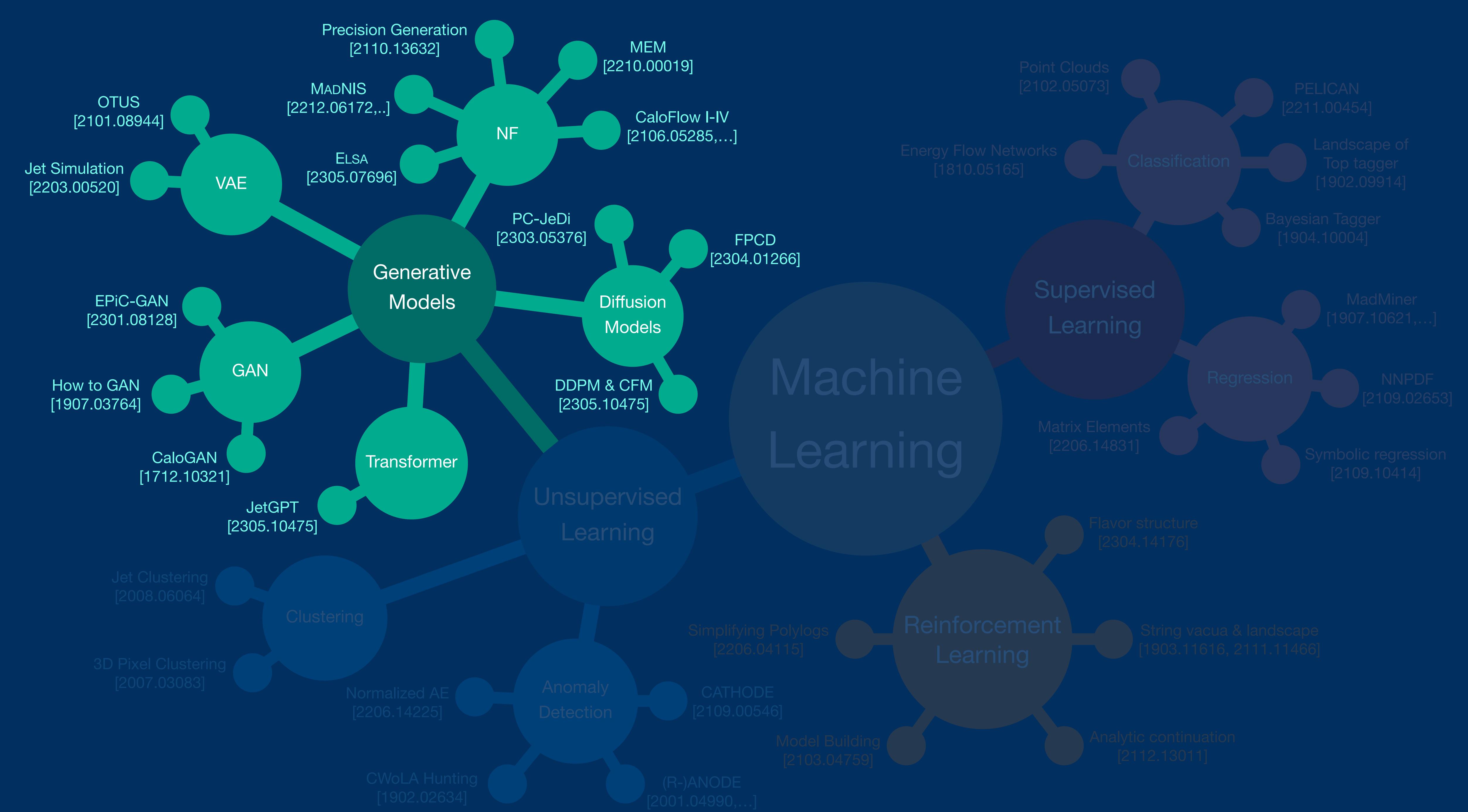
Machine Learning

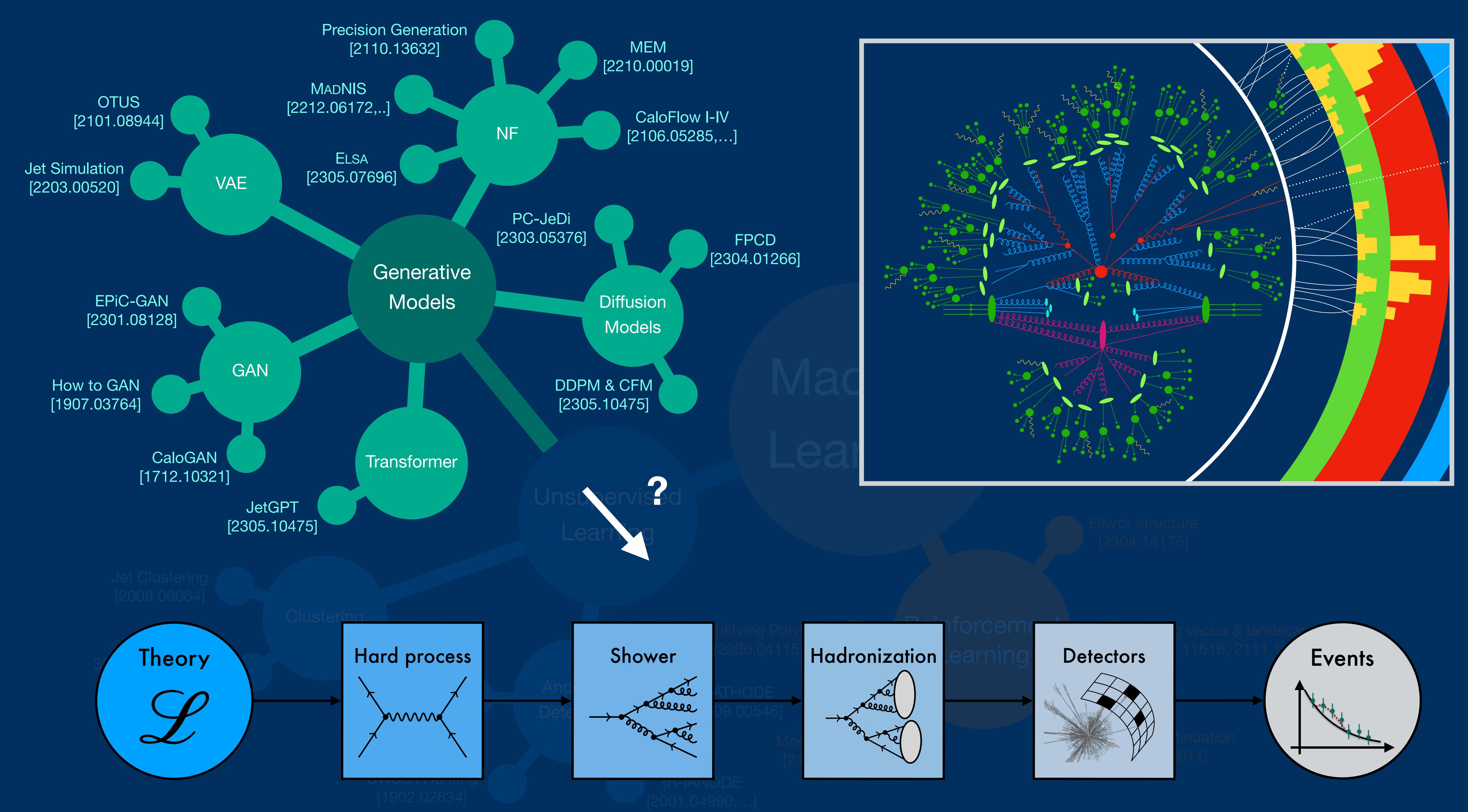
Unsupervised Learning



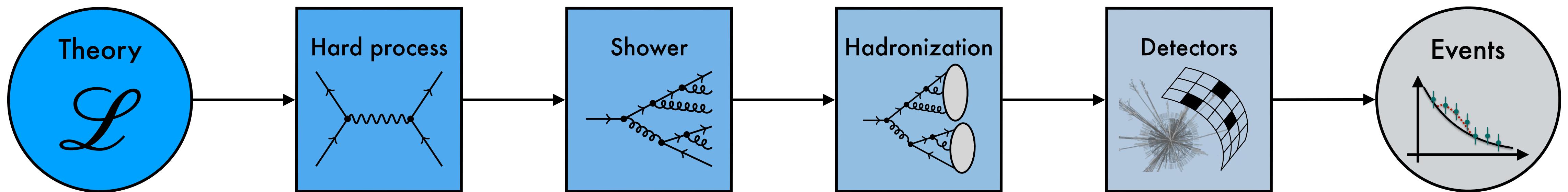
Reinforcement Learning





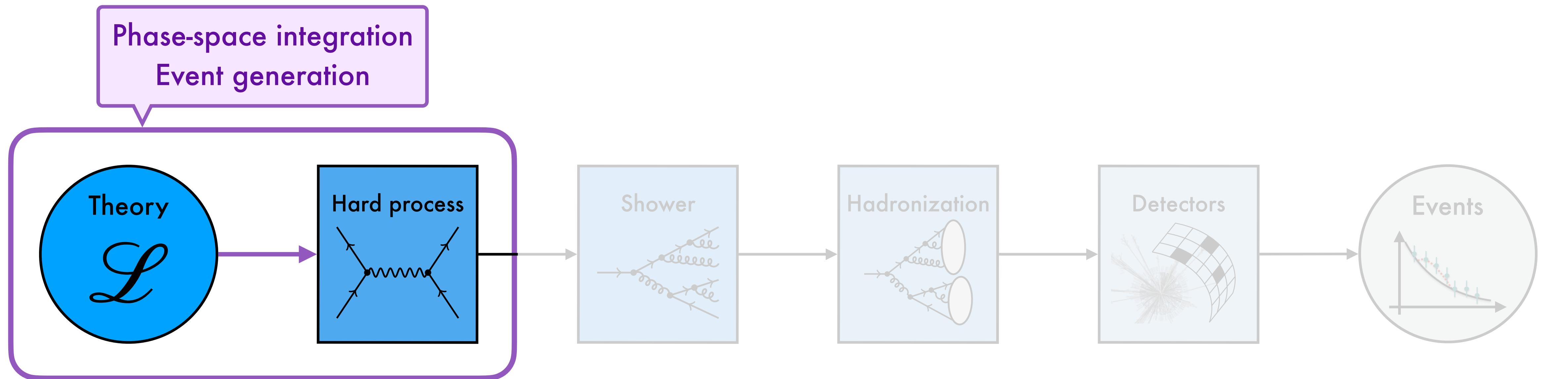


The LHC simulation chain



The LHC simulation chain + ML

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Importance sampling

BDT [1707.00028], NN [1810.11509, 2009.07819]
NF [2001.05486, 2001.05478, 2001.10028, 2005.12719,
2112.09145, 2212.06172, 2311.01548]
Chili [2302.10449]

Surrogate regression

Full weight [2109.11964],
Matrix element [1912.11055, 2002.07516,
2006.16273, 2106.09474, 2107.06625, 2109.11964,
2206.14831, 2301.13562, 2302.04005, 2306.07726]

Example I

Neural importance sampling with MadNIS

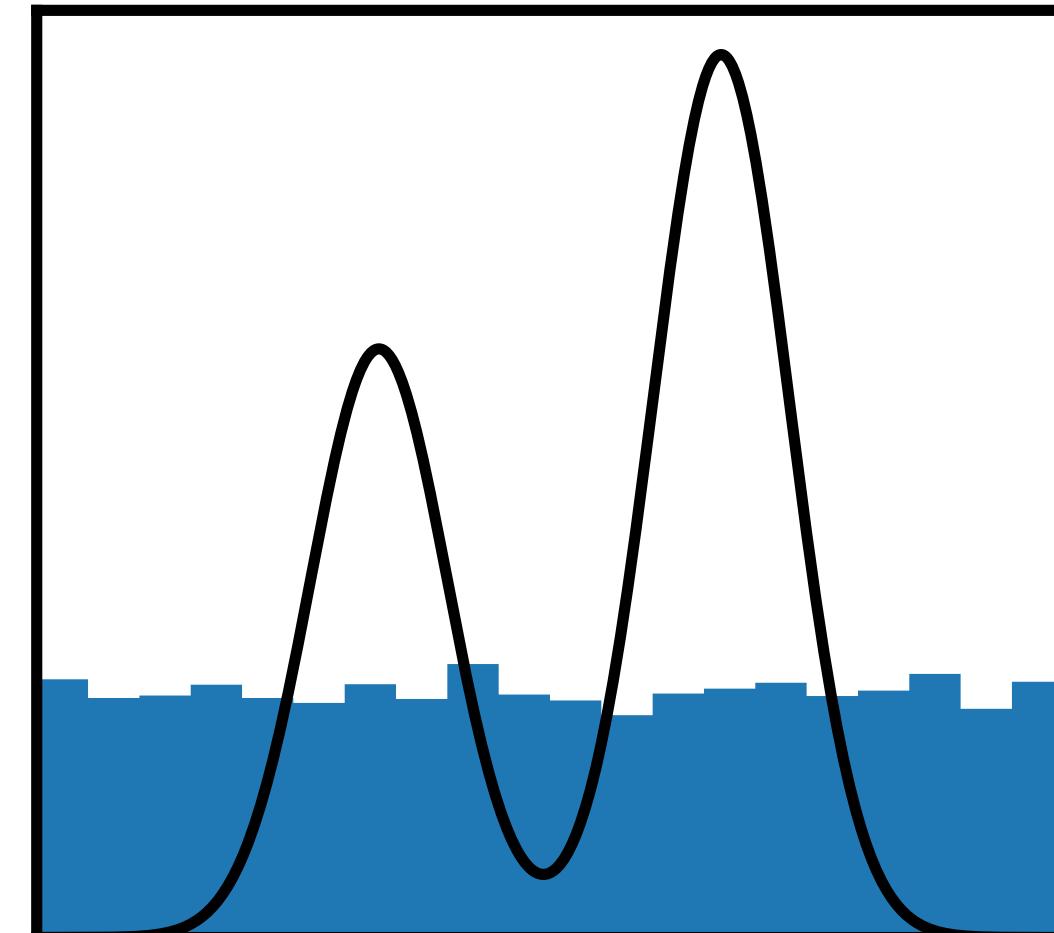
Heimel, Huetsch, Maltoni, Mattelaer, Plehn, RW [2311.01548]

Heimel, RW, Butter, Isaacson, Krause, Maltoni, Mattelaer, Plehn [2212.06172]

Monte Carlo integration

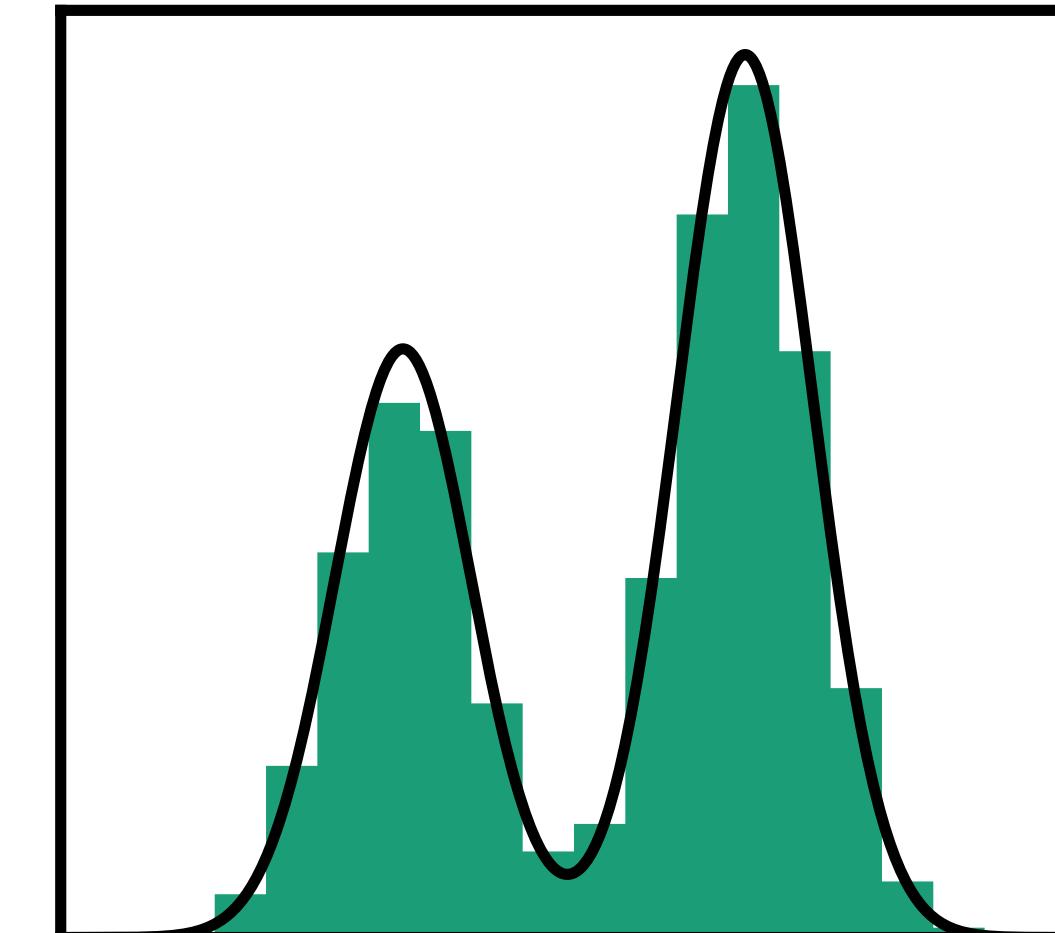
Calculate (differential) cross sections

$$d\sigma = \frac{1}{\text{flux}} dx_a dx_b f(x_a) f(x_b) d\Phi_n \left\langle |M_{\lambda,c,\dots}(p_a, p_b | p_1, \dots, p_n)|^2 \right\rangle$$



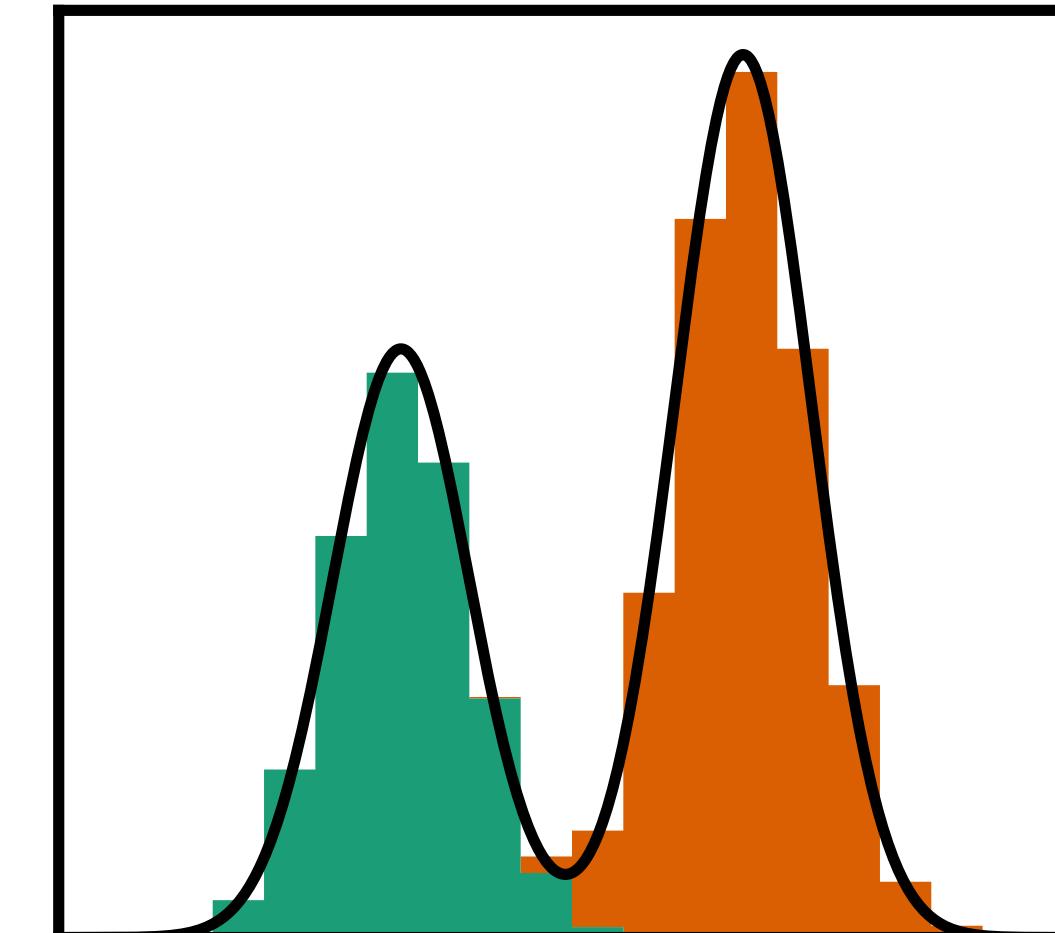
Flat sampling:
inefficient

$$I = \langle f(x) \rangle_{x \sim \text{unif}}$$



Importance sampling:
find p close to f

$$I = \left\langle \frac{f(x)}{p(x)} \right\rangle_{x \sim p(x)}$$



Multi-channel:
one map for each channel

$$I = \sum_i \left\langle \alpha_i(x) \frac{f(x)}{p_i(x)} \right\rangle_{x \sim p_i(x)}$$

Event generation

Calculate (differential) cross sections

$$d\sigma = \frac{1}{\text{flux}} dx_a dx_b f(x_a) f(x_b) d\Phi_n \left\langle |M_{\lambda,c,\dots}(p_a, p_b | p_1, \dots, p_n)|^2 \right\rangle$$

Sum over channels

MadGraph: build channels from Feynman diagrams

Channel weights

MadGraph: $\alpha_i^{\text{MG}}(x) \sim |M_i|^2$

$$I = \sum_i \left\langle \alpha_i(x) \frac{f(x)}{p_i(x)} \right\rangle_{x \sim p_i(x)}$$

Integrand

MadGraph: $d\sigma/dx$

Channel mappings

MadGraph: use amplitude structure, ...
Analytic mappings + refine with **VEGAS**

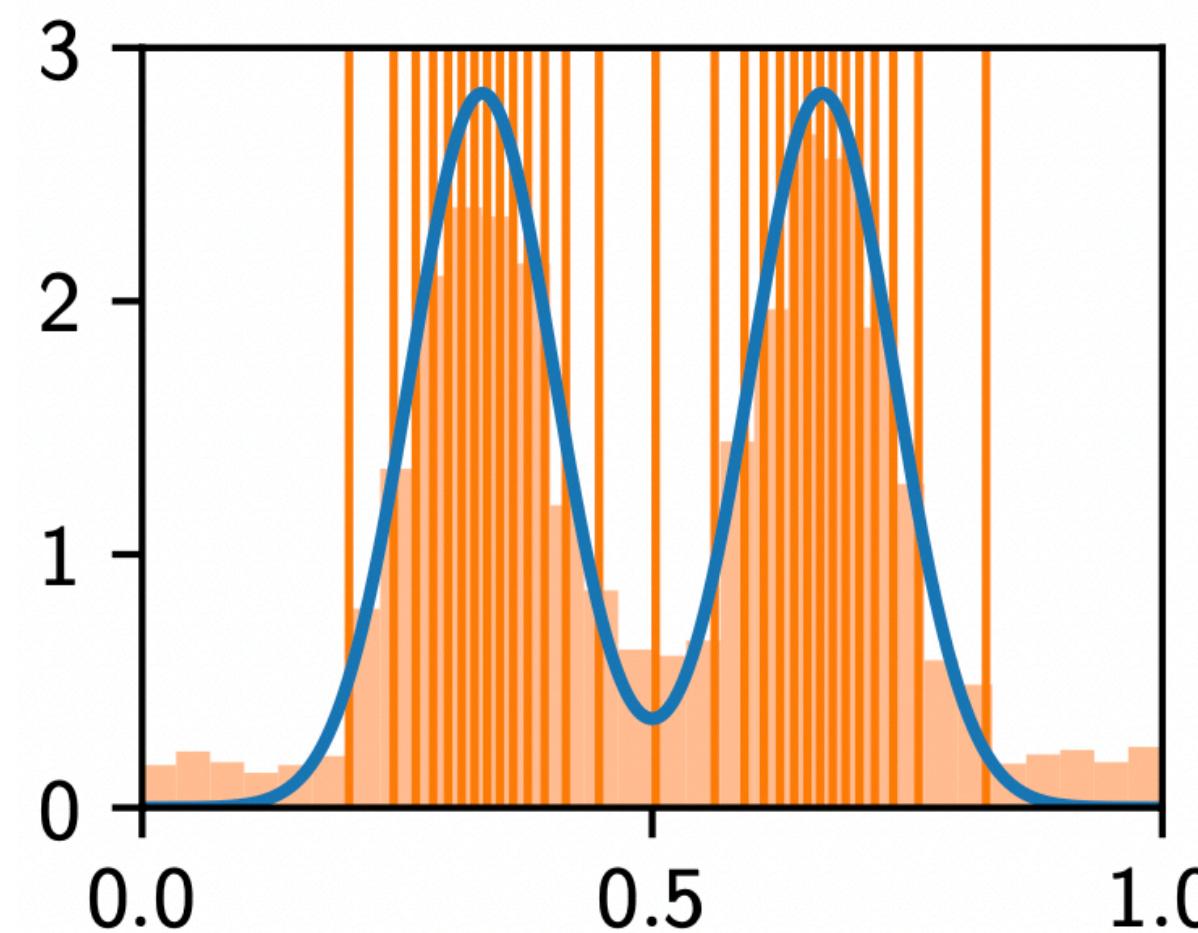
Importance sampling – VEGAS

Factorize probability

$$p(x) = p(x_1) \cdots p(x_n)$$



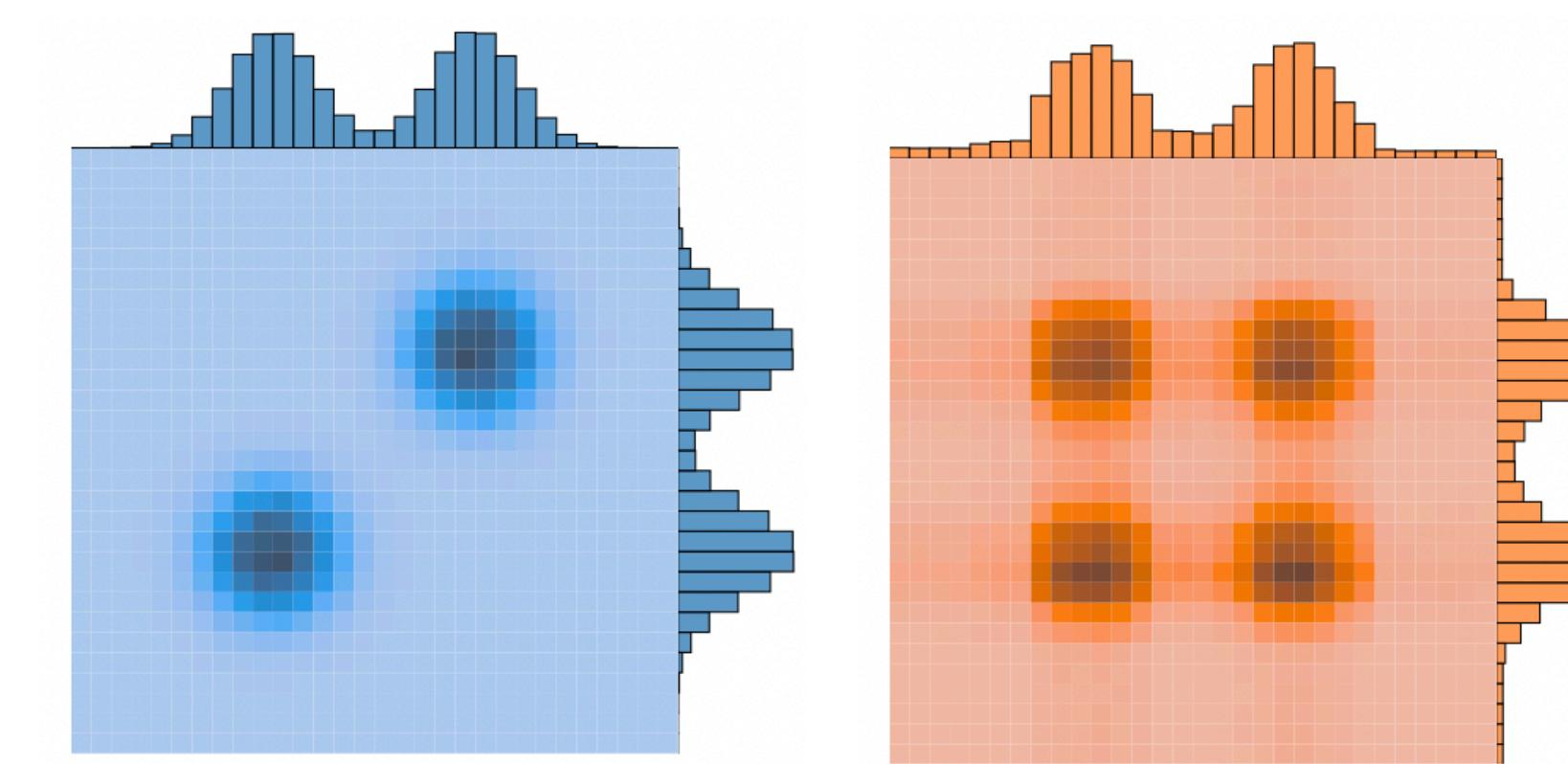
Fit bins with equal probability
and varying width



⊕ Computationally cheap

⊖ High-dim and rich peaking functions
→ **slow convergence**

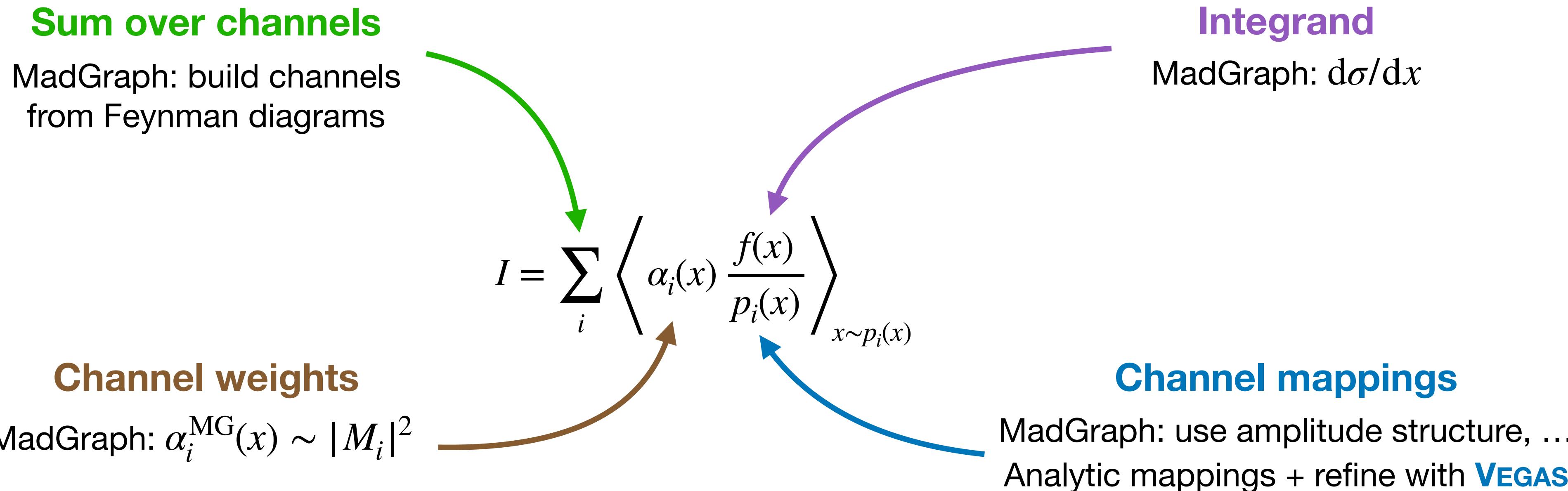
⊖ Peaks not aligned with grid axes
→ **phantom peaks**



Event generation

Calculate (differential) cross sections

$$d\sigma = \frac{1}{\text{flux}} dx_a dx_b f(x_a) f(x_b) d\Phi_n \left\langle |M_{\lambda,c,\dots}(p_a, p_b | p_1, \dots, p_n)|^2 \right\rangle$$



Event generation + MadNIS

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Calculate (differential) cross sections

$$d\sigma = \frac{1}{\text{flux}} dx_a dx_b f(x_a) f(x_b) d\Phi_n \left\langle |M_{\lambda,c,\dots}(p_a, p_b | p_1, \dots, p_n)|^2 \right\rangle$$

Sum over channels

MadGraph: build channels from Feynman diagrams

Channel weights

MadGraph: $\alpha_i^{\text{MG}}(x) \sim |M_i|^2$

$$I = \sum_i \left\langle \alpha_i(x) \frac{f(x)}{p_i^\omega(x)} \right\rangle_{x \sim p_i^\omega(x)}$$

Integrand

MadGraph: $d\sigma/dx$

Learned channel mappings

MadGraph: use amplitude structure, ...
Analytic mappings + refine with ~~VEGAS~~

refine with **NF**

Event generation + MadNIS

Calculate (differential) cross sections

$$d\sigma = \frac{1}{\text{flux}} dx_a dx_b f(x_a) f(x_b) d\Phi_n \left\langle |M_{\lambda,c,\dots}(p_a, p_b | p_1, \dots, p_n)|^2 \right\rangle$$

Sum over channels

MadGraph: build channels from Feynman diagrams

Learned Channel weights

MadGraph: $\alpha_i^{\text{MG}}(x) \sim |M_i|^2$

$$\alpha_i(x) \rightarrow \alpha_i^\xi(x) = \alpha_i^{\text{MG}}(x) \cdot K_i^\xi(x)$$

parametrize with **NN**

$$I = \sum_i \left\langle \alpha_i^\xi(x) \frac{f(x)}{p_i^\omega(x)} \right\rangle_{x \sim p_i^\omega(x)}$$

Integrand

MadGraph: $d\sigma/dx$

Learned channel mappings

MadGraph: use amplitude structure, ...
Analytic mappings + refine with **VEGAS**



refine with **NF**

MadNIS – Overview

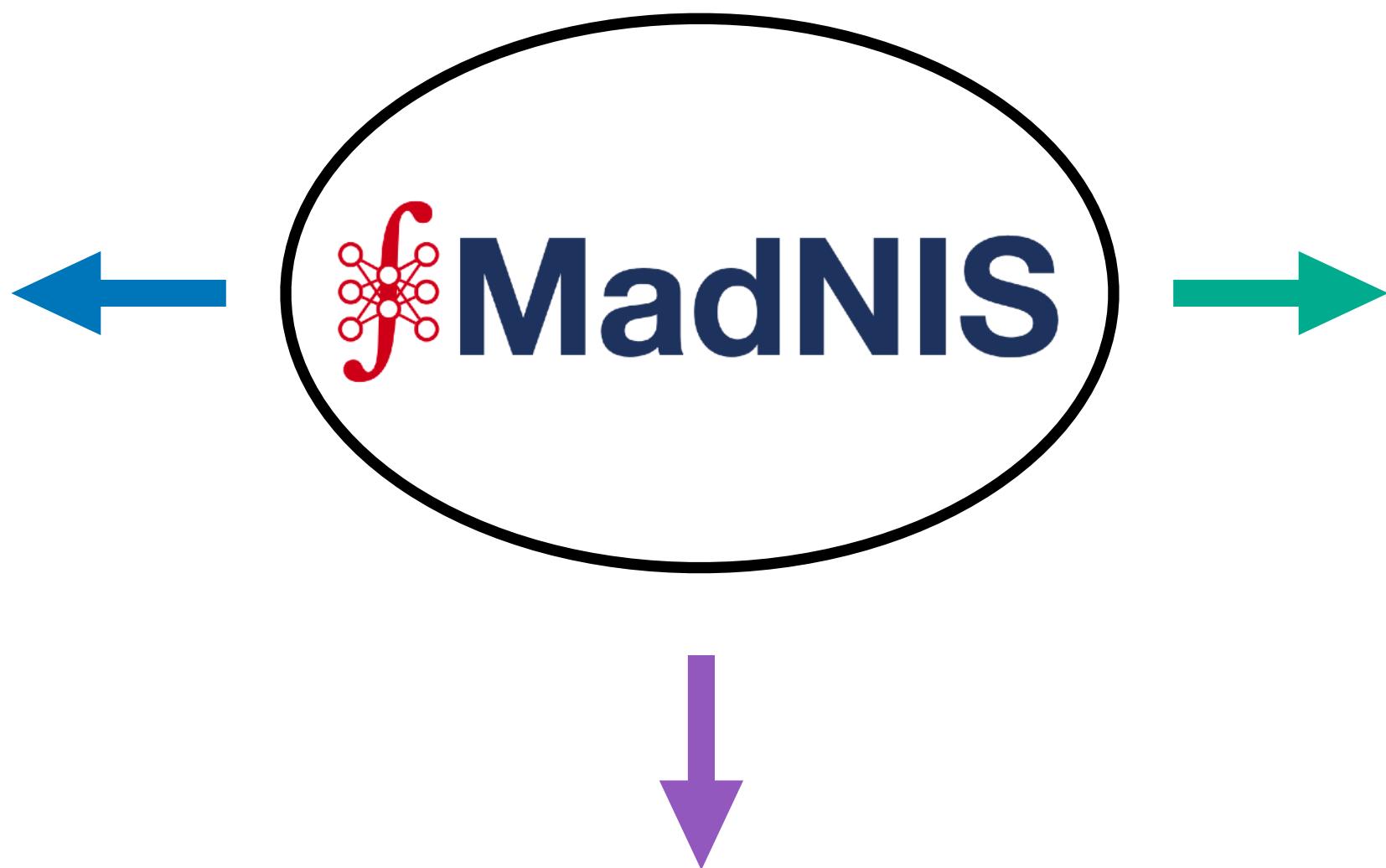
Basic functionality

Neural
Channel
Weights

Normalizing
Flow

MadGraph
matrix
elements

MadEvent
channel
mappings



Improved multi-channeling

Stratified
sampling/
training

Symmetries
between
channels

Channel
Dropping

Partial weight
buffering

Improved training

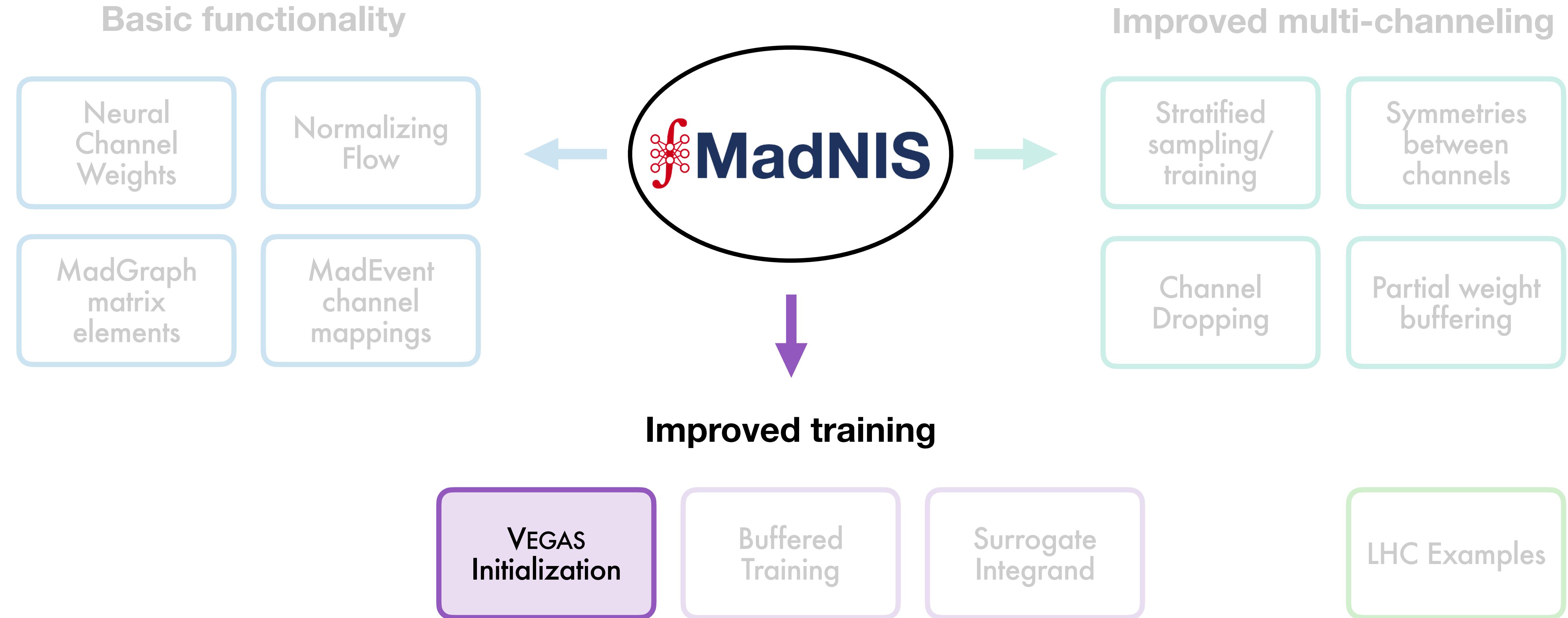
VEGAS
Initialization

Buffered
Training

Surrogate
Integrand

LHC Examples

MadNIS – Overview

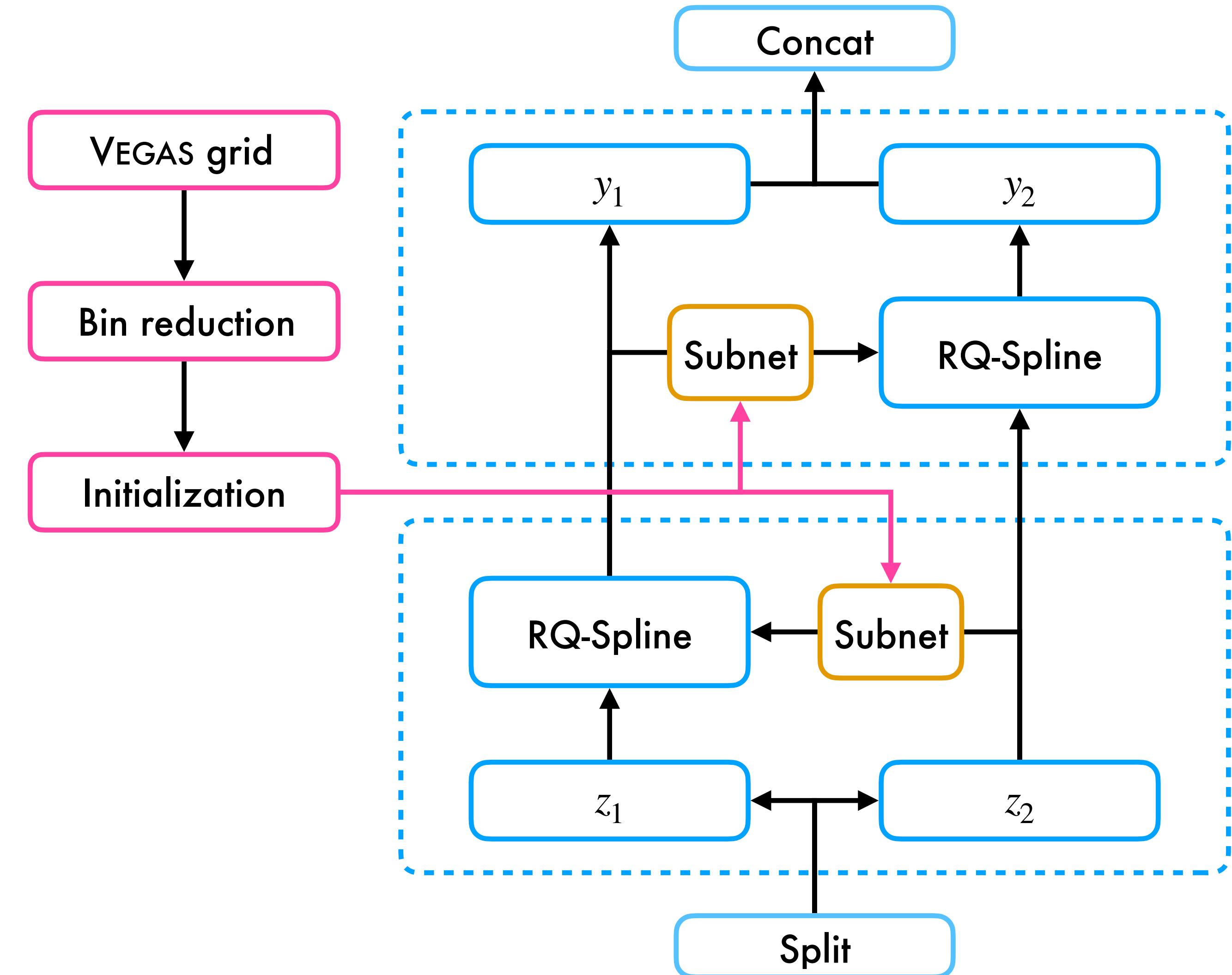


VEGAS initialization

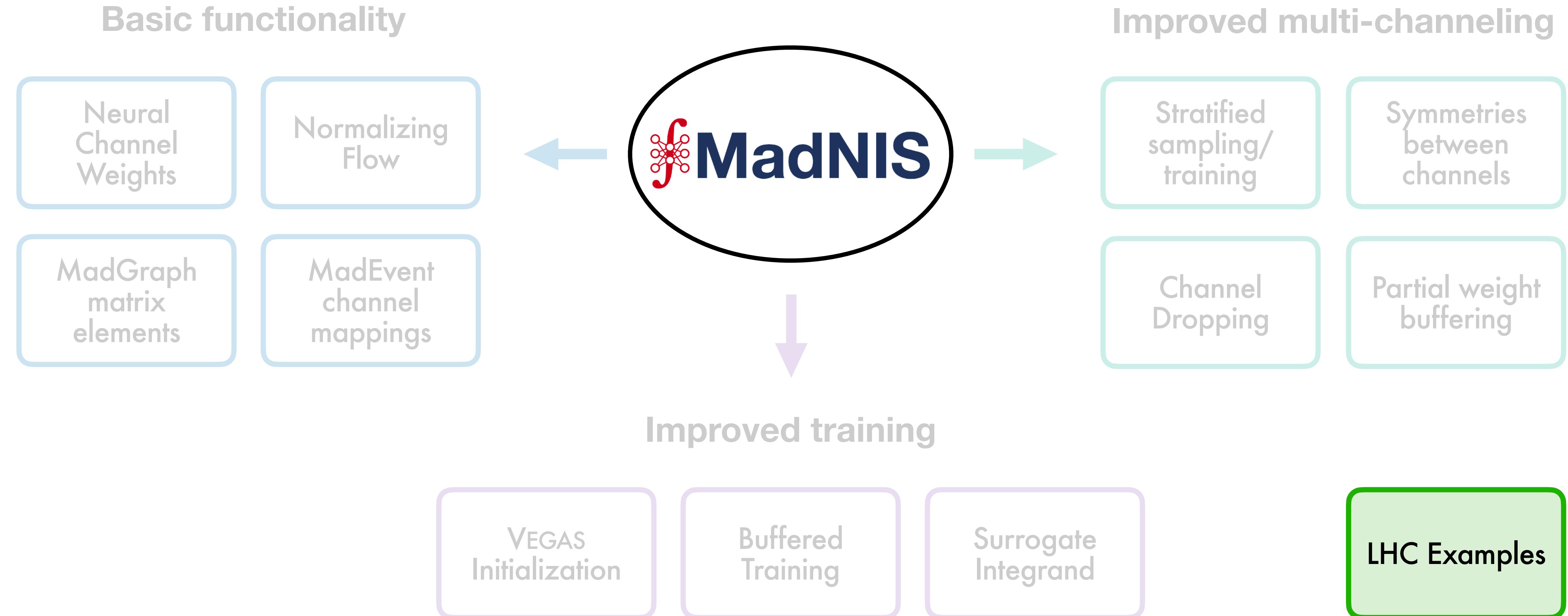
	VEGAS	Flow
Training	Fast	Slow
Correlations	No	Yes



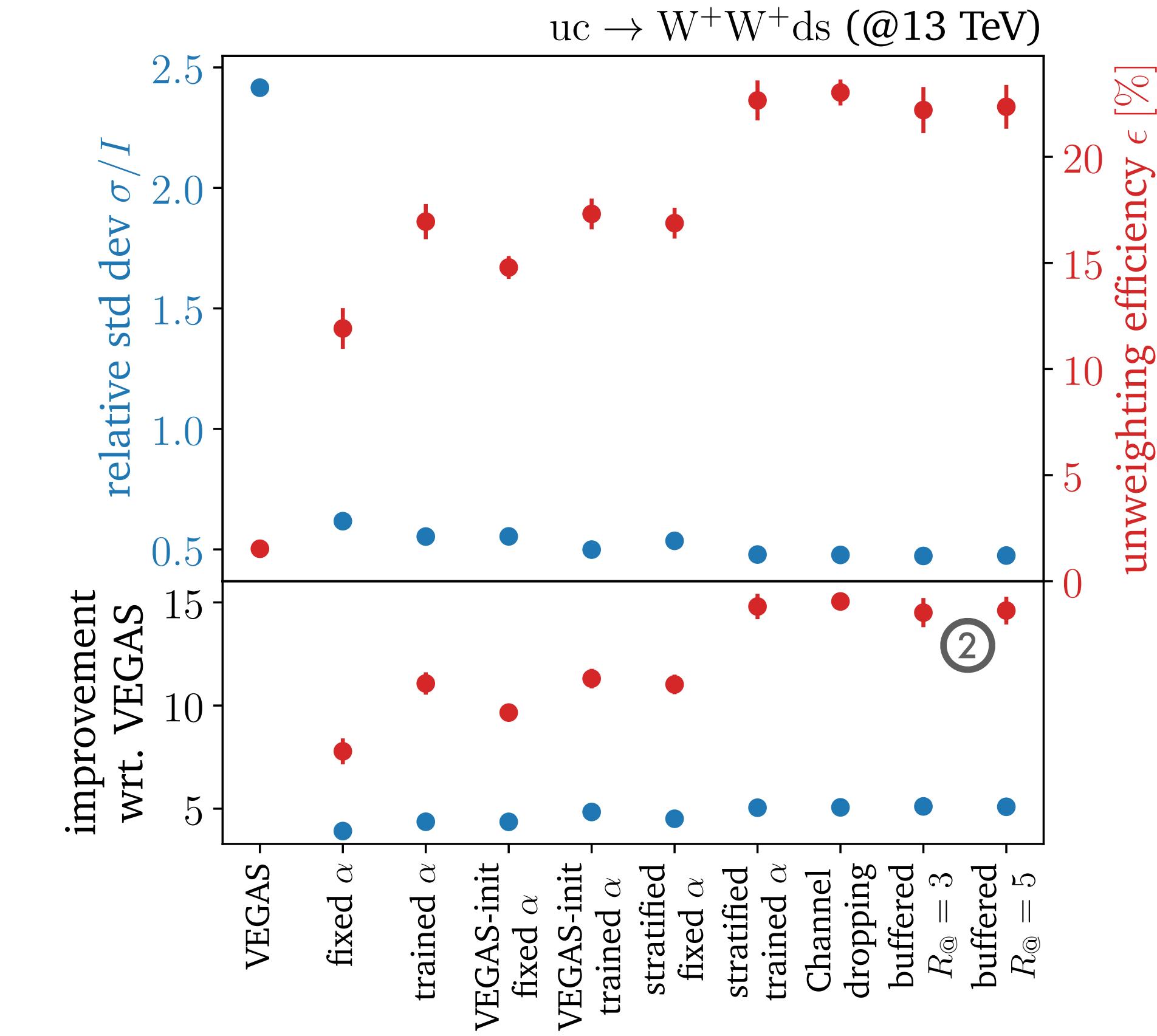
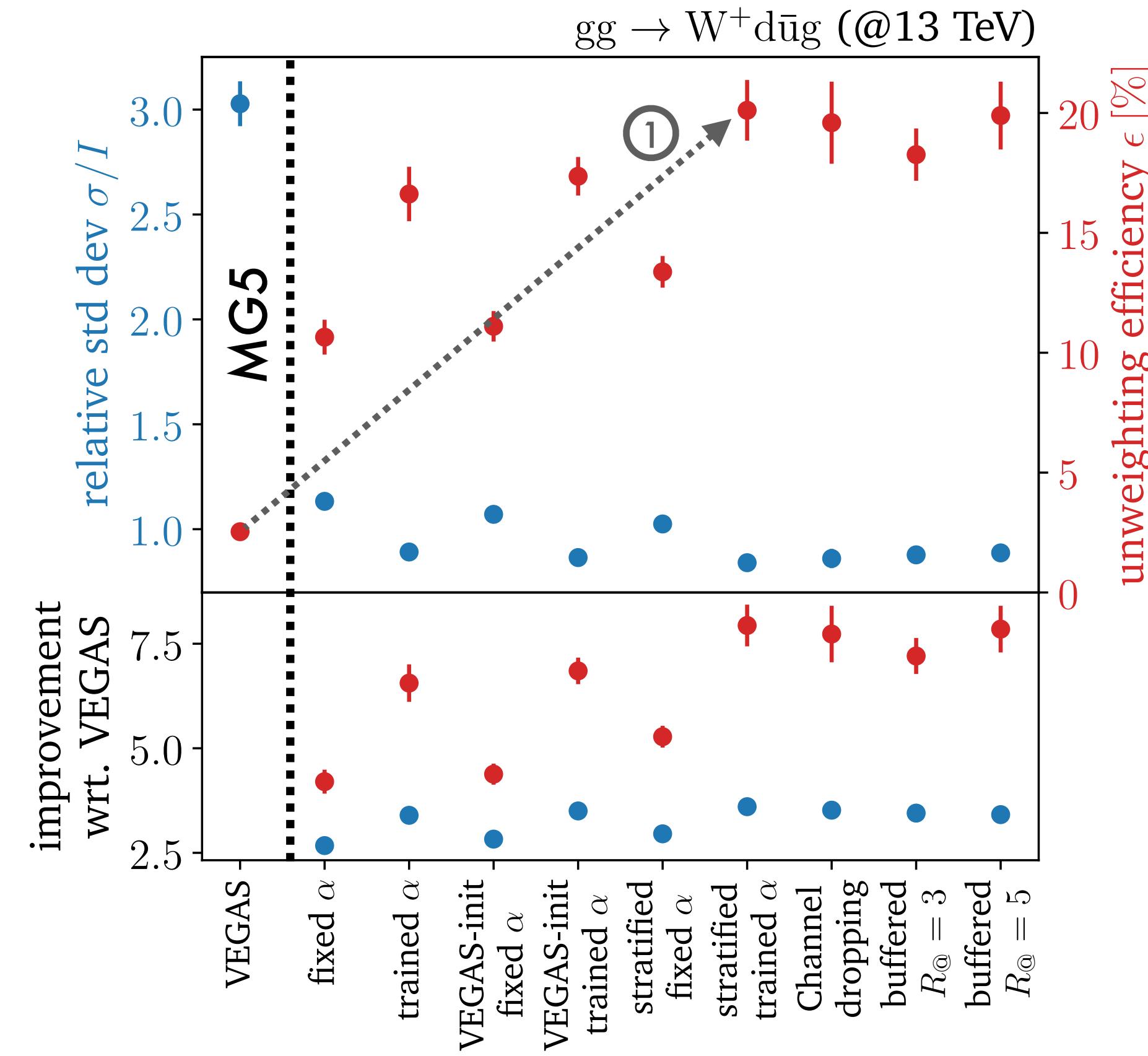
Combine advantages:
Pre-trained VEGAS grid as
starting point for flow training



MadNIS – Overview



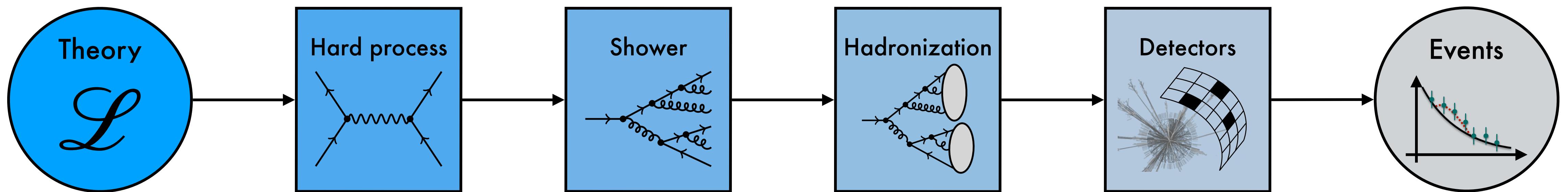
LHC processes



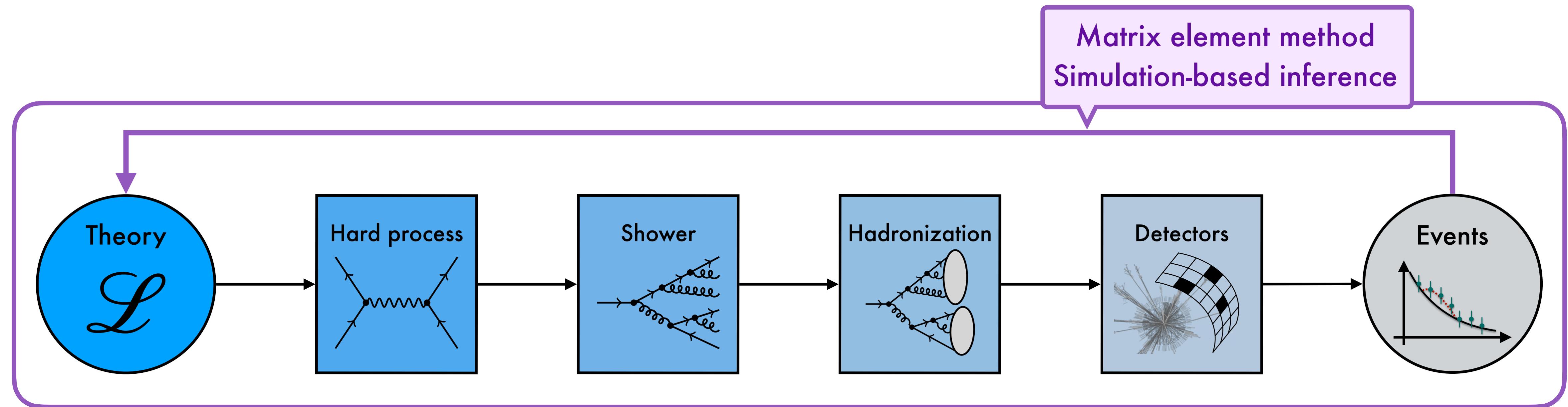
1. excellent results with all improvements

2. Larger improvements for processes with large interference terms

The LHC simulation chain



The LHC simulation chain + ML



Matrix element method (MEM)

[hep-ex/9808029, hep-ex/0406031, hep-ex/0605118,
 1003.1316, 1007.3300, 1010.2263, 1211.3011, 1304.6414,
 1502.02485, 1511.05980, 1511.06170, 1512.03429,
 1606.03107, 1710.10699, 1712.03266, 1805.08555,
 2008.10949, 2210.00019, 2310.07752,.....]

Simulation-based inference

[1506.02169, 1601.07913, 1805.00013, 1805.00020,
 1805.12244, 1907.10621, 2101.07263, 2210.01680,
 2305.10500, 2308.05704,...]

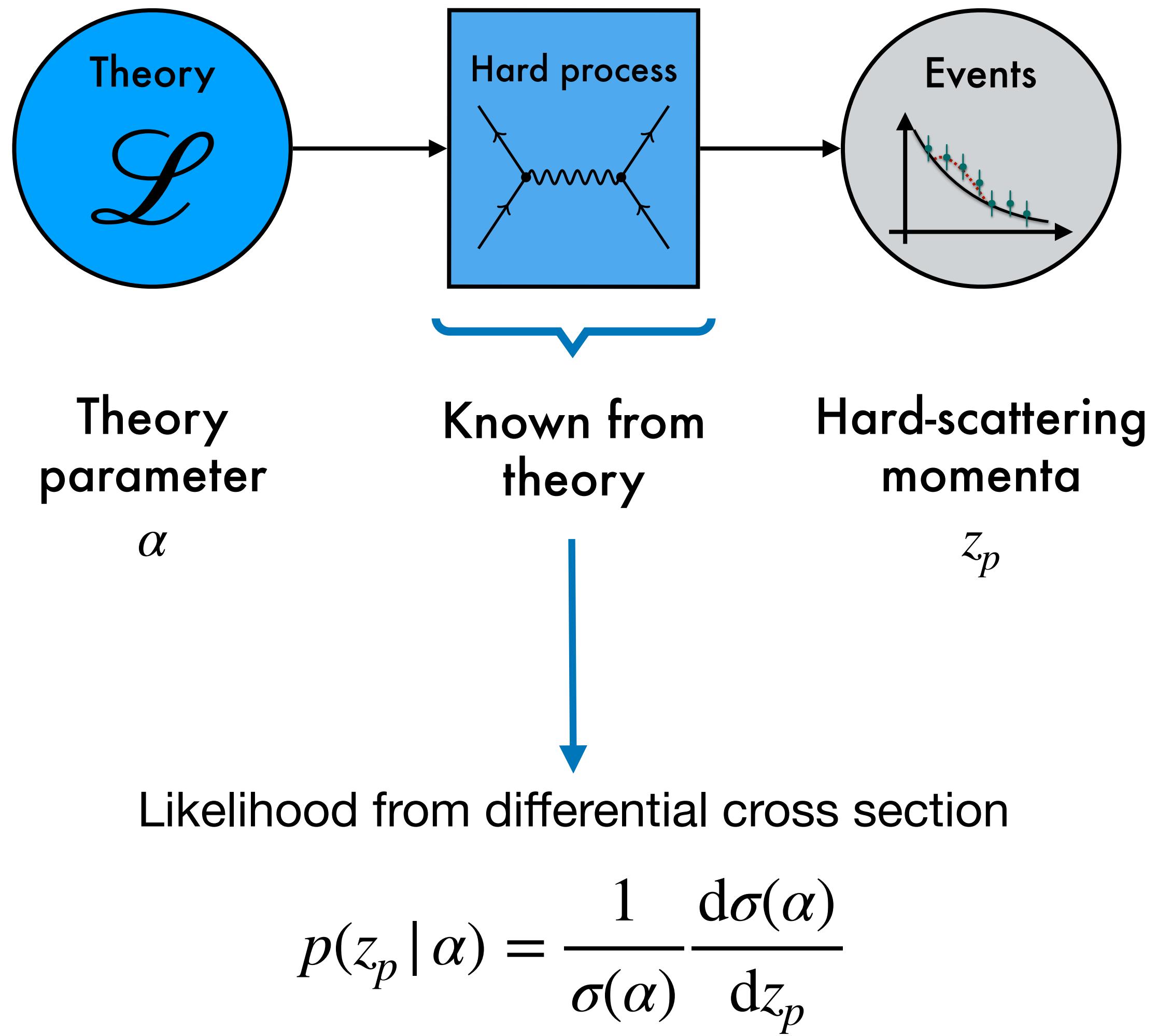
Example II

Matrix Element Method

Heimel, Huetsch, RW, Plehn, Butter [2310.07752]

Butter, Heimel, Martini, Peitzsch, Plehn [2210.00019]

Matrix Element Method – Theory



$$p(z_p | \alpha) = \frac{1}{\sigma(\alpha)} \frac{d\sigma(\alpha)}{dz_p}$$

Classical analysis

- ⊖ hand-crafted observables
- ⊖ binned data

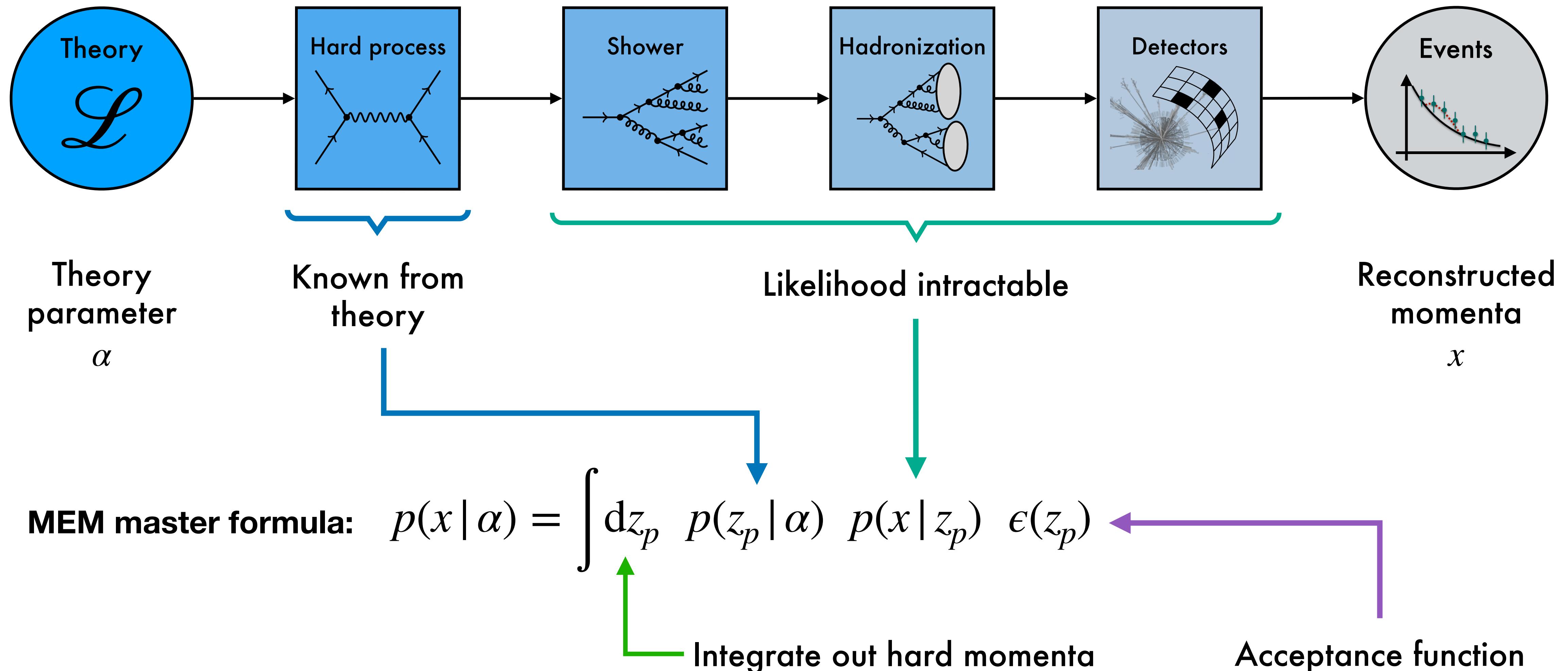
→ not all information is used 😞

Matrix Element Method (MEM)

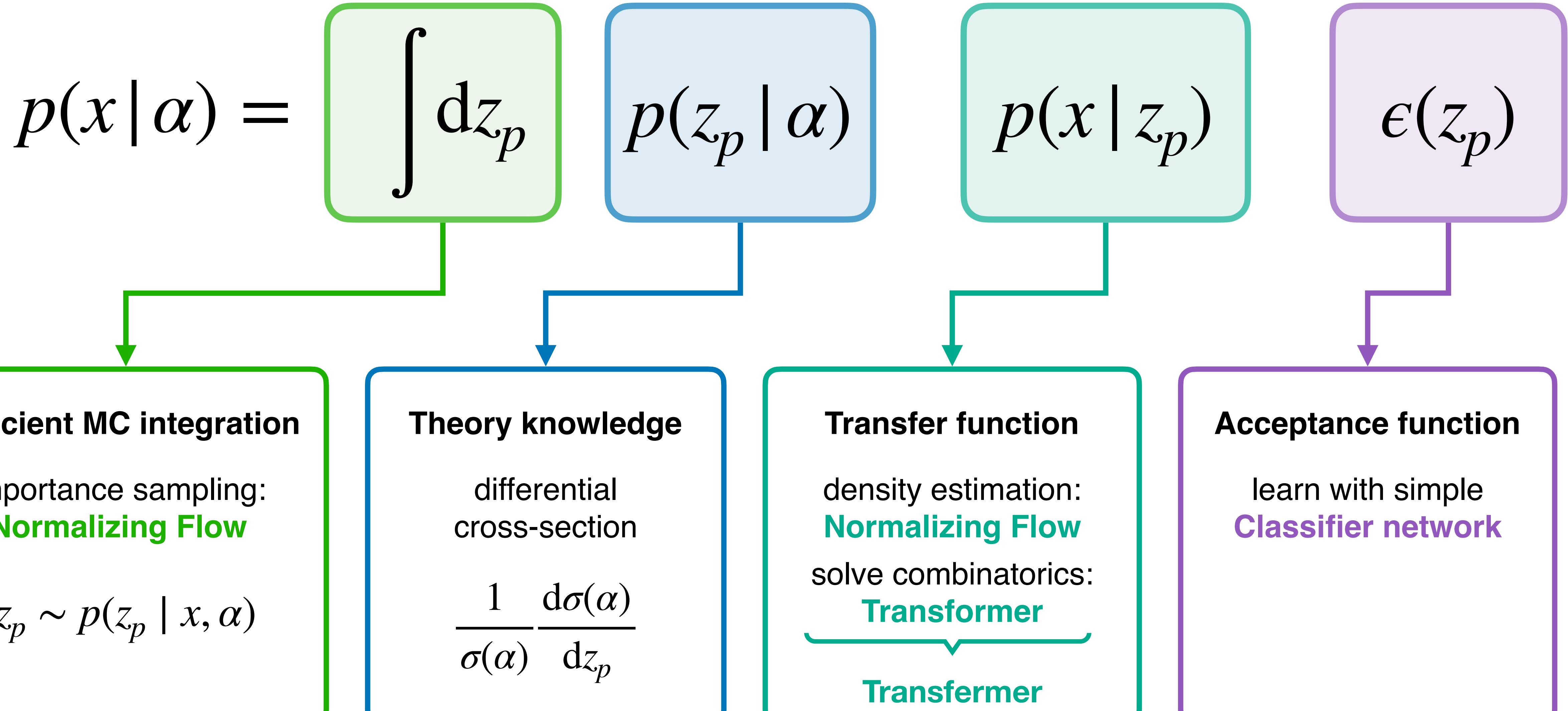
- ⊕ based on first principles
- ⊕ estimates uncertainties reliably
- ⊕ optimal use of information

→ perfect for processes with few events ☺

Matrix Element Method – Reality



Matrix Element Method + ML



LHC example

Single Higgs production with anomalous non-CP-conserving Higgs coupling



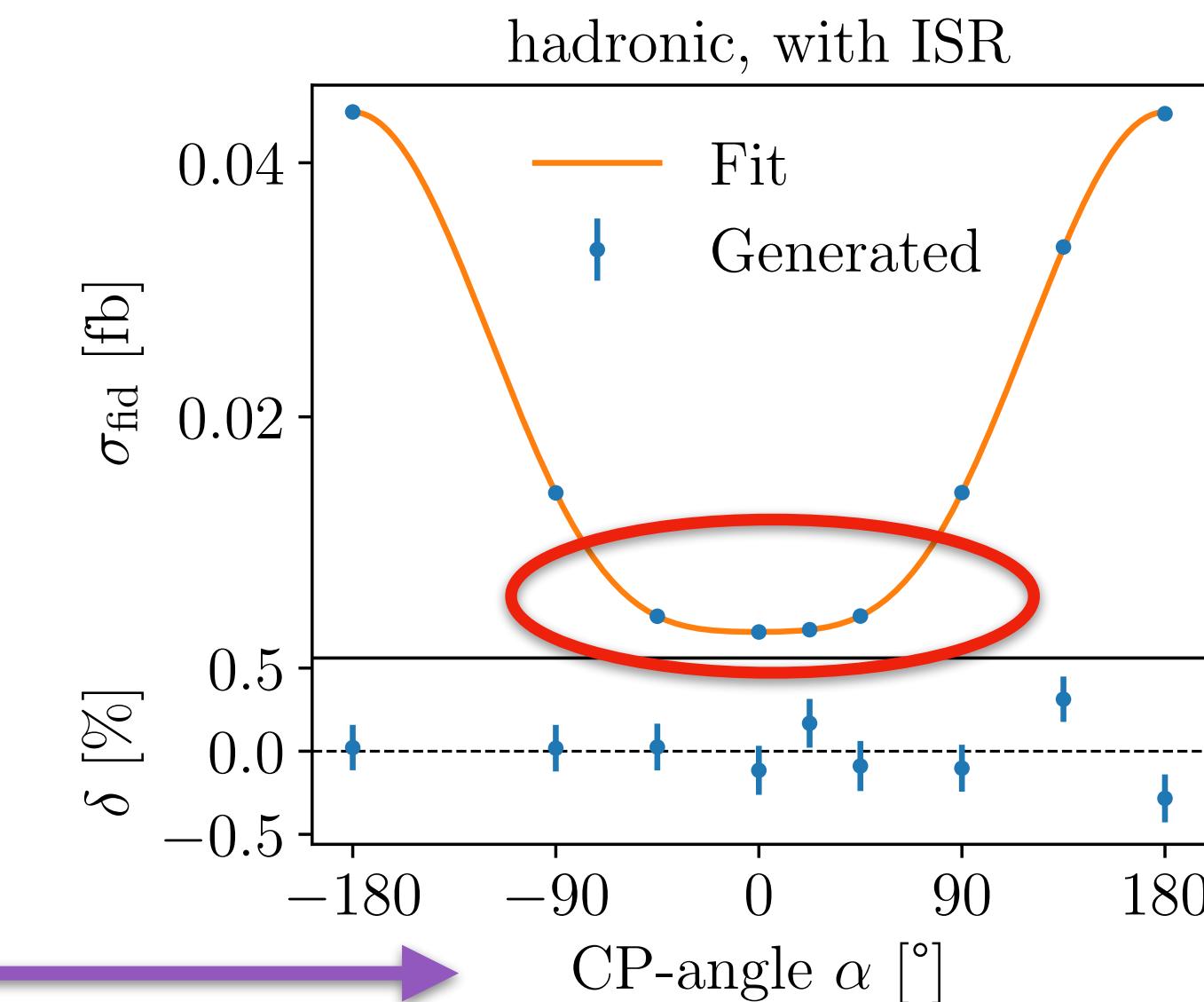
Hadronic decay of top + ISR

$$tHq \rightarrow (bjj) (\gamma\gamma) j + \text{QCD jets}$$

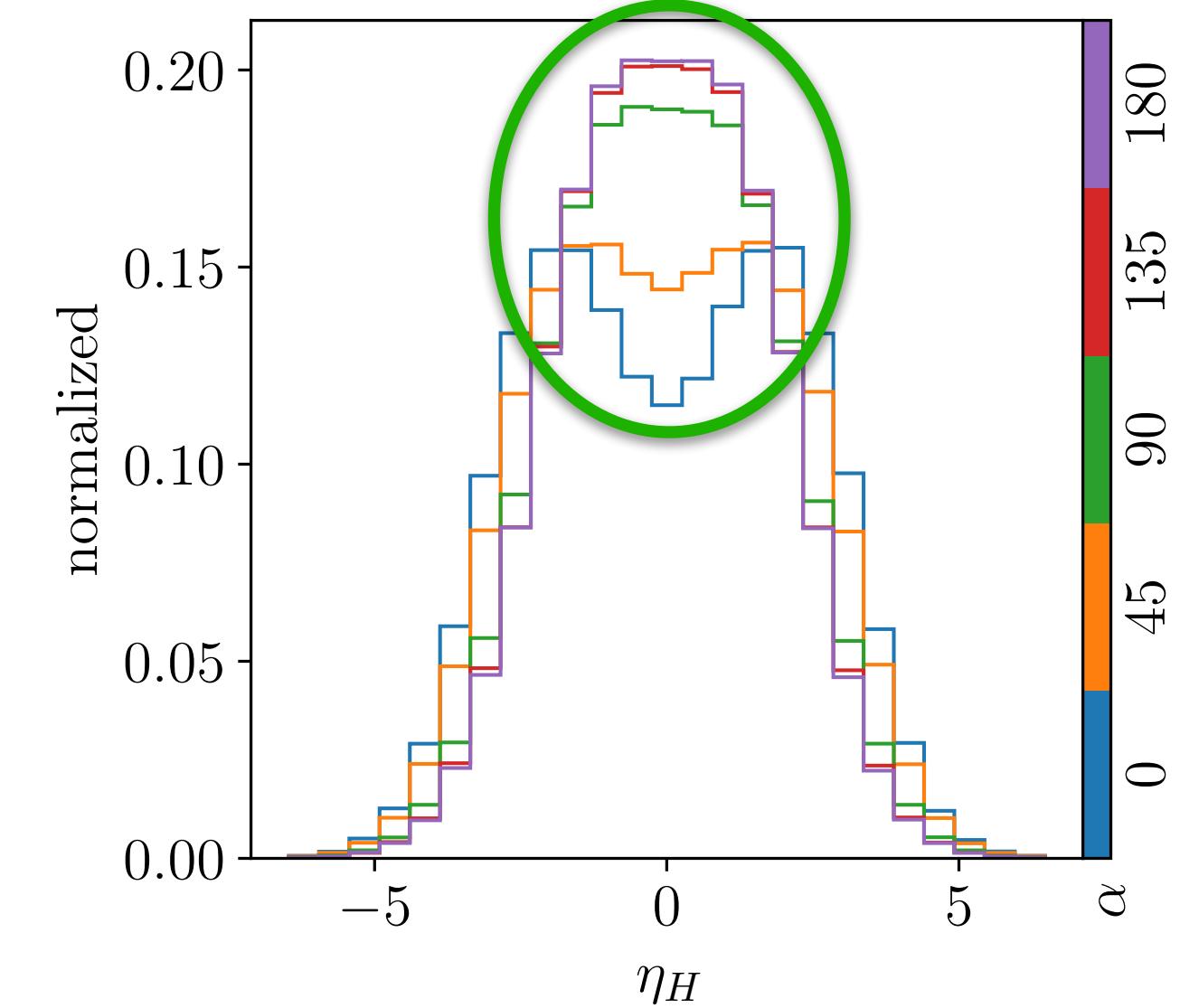


$$\mathcal{L}_{t\bar{t}H} = -\frac{y_t}{\sqrt{2}} \left[\cos \alpha \bar{t}t + \frac{2}{3} i \sin \alpha \bar{t}\gamma_5 t \right] H$$

Anomalous coupling with **CP-angle** α



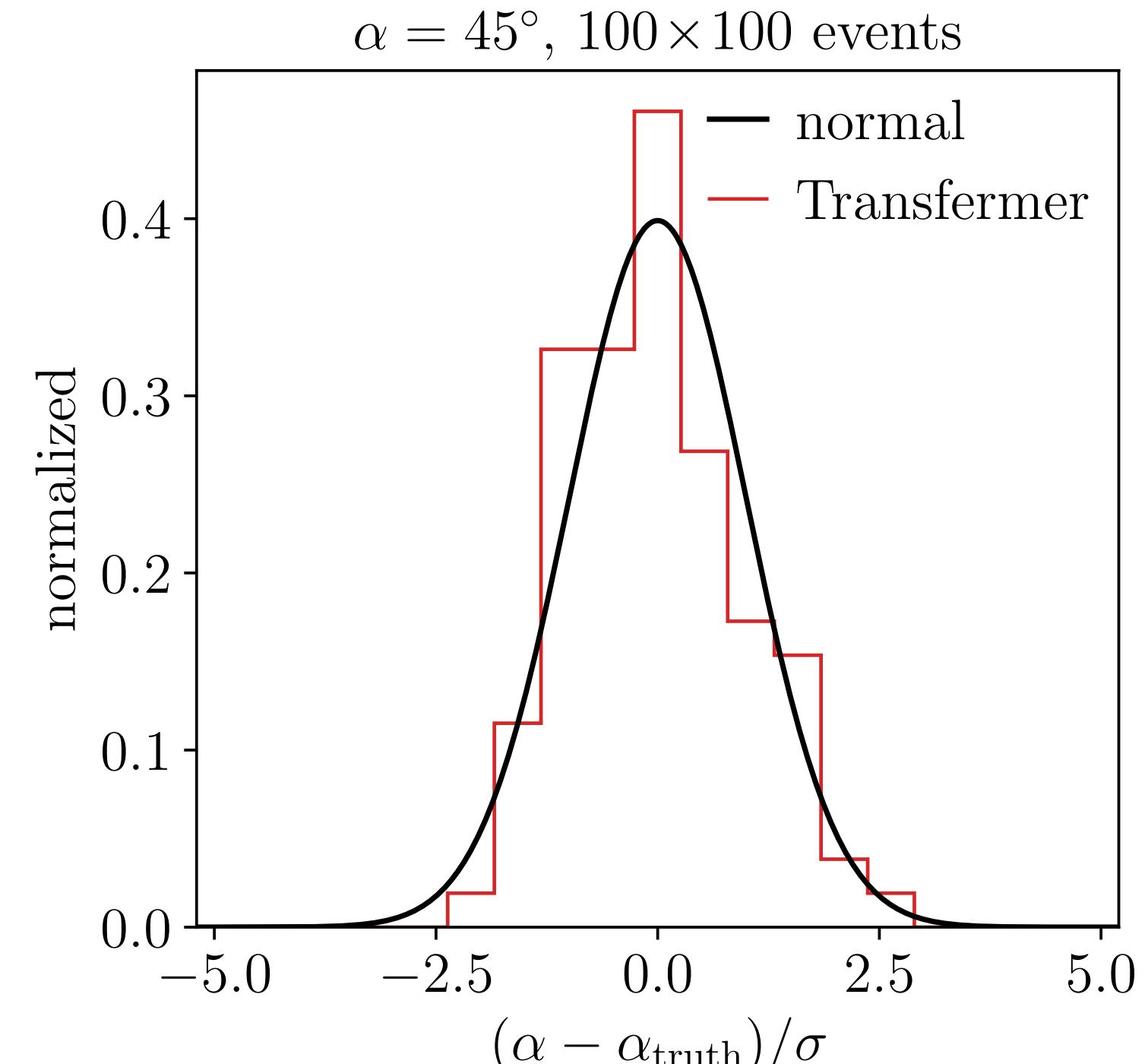
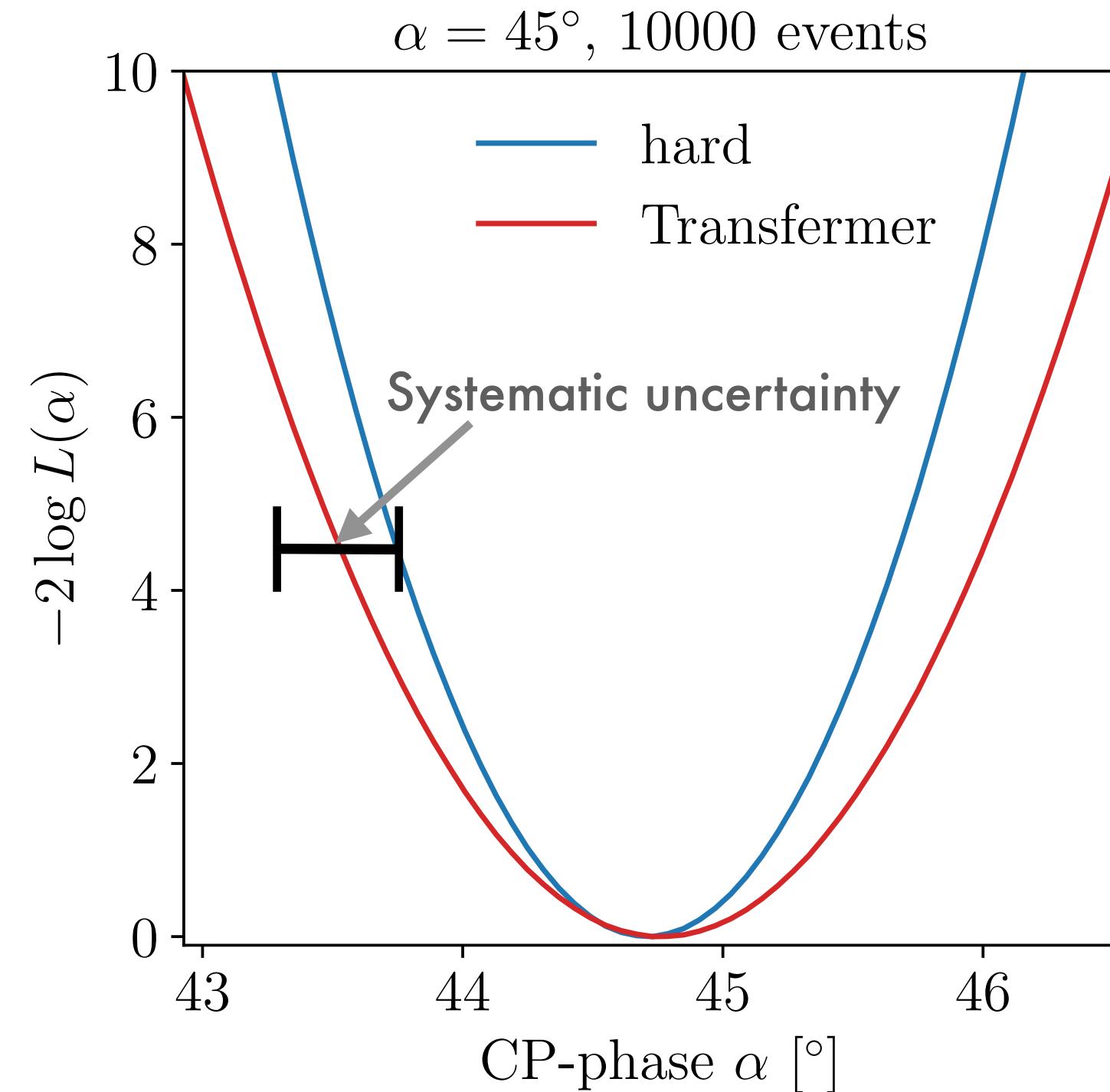
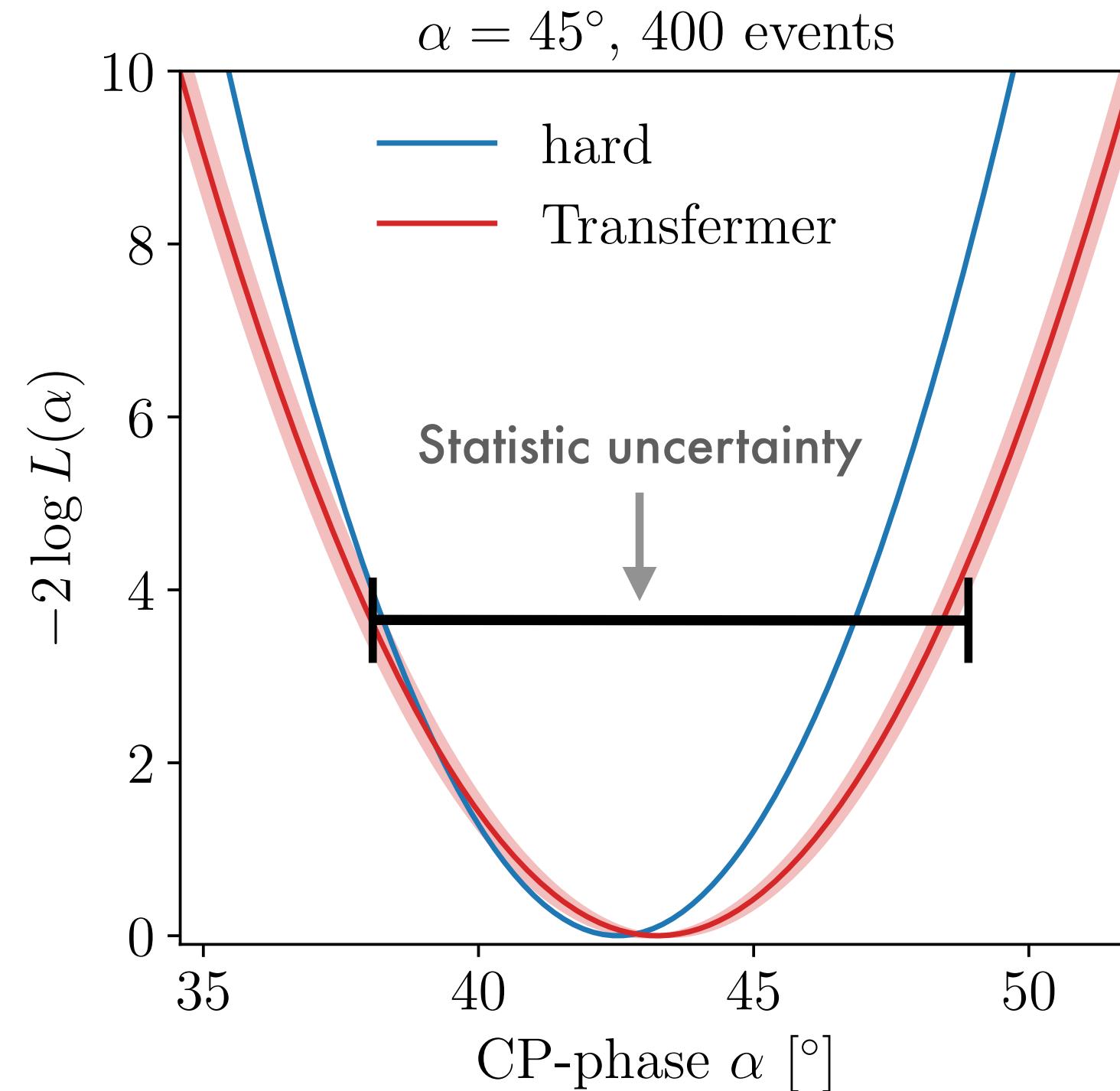
- ⊖ low total cross section
- ⊖ low variation of rate



- ⊕ kinematics sensitive

→ ideal use case for **MEM**

Inference results



- **well-calibrated likelihoods**, both for low and high event count
- Uncertainty bands: **MC integration error** & **syst. error from limited training statistics** (Bayesian NN)