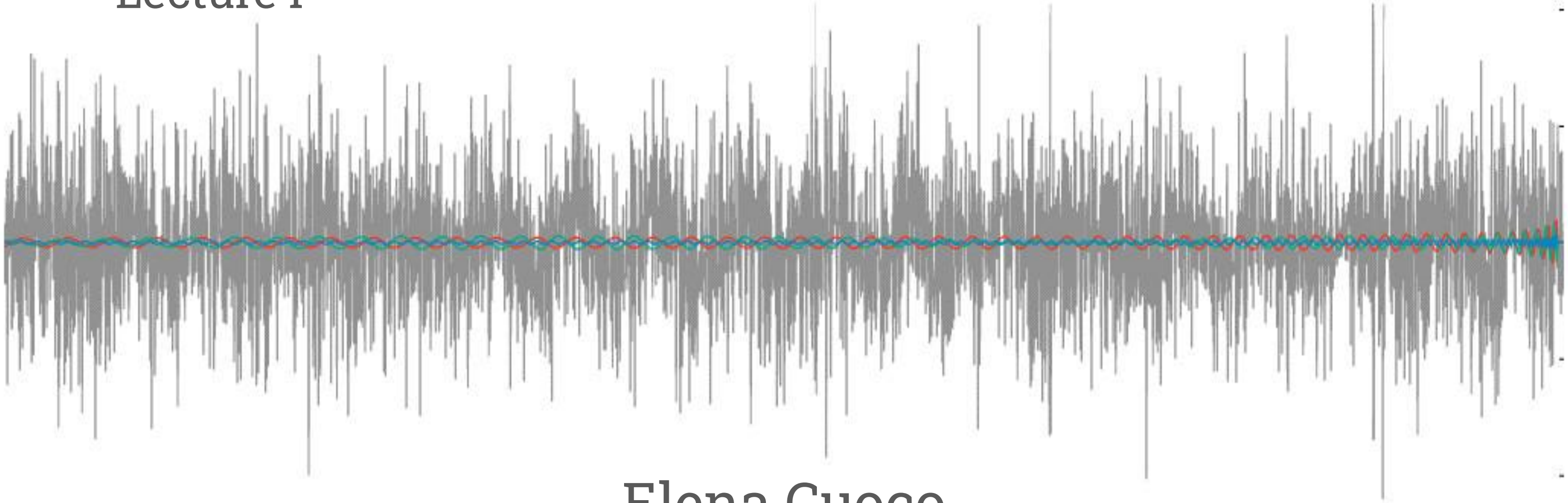
Gravitational wave data analysis

Lecture I



Elena Cuoco
European Gravitational Observatory

BND School 2024 - Blankenberge, Belgium 2 – 12 Sep 2024

EGO - Virgo

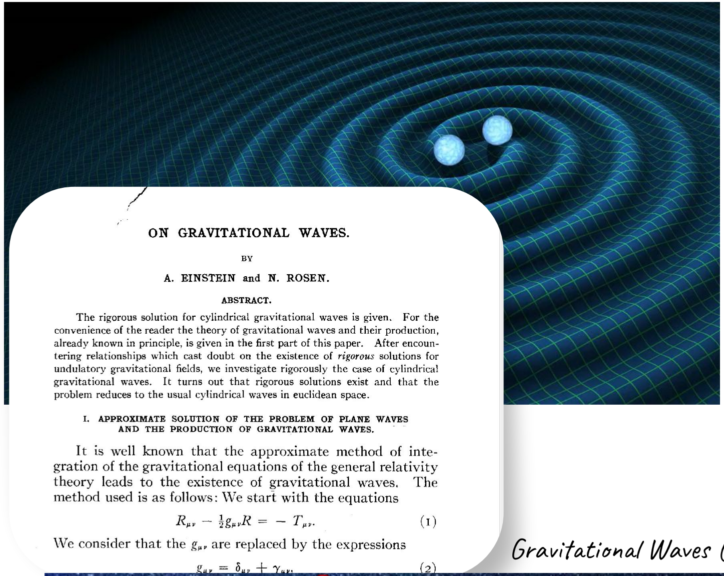
Disclaimer

- I won't talk about general relativity
- I won't talk about technical details of detectors
- I won't go through all the techniques for different sources

I will go trough:

- Data conditioning techniques
- Optimal detection filter
- Transient signal search
- Application of Machine Learning techniques to GW

Introduction to GW



ON GRAVITATIONAL WAVES.

BY

A. EINSTEIN and N. ROSEN.

ABSTRACT.

The rigorous solution for cylindrical gravitational waves is given. For the convenience of the reader the theory of gravitational waves and their production, already known in principle, is given in the first part of this paper. After encountering relationships which cast doubt on the existence of *rigorous* solutions for undulatory gravitational fields, we investigate rigorously the case of cylindrical gravitational waves. It turns out that rigorous solutions exist and that the problem reduces to the usual cylindrical waves in euclidean space.

I. APPROXIMATE SOLUTION OF THE PROBLEM OF PLANE WAVES AND THE PRODUCTION OF GRAVITATIONAL WAVES.

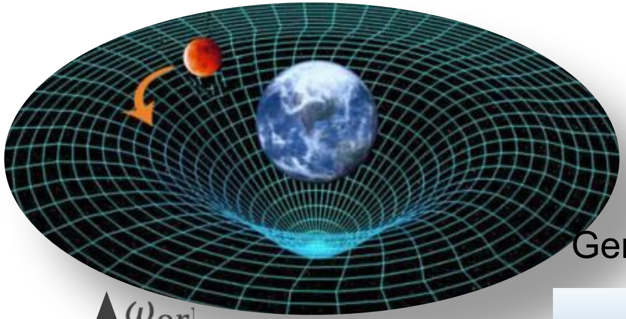
It is well known that the approximate method of integration of the gravitational equations of the general relativity theory leads to the existence of gravitational waves. The method used is as follows: We start with the equations

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = - T_{\mu\nu}. \qquad (1)$$

We consider that the $g_{\mu\nu}$ are replaced by the expressions

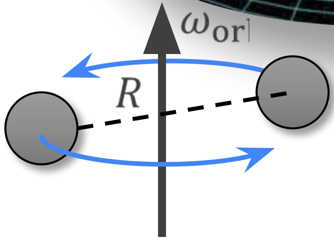
$$g_{\mu\nu} = \delta_{\mu\nu} + \gamma_{\mu\nu}. \qquad (2)$$

Gravitational Waves (1916)



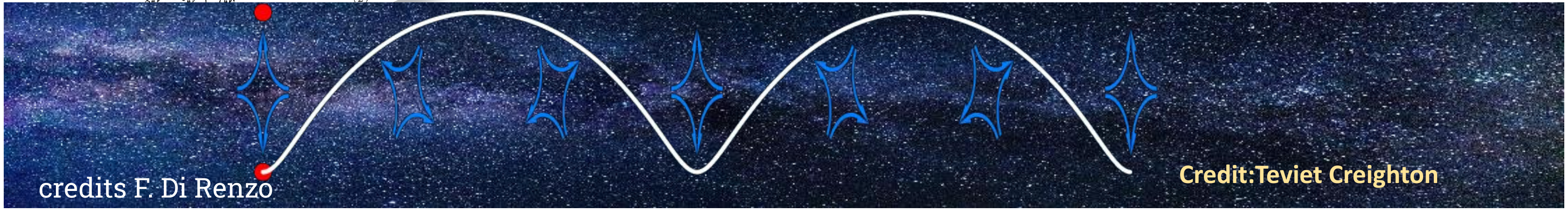
General Relativity (1915)

$$\mathbf{G}_{\mu\nu} = \frac{8\pi G}{c^4} \mathbf{T}_{\mu\nu}$$

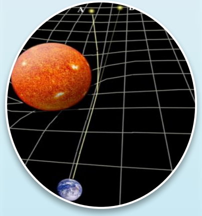


Free propagation along "z-axis" in vacuum ($T_{\mu\nu} = 0$):

$$\Box h_{\mu\nu} = 0 \;\Rightarrow\; h_{\mu\nu}(t,z) = h_{\mu\nu} e^{i(kz-\omega t)} \quad \text{with } \omega/c = k$$

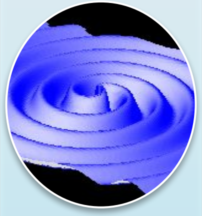


The GW search: a long history



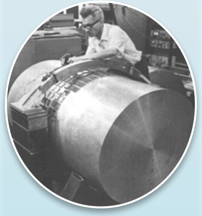
1915

- General Relativity



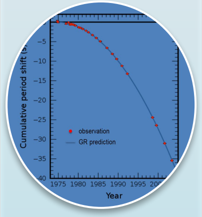
1916

- Gravitational Waves



1966

- Weber and Resonant Bars



1974

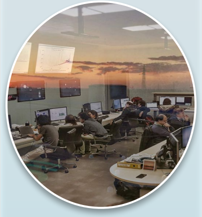
- Hulse-Taylor:Observing the pulsar binary PSR B1913+16

1980-1990

- Cryogenic Resonant Bars

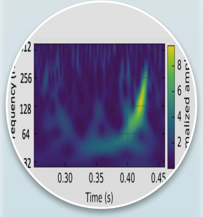
1993

- The approval of Virgo Experiment



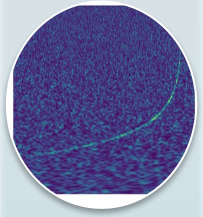
1999+

- Data taking from LIGO an Virgo



2015

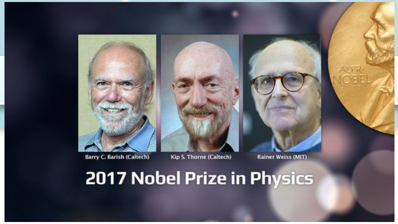
- GW BHBH detection



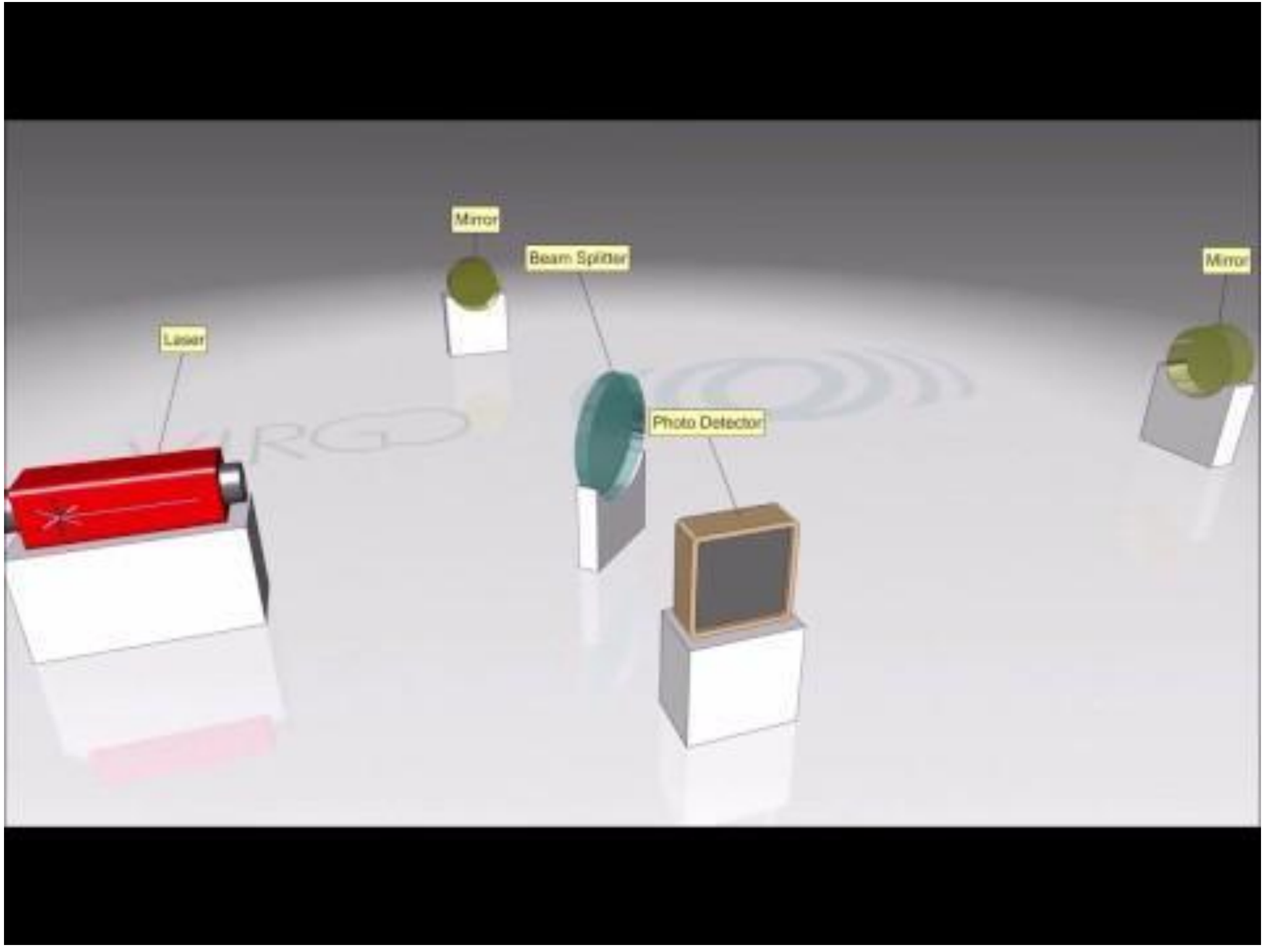
2017

- BNS detection: Multi-messenger Astronomy

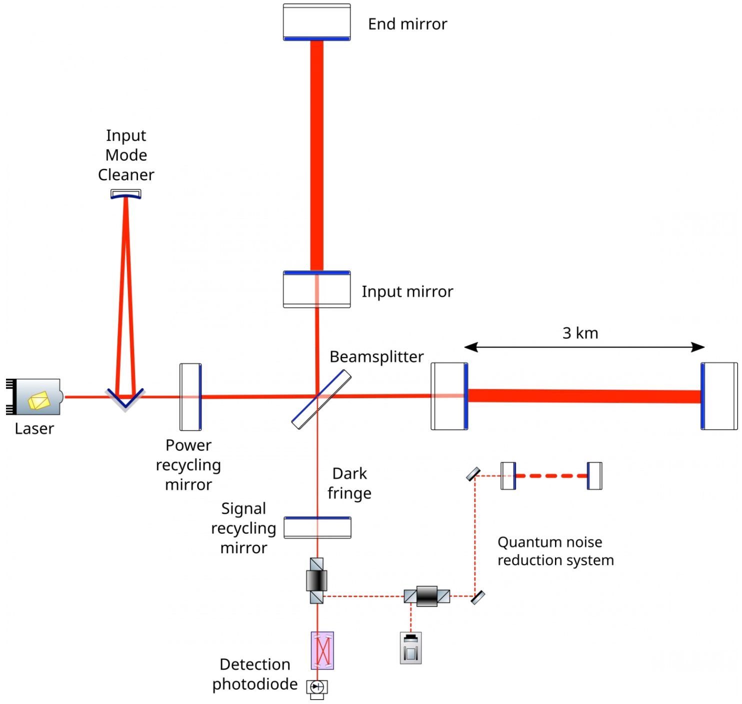
~100 years

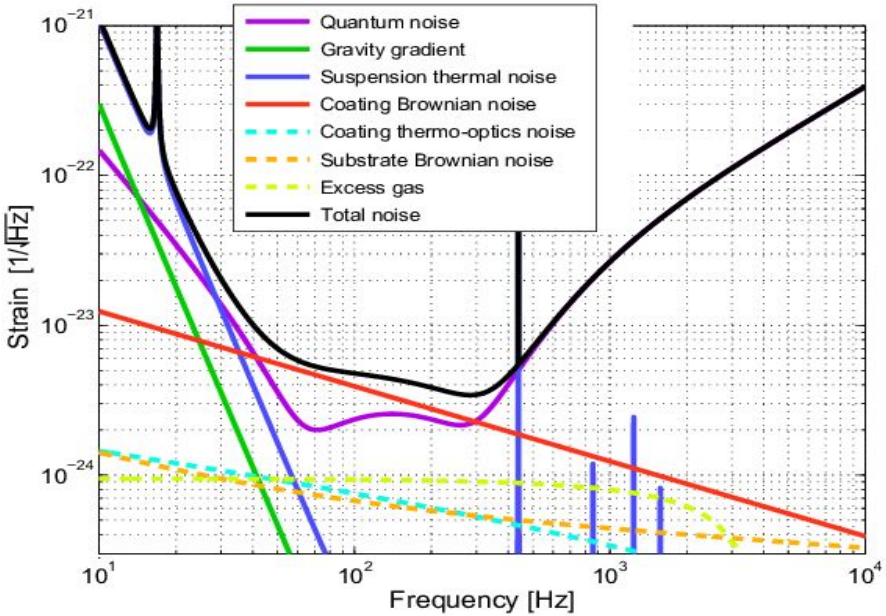


The detector



ITF detector and their sensitivity



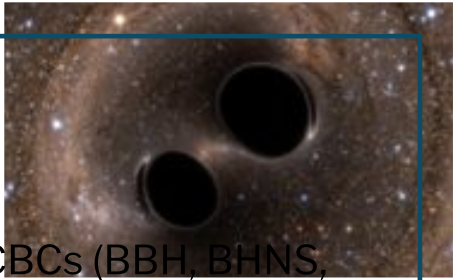



LIGO Hanford
Virgo
LIGO Livingston
KAGRA

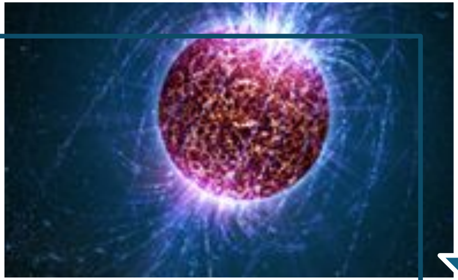
GW astrophysical sources

Short □ Long

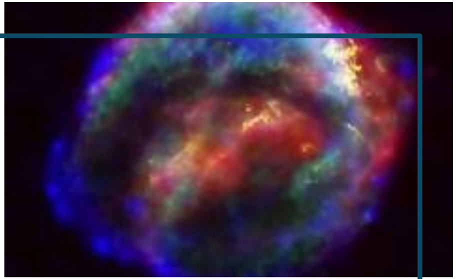
Known □ Unknown



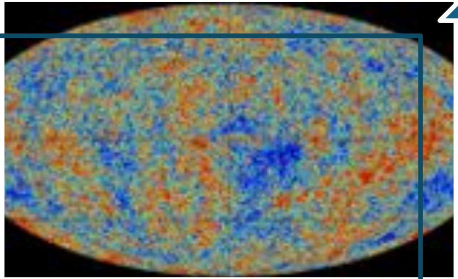
CBCs (BBH, BHNS, BNS)



Pulsars



CCSNe



GW Background

Do we know their Waveforms ?

Which is the signal duration?

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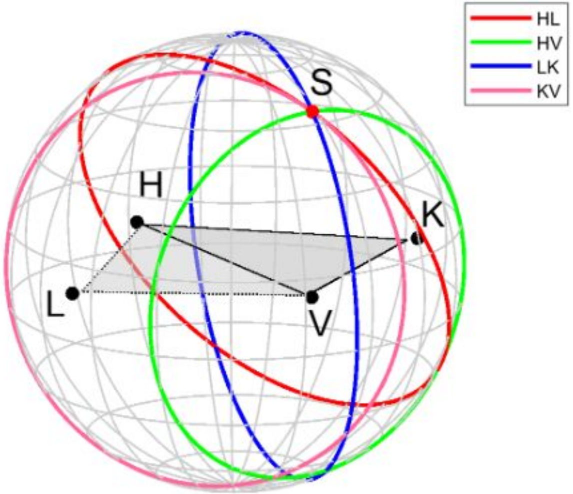
Why more than 1 detector?

Source localization using only timing for a two-site network yields an **annulus** on the sky.

For three detectors, the time delays restrict the source to **two sky regions** which are mirror images with respect to the plane passing through the three sites.

With four or more detectors, timing information alone is sufficient to localize to a single sky region, <10 deg^2 for some signals.



arXiv:1304.0670

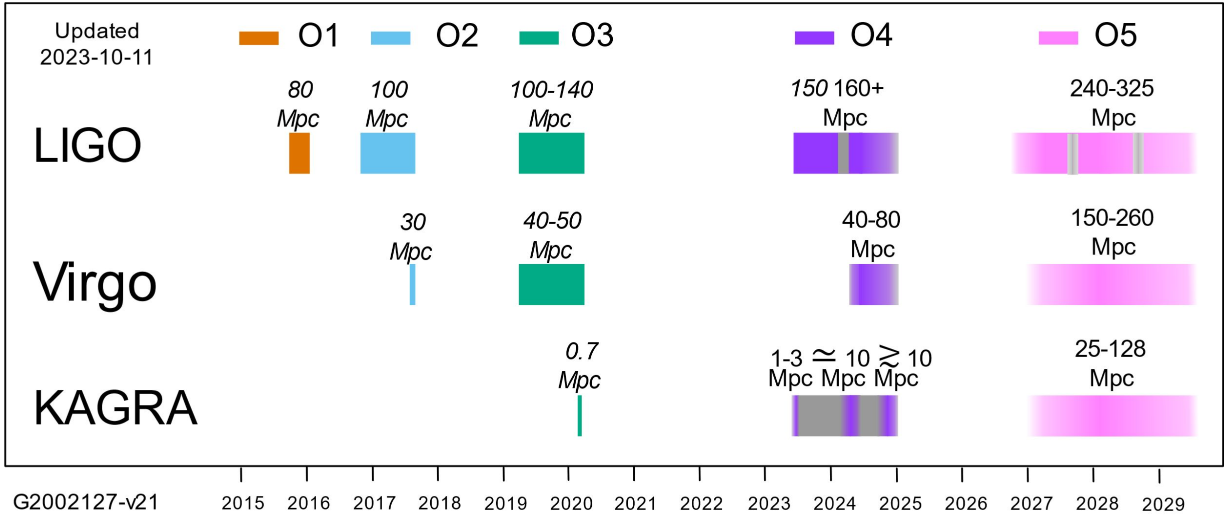
- 2 detector → 100 -1000 deg^2
- 3 detector → 10 - 100 deg^2
- 4 detector → < 10 deg^2

The O-run timeline

The detector **strain sensitivity** is the minimum *detectable* value of the strain produced by an incoming GW:

⇒ It is determined by the **detector noise**.

BNS inspiral range: the distance, averaged over GW polarizations and directions in the sky, at which a single detector can observe with matched-filter Signal-to-noise Ratio (SNR) of 8 the inspiral of two neutron stars.



https://observing.docs.ligo.org/plan/

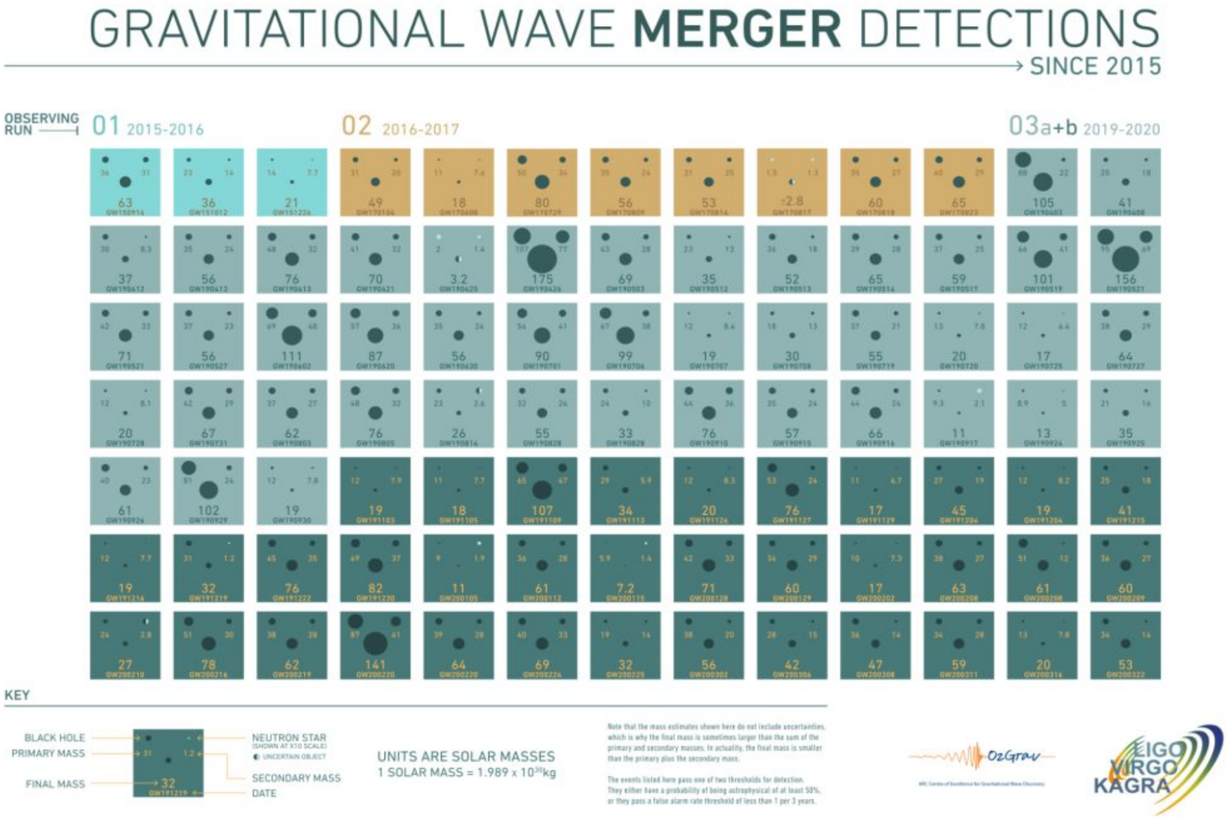
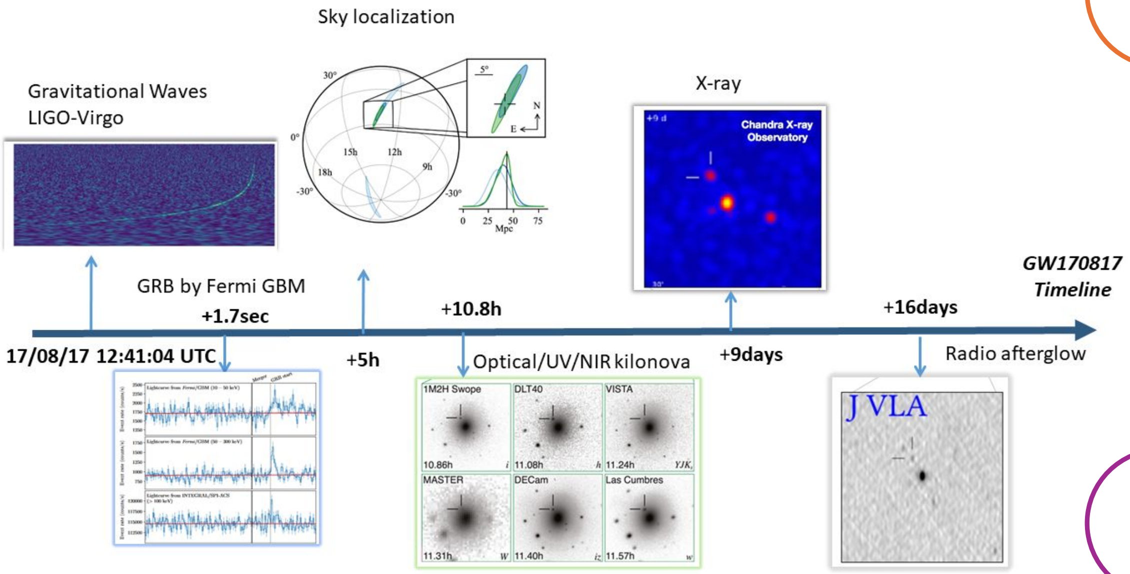


Image credit: Carl Knox,Hannah Middleton, Federica Grigoletto, LVK

GW170817: the first multi-messenger event



Abbott et al. 2017 and refs. therein

*coincident short GRBs detected in **gamma rays***
- *first direct evidence that at least some BNS mergers are progenitors of short GRBs*

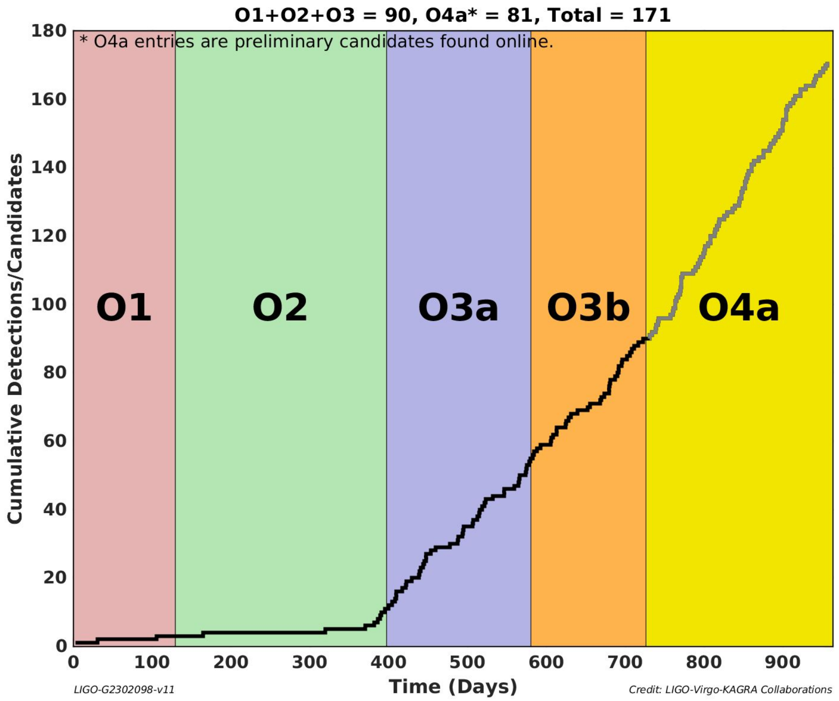
the host galaxy has been identified: NGC 4993

*an **optical/infrared/UV** counterpart (AT2017gfo) has been detected*
- *first spectroscopic identification of a kilonova*

An X-ray and a radio counterparts have been identified
- *off-axis afterglow from a structured jet*

GW Detections

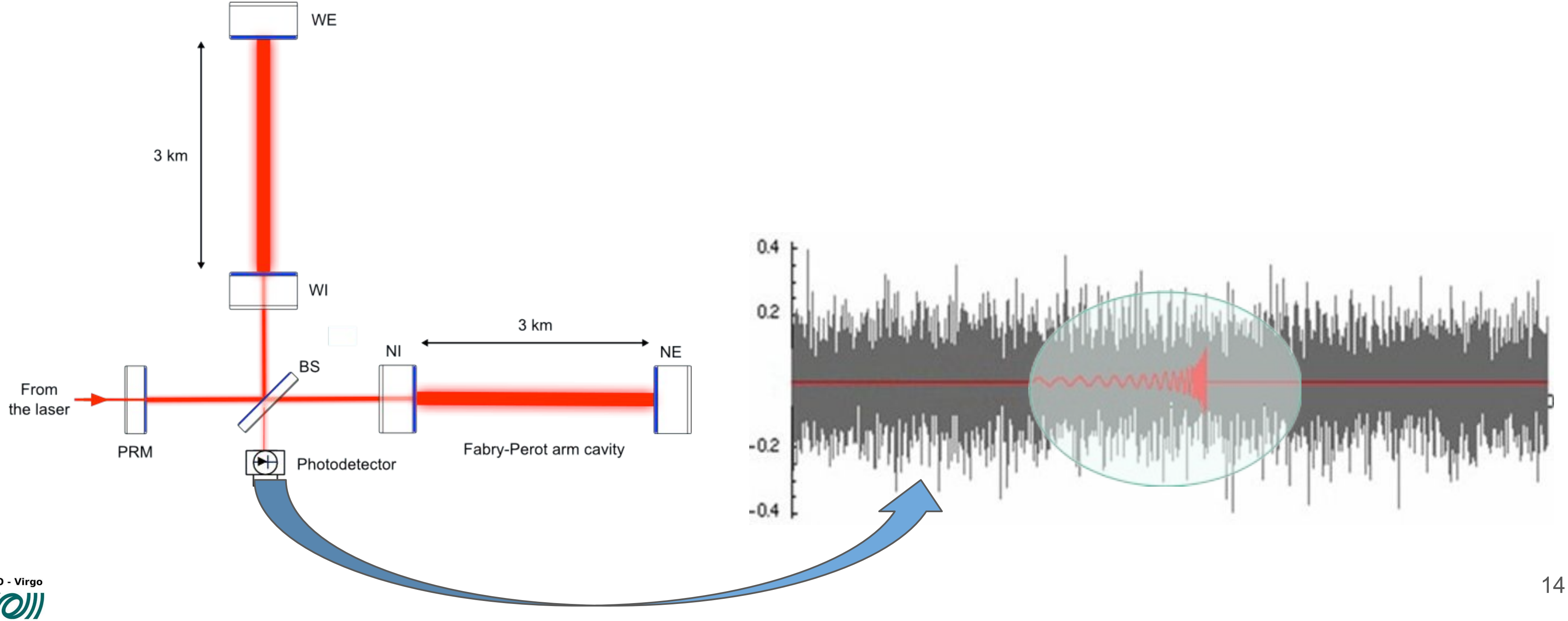
O4 Significant Detection Candidates:
81 (92 Total - 11 Retracted)
O4 Low Significance Detection Candidates:
1610 (Total)



https://gracedb.ligo.org/superevents/public/O4/

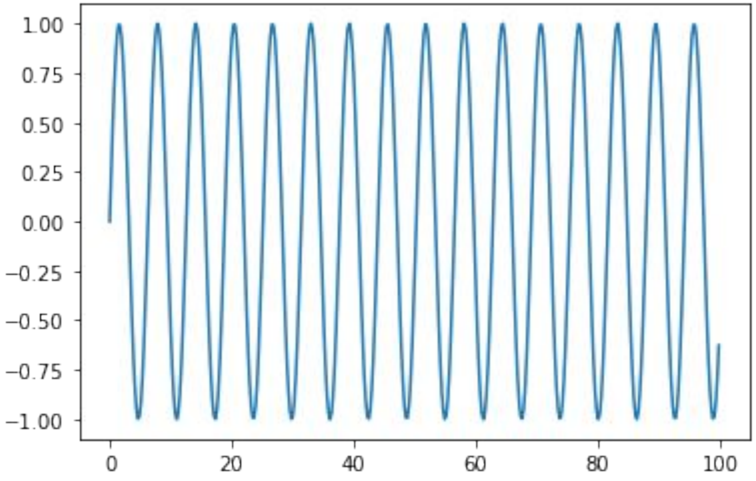
GW detector data

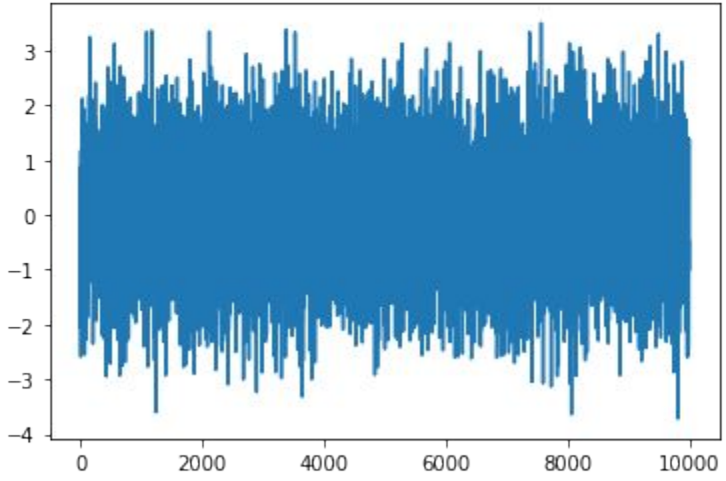
- Time series sequences… noisy time series with low amplitude GW signal buried in



Time series

- A time series x[n] is a sequence of data points measuring a physical quantity at successive times spaced at uniform time intervals.
- We say that x[n] is a stationary process, if its statistical description does not depend on n.





Signal processing utilities

Encapsulating the data information

Autocorrelation function

Definition

Given a discrete random process $x[n]$ we define the *mean* as

$$\mathscr{E}\{x[n]\} = \mu_x$$

Definition

The autocorrelation function (ACF)

$$r_{xx}[k] = \mathscr{E}\{x^*[n]x[n+k]\}$$

Autocovariance function

Definition

The *autocovariance* function is defined as

$$c_{xx}[k] = \mathscr{E}\{(x^*[n] - \mu_x)(x[n+k] - \mu_x)\} = r_{xx}[k] - |\mu_x|^2$$

Similar definition for cross-correlation bewteen $x[n]$ and $y[n]$.
Some properties of ACF:

$$r_{xx}[0] \geq |r_{xx}[k]| \qquad r_{xx}[-k] = r^*_{xx}[k] \qquad r_{xy}[-k] = r^*_{yx}[k]$$

Power Spectral Density

Definition

We define the *Power Spectral Density* (PSD)

$$P_{xx}(f) = \sum_{k=-\infty}^{k=\infty} r_{xx}[k]\exp(-i2\pi fk) \quad P_{xy}(f) = \sum_{k=-\infty}^{k=\infty} r_{xy}[k]\exp(-i2\pi fk)$$

This relationship between PSD and ACF is often known as Wiener-Khinchin theorem.
The PSD describe the content in frequency in power of the signal $x[n]$.
In the following we will refer to $P_{xx}(f)$ as PSD
The PSD is periodic with period 1. The frequency interval $-1/2 \leq f \leq 1/2$ will be considered as the fundamental period.
The ACF is the inverse Fourier transform of the PSD and hence

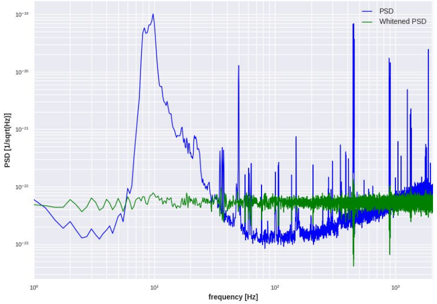
$$r_{xx}[0] = \int_{-1/2}^{1/2} P_{xx}(f)df$$

White noise

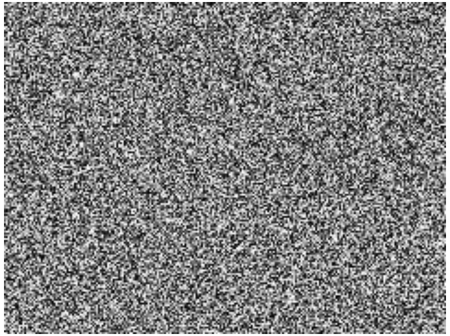
One particular process is the discrete white noise. It si defined as a process having as ACF

$$r_{xx}[k] = \sigma_x^2 \delta[k]$$

where $\delta[k]$ is the delta function.
The PSD of such a process is a flat function with the same value for all the frequency f



$$P_{xx}(f) = \sigma_x^2$$



Gaussian random process

A Gaussian sthocastic process is one for which each set $\{x[n_0], x[n_1] \ldots x[n_{N-1}]\}$ is distribuited as a multivariate Gaussian PDF. If we assume that the process is stationary with zero-mean, then the covariance matrix is the autocorrelation matrix $\mathbf{r}_{xx}$

$$\mathbf{r}_{xx} = \begin{bmatrix} r_{xx}[0] & r_{xx}[-1] & \ldots & r_{xx}[-(N-1)] \\ r_{xx}[1] & r_{xx}[0] & \ldots & r_{xx}[-(N-2)] \\ \vdots & \vdots & \ddots & \vdots \\ r_{xx}[N-1] & r_{xx}[N-2] & \ldots & r_{xx}[0] \end{bmatrix} \quad (1)$$

$$r_{xx}[k] = \mathcal{E}\{x^*[n]x[n+k]\} \,. \quad (2)$$

We can write the probability density function of a real random gaussian process a

$$P[\mathbf{x}] = \frac{1}{(2\pi)^{N/2}|\mathbf{r}_{xx}|^{1/2}} e^{\mathbf{x}^T \mathbf{r}_{xx}^{-1} \mathbf{x}} \,. \quad (3)$$

White random Gaussian process

It is a process $x[n]$ with mean zero and variance σ_x^2 for which

$$x[n] \sim N(0, \sigma_x^2) \qquad -\infty < n < \infty$$

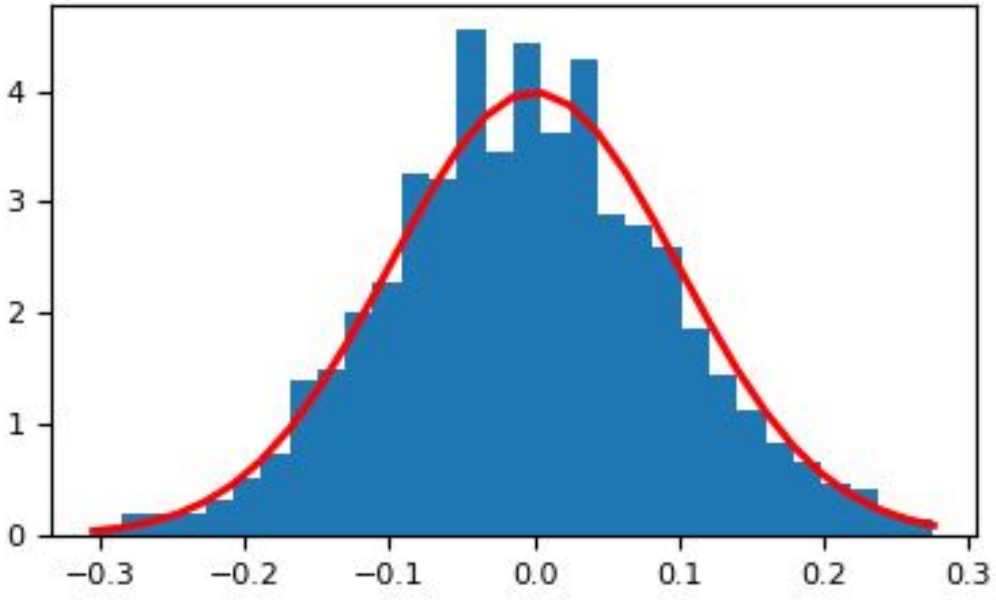
$$r_{xx}[m-n] = \mathscr{E}(x[n]x[m])) = 0 \qquad m \neq n$$

where $x \sim N(\mu_x, \sigma_x^2)$ means that $x[n]$ is Gaussian distributed with a probability density function

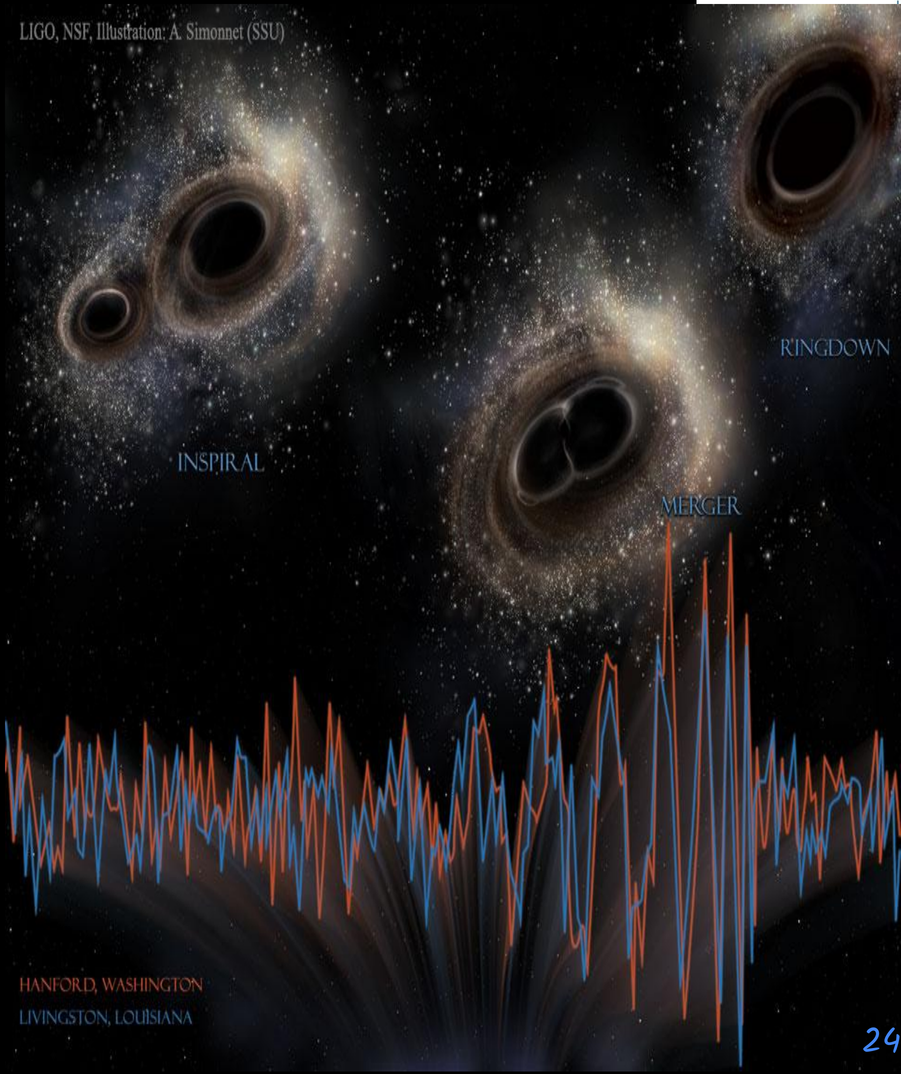
$$p(x) = \frac{1}{\sqrt{2\pi\sigma_x}} \exp[-\frac{1}{2}(\frac{x-\mu_x}{\sigma_x})^2] \qquad -\infty < x < \infty$$

Gaussian noise distribution

The distribution is characterized by its bell-shaped curve, which is symmetrical around the mean value. The mean, median, and mode of the distribution are all equal, and the standard deviation determines the width of the curve.



Gravitational Wave signal detection



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GW Signal Detection and Matched Filter for known waveforms

- Defining the problem
- The Neyman Pearson Criteria
- The Matched Filter

Switch to pdf slides...:)

Optimal Filter is Matched Filter, if the noise is gaussian distributed

Maximizing the likelihood

Data

Template

$$\rho(t) = 4 \int_0^\infty \frac{\tilde{x}(f)\ \tilde{h}^*(f)}{S_n(f)}\ e^{2\pi i f t}\ df$$

Noise power spectral density

Look for maxima of $|\rho(t)|$ above some threshold ➔ trigger

How we detect transient signals: modeled search

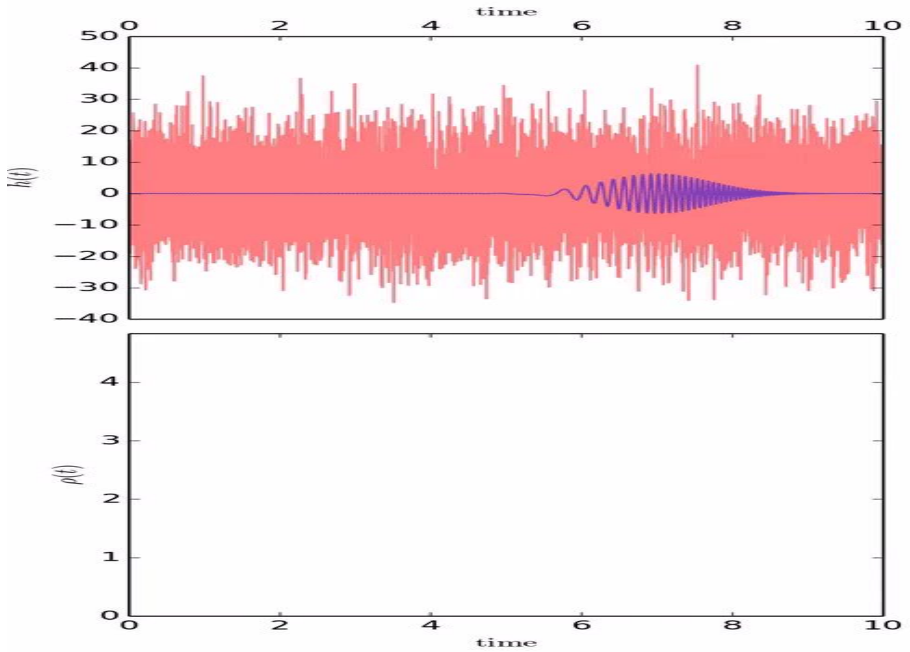
- pyCBC (Usman et al, 2015)
- MBTA (Adams et al. 2015)
- gstlal-SVD (Cannon et al. 2012)

Matched-filter

Data → $\tilde{x}(f)$; Template → $\tilde{h}^*(f)$

$$\rho(t) = 4 \int_0^\infty \frac{\tilde{x}(f)\, \tilde{h}^*(f)}{S_n(f)} e^{2\pi i f t}\, df$$

Noise power spectral density → $S_n(f)$

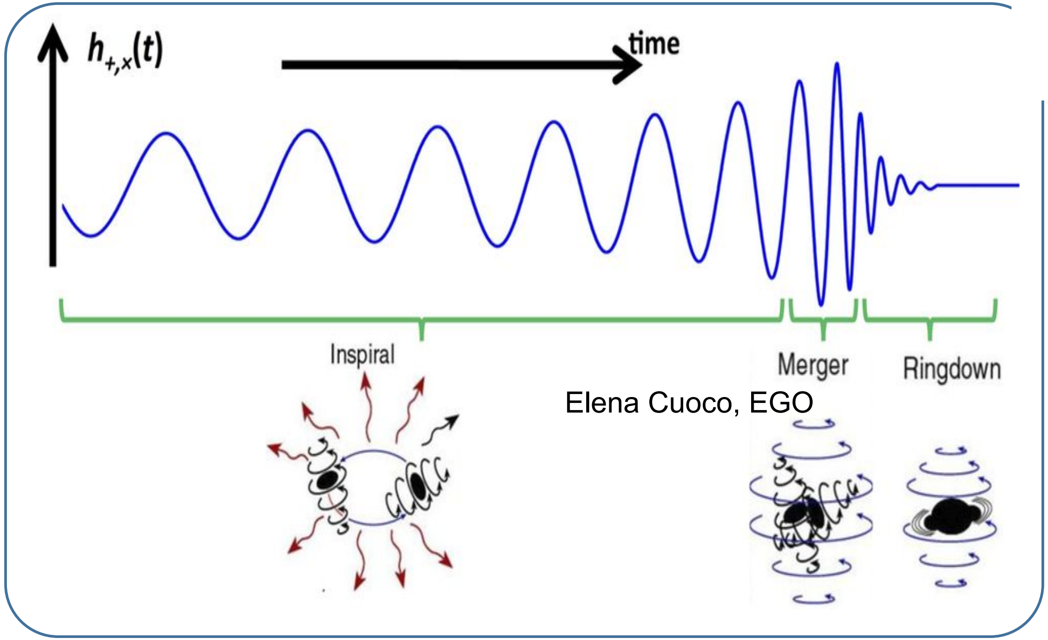


CBC template generation

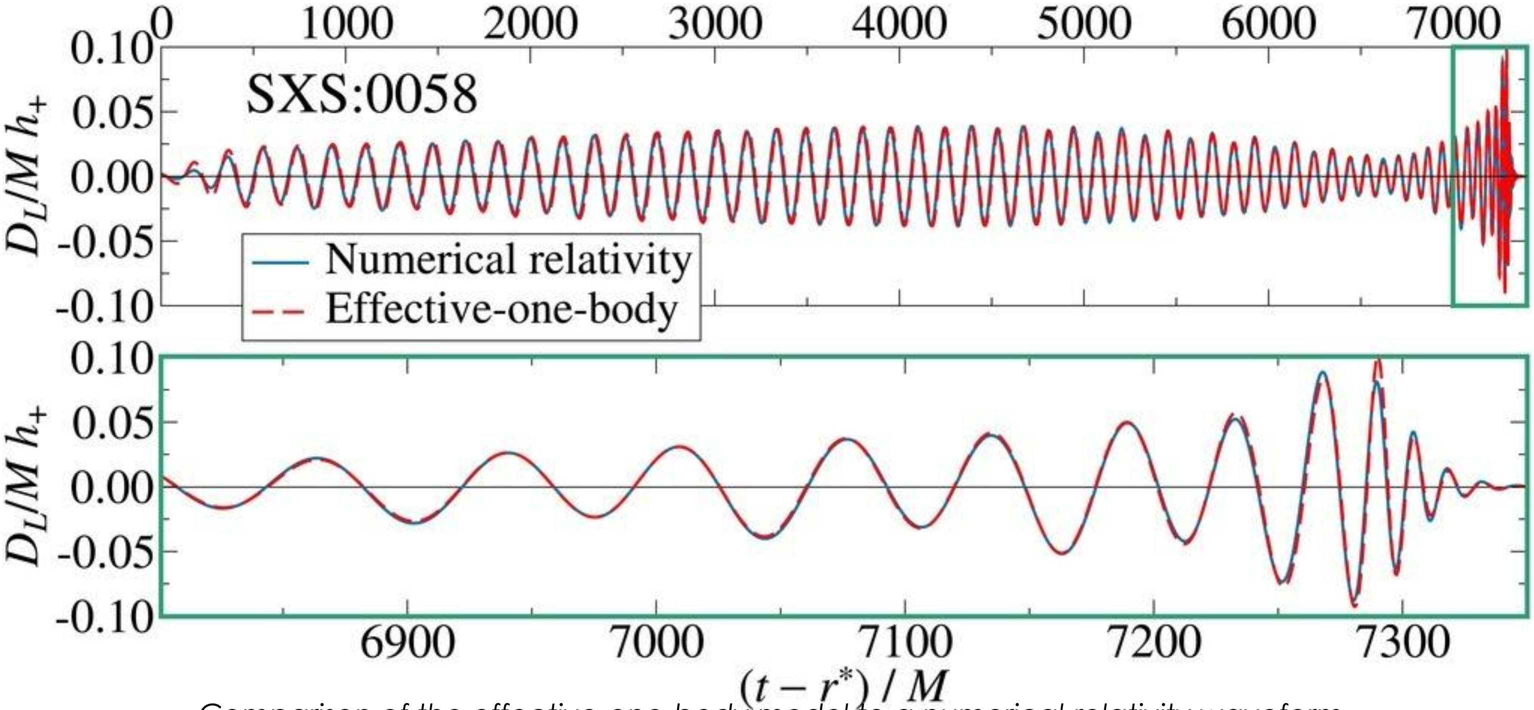
$$h(f) = A(f)e^{i\Phi(f)}$$

$$\Phi(f) = \sum_{k=1}^{7}(\varphi_k + \varphi_k^l \log(f))f^{(5-k)/3} + \sum_{i\neq k}\varphi_i f^i$$

$$\varphi_j \equiv \varphi_j(m_1, m_2, \vec{s}_1, \vec{s}_2)\ \forall j = k, i$$



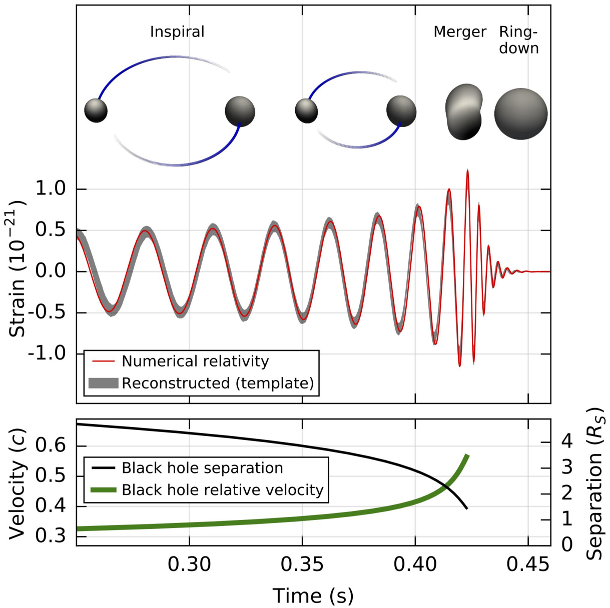
CBC template generation

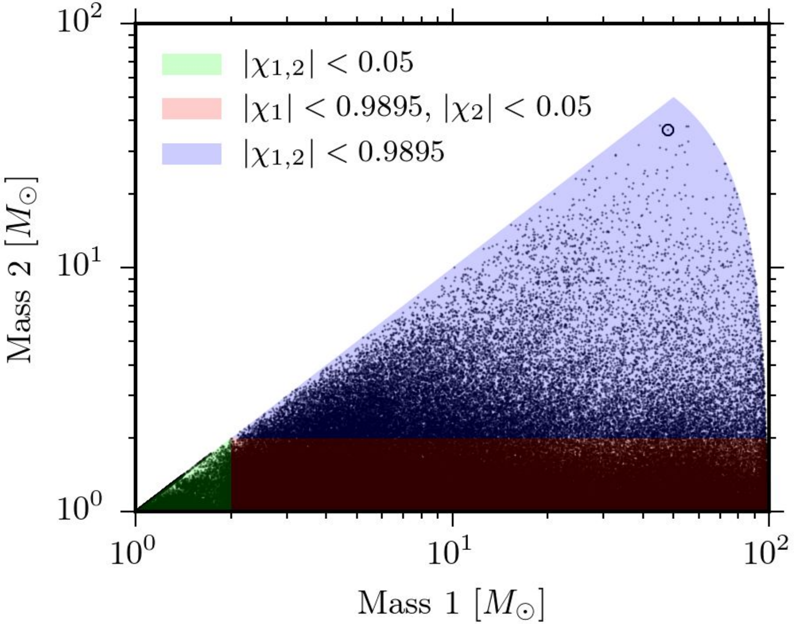


Comparison of the effective-one-body model to a numerical-relativity waveform of a precessing black-hole binary. © A. Taracchini/AEI

How many templates?

To cover in efficient way the parameters space, we build a templates bank requiring that the signal can be detected with a maximum loss of 3% of its SNR





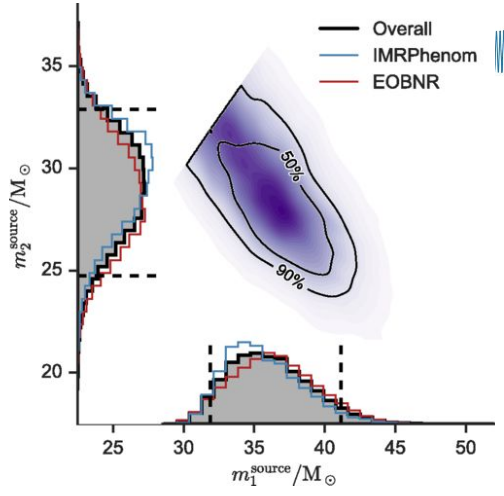
~250000 waveforms used for GW150914

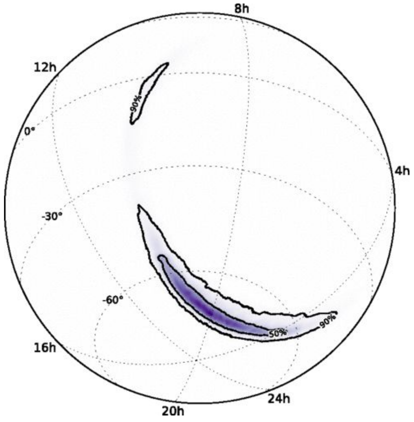
LVC Phys. Rev. X 6 (2016)

Parameter estimation

$$p(\theta|d, H) = \frac{p(\theta|H)p(d|\theta, H)}{p(d|H)}.$$

- *MCMC and Nested Sampling*
    - *MCMC Random steps are taken in parameter space, according to a proposal distribution, and accepted or rejected according to the Metropolis-Hastings algorithm.*
    - *Nested sampling can also compute evidences for model selection.*





Parameter estimation for compact binaries with ground-based gravitational-wave observations using the LALInference software library

J. Veitch et al. Phys. Rev. D 91, 042003

LVC (PRL:116, 241102)

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Data mapping, preserving the info

01 Time-domain

- Time-series at the output of the detector (be careful with the sampling theorem)

02 Frequency-domain

- Fourier transform
  - Useful for stationary data
  - Useful for persistent signals
  - It captures the global frequency information

03 Time-frequency domain

- Short Fourier Transform
  - Useful for not stationary data
  - Useful for transient signals

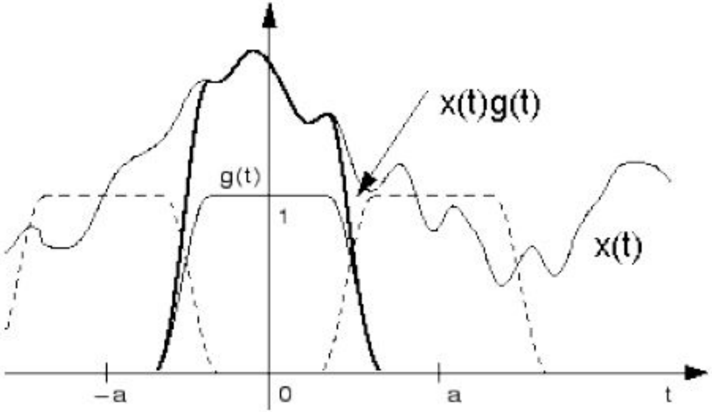
04 Others

- Wavelet decomposition (useful for multiresolution analysis)
- Q-Transform (useful for transient)
- Hough-transform (useful for lines)

Time-Frequency domain: STFT

The short time Fourier transform (STFT) function is simply the Fourier transform operating on a small section of the data. Here a moving window is applied to the signal and the fourier transform is applied to the signal within the window as the window is moved.

$$STFT\{x(t)\} = X(\tau, f) = \int_{-\infty}^{\infty} x(t) g(t-\tau) \exp(-2i\pi f t) dt$$

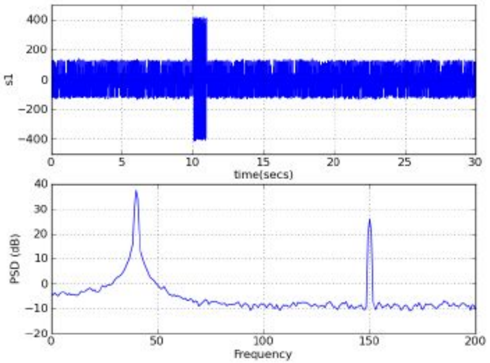


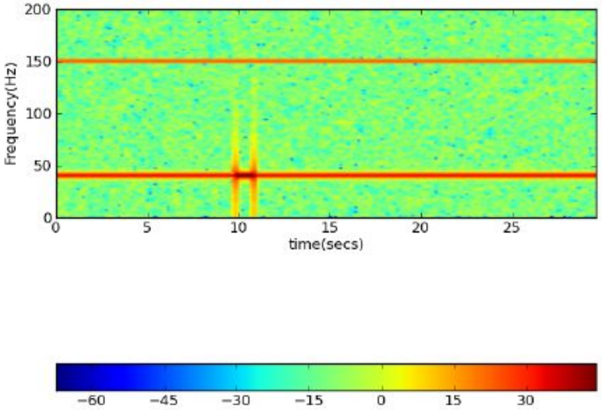
Spectrogram

To have easy access to the information of the STFT we can plot the spectrogram.
It is defined as

$$Spectrogram(\tau, f) = |X(\tau, f)|^2$$

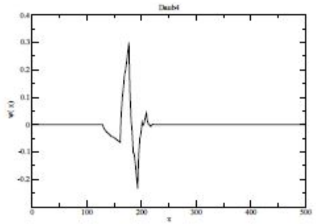
So we will have a bidimensional plot where on x-axis usually is plotted the time, on y-axis the frequency, while the color of the map is the the amplitude of a particular frequency at a particular time.

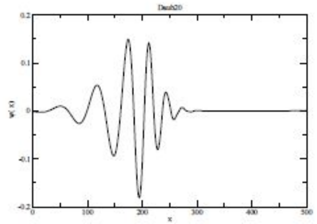




Wavelet decomposition of time series

The wavelet transform replaces the Fourier transform sinusoidal waves by a family generated by translations and dilations of a window called a wavelet.





$$Wf(a,b) = < f, \psi_{a,b} > = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{b}} \psi^{*}\left(\frac{t-a}{b}\right) \, dt$$

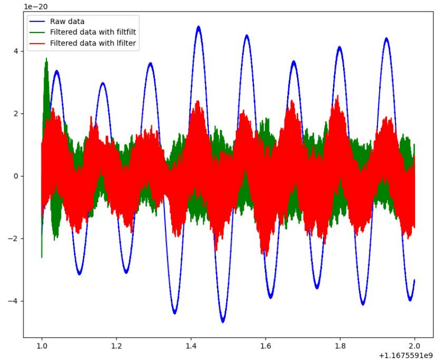
The scale of the wavelet is determined by the parameter **b**.

- When **b** is decreased, the wavelet appears more compressed, allowing it to capture high-frequency information.
- Increasing the value of **b** elongates the wavelet, enabling it to capture low-frequency information.

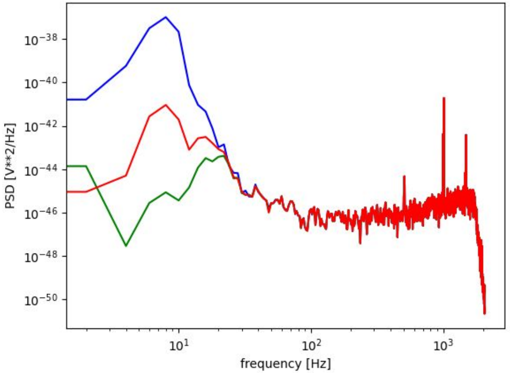
The location of the wavelet is determined by the parameter **a**.

- If we decrease the value of **a**, the wavelet will be shifted to the left, whereas an increase in **a** will shift it to the right.
- Note that the location of the wavelet is crucial because, unlike waves, wavelets are only non-zero within a short interval.

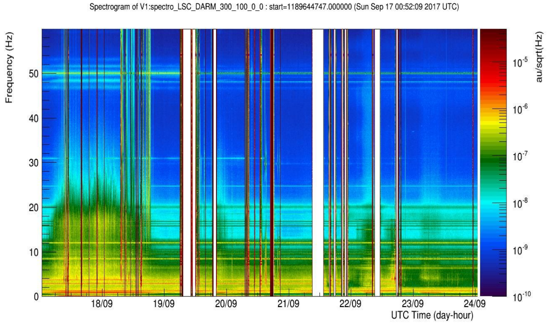
Data representations



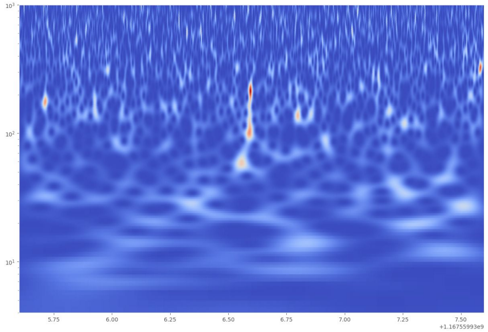
Time-domain



Frequency-domain



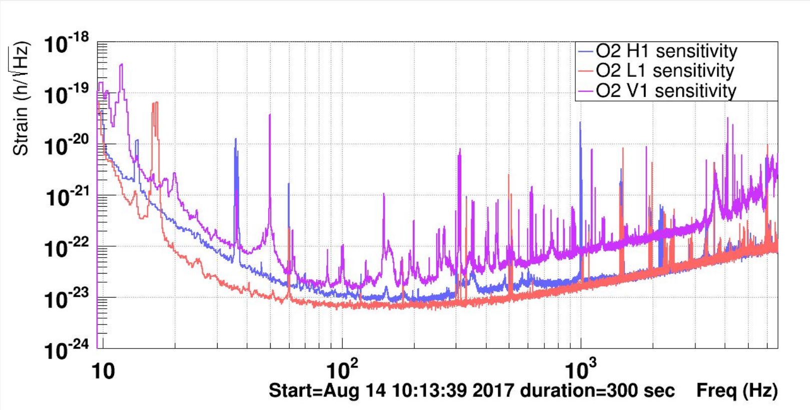
Time-frequency-domain

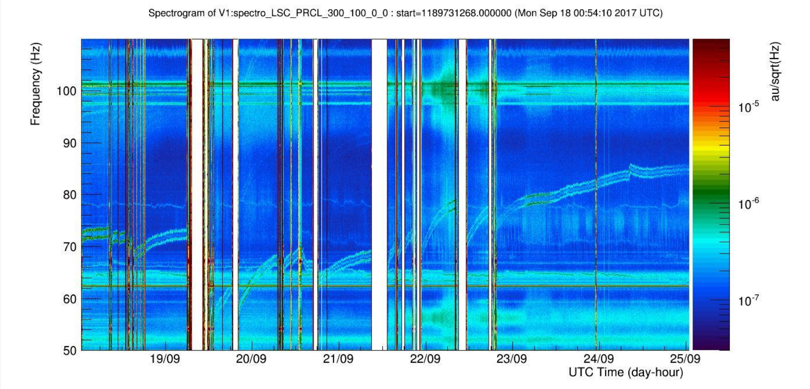


Wavelet-domain

EGO - Virgo

Data preprocessing



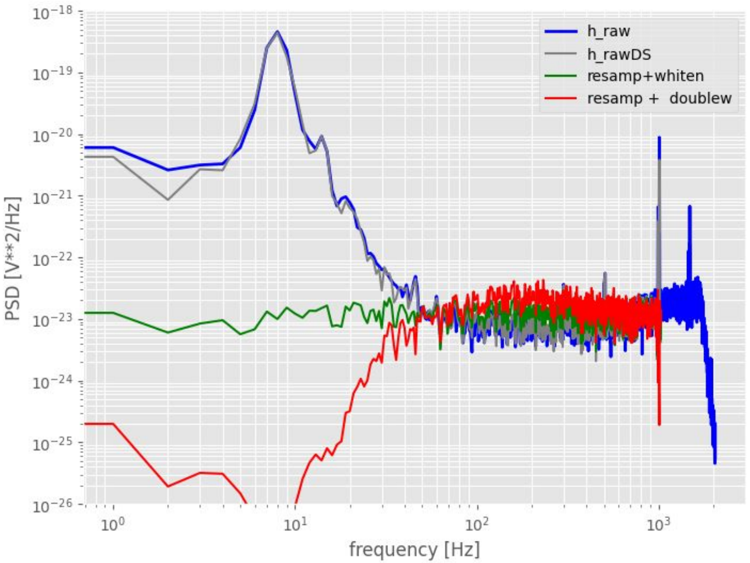


Whitening

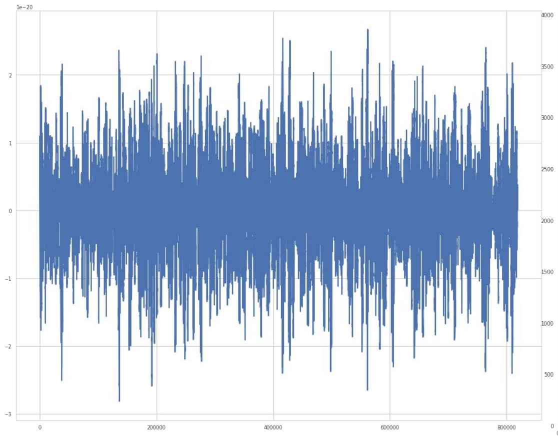
$$\rho(t) = 4\int_0^\infty \frac{\tilde{x}(f)\ \tilde{h}^*(f)}{S_n(f)}\ e^{2\pi i f t}\ df$$

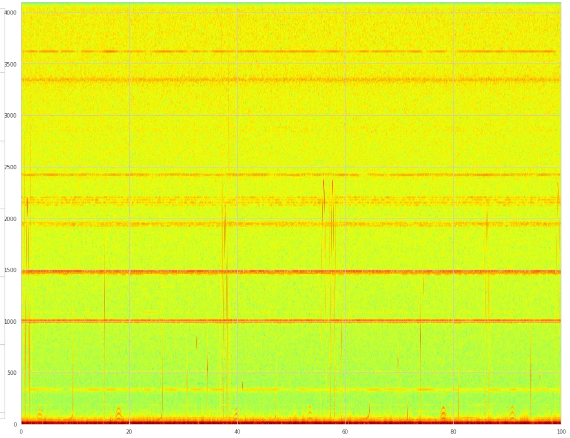
Whitening

We can do in frequency domain estimating the PSD

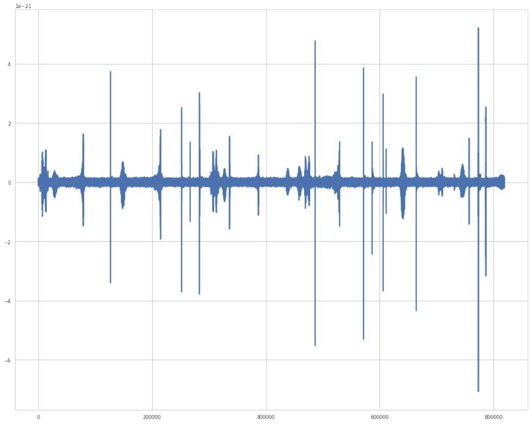


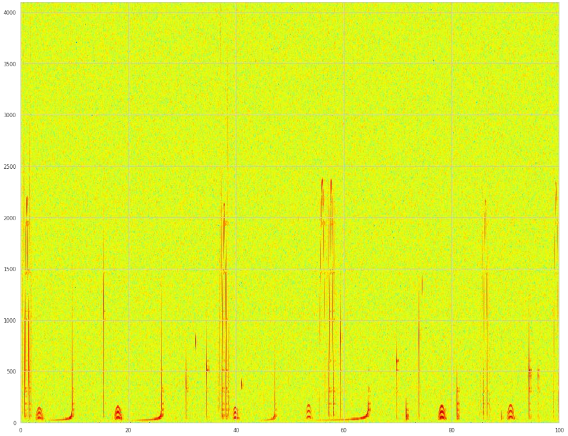
Signals in whitened data





Not Whitened





Whitened

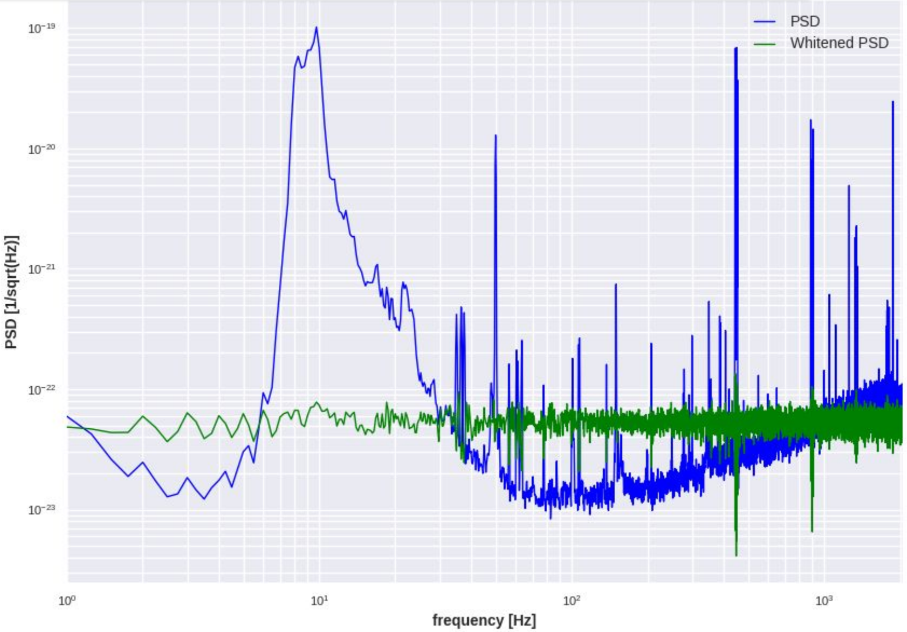
Whitening in time domain

We need parametric modeling

It can be useful for on-line application

It can be implemented for non stationary noise

It can catch the autocorrelation function to larger lags



AR parametric modeling

An AutoRegressive process is governed by this relation

$$x[n] = -\sum_{k=1}^{p} a[k]x[n-k] + w[n],$$

and its PSD for a process of order P is given by

$$P_{AR}(f) = \frac{\sigma^2}{|1 + \sum_{k=1}^{P} a_k \exp(-i2\pi k f)|^2}$$

Kay S 1988 Modern spectral estimation: Theory and Application Prentice Hall Englewood Cliffs

Advantages of AR modeling

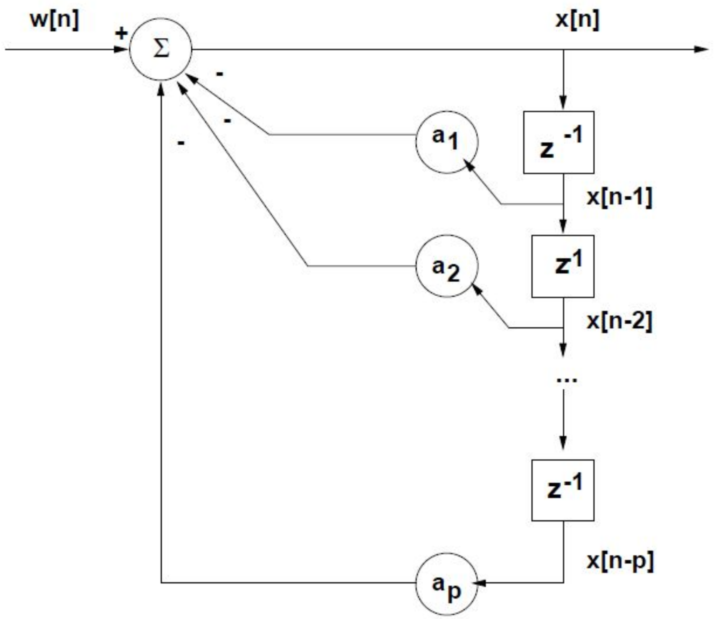
- Stable and causal filter: same solution of **linear predictor filter**

$$\hat{x}[n] = \sum_{k=1}^{P} w_k x[n-k].$$

$$e[n] = x[n] - \hat{x}[n]$$

$$\mathcal{E}_{min} = r_{xx}[0] - \sum_{k=1}^{P} w_k r_{xx}[-k],$$

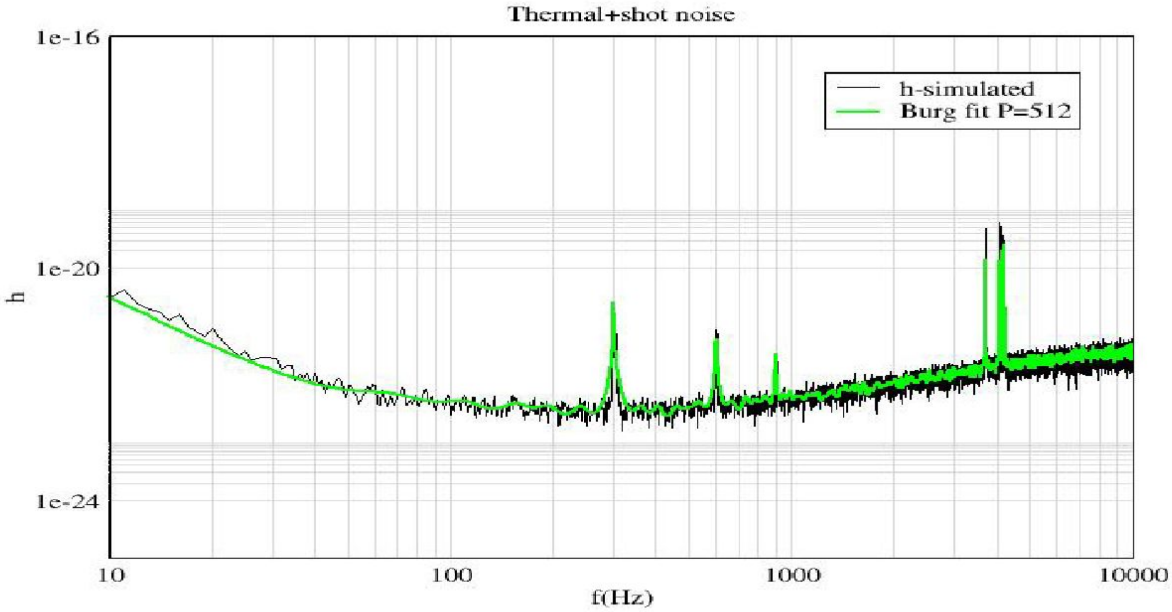
$$\begin{aligned} w_k &= -a_k \\ \mathcal{E}_{min} &= \sigma^2 \end{aligned}$$



Wiener-Hopf equations

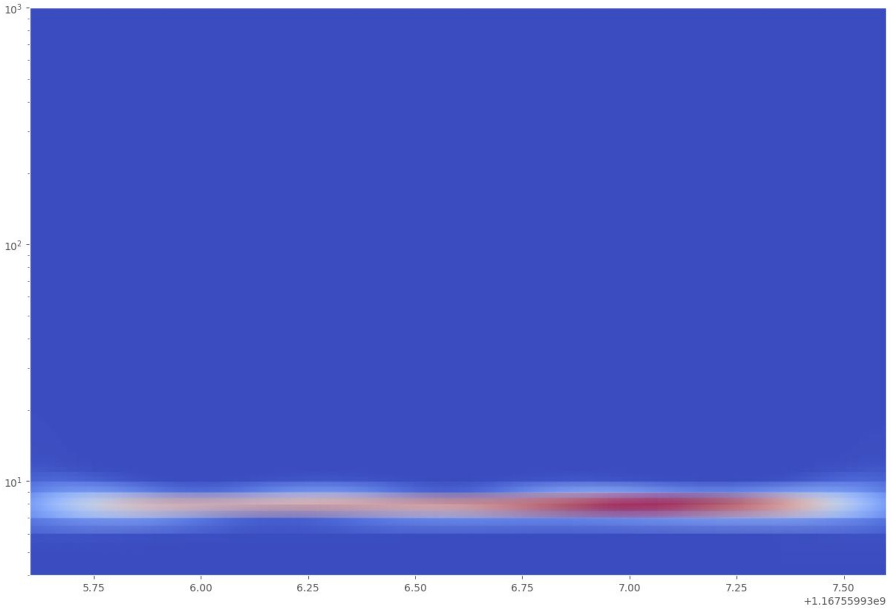
EGO - Virgo

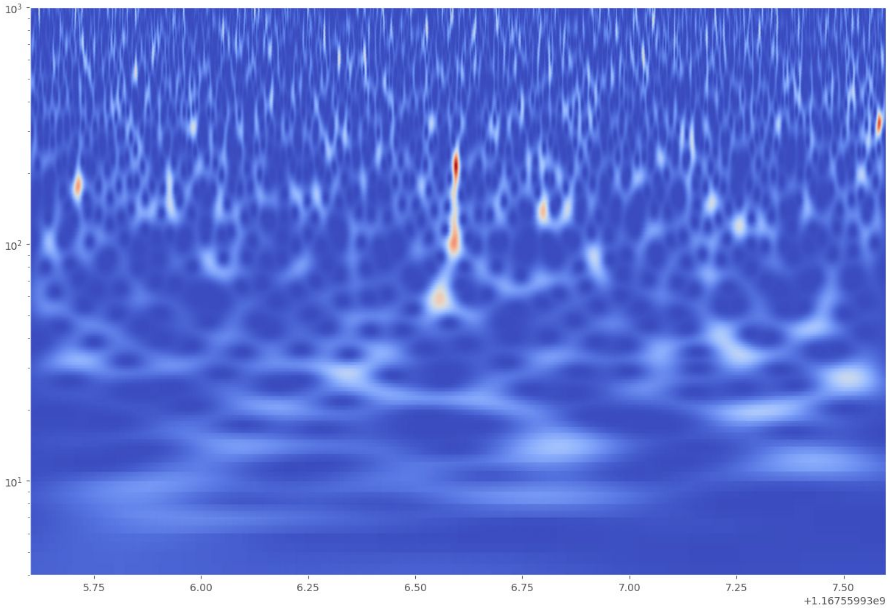
PSD AR(P) Fit



Cuoco et al. Class.Quant.Grav. 18 (2001) 1727-1752 and
Cuoco et al.Phys.Rev.D64:122002,2001

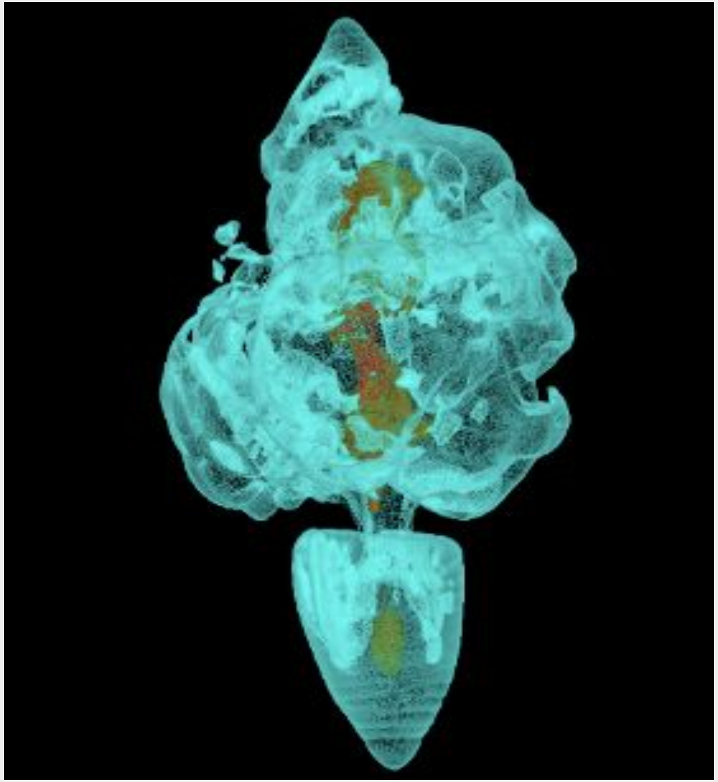
The effect of whitening





Searches for unmodeled signals

What we do for signals with unknown waveforms



Computer simulation of gravitational waves emitted by a supernova. Credit: J Powell / B Mueller

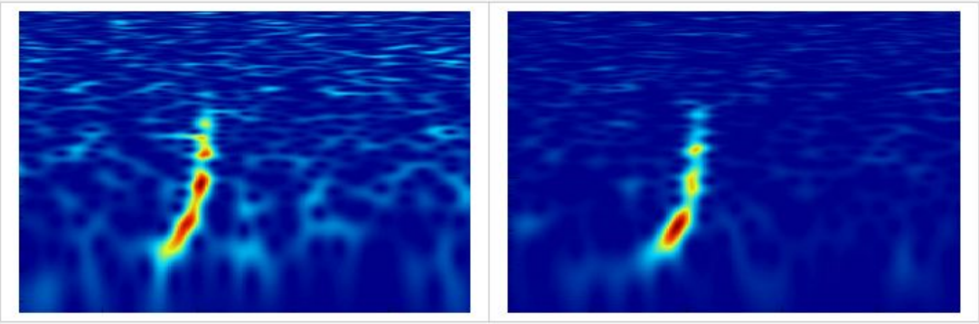
How we detect transient signals: un-modeled search

- *Strategy: look for excess power in single detector or coherent/coincident in network data*

- *Example cWB*

  (https://gwburst.gitlab.io/)
  - *Time-domain data preprocessed*
  - *Wavelet decompostion*
  - *Event reconstruction*

Coherent WaveBurst was used in the first direct detection of gravitational waves (GW150914) by LIGO and is used in the ongoing analyses on LIGO and Virgo data.



Time-Frequency maps of GW150914: Livingston data (left), Hanford data (right)
First screenshot of GW150914 event

Burst search

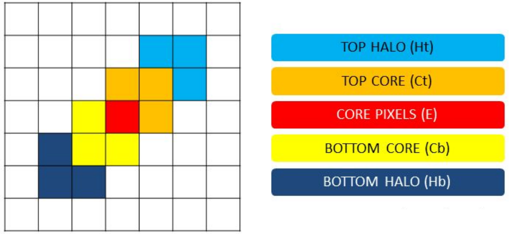
Phys. Rev. D 93, 042004 (2016)
Class.Quant.Grav.25:114029,2008

Coherent WaveBurst

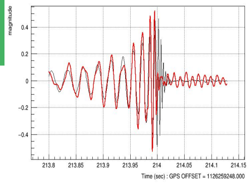
Excess power are selected from a set of wavelet time-frequency maps

Data from both detector are combined together

Triggers are analyzed coherently to estimate signal waveform, wave polarization, source location, using the constrained likelihood method



Selects the best fit waveform which corresponds to the maximum likelihood statistic over a 200000 sky positions



The event are ranked using a variable η_c

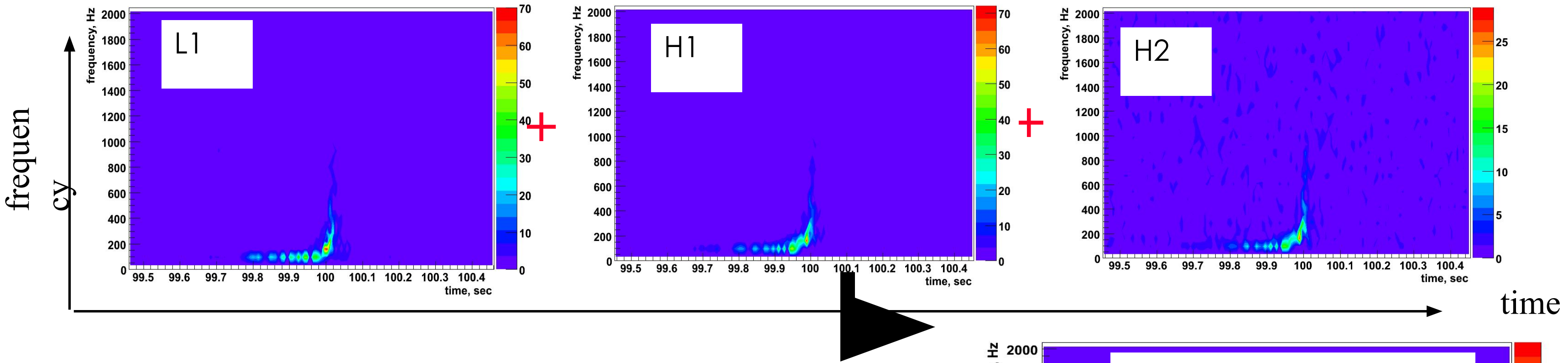
E_c → Normalized coherent energy between the two detectors

E_n → normalized noise energy derived by subtracting the reconstructed signal from the data

$$\eta_c = \sqrt{\frac{2E_c}{(1 + E_n/E_c)}}$$

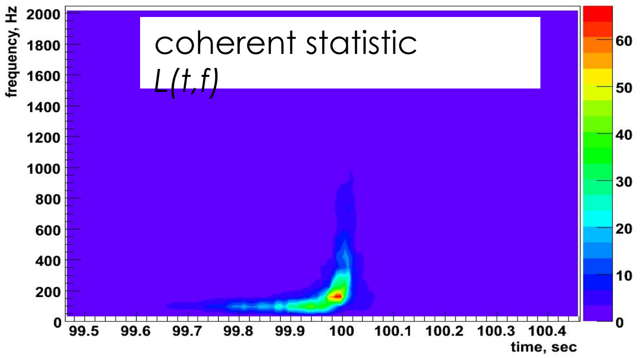
Coherent WaveBurst

- End-to-end multi-detector coherent pipeline
  - construct coherent statistics for detection and rejection of artifacts
  - performs search over the entire sky
  - estimates background with time shifts



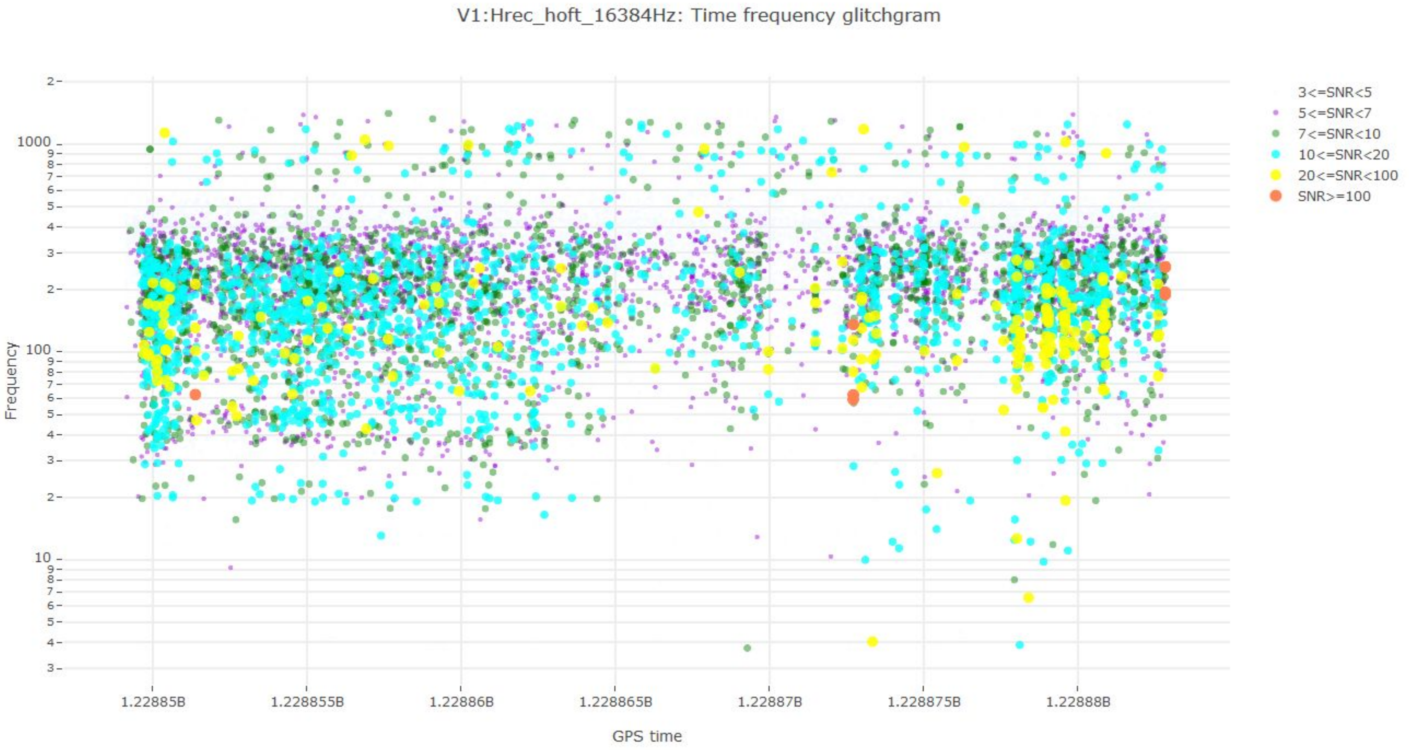
$$\xi_k = h_+ F_{+k} + h_x F_{xk}$$

$$L(t,f) = \max_{h_+ h_\times \theta\varphi} \sum_k \frac{x_k^2[t,f] - \left(x_k[t,f] - \xi_k[t,f]\right)^2}{\sigma_k^2(f)}$$



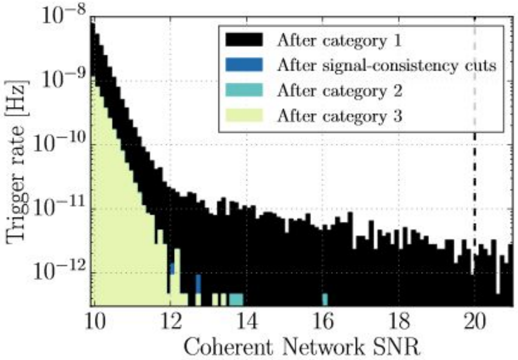
Glitch-gram

Time-Frequency distribution by SNR slice

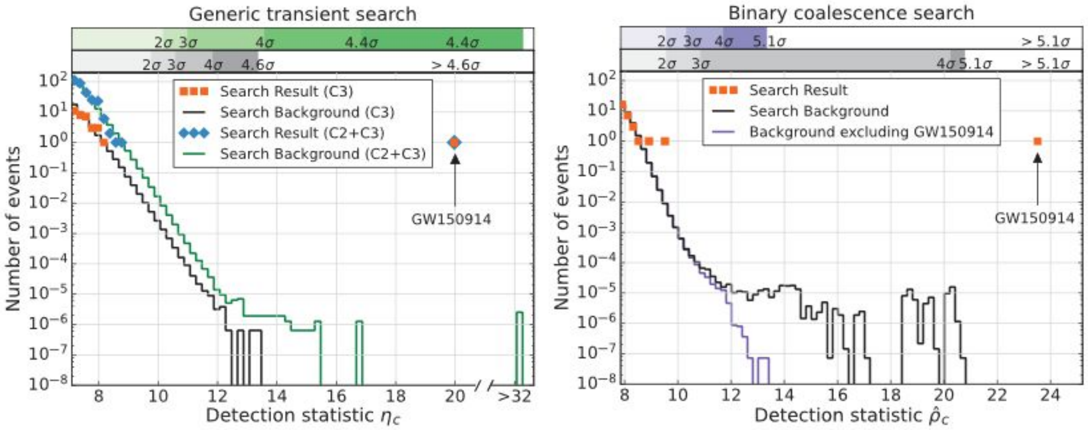


From a glitch-gram to Event selection

- Select the trigger in coincidence among the detector
- Perform data quality check
- Apply veto procedure
- Define the coincidence level of detection

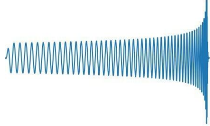


arXiv: 1602.03844



PhysRevLett.116.061102

Low latency analysis



Pipelines running real time → **Pipelines assess the significance of candidate** → **Data Quality evaluated autonomously for initial alert** → **Initial alert released on order of 1 minute; Notice on order of 10 minutes**

- 4 low-latency CBC search pipelines: GstLAL, MBTAOnline, PyCBC Live, and SPIIR
- 1 GW burst search pipeline: cWB (Coherent WaveBurst)

- False Alarm Rate (FAR) based on empirically measured noise properties
- The initial searches focus on detection, not on estimating the parameters of the source

GCN notice

Root		
IVORN	ivo://nasa.gsfc.gcn/LVC#[{T,M}]SYYMMDDabc-{1,2,3}-{Preliminary,Initial,Update,Preliminary-Retraction}	
Role	{observation,test}	
Who		
Date	Time sent (UTC, ISO-8601), e.g. 2018-11-01T22:34:49	
Author	LIGO Scientific Collaboration and Virgo Collaboration	
WhereWhen	Time of signal (UTC, ISO-8601), e.g. 2018-11-01T22:22:46.654437	
What		
GraceID	GraceDb ID: [{T,M}]SYYMMDDabc. Example: MS181101abc	
Packet Type	GCN Notice type: {Preliminary,Initial,Update}	
Notice Type	Numerical equivalent of GCN Notice type: {150,151,152}	
FAR	Estimated false alarm rate in Hz	
Sky Map	URL of HEALPix FITS localization file	
Group	CBC	Burst
Pipeline	{GstLal,MBTAOnline,PyCBC,SPIIR}	{cWB,oLIB}
CentralFreq	N/A	Central frequency in Hz
Duration		Duration of burst in s
Fluence		Gravitational-wave fluence in erg cm^{-2}
BNS, NSBH, BBH, Noise	Probability that the source is a BNS, NSBH, NSBH merger, or terrestrial (i.e., noise) respectively	N/A
HasNS, HasRemnant	Probability, under the assumption that the source is not noise, that at least one of the compact objects was a neutron star, and that the system ejected a nonzero amount of neutron star matter, respectively.	

Public Alert

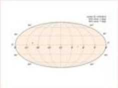
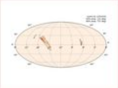
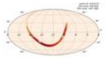
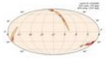
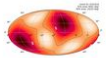
GraceDB — Gravitational-Wave Candidate Event Database

HOME | PUBLIC ALERTS | SEARCH | LATEST | DOCUMENTATION

LIGO/Virgo O3 Public Alerts

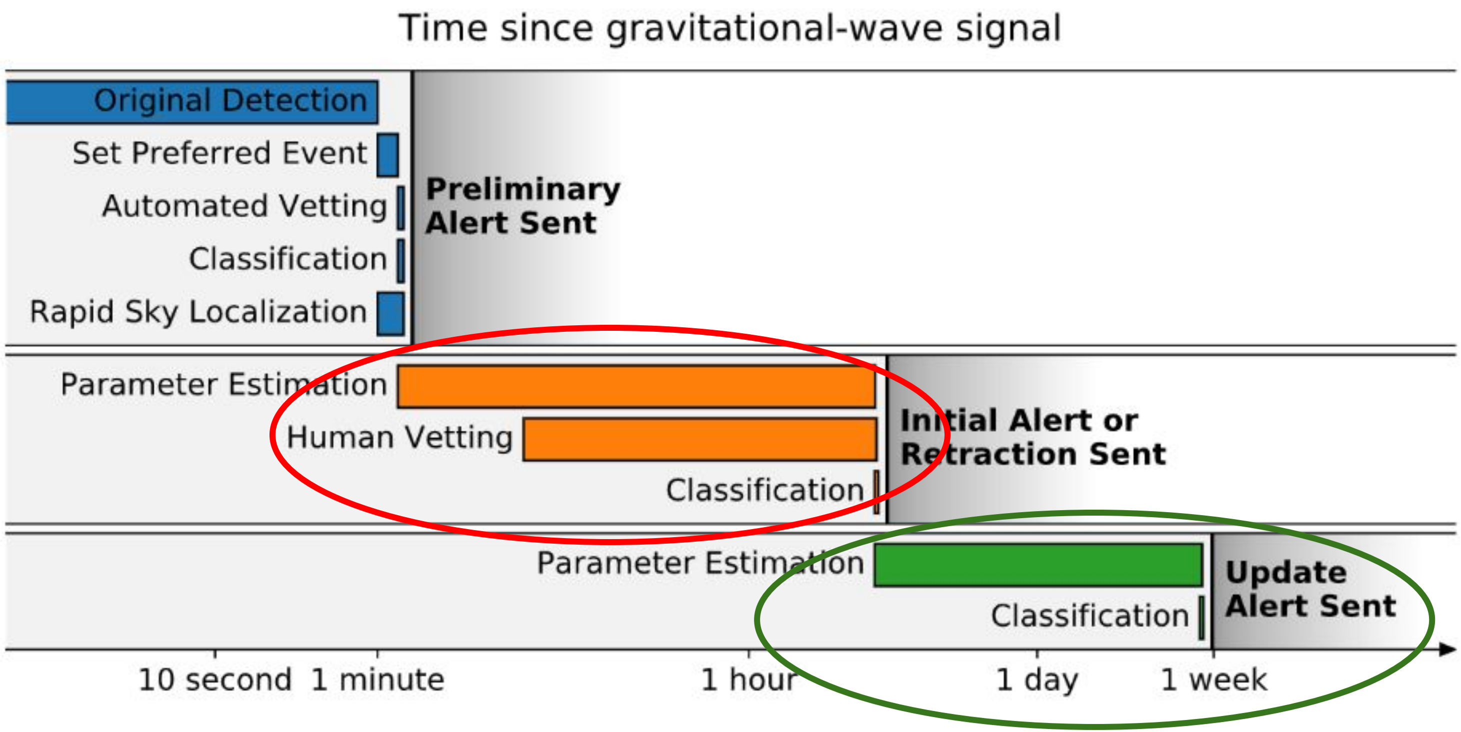
Detection candidates: 35

SORT: EVENT ID (A-Z)

Event ID	Possible Source (Probability)	UTC	GCN	Location	FAR	Comments
S191117j	NSBH (>99%)	Nov. 17, 2019 06:08:22 UTC	GCN Circulars Notices | VOE		1 per 2.8433e+10 years	RETRACTED
S191110af		Nov. 10, 2019 23:06:44 UTC	GCN Circulars Notices | VOE	No public skymap image found.	1 per 12.681 years	RETRACTED
S191110x	MassGap (>99%)	Nov. 10, 2019 18:08:42 UTC	GCN Circulars Notices | VOE		1 per 1081.7 years	RETRACTED
S191109d	BBH (>99%)	Nov. 9, 2019 01:07:17 UTC	GCN Circulars Notices | VOE		1 per 2.062e+05 years	
S191105e	BBH (95%), Terrestrial (5%)	Nov. 5, 2019 14:35:21 UTC	GCN Circulars Notices | VOE		1 per 1.3881 years	
S190930t	NSBH (74%), Terrestrial (26%)	Sept. 30, 2019 14:34:07 UTC	GCN Circulars Notices | VOE		1 per 2.0536 years	

https://gracedb.ligo.org/superevents/public/O3/

GW alert system



Can Machine learning help?