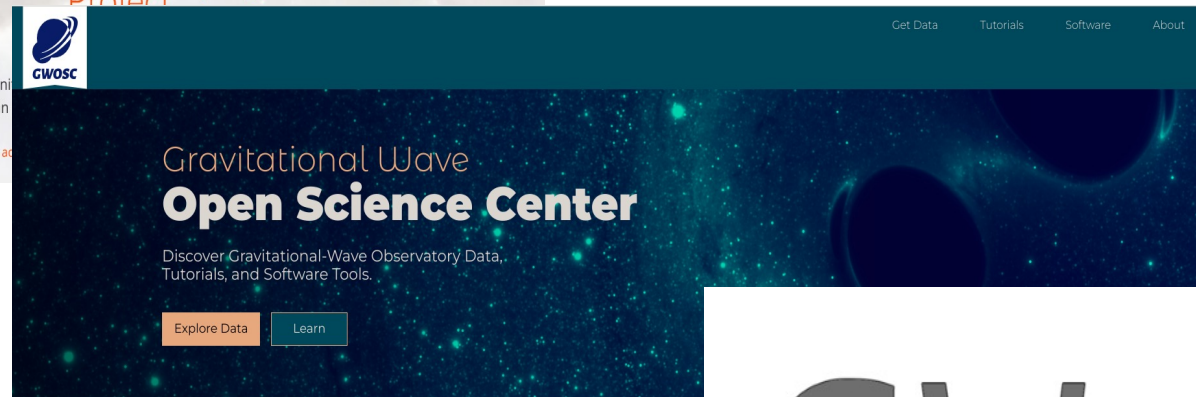
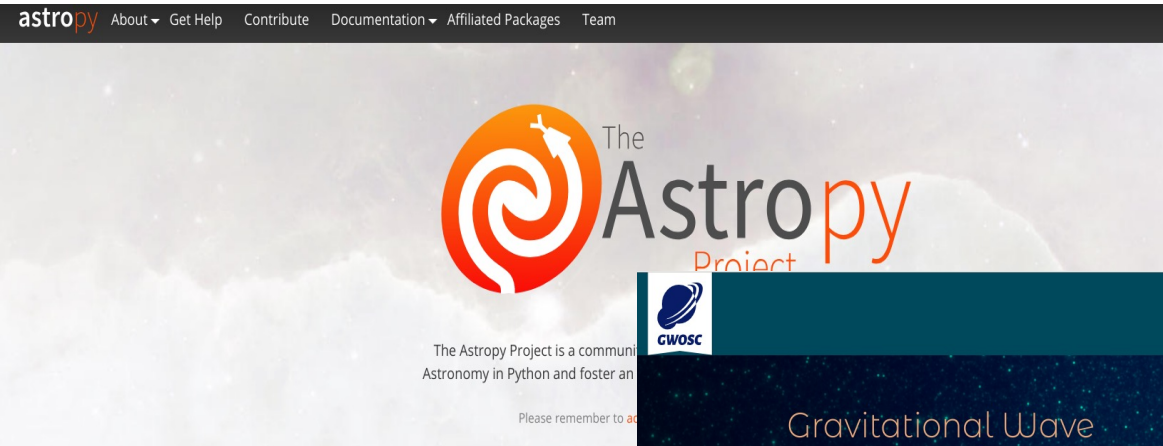


# Gravitational Wave Data Analysis From CBC to pulsars

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BND School 2024 - Blankenberge, Belgium 2 – 12 Sep 2024

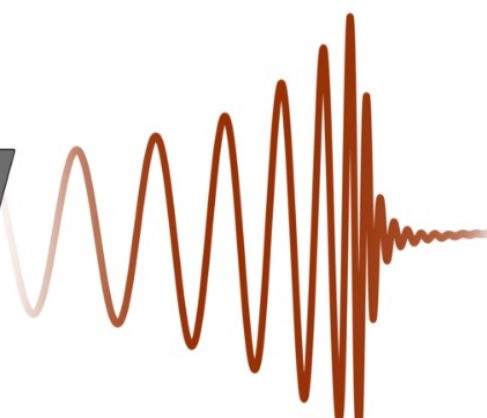
# Previously...



  
**Event Catalog**  
The Gravitational-wave Transient Catalog (GWTC) is a cumulative set of events detected by LIGO, Virgo, and

  
**Open Data Workshop**  
Participants will receive a crash-course in gravitational-wave data analysis that includes lectures, software

# Gwpy



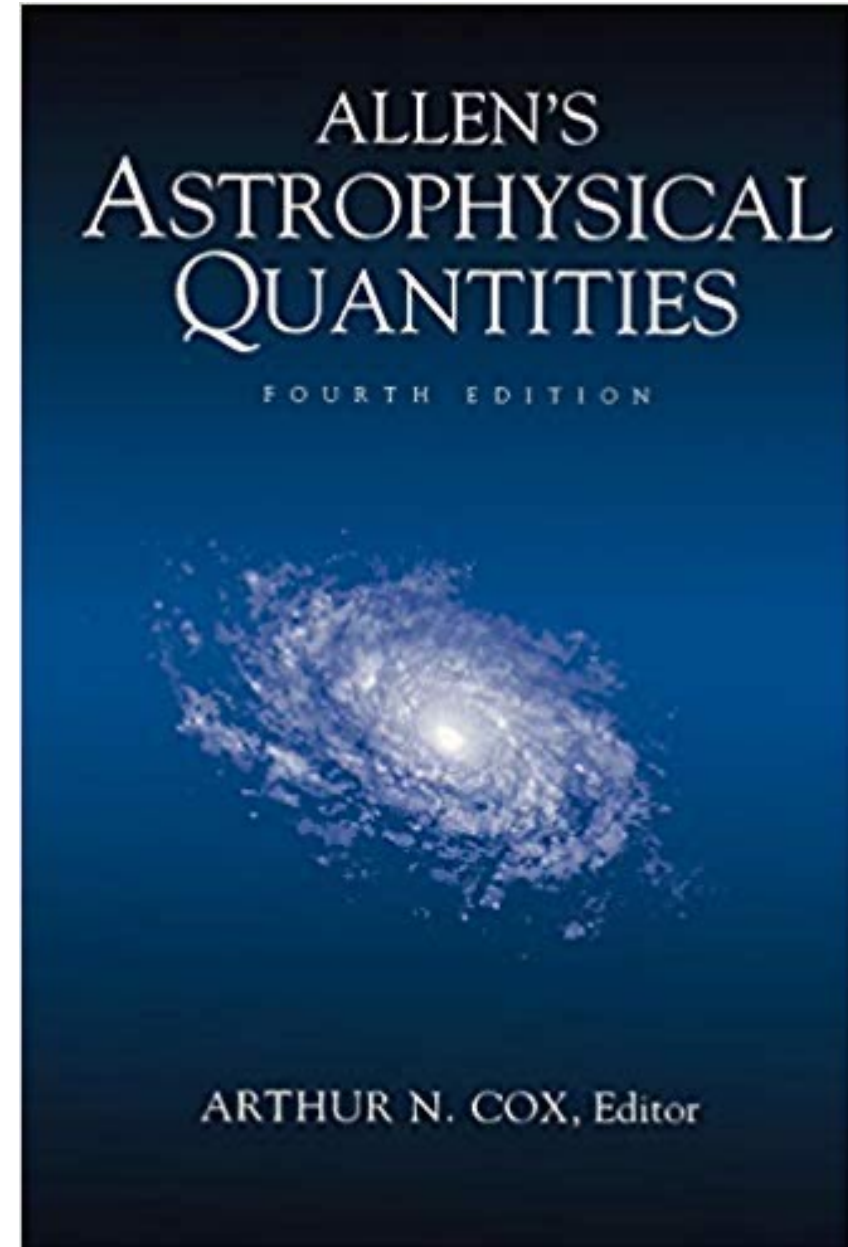
a python package for gravitational-wave astrophysics.

```
>>> help(gwpy)
```

# A quick recap on astrophysical quantities

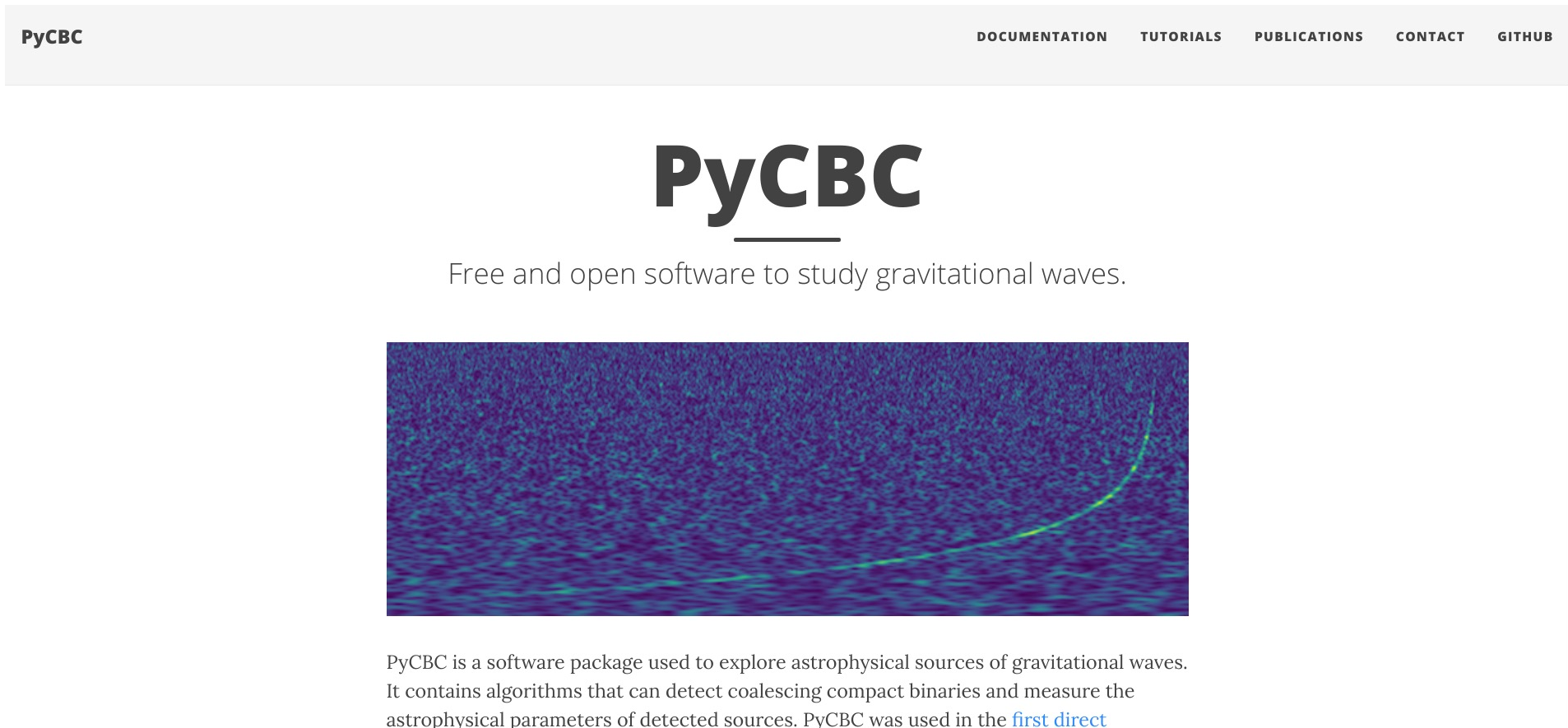
- Useful not just for Gravitational Waves, but also for **multimessenger analysis**
- **Time** → use case: How can I convert the time of a GW event from UTC ?
- **Sky Coordinates** → use case: Can I find all galaxies within the 95% LIGO-Virgo-KAGRA localization ?
- Lots of calculations (ephemeris table, spherical astronomy...)

...but...



# The tools - pyCBC

- [www.pycbc.org](http://www.pycbc.org)
- Software suite for signal detection
- (template matching)
- Install via PIP or conda



The screenshot shows the pyCBC website homepage. At the top, there is a navigation bar with the pyCBC logo on the left and links for DOCUMENTATION, TUTORIALS, PUBLICATIONS, CONTACT, and GITHUB on the right. The main content area features the pyCBC logo in a large, bold font, with a horizontal line under the 'y'. Below the logo is the tagline "Free and open software to study gravitational waves." Underneath this is a square image of a gravitational wave chirp, showing a signal that increases in frequency and amplitude over time. At the bottom of the page, there is a paragraph of text describing the software package and its use in the first direct detection of gravitational waves.

PyCBC is a software package used to explore astrophysical sources of gravitational waves. It contains algorithms that can detect coalescing compact binaries and measure the astrophysical parameters of detected sources. PyCBC was used in the [first direct](#)

# pyCBC – getting data

- Let's do a matched filter analysis using real data from GWOSC and pyCBC
- First, we get data as explained yesterday with the gwosc package

```
[6]: import gwpy
      from gwpy.timeseries import TimeSeries
      from gwpy.segments import DataQualityFlag

      # Select a time interval of 30 mins around the event. This is because on GWOSC there is 1 hour data window released around
      dt_win=3600*0.25
      ev_t0_min = ev_gps-dt_win
      ev_t0_max = ev_gps+dt_win

      print("Get data for %s (%s) GPS: %.2f - %.2f" % (ev_name, ev_ifo, ev_t0_min, ev_t0_max))

      #fetch the data. Use cache=True to keep the data in the cache memory (to speed things up)
      data = TimeSeries.fetch_open_data(ev_ifo, ev_t0_min, ev_t0_max, cache=True)

      #get the segments in a larger time window (just to have a bigger time span to look over)
      segments = DataQualityFlag.fetch_open_data(ev_ifo+"_DATA", ev_t0_min-dt_win, ev_t0_max+dt_win)
      print("Done")
```

```
Get data for GW150914 (H1) GPS: 1126258562.40 - 1126260362.40
```

```
Done
```

# pyCBC – basic manipulation

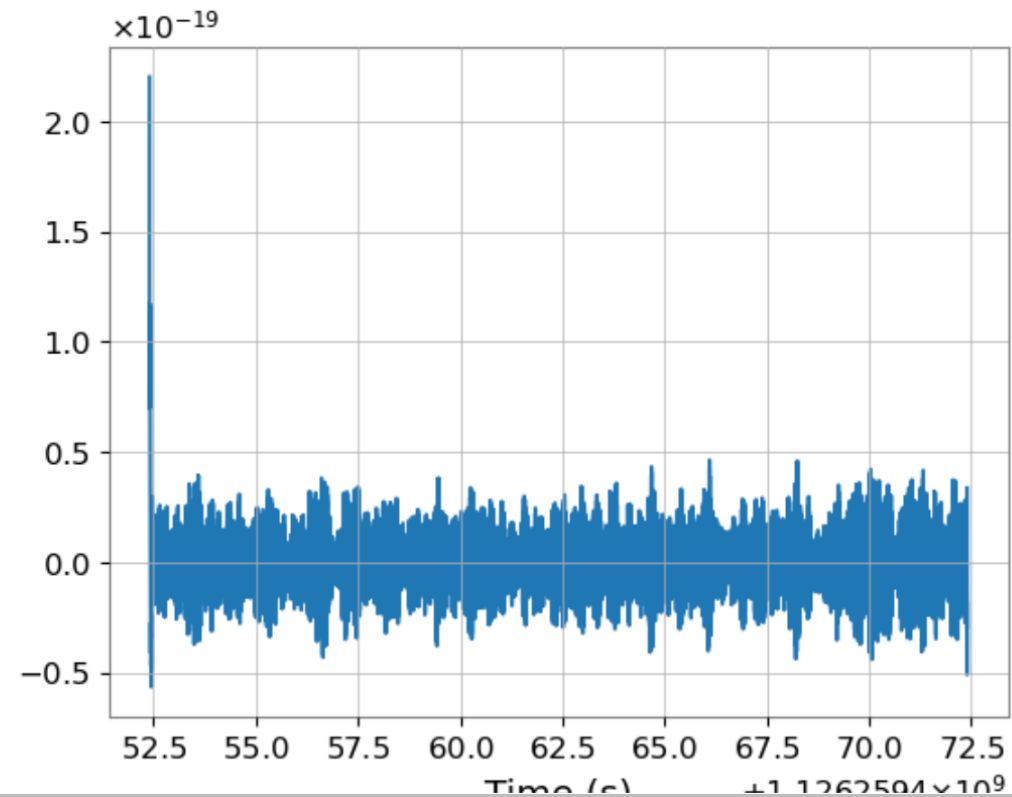
- Now we:
- Convert from gwpy to pyCBC
- Apply a high pass filter
- Resample to 2048Hz

```
[33]: import pylab
      from pycbc.filter import resample_to_delta_t, highpass

      # Convert the data from gwpy to pyCBC format
      strain = data.to_pycbc()

      # Remove the frequencies below 15 Hz and downsample the data to 2048Hz
      strain = highpass(strain, 15.0)
      strain = resample_to_delta_t(strain, 1.0/2048)

      pylab.plot(strain.sample_times, strain)
      pylab.xlabel('Time (s)')
      pylab.show()
```

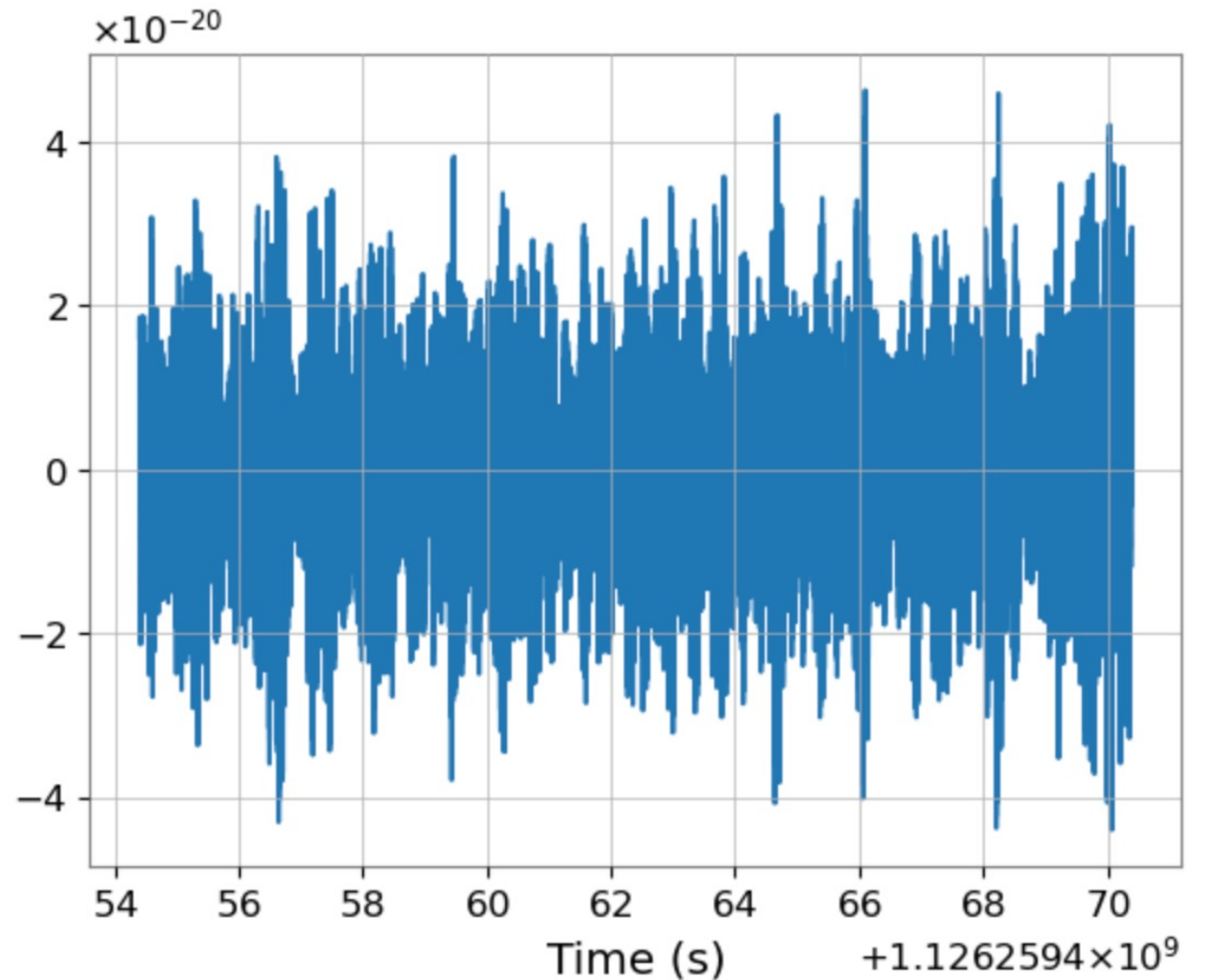


# pyCBC – data conditioning

```
conditioned = strain.crop(2, 2)
```

```
pylab.plot(conditioned.sample_times, conditioned)  
pylab.xlabel('Time (s)')  
pylab.show()
```

- Let's remove the first and last 2 seconds to delete ringing artifacts

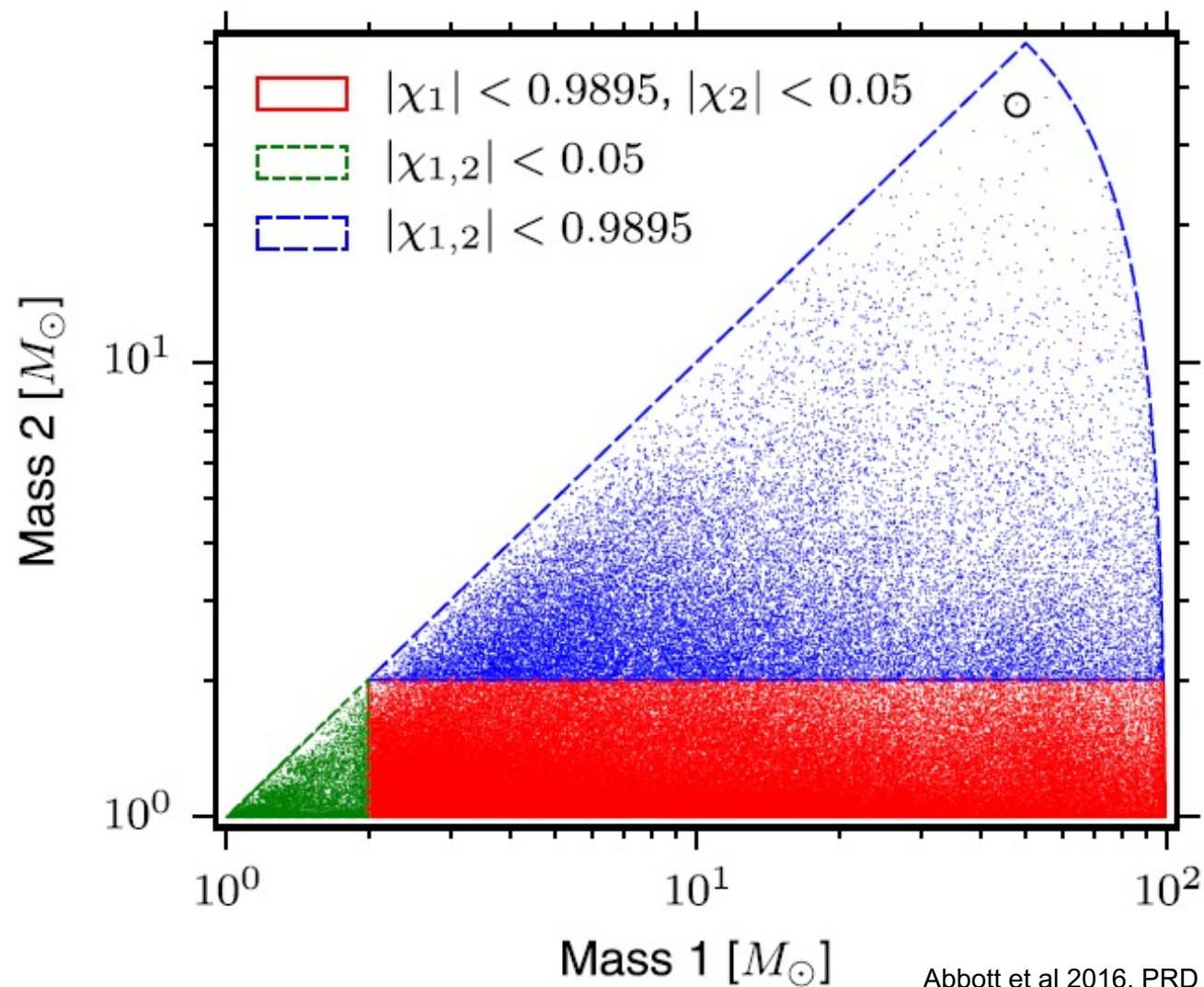


# Template-based searches

Search parameter space  
Used for O1  
Convention:  $m_1 > m_2$

## Three binary types

- **BH-BH**
- **NS-NS**
- **NS-BH**





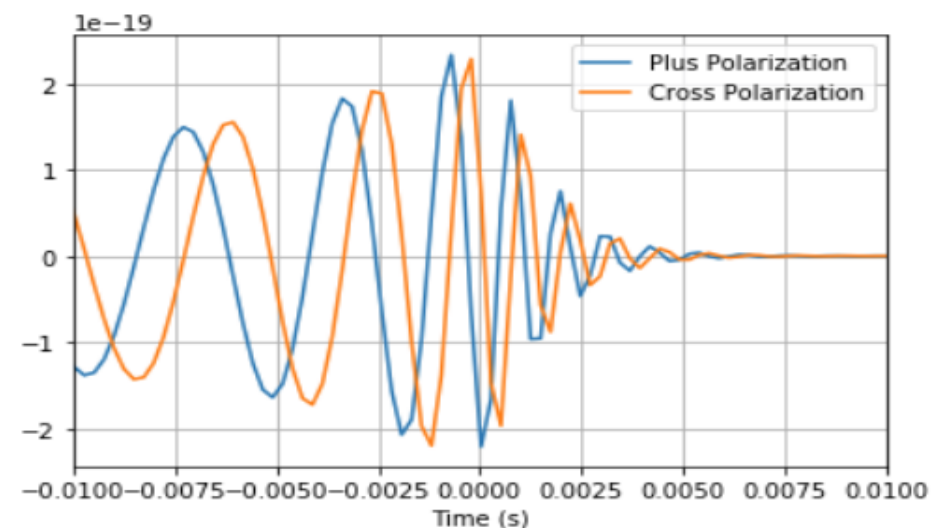
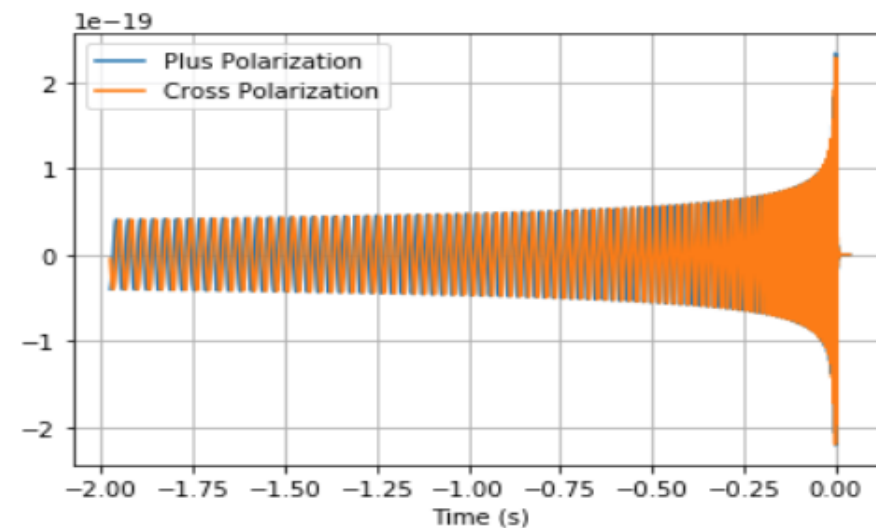
# pyCBC – getting waveform templates

```
from pycbc.waveform import get_td_waveform
import pylab

# The output of this function are the "plus" and "cross" polarizations of the
# as viewed from the line of sight at a given source inclination (assumed face
hp, hc = get_td_waveform(approximant="SEOBNRv4_opt",
                        mass1=10,
                        mass2=10,
                        delta_t=1.0/4096,
                        f_lower=30)

pylab.plot(hp.sample_times, hp, label='Plus Polarization')
pylab.plot(hc.sample_times, hc, label='Cross Polarization')
pylab.xlabel('Time (s)')
pylab.legend()
pylab.grid()
pylab.show()

# Zoom in near the merger time#
pylab.plot(hp.sample_times, hp, label='Plus Polarization')
pylab.plot(hc.sample_times, hc, label='Cross Polarization')
pylab.xlabel('Time (s)')
pylab.xlim(-.01, .01)
pylab.legend()
pylab.grid()
pylab.show()
```



# pyCBC – matched filter

- pyCBC can do automatically the matched filter
- (here from pyCBC official tutorials)

```
from pycbc.filter import matched_filter
import numpy

snr = matched_filter(template, conditioned,
                    psd=psd, low_frequency_cutoff=20)

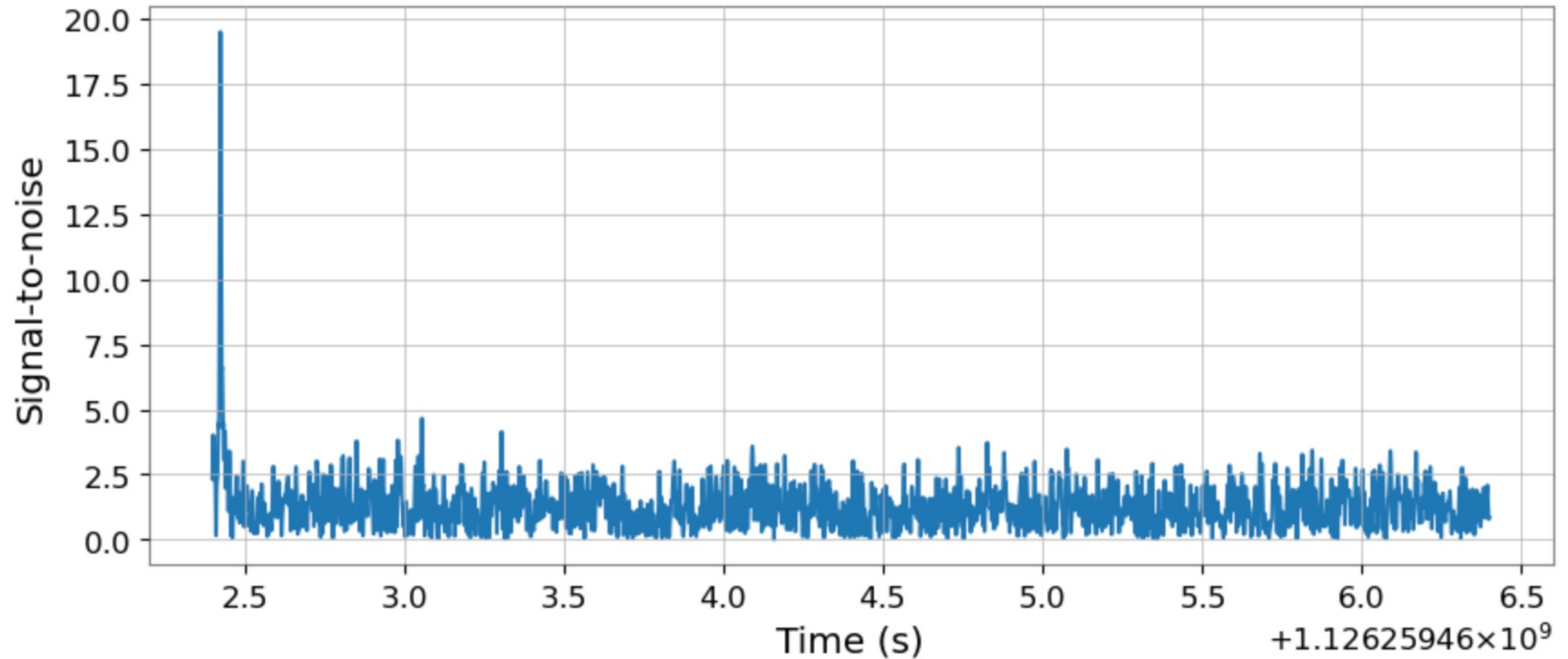
# Remove time corrupted by the template filter and the psd filter
# We remove 4 seconds at the beginning and end for the PSD filtering
# And we remove 4 additional seconds at the beginning to account for
# the template length (this is somewhat generous for
# so short a template). A longer signal such as from a BNS, would
# require much more padding at the beginning of the vector.
snr = snr.crop(4 + 4, 4)

# Why are we taking an abs() here?
# The `matched_filter` function actually returns a 'complex' SNR.
# What that means is that the real portion corresponds to the SNR
# associated with directly filtering the template with the data.
# The imaginary portion corresponds to filtering with a template that
# is 90 degrees out of phase. Since the phase of a signal may be
# anything, we choose to maximize over the phase of the signal.
pylab.figure(figsize=[10, 4])
pylab.plot(snr.sample_times, abs(snr))
pylab.ylabel('Signal-to-noise')
pylab.xlabel('Time (s)')
pylab.show()

peak = abs(snr).numpy().argmax()
snrp = snr[peak]
time = snr.sample_times[peak]

print("We found a signal at {}s with SNR {}".format(time,
                                                  abs(snrp)))
```

# pyCBC – matched filter



We found a signal at 1126259462.425293s with SNR 19.500135766825494

# pyCBC – scaling and filtering the data

- We can now align the template and the data, and scale its amplitude accordingly

```
from pycbc.filter import sigma
# The time, amplitude, and phase of the SNR peak tell us how to align
# our proposed signal with the data.

# Shift the template to the peak time
dt = time - conditioned.start_time
aligned = template.cyclic_time_shift(dt)

# scale the template so that it would have SNR 1 in this data
aligned /= sigma(aligned, psd=psd, low_frequency_cutoff=20.0)

# Scale the template amplitude and phase to the peak value
aligned = (aligned.to_frequencyseries() * snrp).to_timeseries()
aligned.start_time = conditioned.start_time
```

# pyCBC – comparing data and template

- We do some filtering again on data

```
# We do it this way so that we can whiten both the template and the data
white_data = (conditioned.to_frequencyseries() / psd**0.5).to_timeseries()

# apply a smoothing of the turnon of the template to avoid a transient
# from the sharp turn on in the waveform.
tapered = aligned.highpass_fir(30, 512, remove_corrupted=False)
white_template = (tapered.to_frequencyseries() / psd**0.5).to_timeseries()

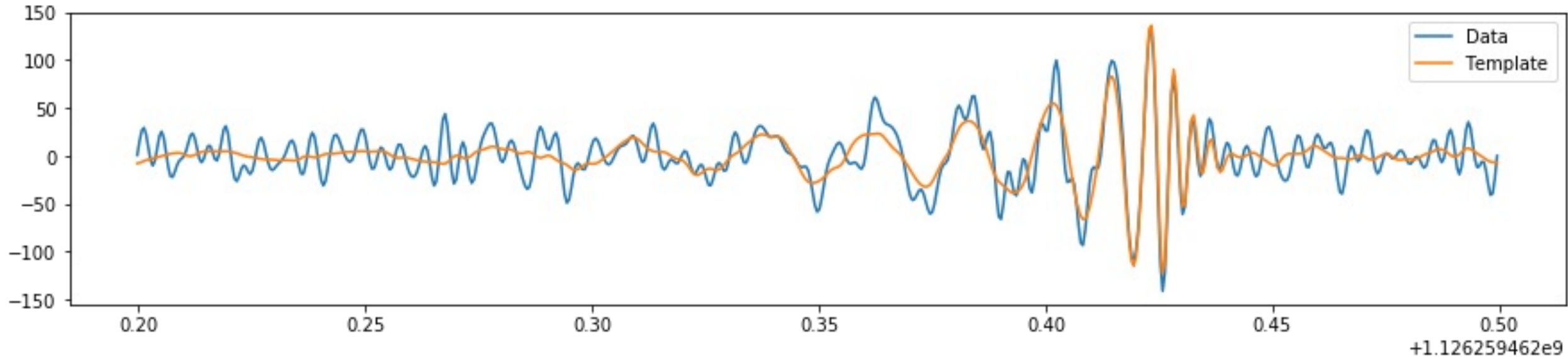
white_data = white_data.highpass_fir(30., 512).lowpass_fir(300, 512)
white_template = white_template.highpass_fir(30, 512).lowpass_fir(300, 512)

# Select the time around the merger
white_data = white_data.time_slice(merger.time-.2, merger.time+.1)
white_template = white_template.time_slice(merger.time-.2, merger.time+.1)

pylab.figure(figsize=[15, 3])
pylab.plot(white_data.sample_times, white_data, label="Data")
pylab.plot(white_template.sample_times, white_template, label="Template")
pylab.legend()
pylab.show()
```

# pyCBC – comparing data and template

- And we have the comparison between data and template!



# pyCBC – subtract and Q-transform

```
subtracted = conditioned - aligned

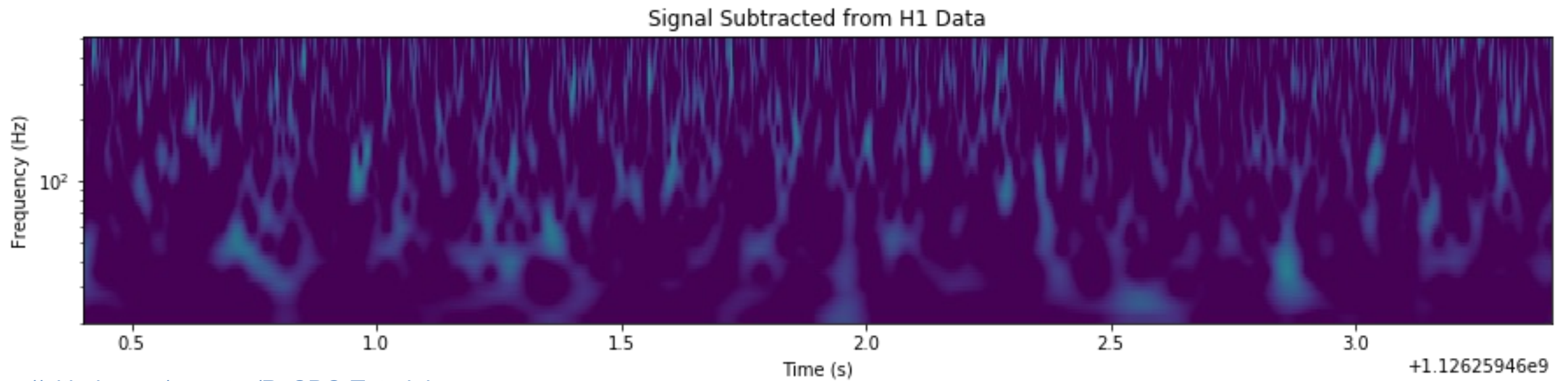
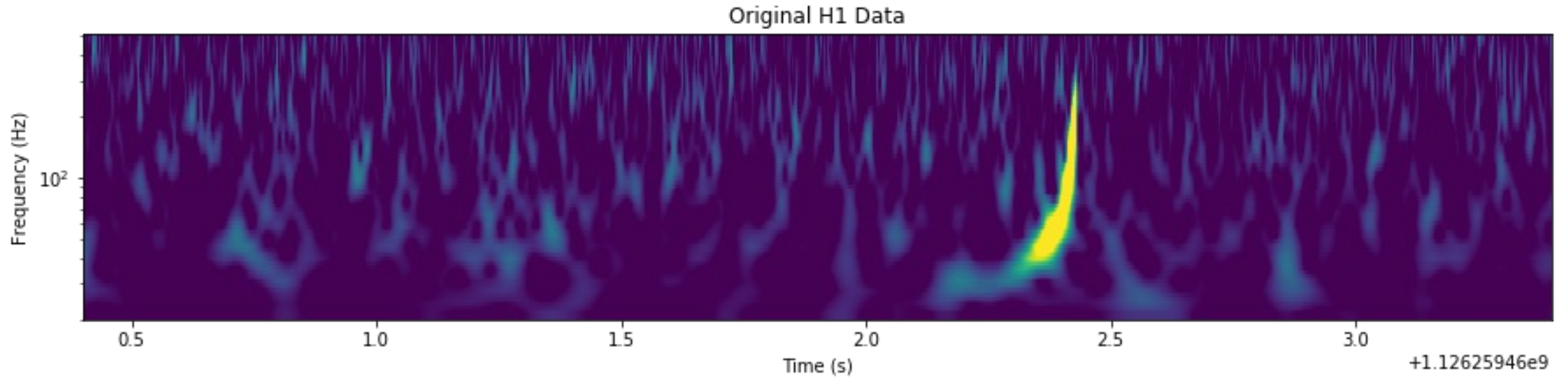
# Plot the original data and the subtracted signal data

for data, title in [(conditioned, 'Original H1 Data'),
                    (subtracted, 'Signal Subtracted from H1 Data')]:

    t, f, p = data.whiten(4, 4).qtransform(.001,
                                           logfsteps=100,
                                           qrange=(8, 8),
                                           frange=(20, 512))

    pylab.figure(figsize=[15, 3])
    pylab.title(title)
    pylab.pcolormesh(t, f, p**0.5, vmin=1, vmax=6)
    pylab.yscale('log')
    pylab.xlabel('Time (s)')
    pylab.ylabel('Frequency (Hz)')
    pylab.xlim(merger.time - 2, merger.time + 1)
    pylab.show()
```

# pyCBC – subtract and Q-transform

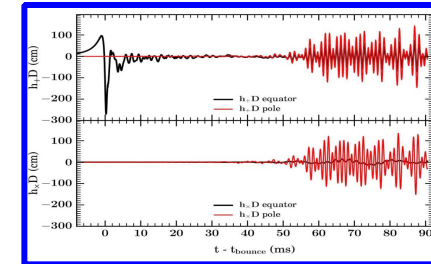
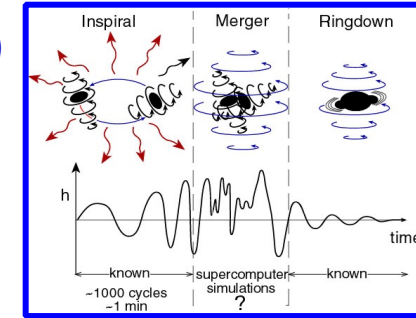
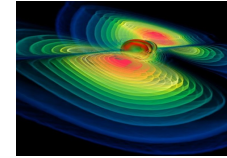




# Not just CBC: Expected sources detectable by LIGO/Virgo

## Transients

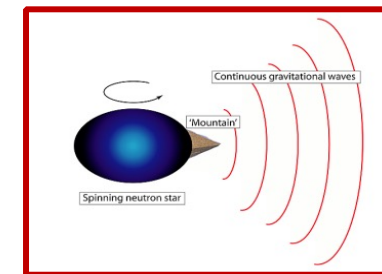
- **Coalescence of compact binary systems (NSs and/or BHs)**
  - Known waveforms (template banks)
  - $E_{\text{gw}} \sim 10^{-2} \text{ Mc}^2$
- **Core-collapse of massive stars**
  - Uncertain waveforms
  - $E_{\text{gw}} \sim 10^{-8} - 10^{-4} \text{ Mc}^2$



Ott, C. 2009

## Non transients

- **Rotating neutron stars**
  - Quadrupole emission from star's asymmetry
  - Continuous and Periodic
- **Stochastic background**
  - Superposition of many signals (mergers, cosmological, etc)
  - Low frequency



# Periodic sources: pulsars

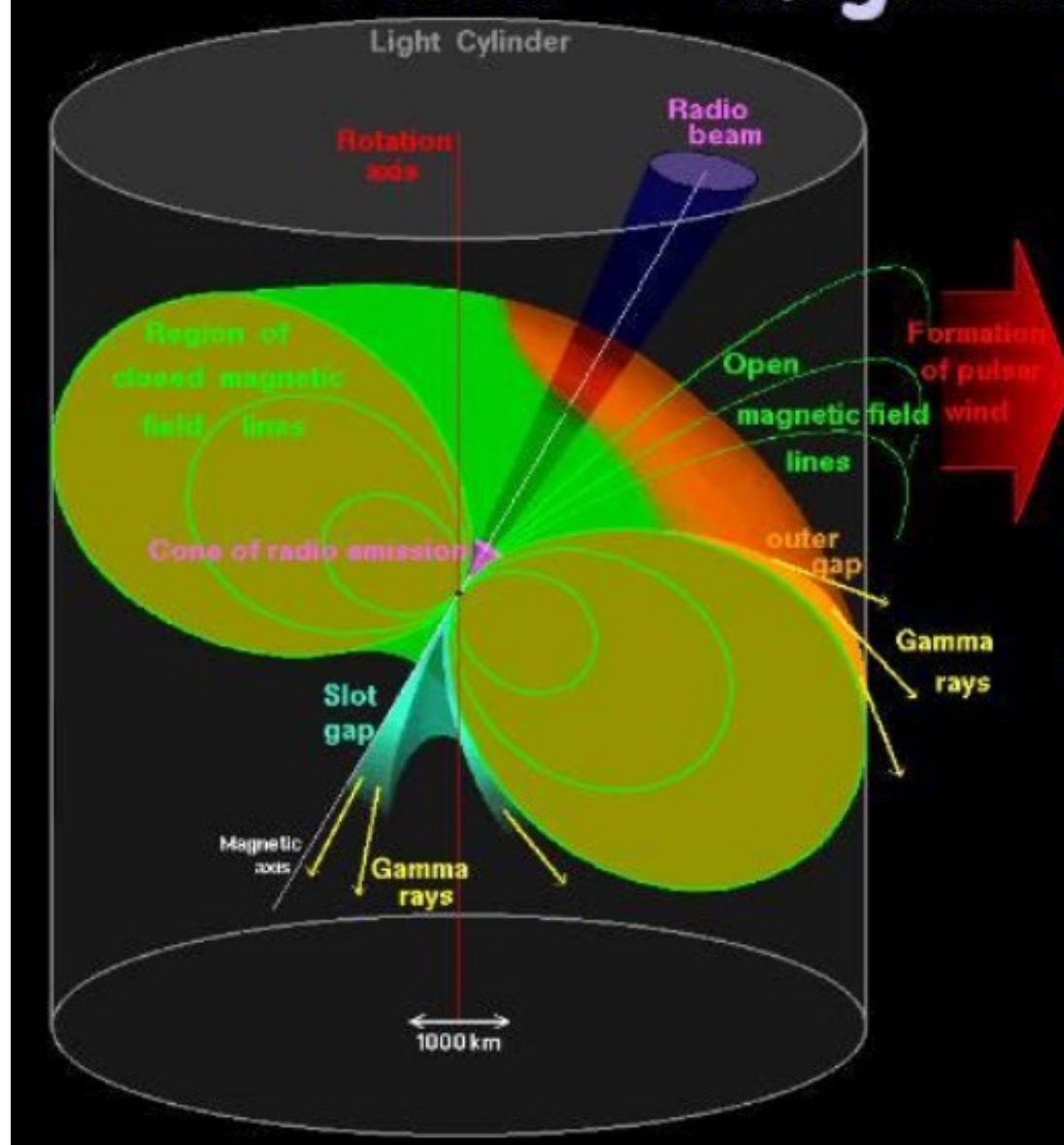


Credits: NASA/HST/ASU/Hester et al/CXC/ASU/ASU/Hester et al

# Neutron stars: historical perspective

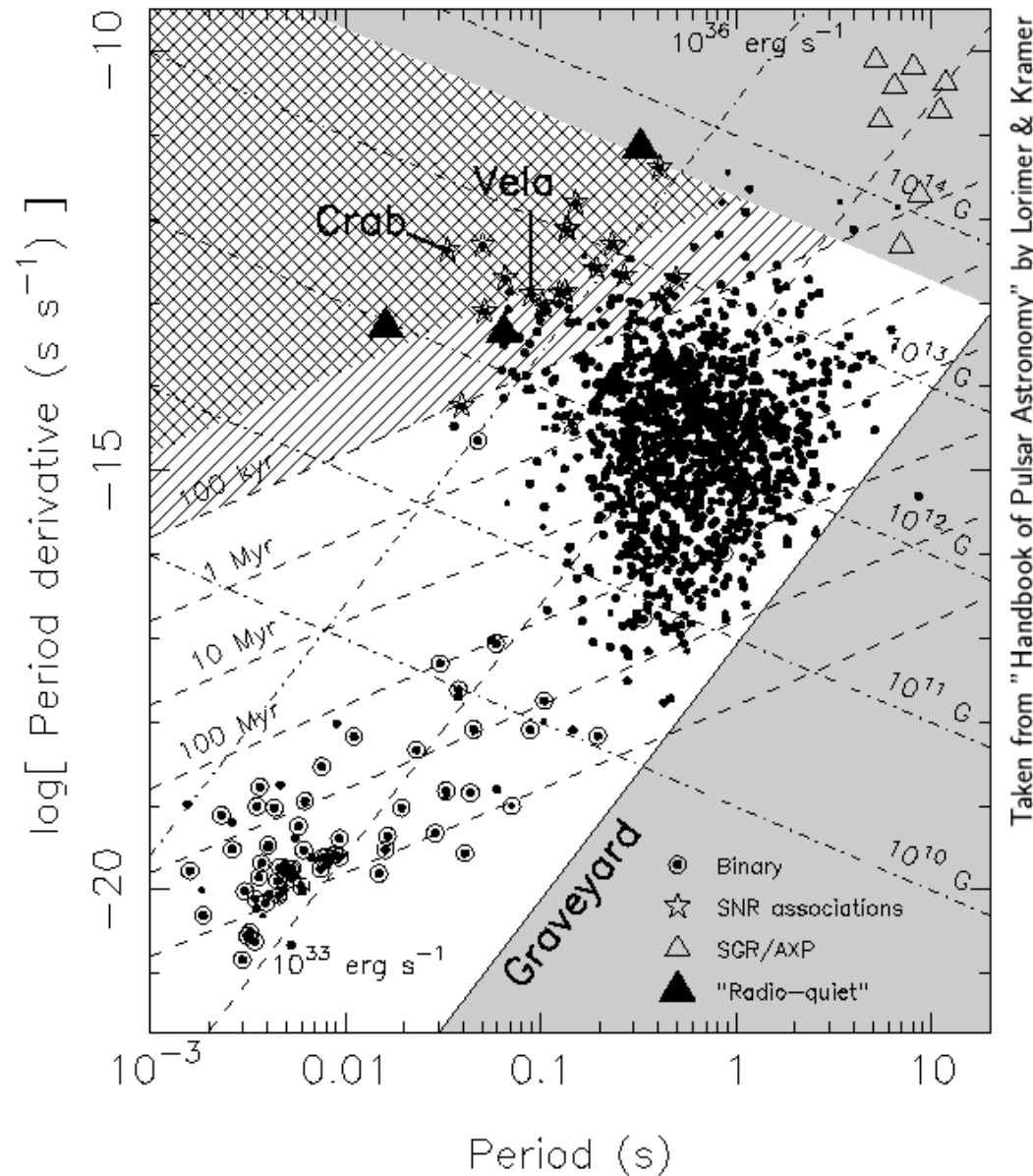
- 1932: J. Chadwick discovers the neutron
- 1934 W. Baade and F. Zwicky suggest the existence of neutron stars, that could be produced in supernovae explosions
- 1939: R. Oppenheimer and N. Volkoff make first calculation of equation of state of neutron stars
- Later works focused on studying how neutrons impact on stellar energy. With discovery of fusion as source for stellar energy, interest in neutron stars faded out
- However, theoretical works by Wheeler, Harrison in 50's and 60's on equation of state
- Calculations showed extremely low fluxes, too low to be measured. Small interest
- Discoveries in 60's by X-ray sources (Giacconi) and quasars (Schmidt) suggested that they could be neutron stars. Again, no evidence
- Interest faded again until 1967...

# Electromagnetic radiation



Credits: A. Spitkovsky

# The P-Pdot diagram



Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Young ( $t < 10^5$  yr)

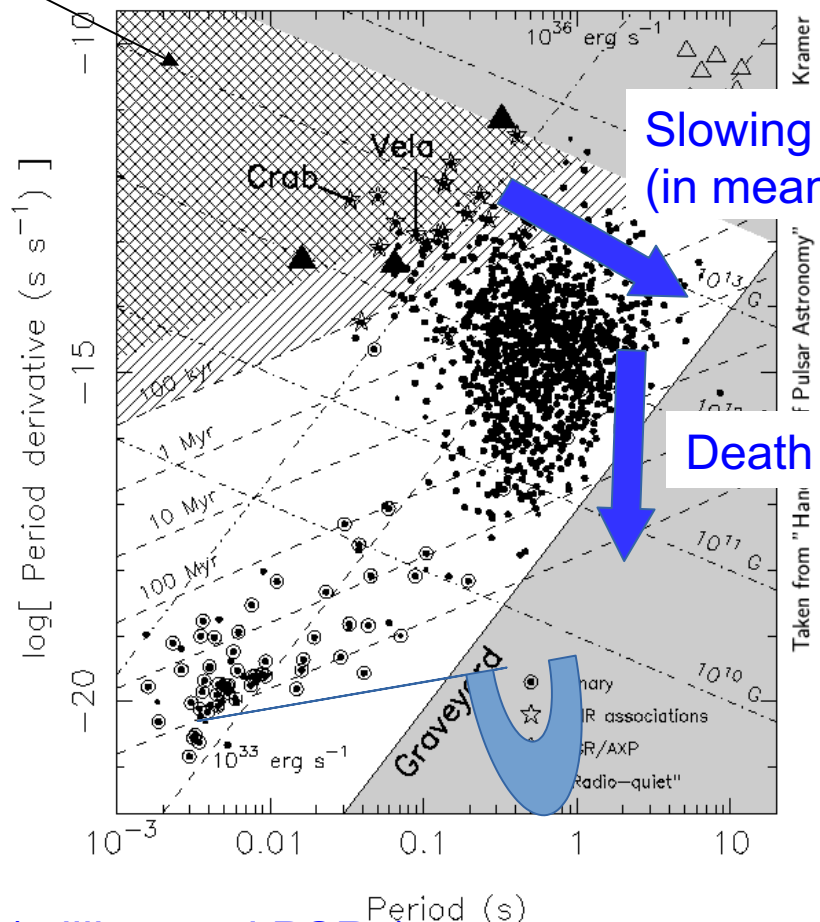
Middle-aged ( $10^6$ - $10^8$  yr)

Old ( $>10^8$ )

Credits: Handbook of Pulsar Astronomy

# Pulsar life

Birth in Supernova



Slowing down  
(in meantime, no more SNR)

Death

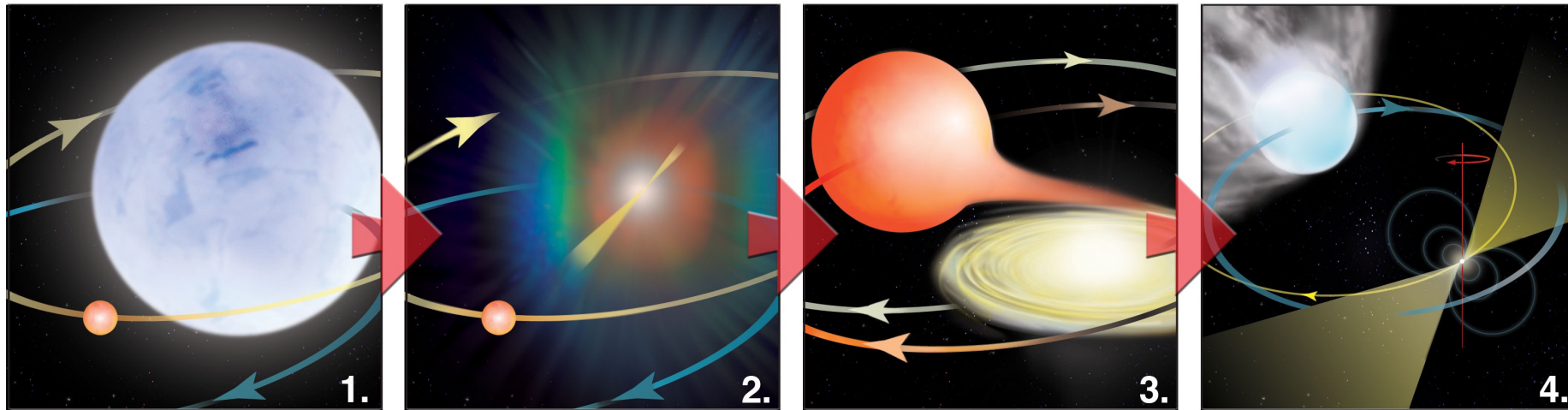
Back to life (millisecond PSRs)

Credits: Handbook of Pulsar Astronomy

# Millisecond pulsars

Low  $\dot{P}$ , B-fields ( $10^8$  G), high characteristic age  $\rightarrow$  old  
However, high period  $\rightarrow$  young?

Recycling mechanism



Credits: NRAO

# The $h(t)$ from sources

We have

$$h_{ij}^{\text{TT}}(t) \simeq \frac{2G}{c^4 r} \ddot{I}_{ij}^{\text{TT}}(t - r/c)$$

$G/c^4 \sim 8 \times 10^{-45} \text{ s}^2\text{m}^{-1}\text{Kg}^{-1}$

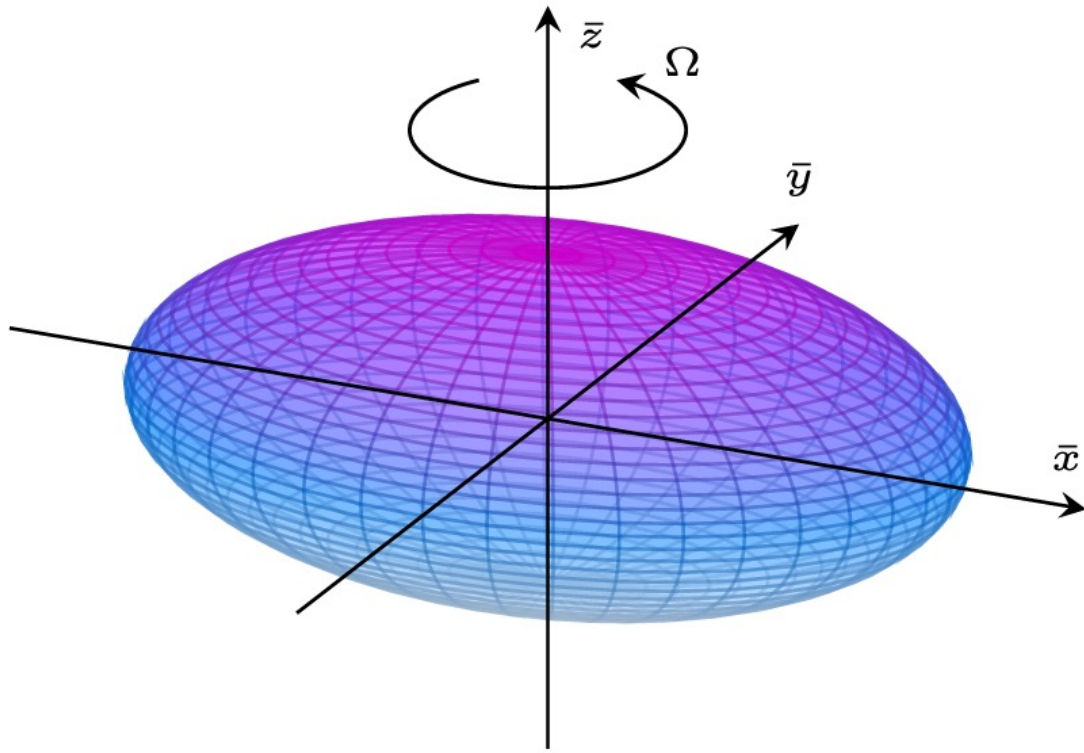
GW effect is small !

Quadrupole tensor  
We can calculate it close to the  
source

Amplitude decrease



# Gravitational waves from pulsars



Modeled as a rotating ellipsoid

$$\mathbf{I} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} .$$

We define  $I = I_1 + I_2$

And ellipticity  $\varepsilon = (I_1 - I_2) / I_3$

Gittins, 2024, CQG, 41, 043001

# Gravitational waves from pulsars

If ellipsoid is rotating with angular speed  $\Omega$ , we can transform coordinates to our frame of reference using a rotation matrix

$$\begin{aligned} \mathbf{I}(t) &= \mathbf{R}_3(\omega t) \mathbf{I}_0 \mathbf{R}_3^{-1}(\omega t) \\ &= \begin{bmatrix} \cos \omega t & \sin \omega t & 0 \\ -\sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \cos \omega t & -\sin \omega t & 0 \\ \sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2}I + \frac{1}{2}\varepsilon I_3 \cos 2\omega t & -\frac{1}{2}\varepsilon I_3 \sin 2\omega t & 0 \\ -\frac{1}{2}\varepsilon I_3 \sin 2\omega t & \frac{1}{2}I - \frac{1}{2}\varepsilon I_3 \cos 2\omega t & 0 \\ 0 & 0 & I_3 \end{bmatrix}, \end{aligned}$$

The second and third derivatives are:

$$\ddot{\mathbf{I}}(t) = 2\varepsilon I_3 \omega^2 \begin{bmatrix} -\cos 2\omega t & \sin 2\omega t & 0 \\ \sin 2\omega t & \cos 2\omega t & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \ddot{\mathbf{I}}(t) = 4\varepsilon I_3 \omega^3 \begin{bmatrix} \sin 2\omega t & \cos 2\omega t & 0 \\ \cos 2\omega t & -\sin 2\omega t & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

# Gravitational waves from pulsars

We can then build the  $h^{\text{TT}}$

$$h_{ij}^{\text{TT}} = \frac{2G}{c^4 r} \ddot{I}_{ij} = \frac{4G \varepsilon I_3 \omega^2}{c^4 r} \begin{bmatrix} -\cos 2\omega t & \sin 2\omega t & 0 \\ \sin 2\omega t & \cos 2\omega t & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$h_+ = -\frac{4G \varepsilon I_3 \omega^2}{c^4 r} \cos 2\omega t \quad h_{\times} = \frac{4G \varepsilon I_3 \omega^2}{c^4 r} \sin 2\omega t .$$

The emitted power is:

$$L_{\text{GW}} = \frac{1}{5} \frac{G}{c^5} \langle \ddot{\ddot{I}}_{ij} \ddot{\ddot{I}}^{ij} \rangle = \frac{32}{5} \frac{G}{c^5} \varepsilon^2 I_3^2 \omega^6 .$$

# Gravitational waves from pulsars

What if the pulsar rotational axis is inclined by an angle  $i$ ?

We use this rotation matrix:

$$\mathbf{R}_1(i) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix}.$$

$$\ddot{\mathbf{I}}^{\text{TT}} = 2\varepsilon I_3 \omega^2 \begin{bmatrix} -\frac{1}{2}(1 + \cos^2 i) \cos 2\omega t & \cos i \sin 2\omega t & 0 \\ \cos i \sin 2\omega t & \frac{1}{2}(1 + \cos^2 i) \cos 2\omega t & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The  $h(t)$  will be:

$$h_+ = -\frac{4G\varepsilon I_3 \omega^2}{c^4 r} \frac{1 + \cos^2 i}{2} \cos 2\omega t$$

$$h_\times = \frac{4G\varepsilon I_3 \omega^2}{c^4 r} \cos i \sin 2\omega t.$$

- $i=0, i=\pi \rightarrow$  Circular polarization
- $i=\pi/2 \rightarrow$  linear polarization
- $I_1=I_2 \rightarrow \varepsilon=0 \rightarrow$  no radiation

# Spin-down limit

We can compute the GW luminosity which is

$$L_{\text{GW}} = \frac{32}{5} \frac{G}{c^5} m^2 a^4 \omega^6 .$$

We can ask what would be the  $h_0$  of a pulsar IF it emitted all its power in GW?

We obtain the spindown limit

$$h_{sd} = 8.06 \cdot 10^{-19} I_{38}^{1/2} \left[ \frac{\text{1kpc}}{d} \right] \left[ \frac{\dot{f}_{\text{rot}}}{\text{Hz/s}} \right]^{1/2} \left[ \frac{\text{Hz}}{f_{\text{rot}}} \right]^{1/2}$$

This can be converted to ellipticity

$$\epsilon_{sd} = 0.237 I_{38}^{-1} \left[ \frac{h_{sd}}{10^{-24}} \right] \left[ \frac{\text{Hz}}{f_{\text{rot}}} \right]^2 \left[ \frac{d}{\text{1kpc}} \right] .$$

# Elastic deformations

Ellipticity depends on the internal structure of the NS, i.e. on its EoS  
Various models, including breaking limits of the NS crust (Owen, 2005)

$$\epsilon_{max}^{NS} = 3.4 \times 10^{-7} \left( \frac{\sigma_{max}}{10^{-2}} \right) \left( \frac{1.4 M_{\odot}}{M} \right)^{2.2} \left( \frac{R}{10 \text{ km}} \right)^{4.26} \left[ 1 + 0.7 \left( \frac{M}{1.4 M_{\odot}} \right) \left( \frac{10 \text{ km}}{R} \right) \right]^{-1}$$

Which can be approximated as (Ushomirsky et al. 2000)

$$\epsilon_{max}^{NS} \approx 5 \times 10^{-7} \left( \frac{\sigma_{max}}{10^{-2}} \right).$$

$\sigma_{max}$  can go from  $10^{-5}$  to 0.1, therefore leading to values  $< 5 \times 10^{-6}$

We can also consider more exotic strange stars, made of deconfined quarks (e.g. Itoh 1970), but their existence is still debated.

Ellipticity larger t

$$\epsilon_{max}^{SS} = 2 \times 10^{-4} \left( \frac{\sigma_{max}}{10^{-2}} \right) \left( \frac{1.4 M_{\odot}}{M} \right)^3 \left( \frac{R}{10 \text{ km}} \right)^3 \left[ 1 + 0.14 \left( \frac{M}{1.4 M_{\odot}} \right) \left( \frac{10 \text{ km}}{R} \right) \right]^{-1}$$

**Powerful test!**

# Magnetically-induced deformations

High magnetic fields are believed to deform the structure of NSs (since Chandraseckar & Fermi 1954). One simplified model using inclination angle  $\chi$  (Bonazzola&Gourgoulhon 1996) suggest that:

$$h_0 = 6.48 \times 10^{-30} \frac{\beta}{\sin^2(\chi)} \left( \frac{R}{10 \text{ km}} \right)^2 \left( \frac{1 \text{ kpc}}{d} \right) \left( \frac{1 \text{ ms}}{P} \right) \left( \frac{\dot{P}}{10^{-13}} \right)$$

Leading to ellipticities

$$\epsilon \approx A \left( \frac{R}{10 \text{ km}} \right)^4 \left( \frac{M}{1.4 M_\odot} \right)^{-2} \left( \frac{\bar{B}}{10^{12} \text{ G}} \right)^2$$

Considering a mix of poloidal and toroidal component in the B-field (Mastrano et al. 2011)

$$\epsilon_{mixed} \approx 4.5 \times 10^{-7} \left( \frac{B_{poloidal}}{10^{14} \text{ G}} \right)^2 \left( 1 - \frac{0.389}{\Xi} \right)$$

# Spin-down limit

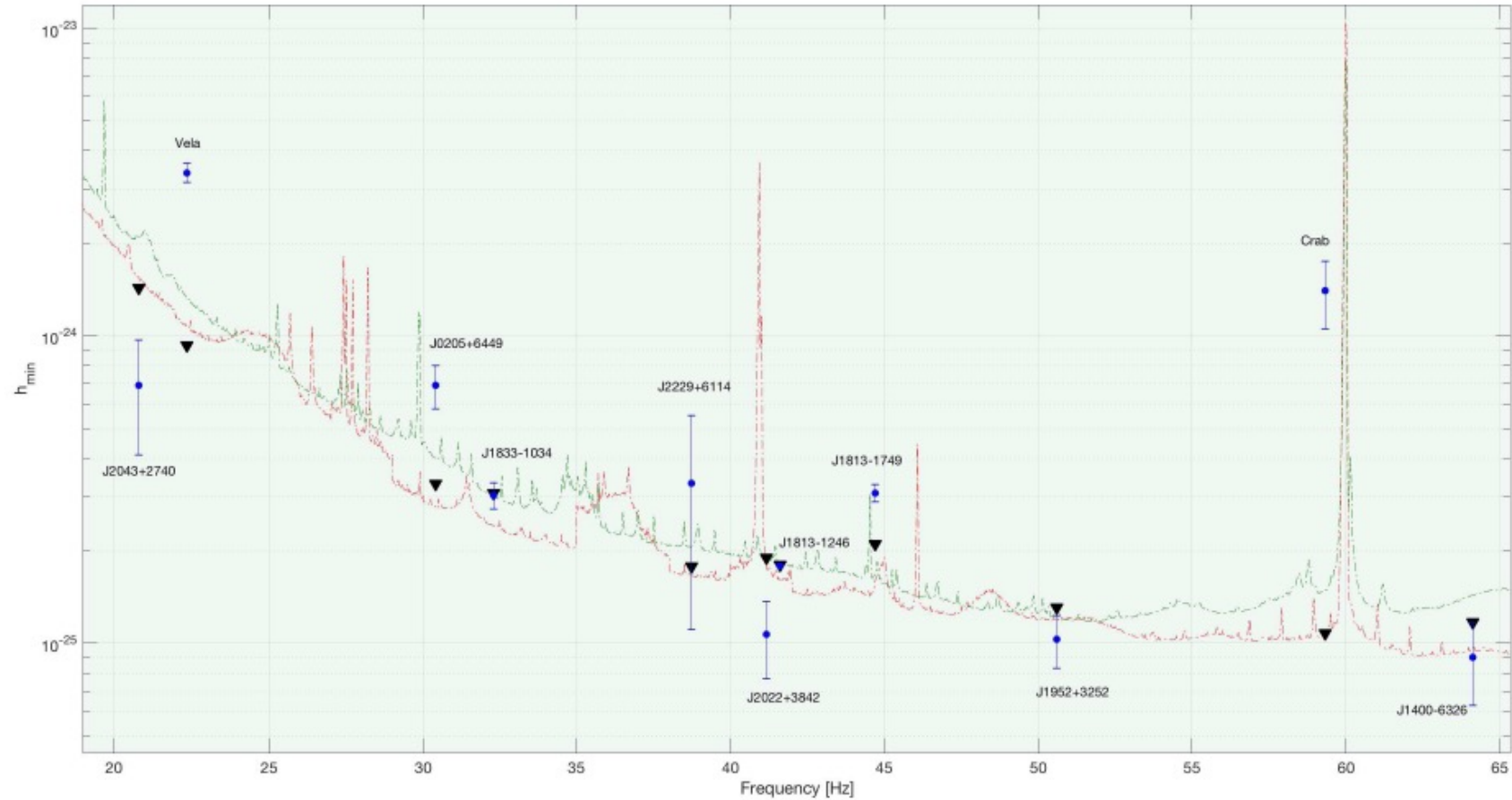


FIG. 2. Blue points: Value of the theoretical spin-down limit computed for the 11 known pulsars in our analysis, corresponding to Tab. I, error bars correspond to  $1\sigma$  confidence level. Black triangles: median over the analysed frequency band of the upper-limits on the GW amplitude, corresponding to Tab. IV. Red dashed line: Estimated sensitivity at 95% confidence level of a narrow-band search using data from LIGO H. Green dashed line: Estimated sensitivity at 95% confidence level of a narrow-band search using data from LIGO L.

Abbott+17



# Spin-down limit

TABLE I.

Distance and spin-down limit on the GW amplitude and ellipticity for the 11 selected pulsars. Distance and spin-down limit uncertainties refer to  $1\sigma$  confidence level.

Name	distance[kpc]	$h_{sd} \cdot 10^{-25}$	$\epsilon_{sd} \cdot 10^{-4}$
J0205+6449 <sup>a</sup>	$2.0 \pm 0.3^b$	$6.9 \pm 1.1$	14
J0534+2200 (Crab)	$2.0 \pm 0.5^c$	$14 \pm 3.5$	7.6
J0835-4510 (Vela)	$0.28 \pm 0.02^c$	$34 \pm 2.4$	18
J1400-6326	$10 \pm 3^d$	$0.90 \pm 0.27$	2.1
J1813-1246	$> 2.5^e$	$< 1.8$	$< 2.4$
J1813-1749	$4.8 \pm 0.3^f$	$3.0 \pm 0.2$	7.0
J1833-1034	$4.8 \pm 0.4^g$	$3.1 \pm 0.3$	13
J1952+3252	$3.0 \pm 0.5^h$	$1.0 \pm 0.2$	1.1
J2022+3842	$10 \pm 2^i$	$1.0 \pm 0.3$	6.0
J2043+2740	$1.5 \pm 0.6^j$	$6.9 \pm 2.8$	23
J2229+6114	$3.0 \pm 2^c$	$3.4 \pm 2.2$	6.2

<sup>a</sup> This pulsar had a glitch on November 11th 2015

<sup>b</sup> Distance from neutral Hydrogen absorption of pulsar wind nebula 3C 58 [14]

<sup>c</sup> Distance taken from independent measures reported in ATNF catalog, see text for references

<sup>d</sup> Distance from dispersion measures [21]

<sup>e</sup> Lower limit of [15]

<sup>f</sup> Distance from Chandra and XMM-Newton from [22]

<sup>g</sup> Distance from Parkes telescope [23]

<sup>h</sup> Distance from kinematic distance of the associated supernova remnant [12]

<sup>i</sup> Distance of the hosting supernova remnant [25]. In some papers a distance value of  $\sim 10$  kpc is considered [24].

<sup>j</sup> Distances taken from v1.56 of the ATNF Pulsar Catalog[11]

Name	$h_{ul} \cdot 10^{-25}$	$\epsilon_{ul} \cdot 10^{-4}$	$h_{ul}/h_{sd}$	$\dot{E}_{rot}/\dot{E}_{GW}$
J0205+6449	3.76	7.7	$0.54 \pm 0.09$	0.29
J0534+2200 (Crab)	1.08	0.58	$0.07 \pm 0.02$	0.005
J0835-4510 (Vela)	9.28	5.3	$0.27 \pm 0.02$	0.07
J1400-6326	1.17	2.7	$1.3 \pm 0.4$	-
J1813-1246	1.80	2.5	$> 1.0$	-
J1813-1749	1.9	4.8	$0.64 \pm 0.04$	0.41
J1833-1034	3.08	13	$0.99 \pm 0.09$	-
J1952+3252	1.31	1.4	$1.31 \pm 0.22$	-
J2022+3842	1.90	11	$1.77 \pm 0.35$	-
J2043+2740	14.4	47	$2.07 \pm 0.83$	-
J2229+6114	1.78	3.4	$0.54 \pm 0.35$	0.30

# Torque balance

- Start from observational fact: no accreting pulsars with frequencies higher than 700 Hz (Patruno et al. 2010)
- However, transfer of angular momentum should lead to frequencies up to 1 KHz (break-up).
- Why this is not happening?
  - Pulsar is losing angular momentum by GW emission
- Balancing the GW emission with the accretion torque one can estimate the UL for continuous Gws (Wagoner 1984)

$$h_0^{\text{EQ}} = 5.5 \times 10^{-27} \frac{R_{10}^{3/4}}{M_{1.4}^{1/4}} \left( \frac{F_X}{F_\star} \right)^{1/2} \left( \frac{300 \text{ Hz}}{\nu} \right)^{1/2}$$

Brightest X-ray accreting sources should be loudest GW emitters

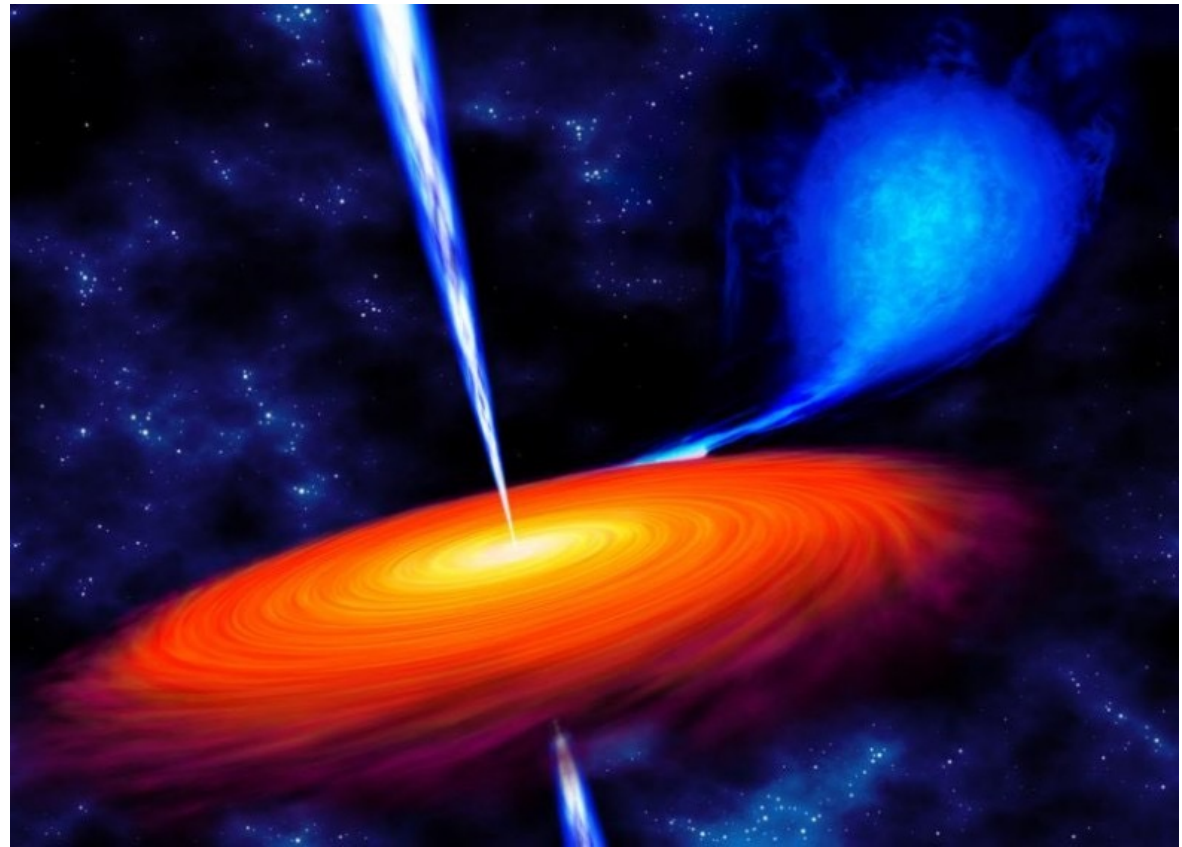
# Scorpius X-1

- Best candidate: brightest X-ray source in the sky
- Distance 2.8 kpc
- NS (unknown period) + low mass ( $1.4 M_{\text{sun}}$ ) star

- Expected  $h_0 = 3.5 \times 10^{-26} (f/300)^{1/2}$

- Best UL  $3.4 \times 10^{-25}$

(LIGO&Virgo, Abbott et al, 2019)



# Searching for GW pulsars

- Searches for continuous GW are a big challenge from computation point of view
  - Signal is very faint (fainter than CBC)
  - Signal is very long
  - Large amount of data to be searched (order of months-years)
- Depending on the pulsar we are looking for, we might know:
- Frequency evolution+sky location (targeted searches)
  - Sky location, NO frequency evolution (directed searches)
    - NO sky location, NO frequency evolution (all-sky searches)

# Searching for GW pulsars

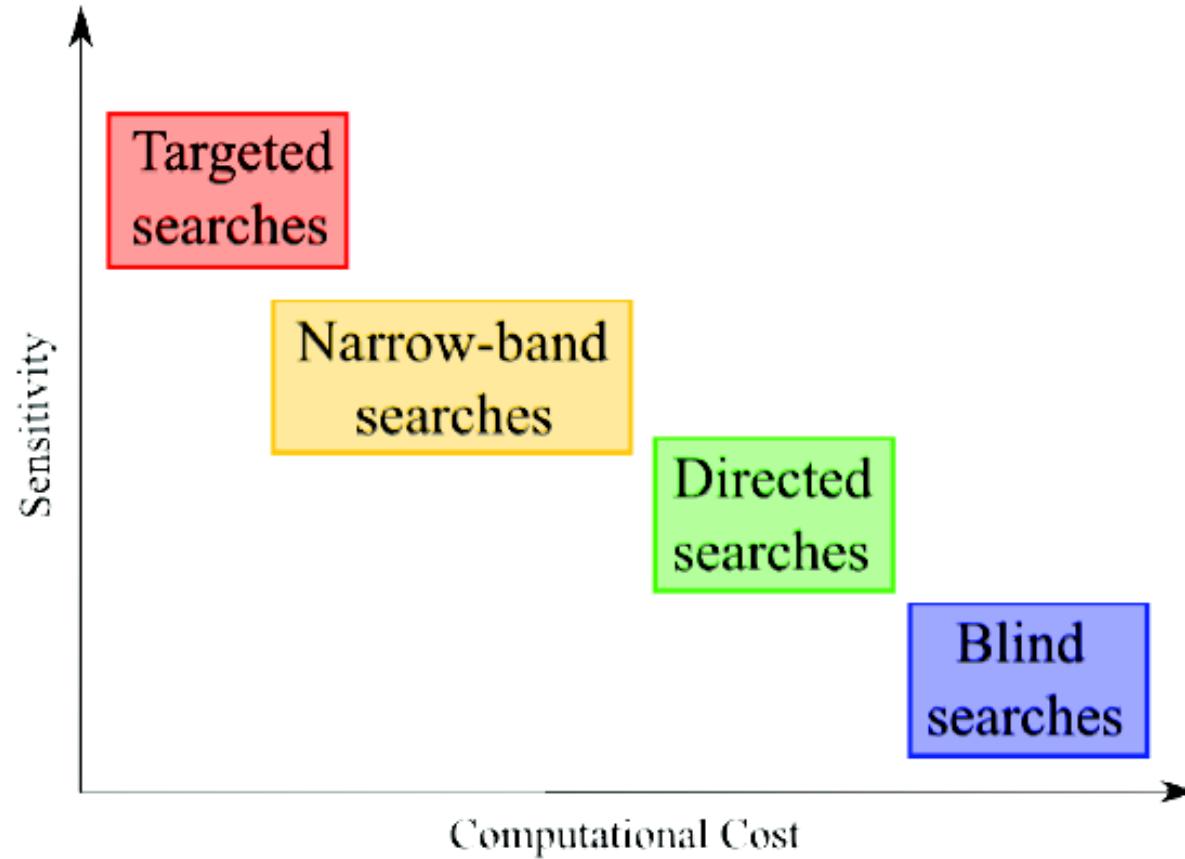


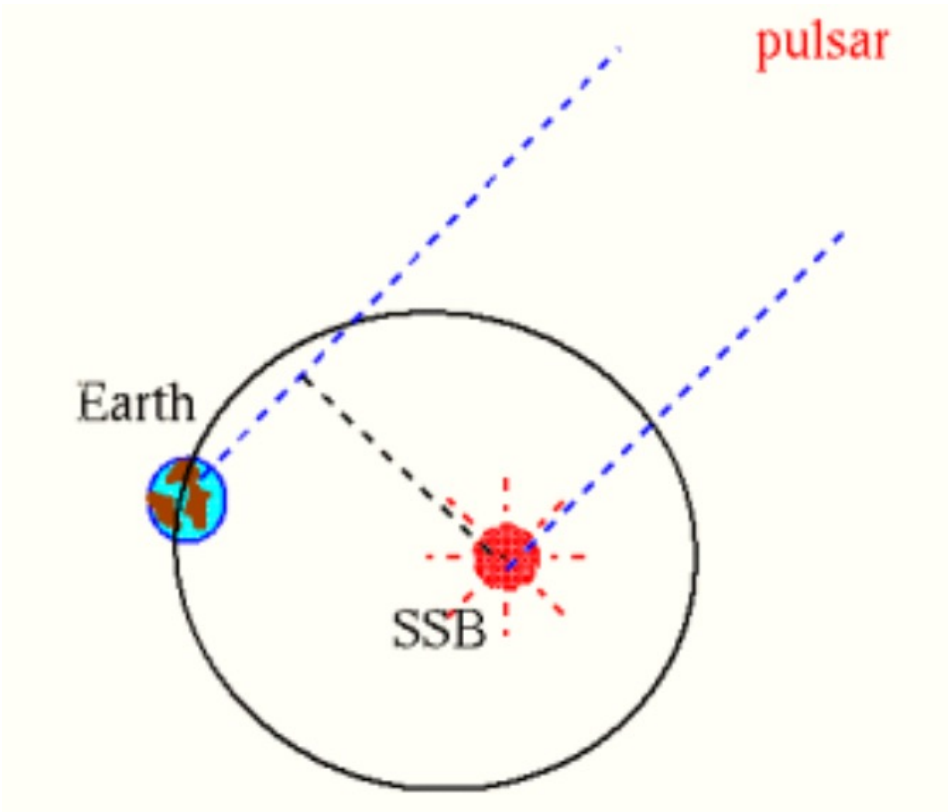
Figure 3: Schematic plot of the sensitivity versus computational cost of the different CGWs searches strategies.

# Barycentric corrections

The analysis procedure on pulsars starts by performing the **barycentering**, i.e. transform the photon arrival times at the spacecraft to the Solar System Barycenter, located near the surface of the Sun

Several effects that contribute to the barycentering, mainly:

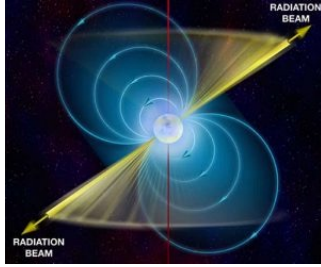
- ✓ Geometrical delays (due to light propagation);
- ✓ Relativistic effects (i.e "Shapiro delay" due to gravitational well of Sun)



Credits: N. Rea

$$T = t + \delta t = t + \Delta_{\text{Roemer}} + \Delta_{\text{Shapiro}} + \Delta_{\text{Einstein}} + \Delta_{\text{Binary}}$$

Annotations: 8.5min points to  $\Delta_{\text{Roemer}}$ , 120us points to  $\Delta_{\text{Shapiro}}$



# Pulsar timing models

*Pulsar period increases with time due to loss of rotational energy:*

We can Taylor expand  $P(t)$  or  $f(t)$

$$f(t) = \frac{1}{P(t)}$$

$$f_0 = f(t_0)$$

$$f_1 = \dot{f}(t_0)$$

$$f_2 = \ddot{f}(t_0)$$

Phase assignment in analysis:

• *# of rotations:*

$$dN = f(t) dt = \left[ f(t_0) + \dot{f}(t_0)(t - t_0) + \frac{1}{2} \ddot{f}(t_0)(t - t_0)^2 + \dots \right] dt$$

• *Integrating and taking the fractional part:*

$$\varphi(t) = \varphi(t_0) + f_0(t - t_0) + \frac{1}{2} f_1(t - t_0)^2 + \frac{1}{6} f_2(t - t_0)^3 + \dots$$

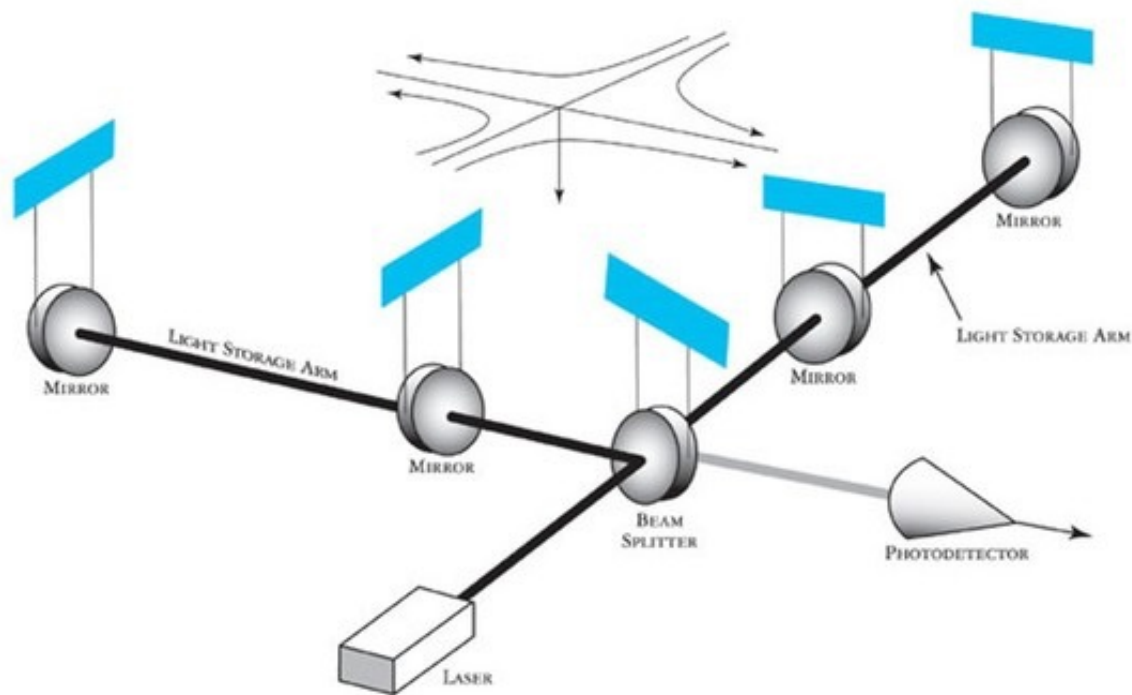

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The timing parameters come:

- from radio astronomers (if the pulsar is radio quiet)
- From gamma rays only (if is a blind search pulsar)

# Interlude: Antenna pattern function

- GW detectors are antenna and therefore have a specific response



From  $h_+(t)$  and  $h_\times(t)$ , how to get the measured strain at the detector?  
We need to use the **Antenna Pattern Functions**  $F_+$  and  $F_\times$  that depend on the polarization angle  $\psi$  and on the sky position

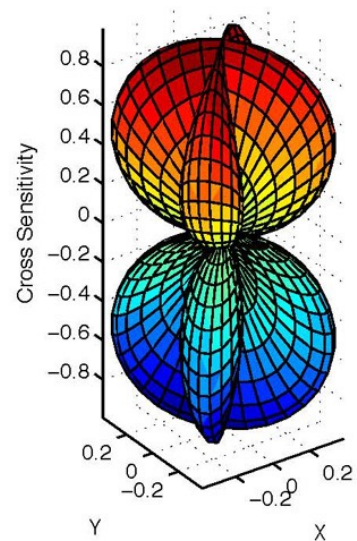
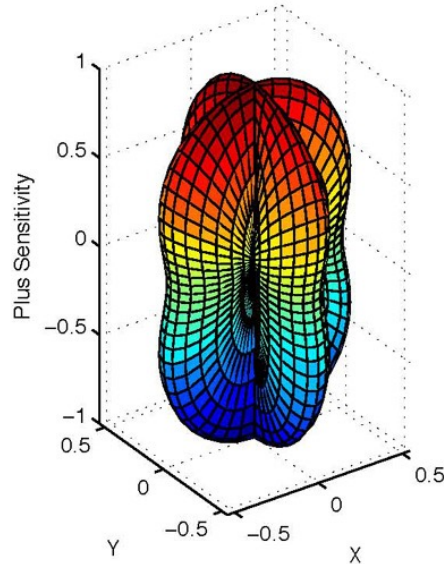
$$h(t) = \frac{1}{2}F_+(t; \psi)h_0(1 + \cos^2\iota) \cos\Phi(t) + F_\times(t; \psi)h_0 \cos\iota \sin\Phi(t),$$

Credits:  
Caltech/MIT/LIGO Lab



# Interlude: Antenna pattern function

- GW detectors are antenna and therefore have a specific response



From  $h_+(t)$  and  $h_x(t)$ , how to get the measured strain at the detector?  
We need to use the **Antenna Pattern Functions**  $F_+$  and  $F_x$  that depend on the polarization angle  $\psi$  and on the sky position

# Searching for GW pulsars

- If the frequency, derivatives and position are known (targeted or narrow-band), one first method is based on the heterodyne (Wohan&Dupuis 2005)

From rotating NS we expect a signal:

$$h(t) = \frac{1}{2}F_+(t; \psi)h_0(1 + \cos^2\iota) \cos\Phi(t) \\ + F_\times(t; \psi)h_0 \cos\iota \sin\Phi(t),$$

Where:

$$\Phi(T) = \phi_0 + 2\pi[f_s(T - T_0) + \frac{1}{2}\dot{f}_s(T - T_0)^2 \\ + \frac{1}{6}\ddot{f}_s(T - T_0)^3],$$

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{I_{zz} f_r^2}{r} \epsilon$$

# Searching for GW pulsars

- We can use Euler's formulae to recast the equation:

$$h(t) = A(t)e^{i\Phi(t)} + A^*(t)e^{-i\Phi(t)}$$

With:

$$A(t) = \frac{1}{4}F_+(t; \psi)h_0(1 + \cos^2\iota) - \frac{i}{2}F_\times(t; \psi)h_0 \cos\iota$$

If we multiply this by  $e^{-i\phi(t)}$  the signal + noise  $s(t) = h(t) + n(t)$  now becomes:

$$\begin{aligned} s_{\text{het}}(t) &= s(t)e^{-i\phi(t)} \\ &= A(t)e^{i\phi_0} + A^*(t)e^{-i\phi_0 - 2i\phi(t)} + n(t)e^{-i\phi(t)} \end{aligned}$$

We can then keep the first term (low-frequency,  $T \sim 1$  day) and resample to lower frequencies ( $\rightarrow$  less data), and check if there is a significant signal

# Searches in O3

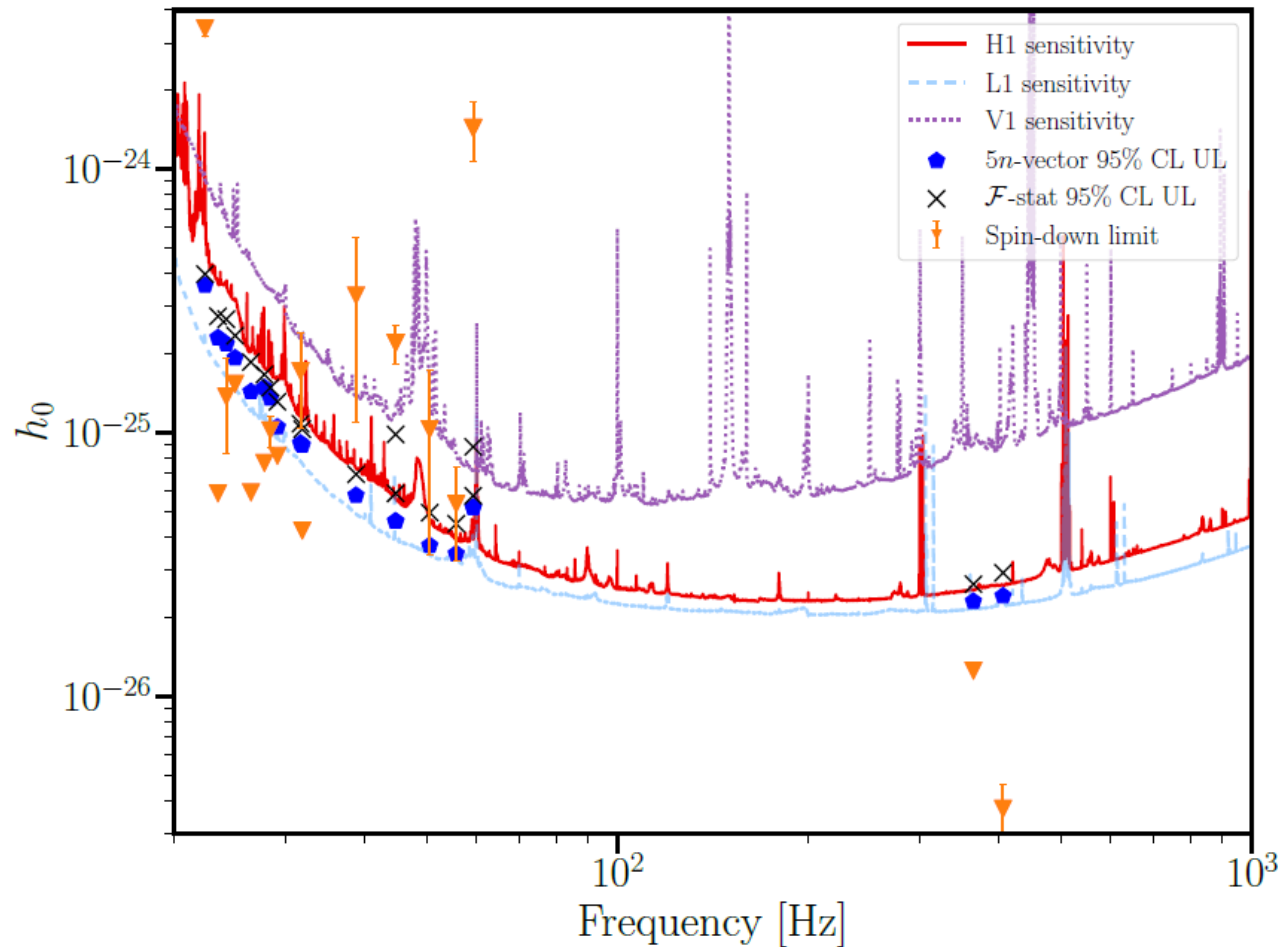
- Targeted and narrowband searches

Name	R.A.	DEC.	$f$	$\Delta f$ (5v)	$\Delta f$ ( $\mathcal{F}$ )	$\dot{f}$	$\Delta \dot{f}$ (5v)	$\Delta \dot{f}$ ( $\mathcal{F}$ )	$\ddot{f}$	$\Delta \ddot{f}$	$n_{\text{total}}^{5v}$	$n_{\text{total}}^{\mathcal{F}}$	Ref.
			Hz	Hz	Hz	$\times 10^{-13}$ Hz s $^{-1}$	$\times 10^{-15}$ Hz s $^{-1}$	$\times 10^{-15}$ Hz s $^{-1}$	$\times 10^{-23}$ Hz s $^{-2}$	$\times 10^{-23}$ Hz s $^{-2}$	$\times 10^7$	$\times 10^7$	
J0534+2200 bg	05 <sup>h</sup> 34 <sup>m</sup> 31.97 <sup>s</sup>	+22°00'52.07''	59.241	...	0.24	-7370.0	...	2900.0	2360.0	5.2	...	983.7	a
J0534+2200 ag	05 <sup>h</sup> 34 <sup>m</sup> 31.97 <sup>s</sup>	+22°00'52.07''	59.241	0.24	0.24	-7370.0	2300.0	2900.0	2360.0	5.2	498.0	9362.0	a
J0711-6830	07 <sup>h</sup> 11 <sup>m</sup> 54.18 <sup>s</sup>	-68°30'47.37''	364.234	0.72	1.5	-0.00989	2.1	0.004	0.0	0.0	4.391	45.85	b
J0835-4510	08 <sup>h</sup> 35 <sup>m</sup> 20.52 <sup>s</sup>	-45°10'34.28''	22.371	0.089	0.089	-313.0	130.0	120.0	504.0	0.0	32.9	281.6	c
J1101-6101	11 <sup>h</sup> 01 <sup>m</sup> 44.96 <sup>s</sup>	-61°01'39.6''	31.846	0.13	0.13	-45.3	19.0	18.0	0.0	0.0	7.026	61.52	d
J1105-6107	11 <sup>h</sup> 05 <sup>m</sup> 25.71 <sup>s</sup>	-61°07'55.63''	31.644	...	0.13	-79.7	...	32.0	554.0	35.0	...	266.0	e
J1809-1917	18 <sup>h</sup> 09 <sup>m</sup> 43.13 <sup>s</sup>	-19°17'38.2''	24.166	0.097	0.097	-74.4	32.0	30.0	3.7	0.14	8.886	152.3	a
J1813-1749 bg	18 <sup>h</sup> 13 <sup>m</sup> 35.11 <sup>s</sup>	-17°49'57.57''	44.703	...	0.18	-1290.0	...	510.0	0.0	0.0	...	92.08	d
J1813-1749 full	18 <sup>h</sup> 13 <sup>m</sup> 35.11 <sup>s</sup>	-17°49'57.57''	44.703	0.18	0.18	-1290.0	520.0	510.0	0.0	0.0	266.4	2270.0	d
J1828-1101	18 <sup>h</sup> 28 <sup>m</sup> 18.85 <sup>s</sup>	-11°01'51.72''	27.754	0.11	0.11	-57.0	23.0	23.0	4.23	9.4	7.481	1162.0	a
J1833-0827	18 <sup>h</sup> 33 <sup>m</sup> 40.26 <sup>s</sup>	-08°27'31.53''	23.449	0.094	0.094	-25.2	11.0	10.0	-0.463	0.082	2.873	242.2	e
J1838-0655	18 <sup>h</sup> 38 <sup>m</sup> 3.13 <sup>s</sup>	-06°55'33.4''	28.363	0.11	0.11	-199.0	83.0	80.0	67.1	7.8	27.13	555.4	d
J1856+0245	18 <sup>h</sup> 56 <sup>m</sup> 50.91 <sup>s</sup>	+02°45'53.17''	24.714	0.099	...	-189.0	79.0	...	0.0	0.0	22.42	...	a
J1913+1011	19 <sup>h</sup> 13 <sup>m</sup> 20.34 <sup>s</sup>	+10°11'22.97''	55.694	0.22	0.22	-52.5	23.0	21.0	-0.321	2.6	15.02	1951.0	a
J1925+1720	19 <sup>h</sup> 25 <sup>m</sup> 27.06 <sup>s</sup>	+17°20'27.42''	26.434	0.11	0.11	-36.6	15.0	15.0	3.93	6.3	4.538	469.2	a
J1928+1746	19 <sup>h</sup> 28 <sup>m</sup> 42.55 <sup>s</sup>	+17°46'29.67''	29.097	0.12	0.12	-55.7	23.0	22.0	0.0	0.0	7.845	68.41	a
J1935+2025	19 <sup>h</sup> 35 <sup>m</sup> 41.94 <sup>s</sup>	+20°25'40.1''	24.955	0.092	0.1	-189.0	79.0	76.0	95.0	3.4	20.99	581.3	a
J1952+3252	19 <sup>h</sup> 52 <sup>m</sup> 58.21 <sup>s</sup>	+32°52'40.51''	50.587	0.2	0.2	-74.9	32.0	30.0	2.92	0.012	18.61	3908.0	f
J2124-3358	21 <sup>h</sup> 24 <sup>m</sup> 43.84 <sup>s</sup>	-33°58'45.06''	405.588	0.91	1.6	-0.0169	2.1	0.0068	0.0	0.0	5.595	48.06	g
J2229+6114	22 <sup>h</sup> 29 <sup>m</sup> 6.57 <sup>s</sup>	+61°14'10.9''	38.709	0.15	0.16	-590.0	240.0	260.0	1170.0	0.0	107.3	1021.0	f

Abbot et al 2021,  
arXiv 2112.10990

# Searches in O3

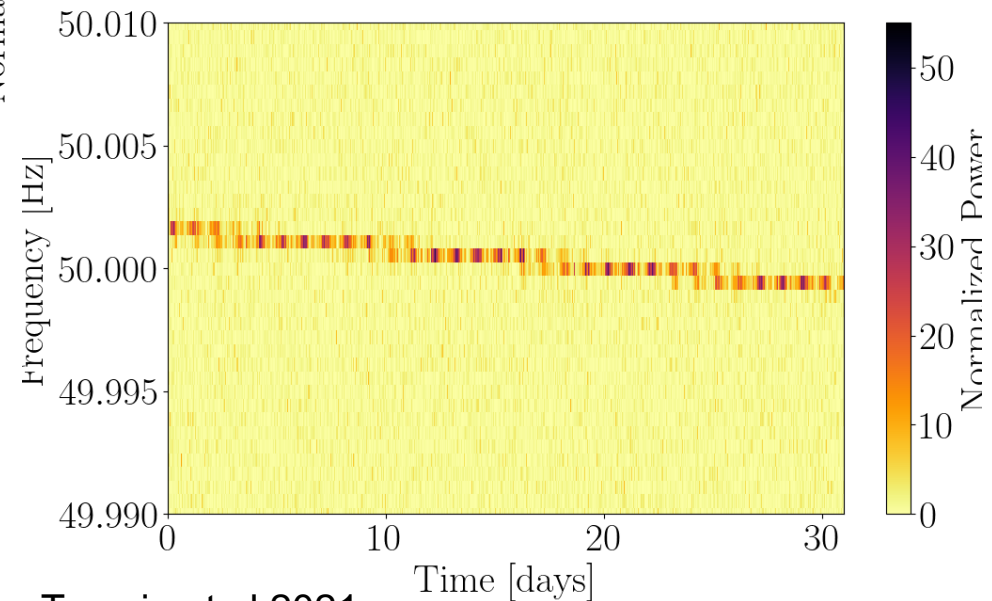
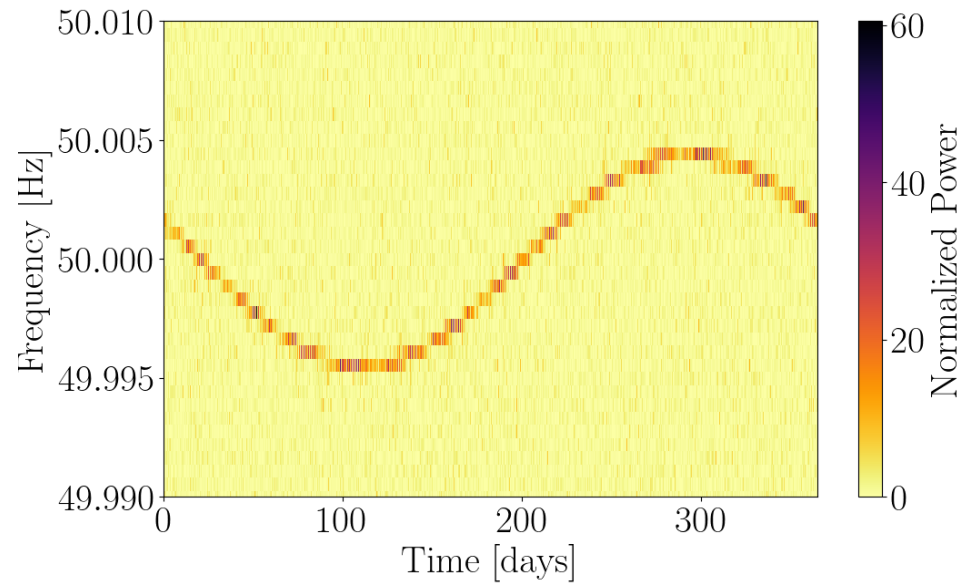
- 2 Methods based on F-statistics and multiple harmonics coherent search



Abbot et al 2021,  
arXiv 2112.10990

# Searches for unknown pulsars

- When location and/or frequency are not known, we follow a different strategy:
  - All-sky searches (wide parameter space, no location nor timing)
  - Spot-light searches (small sky regions)
  - Directed searches (location known, timing unknown)



Tenorio et al 2021

# Searches for unknown pulsars

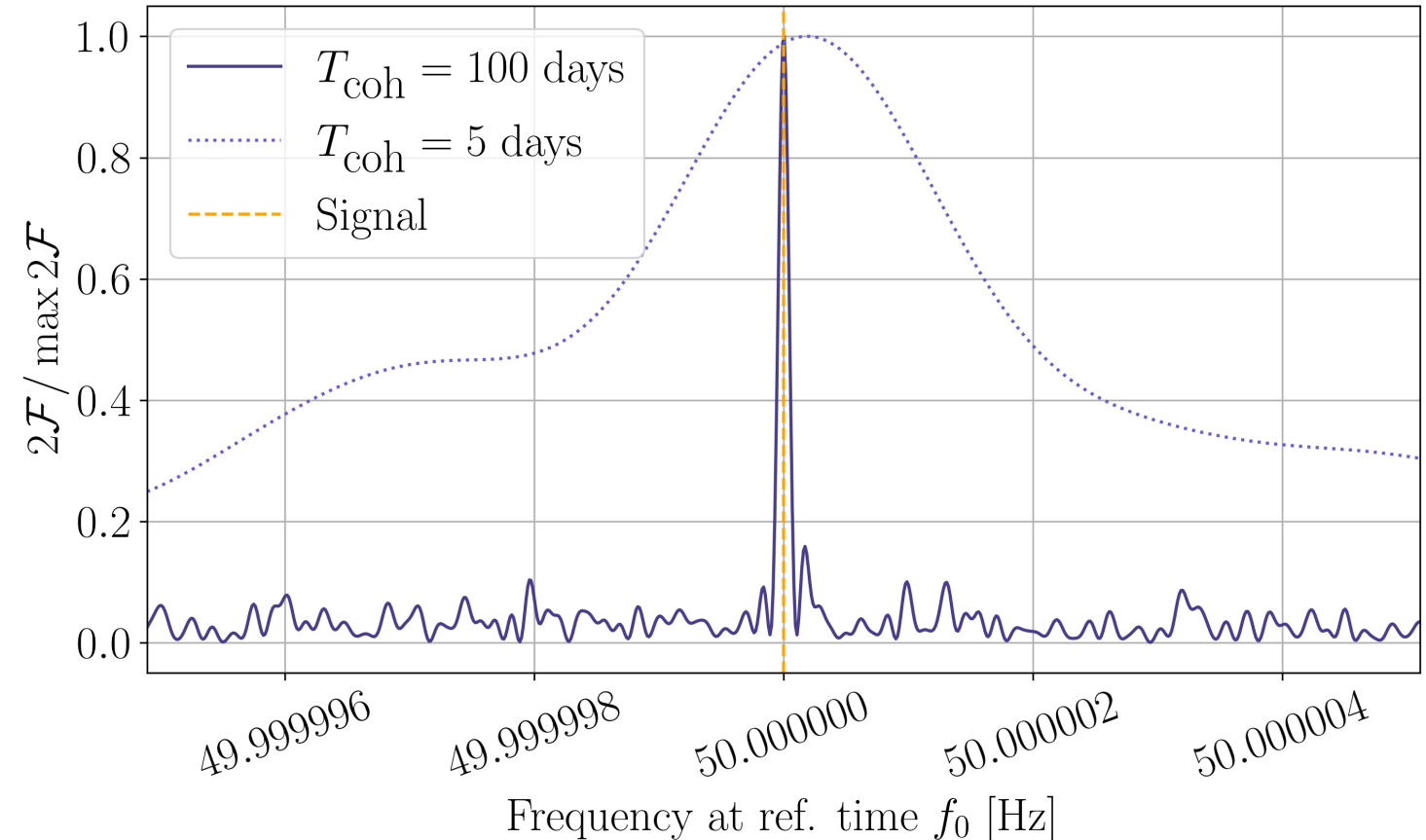
Two approaches can be followed

- Coherent searches
- Semicoherent searches (combination of coherent searches)

**Hierarchical  
Approach**

**Searches based on**

- Fourier transform
- F-stat
- Other methods (MCM, Hough transform, machine learning)



# Searches for unknown pulsars

Table 2. Summary of CW search methods covered by the present review, grouped by scope and observing run.

Search	Pipeline	References
All-sky O1	Einstein@Home	[40]
	Falcon	[43,44]
	FrequencyHough	[41]
	PowerFlux	[41,42]
	SkyHough	[41,42]
All-sky O2	Time-domain $\mathcal{F}$ -statistic	[41,42]
	BinarySkyHough	[50]
	Einstein@Home	[49]
	Falcon	[46–48]
	FrequencyHough	[45]
All-sky O3a	SkyHough	[45]
	Time-domain $\mathcal{F}$ -statistic	[45]
	BinarySkyHough	[52]
	PowerFlux	[54]
Deep exploration O2	Weave	[51]
GC O1	PowerFlux	[55]
GC O2	FrequencyHough + BSD	[56]
SNR O1	Einstein@Home	[61]
SNR O2	Fully-coherent $\mathcal{F}$ -statistic	[59]
	Einstein@Home	[62,69]
SNR O3a	Fully-coherent $\mathcal{F}$ -statistic	[66]
	Viterbi 1.0	[63]
	Viterbi SNR	[65]
	FrequencyHough + BSD	[68]
	Dual-harmonic Viterbi	[68]
Viterbi SNR	[68]	
CDOs in the Solar System O2	Excess power	[9]
Cygnus X-1 O2	Viterbi 1.0	[16]
Scorpius X-1 O1	Cross-Correlation	[58]
	Viterbi 1.0	[57]
Scorpius X-1 O2	Cross-Correlation	[67]
	Viterbi 2.0	[60]
LMXBs O2	Viterbi 2.0	[64]

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# Results from O3 – all sky search

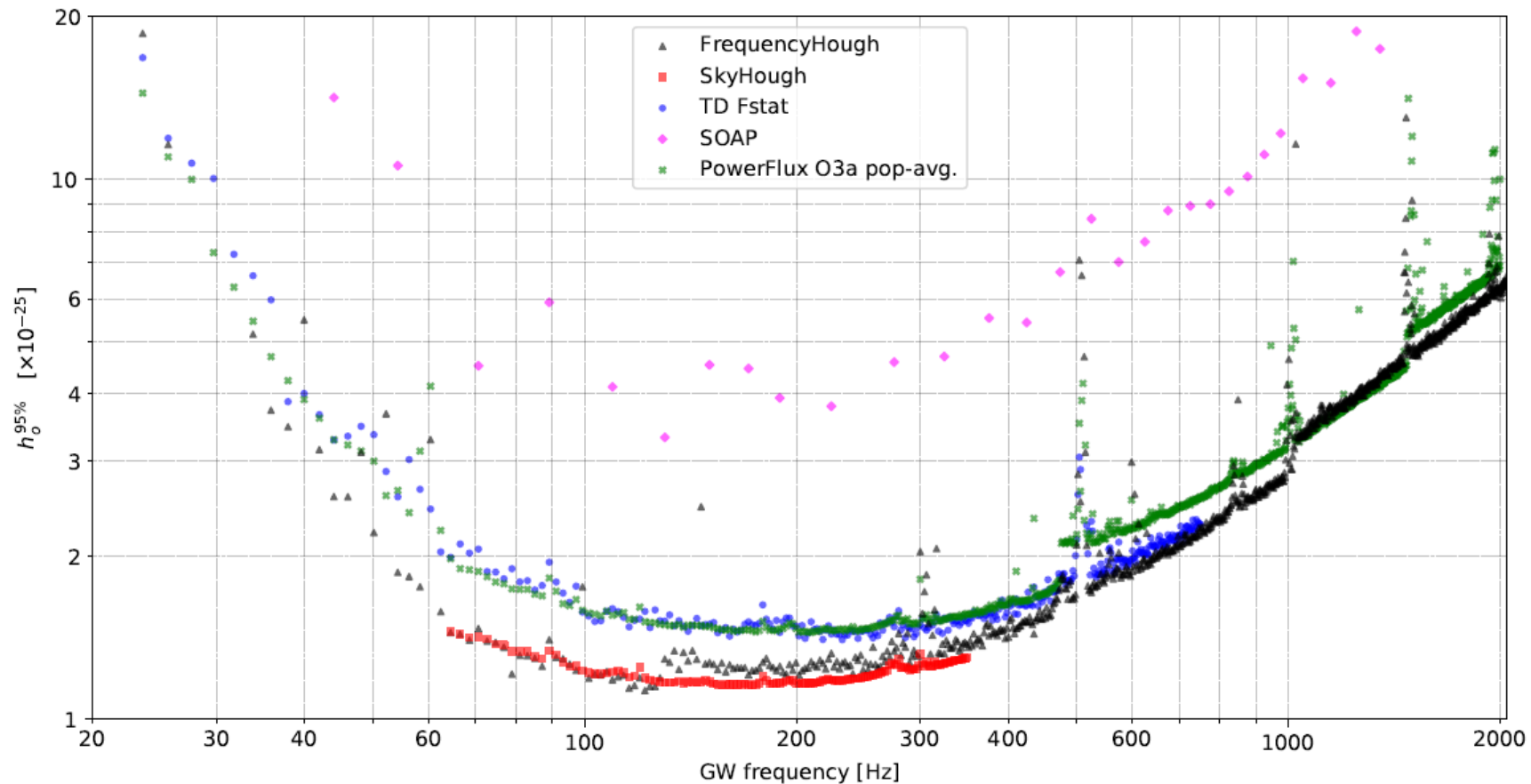


FIG. 15. Comparison of 95% confidence upper limits on GW amplitude  $h_0$  obtained by the *FrequencyHough* pipeline (black triangles), the *SkyHough* pipeline (red squares), the *Time-Domain  $\mathcal{F}$ -statistic* pipeline (blue circles), and the *SOAP* pipeline (magenta diamonds). Population-averaged upper limits obtained in [101] using the O3a data are marked with dark-green crosses. To enhance visibility, we do not show the error estimates of  $h_0$  in this plot; additionally, the data is divided in 2 Hz bins, and the median of  $h_0$  values within each bin is presented.

# Pulsar glitches

- Glitches are abrupt changes in spin characteristics of a pulsar
- They can be detected and followed over the “recovery” time
- Starquakes?
- Fundamental to understand interiors of NS

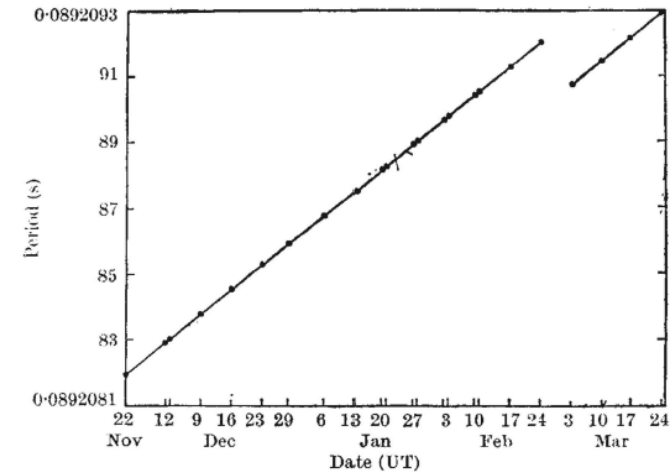
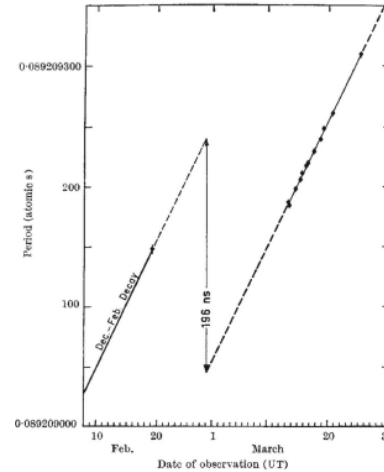
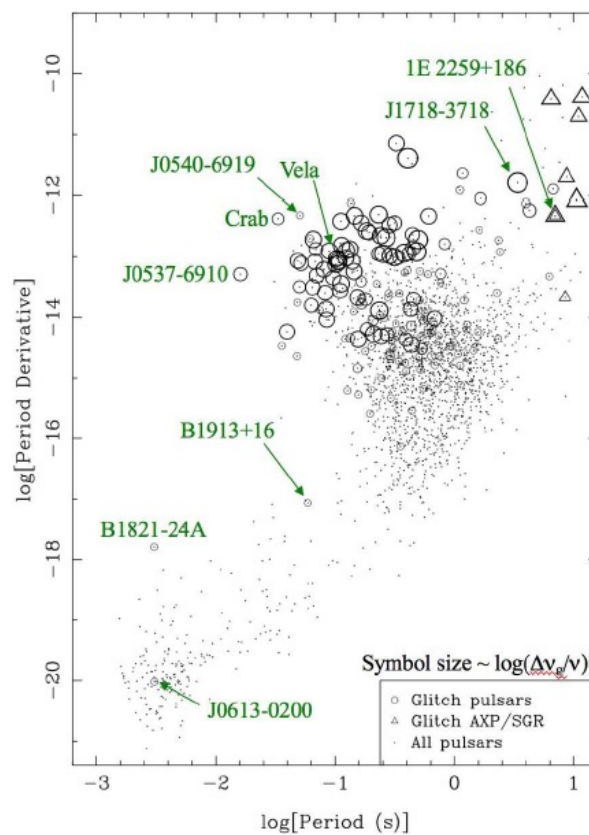


Figure 1. Changes in the Vela pulsar period in late 1968 and early 1969 showing the first detections of a pulsar glitch. The left panel shows the Parkes observations (Radhakrishnan & Manchester 1969) and the right panel shows the JPL Goldstone observations (Reichley & Downs 1969).

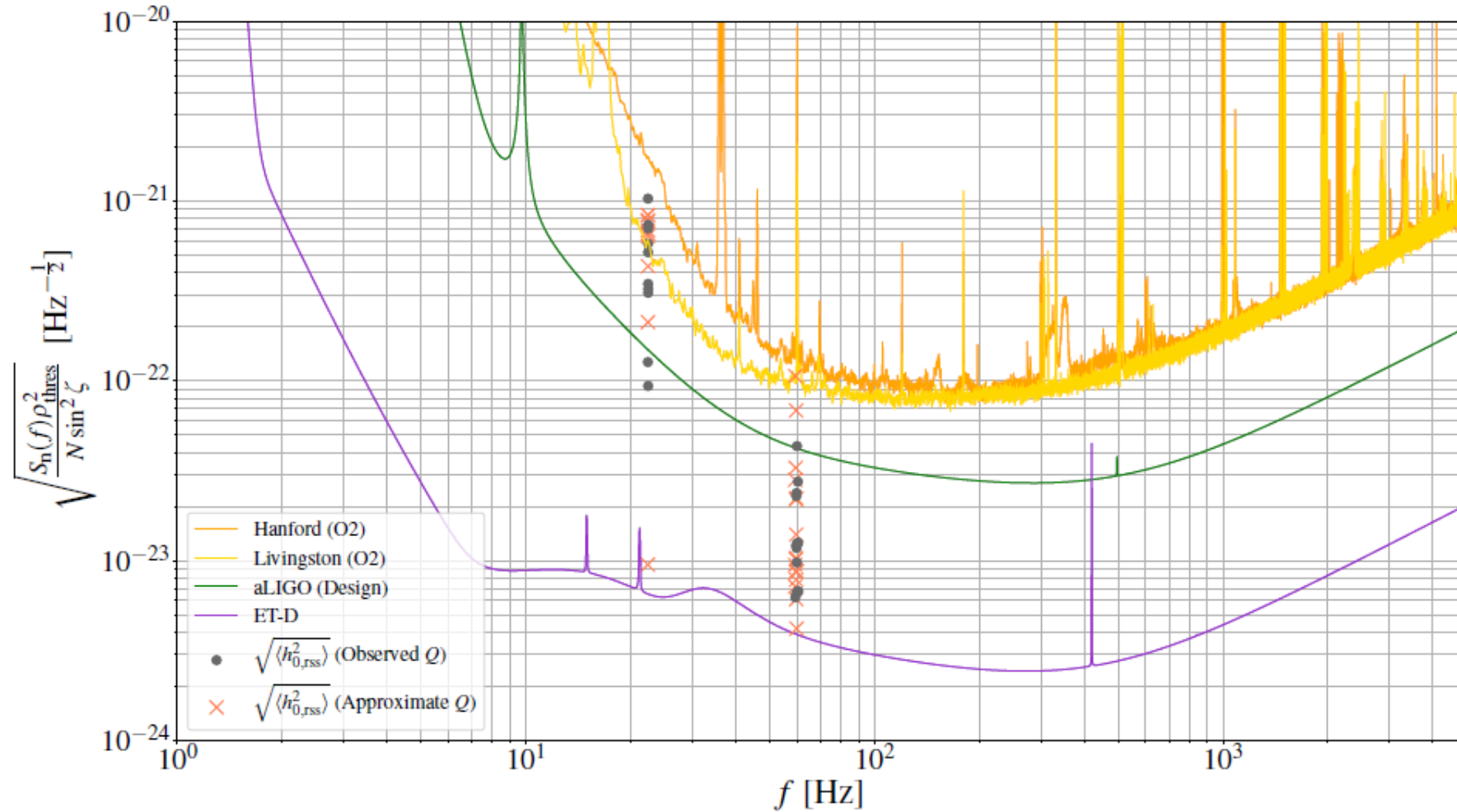
For GWs, they can reduce the coherence time  
Required for a search

## Expected GW Signals

- Short GW burst
- Long transient post-glitch

# Glitches & GWs

Test using the Vela and Crab pulsars glitches





**Thank you!**

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