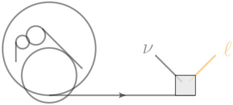
Neutrino Physics (Theory) – 1

2024 BND school, Blankenberge, België

Richard Ruiz

Institute of Nuclear Physics – Polish Academy of Science (IFJ PAN)

2 September 2024



a few plesantries

most important: these lectures are low-key; questions are great

I am literally here to tell you what I know

apologies and disclaimers

Lectures are "Summer School" style

- More material/slides than allowed by time
- Some slides will be skipped (kept for completeness)
- NOT an historical summary (see νExpt lectures by de Roeck)

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Goal: fill in some gaps between courses and research

- Explain what goes into plots often shown in seminars & conferences
- Healthy mixture of math and plots (ν physics is rigorous physics)
- Personally, I have never seen some of the following in a lecture

(sorry also for the typos!)

Lecture Plan (one-day show!)

Lecture I:

- Pt1: The **Standard Model (SM)** neutrino
- Pt2: The neutrino that nature gave us: intro to ν oscillations

Coffee break at 10:30ish

Lecture II:

- Pt1. Consequences of neutrino masses (theory perspective)
- Pt2. Neutrino mass models (highlights)

Lunch at 12:30ish

Pt1. the Standard Model neutrino

Particle Physics: Then and Now

Throughout the 20th century, a chief goal of particle physics was to establish the **particle spectrum**, their **structures**, and their **properties**

possible with many tools, e.g., production at colliders, tabletop measurements of fundamental symm., and rare decays

credit: I. Bigaran

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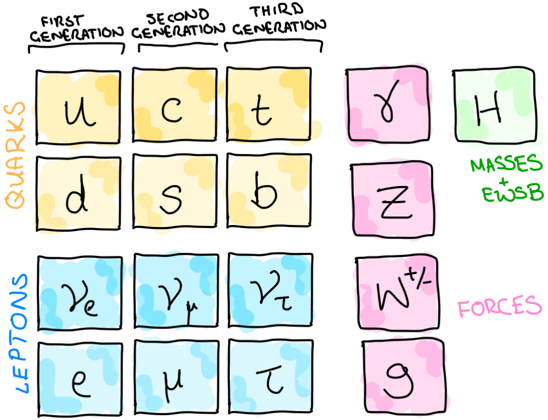
The Standard Model (SM) of particle physics

– position indicates quantum numbers/ charges

(just like in chemistry!)

– e.g., spin, flavor, color, electromagnetic, weak hyper charge



credit: I. Bigaran

Position makes quantum numbers, e.g., gauge charges, manifest

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = \overbrace{\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}}^{T_L^3}$$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$$

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$$Q_f = T^3_{Lf} + \frac{1}{2} Y_f \implies Y_{Q_L} = +1/3$$
$$Q_f = T^3_{Lf} + \frac{1}{2} Y_f \implies Y_{L_L} = -1$$

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Sanity: $(Q^{\rm upper} - Q^{\rm lower}) = (T_L^{\rm upper} - T_L^{\rm lower}) = +1$

Sanity: $(2Nc) \cdot Y_{Q_L} + 2Y_{L_L} = 0$

Exercise: show that $N_c \cdot Y_{u_R} + N_c \cdot Y_{d_R} + Y_{e_R} = 0$

Position makes quantum numbers, e.g., gauge charges, manifest

Species	Symbol	$SU(3)_C \times SU(2)_L \times U(1)_Y$ Rep.	$U(1)_{EM}$ Charge [Units of $e > 0$]
Quark	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})$	$\begin{pmatrix} +2/3 \\ -1/3 \end{pmatrix}$
Quark	u_R	$(\mathbf{3}, \mathbf{1}, +\frac{4}{3})$	$+2/3$
Quark	d_R	$(\mathbf{3}, \mathbf{1}, -\frac{2}{3})$	$-1/3$
Lepton	$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1)$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
Lepton	e_R	$(\mathbf{1}, \mathbf{2}, -2)$	-1

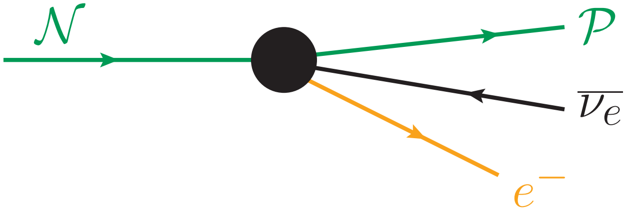
technical note: here, fermions are states in the **gauge/interaction basis** ($\neq$ mass basis)

– not consistent to assign masses need to rotate into mass basis!

question: how do we know that ν carries weak charges?

a few steps back

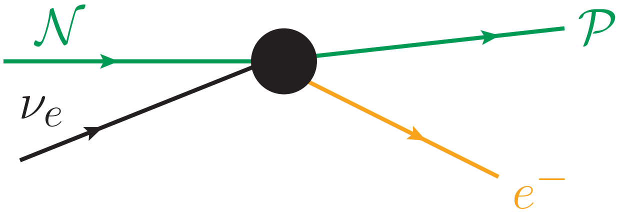
Nuclear β decay is governed by Fermi Theory



$$\mathcal{L}_{\rm Fermi} = G_F \left[\overline{\mathcal{N}}\gamma^\mu P_L \mathcal{P}\right] \cdot \left[\overline{\nu_e}\gamma_\mu P_L e\right]$$

Fermi('31)

Inverting ν_e leg $\Longrightarrow$ inverse β decay **("elastic" ν-nucleus scattering)**



$$-i\mathcal{M}(\nu_e\mathcal{N} \to e^-\mathcal{P}) \sim G_F\ \left[\overline{u}(k_{\mathcal{P}})\gamma^\mu P_L u(k_{\mathcal{N}})\right]\cdot\left[\overline{u}(k_e)\gamma_\mu P_L u(k_{\nu_e})\right] \sim G_F\ E_\nu^2$$

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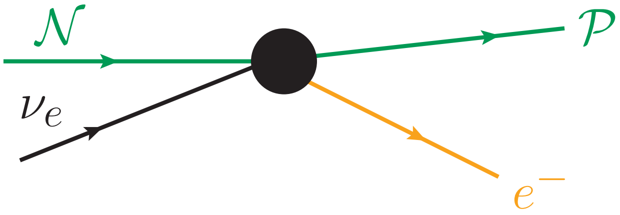

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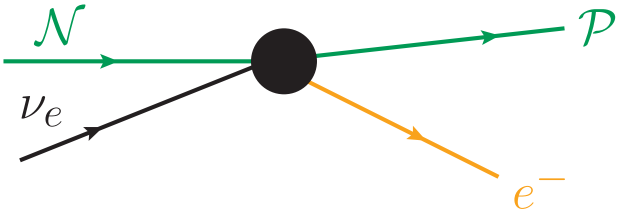
$$\Longrightarrow \sigma(\nu_e\mathcal{N} \to e^-\mathcal{P}) \sim \frac{1}{(\text{flux})} \ \not{\Sigma}_{\text{dof}} \ (\text{phase space}) \times |\mathcal{M}|^2 \sim G_F^2 \frac{E_\nu^4}{\pi E_\nu^2}$$

Inverting ν_e leg $\Longrightarrow$ inverse β decay **(“elastic” ν-nucleus scattering)**



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$\Longrightarrow$ **scatt. rate (σ) grows with scatt. energy without bound**

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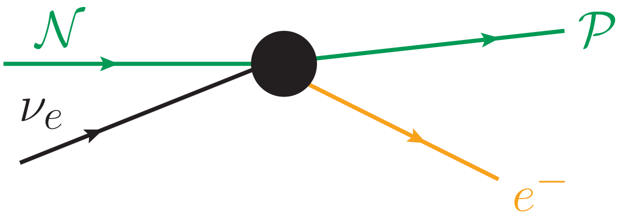

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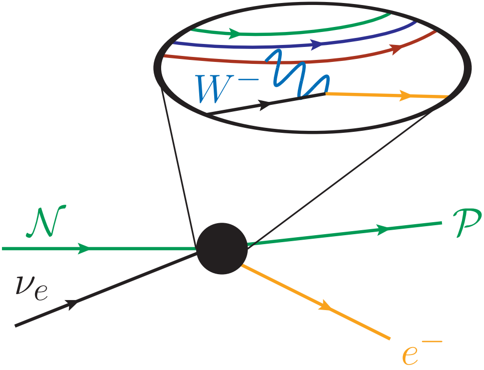
$\implies$ **scatt. rate (σ) grows with scatt. energy without bound**

$\implies$ **violation of unitarity in scattering theory, i.e., $\sum$(prob)≤ 1**

Inverse β decay is a charged-current interaction!



Fermi thry is the low-energy manifestation of the electroweak thry

$$\left(\frac{g_W}{\sqrt{2}}\right)^2 \times \left(\frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2 + i\Gamma_W M_W}\right) \xrightarrow{|q^2| \ll M_W^2} \frac{-g_W^2}{2M_W^2} = -2\sqrt{2}G_F$$

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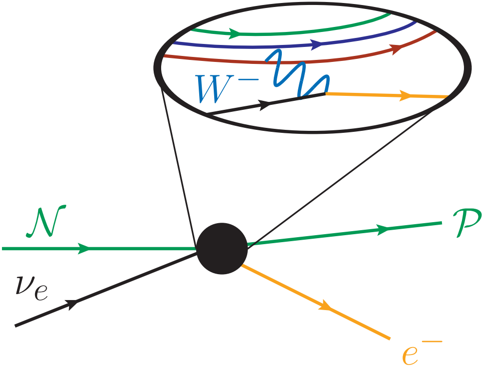

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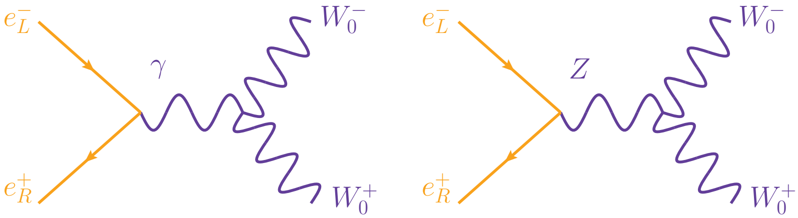
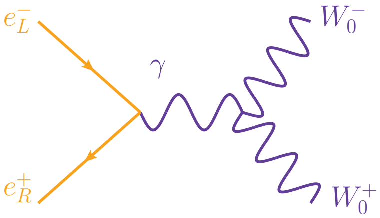
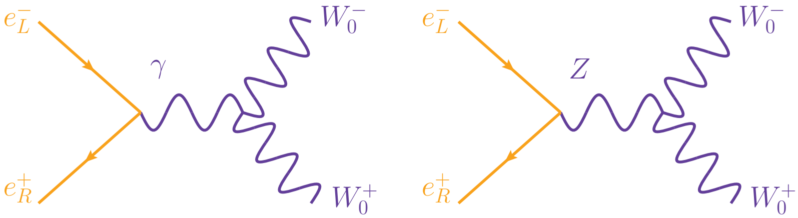
$$\implies \sigma(\nu_e \mathcal{N} \to e^- \mathcal{P}) \sim \frac{g_W^4}{\pi}\frac{E_\nu^2}{(E_\nu^2 - M_W^2)^2}$$ ← high-E behavior is regulated (finite at large E_ν)

question: how do we know that ν carries weak charges?


e_L^-
e_R^+
γ
W_0^-
W_0^+
e_L^-
e_R^+
Z
W_0^-
W_0^+



$$-i\mathcal{M}(e_L^- e_R^+ \xrightarrow{\gamma} W_0^+ W_0^-) \sim \sqrt{E}\sqrt{E}(Qe)\cdot\frac{1}{E^2}\cdot(g\sin\theta_W E)\cdot E^2 = Qg^2\sin^2\theta_W E^2$$



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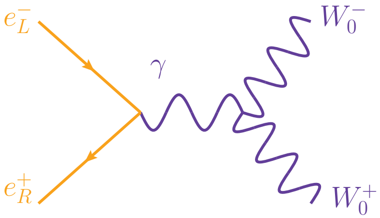
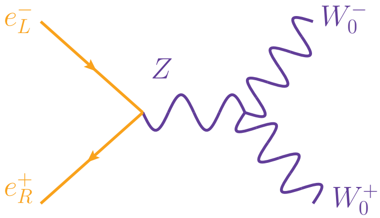
$$= T_{Le}^3 g^2 E^2 - Qg^2\sin^2\theta_W E^2$$





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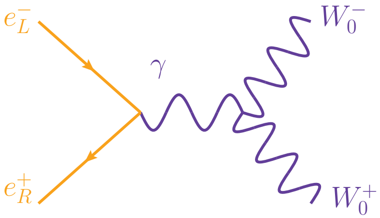
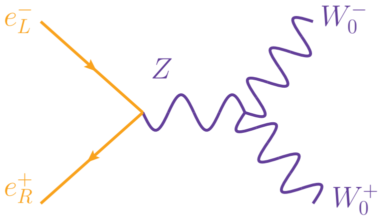
$\Longrightarrow$ **scatt. amplitude** $(\mathcal{M}_{\gamma+Z} \sim E^2)$ **grows without w/o bound!**





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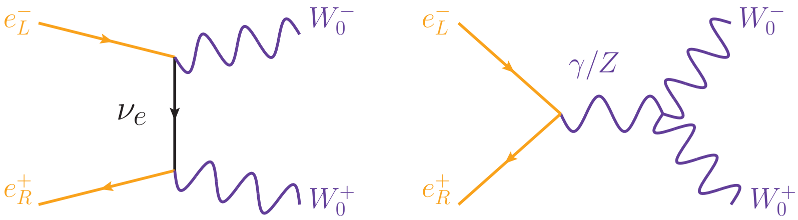
$\Longrightarrow$ **scatt. amplitude** $(\mathcal{M}_{\gamma+Z} \sim E^2)$ **grows without w/o bound!**

$\Longrightarrow$ **violation of unitarity!**



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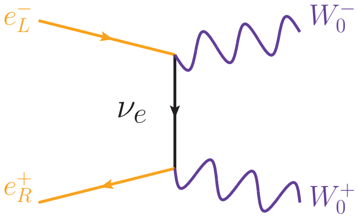

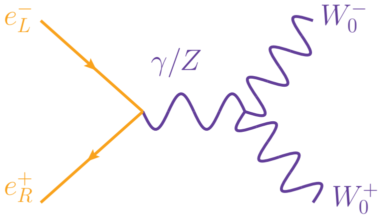

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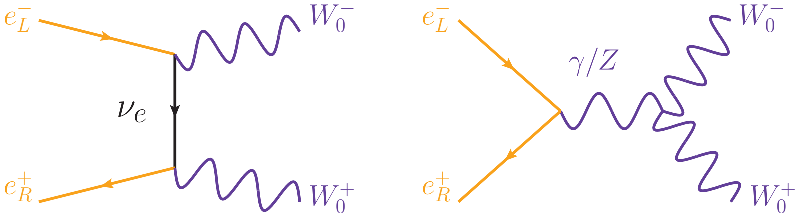
$$-i\mathcal{M}(e_L^- e_R^+ \xrightarrow{\nu} W_0^+ W_0^-) \sim \sqrt{E}\sqrt{E} \cdot \left(\frac{g}{\sqrt{2}}\right)^2 \cdot \frac{E}{E^2} \cdot E^2 \sim +\frac{1}{2} g^2 E^2$$



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$$\implies (T^3_{Le} + 1/2) = 0 \text{ or } T^3_{L\nu} = +1/2 \text{ since } T^3_{L\nu} = -T^3_{Le}.$$

Delicate (structural) cancellations when all particles are included!

Position makes quantum numbers, e.g., gauge charges, manifest

Species	Symbol	$SU(3)_C \times SU(2)_L \times U(1)_Y$ Rep.	$U(1)_{EM}$ Charge [Units of $e > 0$]
Quark	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})$	$\begin{pmatrix} +2/3 \\ -1/3 \end{pmatrix}$
Quark	u_R	$(\mathbf{3}, \mathbf{1}, +\frac{4}{3})$	$+2/3$
Quark	d_R	$(\mathbf{3}, \mathbf{1}, -\frac{2}{3})$	$-1/3$
Lepton	$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1)$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
Lepton	e_R	$(\mathbf{1}, \mathbf{2}, -2)$	-1

how many ν are there?

In the SM **(and nature)** $m_\nu \ll M_Z \implies Z \to \nu_\ell \overline{\nu_\ell}$ possible for all ℓ

$\implies \Gamma_\nu \equiv \Gamma(Z \to \nu_e \overline{\nu_e}) = \Gamma(Z \to \nu_\mu \overline{\nu_\mu}) = \Gamma(Z \to \nu_\tau \overline{\nu_\tau})$ "=" in SM and "≈" in nature

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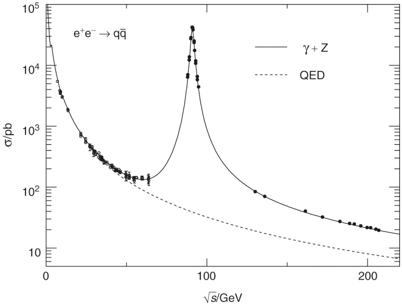
- Paramertize total width of Z as $\Gamma_Z^{\rm Tot.} = \Gamma_{\ell\ell} + \Gamma_{\rm Had.} + N_\nu^{\rm Active} \times \Gamma_\nu$
- Number of light, active ν ($N_\nu^{\rm Active}$) can be determined from $e^+e^- \to Z \to$ had. line shape



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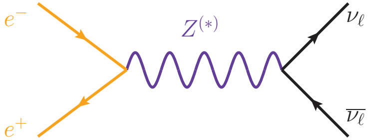

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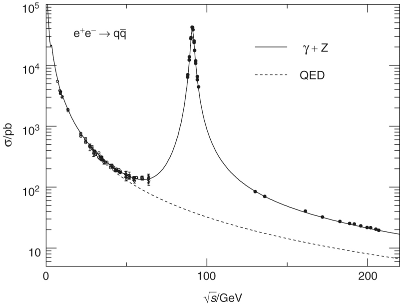
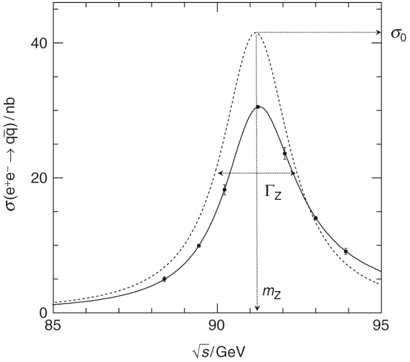
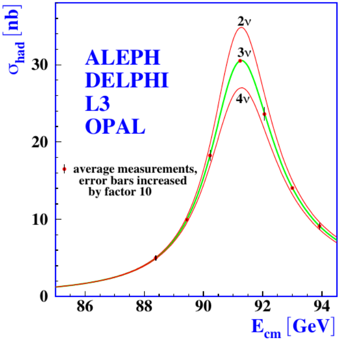


Thomson ('13)



One of the most important (and neatest!) LEP results:

- From line shape, $N_\nu^{\rm Active} = 2.9840 \pm 0.0082$
- From inv. Z decays, $N_\nu^{\rm Active} = 2.92 \pm 0.05$
- 2σ deviations consistent with $Z \to N\nu$ decays [Jarlskog, ('91)]



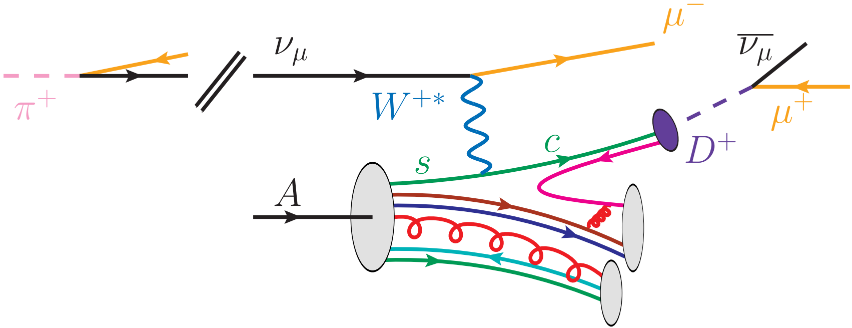
- Helps drive (mild) preference for non-unitarity of 3×3 mixing

See, e.g., Fernandez-Martinez, et al [1605.08774]

Important: e^+e^- colliders under discussion can resolve this tension [1411.5230]

the SM ν: ν scattering at high energies

high-energy ν-hadron scattering probes flavor composition of sea (anti)quarks in hadrons and valence quarks through



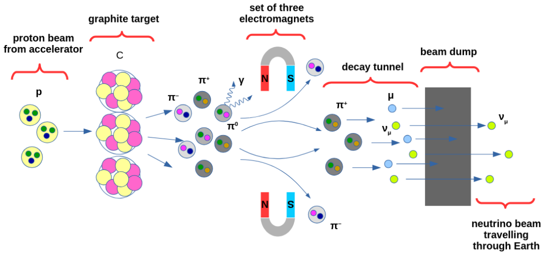
e.g., **charm dimuon** $\nu A \to \mu D + X \to \mu\mu + X'$

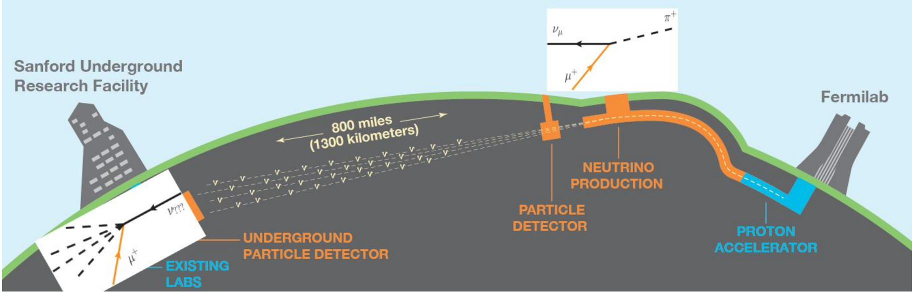
how to make a ν beam?

"accelerator ν's," i.e., high-energy ν's beams, are tertiary/3rd-stage beams:

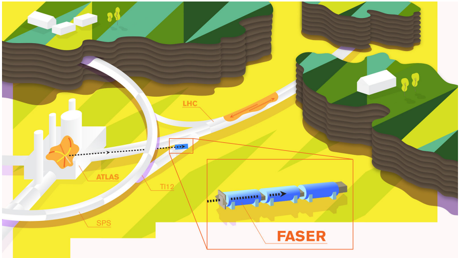
$$pA \to \pi^\pm, K^\pm, \ldots \to \ell^\pm \nu$$

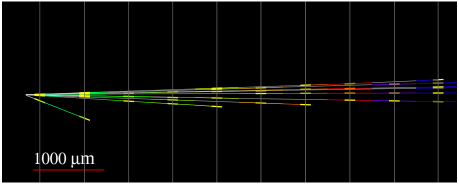
Lederman, Schwartz, Steinberger (PRL'62) ('88)





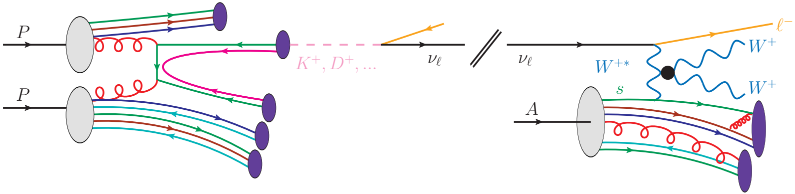
In the past few years, the LHC has been established as an intense (laboratory) source of TeV-scale ν **(a remarkable expt. achievement!)**

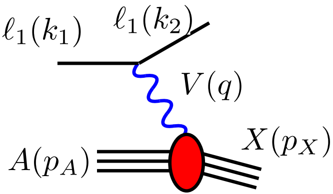


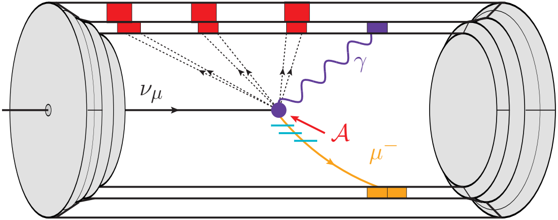


Candidate LHC neutrino event from FASER's pilot run

New programs (FASER, SND@LHC) now collecting ν-nucleus scattering data

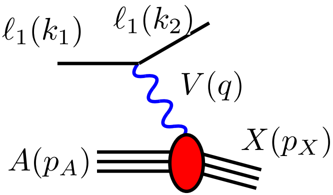


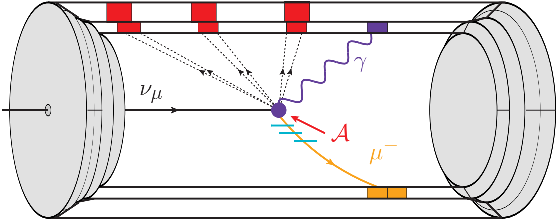




ν **scattering experiments are counting experiments:**

- **count** # of candidate signal events, e.g., $1e^\pm + X$ satisfying criteria
- **estimate** # of background events from data-driven control region
- **calculate** statistical significance





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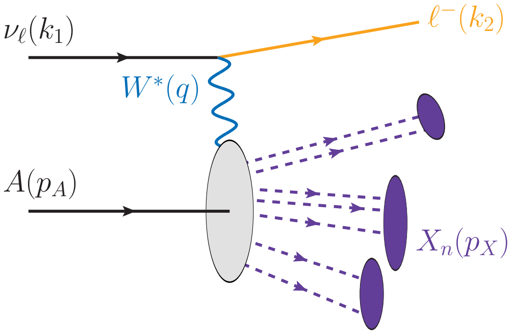
Theory needed to estimate number (and unc.) of signal and bkg events:

$$\underbrace{N_{\rm candidates}}_{\rm hep/nucl-ex} = \underbrace{\mathcal{L}(\text{data!})}_{\rm accelerators} \times \underbrace{\sigma(\text{scattering likelihood})}_{\rm hep/nucl-th/ph}$$

Generically, hard scattering of $\ell \in \{\ell^\pm, \nu, \overline{\nu}\}$ off **nucleons** well-described by **kinematic factor** (lepton bit) and "**structure functions**" (hadron bit)



$$d\sigma(\nu\mathcal{A} \to \ell X) = \sum_i \underbrace{(\text{some function of } p_A, q)_i}_{\text{calculable from first principles}} \times \underbrace{F_i^{\nu\mathcal{A}}(p_A, q)}_{\text{parameterizes response of } \mathcal{A}}$$

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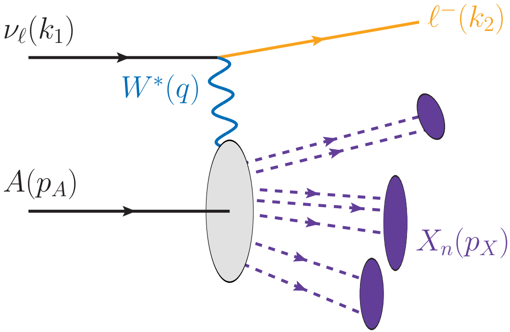

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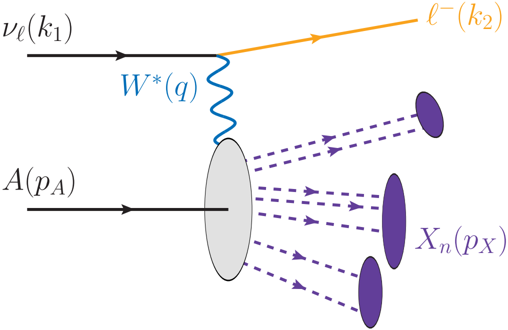
Quark-Parton Model: $F^{\nu\mathcal{A}}(p_A, q) \sim \sum_{k=q,g,\overline{q}} f_k^{\mathcal{A}}(x), \quad x = -q^2/(2p_A \cdot q)$

- $f_k^{\mathcal{A}}$ is the **parton (number) density function (PDF)** of k in $\mathcal{A}$

sketching the scattering formula



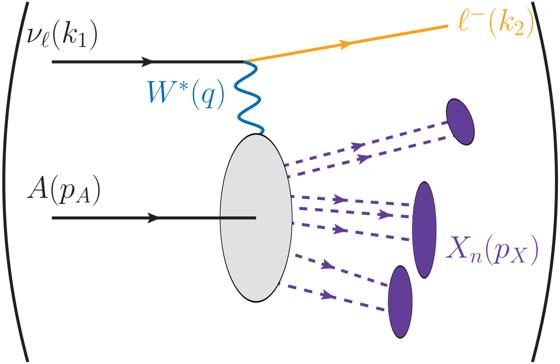
[1]

---

[1] for a more thorough & pedagogical treatment, see appendices of RR, et al [Prog.Part.Nucl.Phys. 136 ('24) 104096]

draw diagrams, currents, and build the matrix element

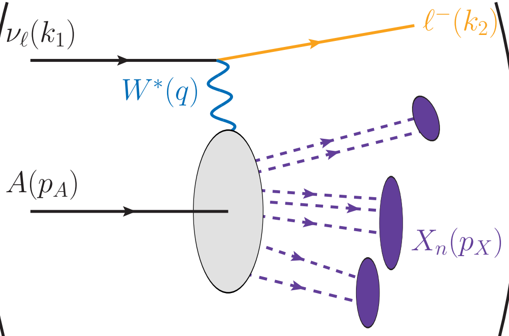
$-i\mathcal{M}$



$$= \; J^{\alpha}_{\ell_f \ell_i} \cdot \Delta_{\alpha\mu} \cdot J^{\nu}_{X_n A}$$

squaring and summing over spins gives us $H_n^{\mu\nu}$ (exclusive, n-body)

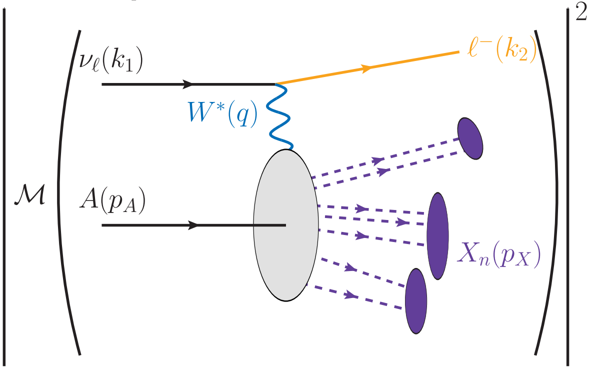
$$\Sigma_{\text{spins}} \left| \mathcal{M} \left(\right. \right.$$



$$\left. \left. \right) \right|^2 \sim L_{\mu\nu} \cdot H_n^{\mu\nu}$$

n-body phase space integral *and* summing over n gives us $W_A^{\mu\nu}$

$\frac{d^3\sigma}{dk_2^3} \sim \int dPS_n \; \Sigma_n \; \Sigma_{\text{spins}}$



$\sim \; L_{\mu\nu} \cdot W_A^{\mu\nu}$

this step sometimes omitted in textbooks, e.g., Halzen & Martin

Summing over X_n ensures “inclusivity” and closure, $1 = \sum_n |X_n\rangle\langle X_n|$

Leptonic currents depend on exchange boson:

$$\sum_{\{\lambda\}} L^{\mu\nu}\Big|_{\text{QED}} = 4e^2\left\{k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - (k_1\cdot k_2)g^{\mu\nu}\right\}$$

$$\sum_{\{\lambda\}} L^{\mu\nu}\Big|_{W} = g_W^2\left\{k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - (k_1\cdot k_2)g^{\mu\nu} + ik_{1\alpha}k_{2\beta}\epsilon^{\mu\nu\alpha\beta}\right\}$$

[2] This is equal to the usual expression $W^A_{\mu\nu} = \frac{1}{4\pi}\int d^4z\ e^{iq\cdot z}\langle A|J^\dagger_{had.\mu}(z)\ J_{had.\nu}(0)|A\rangle$. See Eq. (A.22) and below of Prog.Part.Nucl.Phys. 136 ('24) 104096 [2301.07715].

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Hadronic current is formally defined by[2]:

$$W^A_{\mu\nu} = \frac{1}{4\pi}\sum_{n=1}^{\infty}\int dPS_n \ \sum_{\rm dof}\ H^A_{\mu\nu}$$

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$$= -g_{\mu\nu}\ F_1^A \ + \frac{p_{A\mu} p_{A\nu}}{Q^2}\ 2x_A\ F_2^A \ - i\epsilon_{\mu\nu\rho\sigma}\frac{p_A^\rho q^\sigma}{Q^2}\ x_A F_3^A$$

$$+ \frac{q_\mu q_\nu}{Q^2}\ 2F_4^A \ + \frac{p_{A\mu} q_\nu + p_{A\nu} q_\mu}{Q^2}\ 2x_A\ F_5^A \ + \frac{p_{A\mu} q_\nu - p_{A\nu} q_\mu}{Q^2}\ 2x_A\ F_6^A$$

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point #1: $W^A_{\mu\nu}$ has at most six unknown components $(4\times 4) = \mathbf{1} + \underline{\mathbf{1}} + \mathbf{4} + \underline{\mathbf{4}} + \mathbf{6}$

[2] This is equal to the usual expression $W^A_{\mu\nu} = \frac{1}{4\pi}\int d^4z\ e^{iq\cdot z}\langle A|J^\dagger_{had.\mu}(z)\ J_{had.\nu}(0)|A\rangle$. See Eq. (A.22) and below of Prog.Part.Nucl.Phys. 136 ('24) 104096 [2301.07715].

point #2: "structure functions" $F_i(x, Q^2)$ are well-defined experimentally

point #3: $F_i(x, Q^2)$ are independent of underlying theory[3]

point #4: $W^A_{\mu\nu}$ is defined in the "DIS" limit:

$$x_A = \frac{Q^2}{2p_A \cdot q} \text{ is fixed and } Q^2 \gg \Lambda^2_{\rm NP}, \text{ where } \Lambda_{\rm NP} \sim \mathcal{O}(1-2)\ {\rm GeV}$$

[3] parton model says $F_i = \sum f_{j/p}$; see Collins ('11) for nice discussion on this!

[4] see Sterman [hep-ph/9606312] and see Collins ('11) for nice discussions!

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point #5: in practice, $F_{4,5,6}$ can be neglected, but not always[4]

$$W^A_{\mu\nu} = -g_{\mu\nu}\ F^A_1(x_A)\ + \frac{p_{A\mu}p_{A\nu}}{Q^2}\ 2x_A\ F^A_2(x_A)\ - \mathcal{O}(\mathcal{P})\ x_A F^A_3(x_A)$$

$$+\,\mathcal{O}\left(\frac{m^2_\nu, m^2_\ell}{Q^2}\right)\ 2F^A_4(x_A)\ + \mathcal{O}\left(\frac{m^2_\nu, m^2_\ell}{Q^2}\right)\ 2x_A\ F^A_5(x_A)\ + \mathcal{O}(\mathcal{CP})\ 2x_A\ F^A_6(x_A)$$

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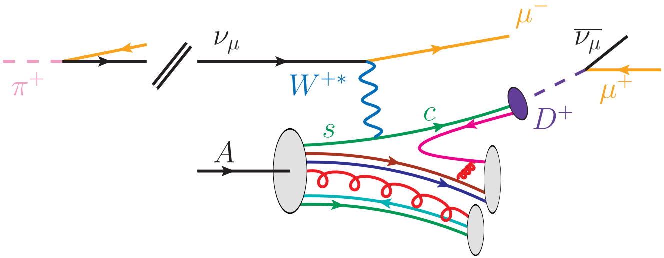
the point of this effort?



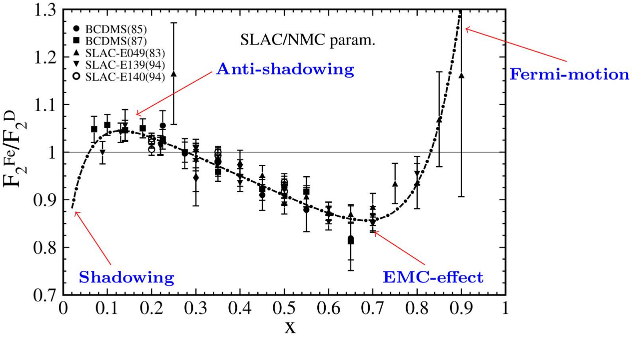
the dark secret of ν scattering experiments

in practice, νDIS needs nuclear targets

1. ν only interact through weak force: targets must be bigger ($\mathcal{O}(10)$tons) and denser (Ar,Fe,Pb) $\implies$ more nuclear

2. fact of life: nuclear dynamics impact hadronic structure

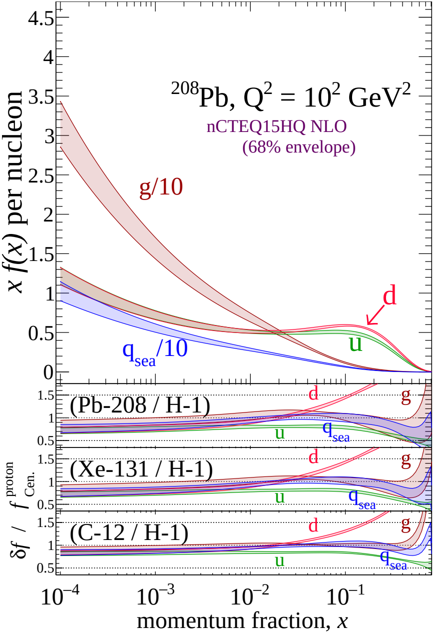
Plotted: $\frac{F_2^{\rm iron}}{F_2^{\rm deuteron}}$ for ℓ-DIS



For non-expert, QED (γ) contribution to F_2: $F_2(\xi) \approx \sum_{i\in\{q,\overline{q},g\}} Q_i^2\ \xi\ f_i^A(\xi)$ Schienbein, et al [0710.4897] w/ Fuks, Marougkas, Sztandera [2405.19399]

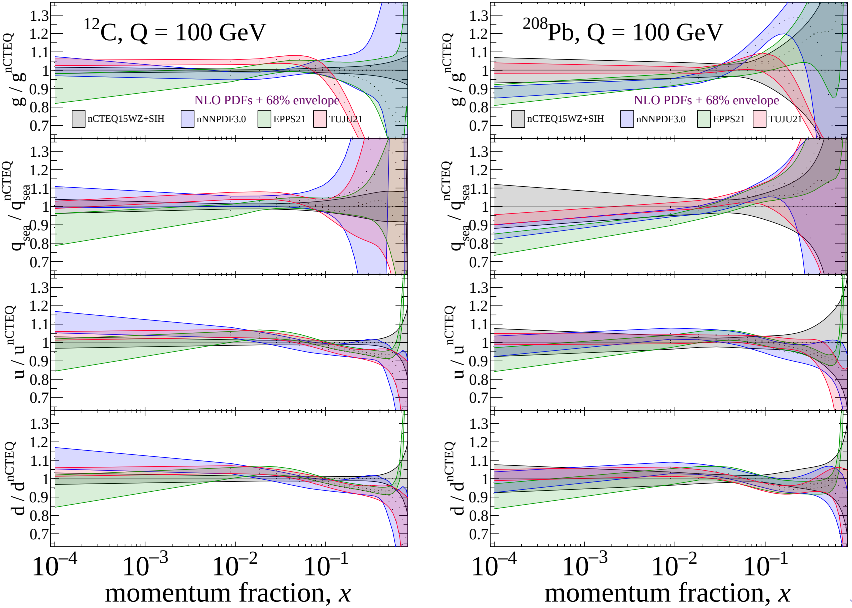
Plotted: PDF of avg. nucleon in ^{208}Pb vs (avg) energy fraction carried by parton

- huge g content (always easy to make more g)
- $q_{\rm sea}$, d, and u content converge for $x \lesssim 10^{-2}$ (dominated by $g^* \to q\bar{q}$ splitting)
- densities smaller (larger) than proton for $x \lesssim 10^{-2} (x \gtrsim 10^{-2})$
- qualitatively different from proton
- smaller $\mathcal{A}$ are more proton-like

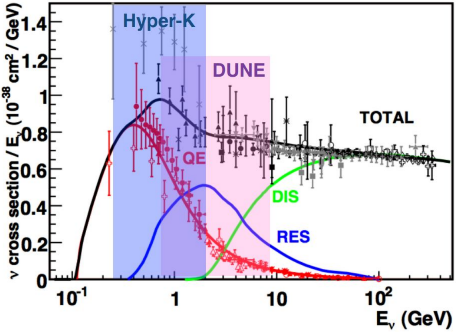


Plotted: Ratios of nuclear PDFs vs (avg) energy fraction carried by parton

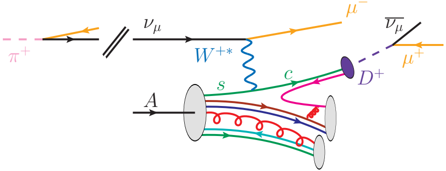
w/ Fuks, Marougkas†, Sztandera† [2405.19399]



big take away νDIS gives complementary access to hadronic structure



little take away parton distribution function (PDF) uncertainties are smaller for νDIS, ℓDIS, pp, and $\mathcal{AA}$ programs thanks to νDIS



nCTEQ[0710.4897] + others

the SM ν: the massless ν hypothesis

The Massless ν Hypothesis

In quantum field theory: we learn about three types of fermions

$$\mathcal{L}_{\text{Kin.}} = \overline{\psi} i \partial\!\!\!/ \psi \qquad \mathcal{L}_{\text{Kin.}} = \overline{\psi} \left(i \partial\!\!\!/ - m \right) \psi \qquad \mathcal{L}_{\text{Kin.}} = \frac{1}{2} \overline{\psi} \left(i \partial\!\!\!/ - m \right) \psi$$

Weyl fermion $(m = 0)$ **Dirac fermion** $(m \neq 0)$ **Majorana fermion** $(m \neq 0)$

In quantum field theory: we learn about three types of fermions

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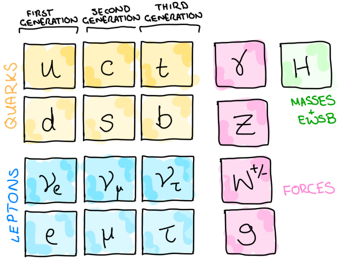
Weyl fermion $(m = 0)$ **Dirac fermion** $(m \neq 0)$ **Majorana fermion** $(m \neq 0)$

- **History:** *Model of Leptons* (Weinberg'67) hypothesizes massless ν (no evidence for $m_\nu \neq 0$)
- **Data:** evidence only for $m_\nu \neq 0$, not whether ν is Dirac or Majorana (more later today!)
- **The** 1/2 **Problem**: What is the Kinetic Lagrangian of the ν realized in nature?

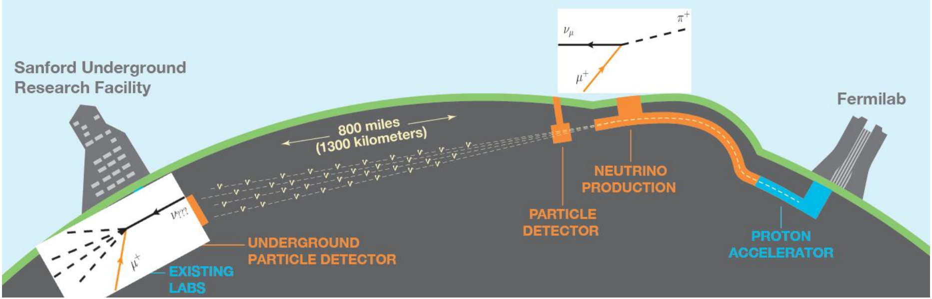


ν masses is physics beyond the SM!!!

The non-Standard Model ν:
the ν that nature gave us

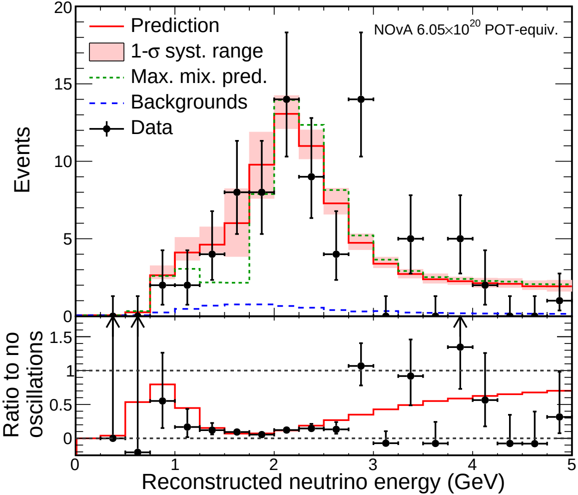
ν Oscillations

Idea: count ν_μ at near detector and compare to # at far detector



Result: **far detector** reports ν_μ deficit + unexpected apperance of ν_e/ν_τ

(focus on the lower panel!)



Interpretation: neutrinos are transitioning between **flavor eigenstates** and **mass eigenstates**:

$$\nu_{\ell_1} \to \nu_{\rm mass} \to \nu_{\ell_2}$$

NOνA ν_μ disappearance [1701.05891]

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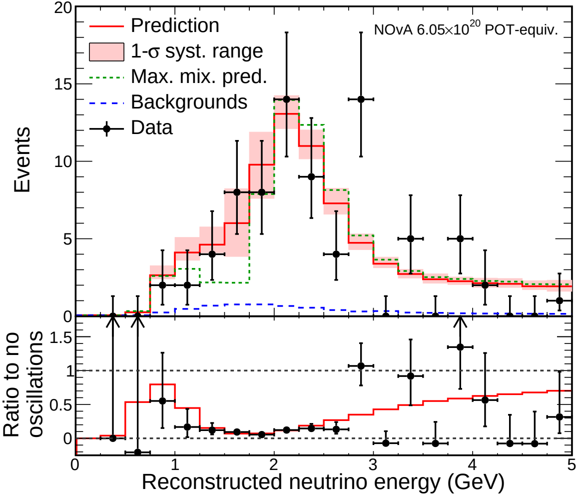

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evidence for ν masses!

('15) SNO, Super-K

NOνA ν_μ disappearance [1701.05891]

the massive ν hypothesis

Consider left-handed (LH), $SU(2)_L$ lepton doublets (**gauge eigenbasis**):

$$L_{aL} = \begin{pmatrix} \nu_a \\ l_a \end{pmatrix}_L, \quad a = 1, 2, 3.$$

The SM $W^\pm$ boson coupling to **leptons** in the **flavor eigenbasis** is

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W_\mu^+ \sum_{l=1}^3 \left[\overline{\nu_{lL}} \gamma^\mu P_L l^-\right] + \text{H.c.}$$

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Supposing $m_\nu \neq 0$, we can rotate ν_l and l into the **mass eigenbasis**:

$$\nu_l = \sum_{m=1}^{3} \Omega_{lm} \nu_m \quad \text{and} \quad l = \sum_{\ell=3}^{\tau} \Omega_{l\ell} \ell$$

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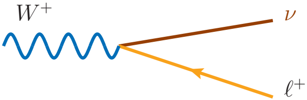
This allows us to describe SM $W^\pm$ boson coupling to ν *with* $m_\nu \neq 0$:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W_\mu^+ \sum_{\ell=e}^{\tau} \sum_{m=1}^{3} \left[\overline{\nu_m} \underbrace{U_{m\ell}^*}_{U_{m\ell}^* \equiv \sum_l \Omega_{ml}^* \Omega_{l\ell}} \gamma^\mu P_L \ell^- \right] + \text{H.c.}$$

Like the CKM, SM Feynman rules are modified by **PMNS** mixing factor:

$$\Gamma^\mu = \frac{-ig}{\sqrt{2}} \gamma^\mu P_L \to \tilde{\Gamma}^\mu = \frac{-ig}{\sqrt{2}} U_{m\ell}^* \gamma^\mu P_L$$



2-State Neutrino Mixing

Generically, mixing between **flavor eigenstates** and **mass eigenstates** is given by unitary transformation/rotation

$$\underbrace{\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}}_{\text{flavor basis}} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} \\ U_{\mu 1} & U_{\mu 2} \end{pmatrix}}_{\text{mixing}} \underbrace{\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}}_{\text{mass basis}} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

2-State Neutrino Mixing

Generically, mixing between **flavor eigenstates** and **mass eigenstates** is given by unitary transformation/rotation

$$\underbrace{\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}}_{\text{flavor basis}} = \underbrace{\begin{pmatrix}U_{e1} & U_{e2}\\ U_{\mu 1} & U_{\mu 2}\end{pmatrix}}_{\text{mixing}} \underbrace{\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}}_{\text{mass basis}} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix} \begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

For a **two-state system**, the state vector for ν_ℓ $(\ell = e, \mu)$ is simply

$$\underbrace{|\nu_e\rangle}_{\text{flavor basis}} = U_{e1}\underbrace{|\nu_1\rangle}_{\textit{light}} + U_{e2}\underbrace{|\nu_2\rangle}_{\textit{heavy}} = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

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If we treat the spacetime propagation of ν_m $(m = 1, 2)$ as a plane wave, then the evolution from $x^\mu = x_a^\mu$ to $x^\mu = x_b^\mu$ is

$$|\nu_\ell(x_b, x_a)\rangle = U_\ell(x_b, x_a)|\nu_\ell\rangle = U_{\ell 1} U_1(x_b, x_a)|\nu_1\rangle + U_{\ell 2} U_2(x_b, x_a)|\nu_2\rangle$$

Evolution through space and time

Assuming $\hat{p}_\nu = \Delta\hat{x}$, the plane wave evolution over $L = |\vec{x}_b - \vec{x}_a]|$ is

$$U_m(x_b, x_a) = e^{-ip_m\cdot(x_b - x_a)}$$

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Now, working in the ultra relativistic limit, where $E_m + |\vec{p}_m| \approx 2E_m$,

$$(E_m\Delta t_m - |\vec{p}_m| L) \approx (E_m - |\vec{p}_m|)\, L$$

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Since $m_2, m_1 \ll E_1, E_2$, the E_m can be approximated as the same:

$$|\nu_e(E,L)\rangle = U_{e1}e^{-im_1^2L/2E}|\nu_1\rangle + U_{e2}e^{-im_2^2L/2E}|\nu_2\rangle$$

$$|\nu_\mu(E,L)\rangle = U_{\mu1}e^{-im_1^2L/2E}|\nu_1\rangle + U_{\mu2}e^{-im_2^2L/2E}|\nu_2\rangle$$

We are now ready to compute oscillation transtions!

Neutrino Oscillation Transitions

To reproduce the ν_μ deficit, consider the $\nu_\mu \to \nu_\mu$ *transition amplitude*:

$$\mathcal{M}(\nu_\mu \to \nu_\mu) \equiv \langle \nu_\mu | \nu_\mu(E, L) \rangle$$

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Since $|\nu_m\rangle$ are mass **eigenstates**, $\langle \nu_{m'} | \nu_m \rangle = \delta_{m'm}$. This implies

$$\mathcal{M}(\nu_\mu \to \nu_\mu) = e^{-i m_1^2 L/2E} |U_{\mu 1}|^2 \langle \nu_1 | \nu_1 \rangle + e^{-i m_2^2 L/2E} |U_{\mu 2}|^2 \langle \nu_2 | \nu_2 \rangle$$

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The $\nu_\mu \to \nu_\mu$ *transition probability* is

$$\Pr(\nu_\mu \to \nu_\mu) = |\mathcal{M}(\nu_\mu \to \nu_\mu)|^2 = |U_{\mu 1}|^4 + |U_{\mu 2}|^4$$

$$+ e^{-i\Delta m_{21}^2 L/2E} |U_{\mu 1}|^2 |U_{\mu 2}|^2 + e^{+i\Delta m_{21}^2 L/2E} |U_{\mu 1}|^2 |U_{\mu 2}|^2$$

note: $\Delta m_{21}^2 \equiv (m_2^2 - m_1^2)$

Some Quick Algebra

Recalling that $U_{e1} = U_{\mu 2} = \cos\theta$ and $U_{e2} = -U_{\mu 1} = \sin\theta$,

$$\Pr(\nu_\mu \to \nu_\mu) = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta\cos\left[\frac{\Delta m_{21}^2 L}{2E}\right]$$

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Lots to unpack:

$$\Pr(\nu_\mu \to \nu_\mu) = \underbrace{1}_{\text{unitarity}} - \underbrace{\sin^2(2\theta)}_{\text{amplitude of dip}} \underbrace{\sin^2\left[\frac{\Delta m^2_{21} L}{4E}\right]}_{\text{spacing between beats}}$$

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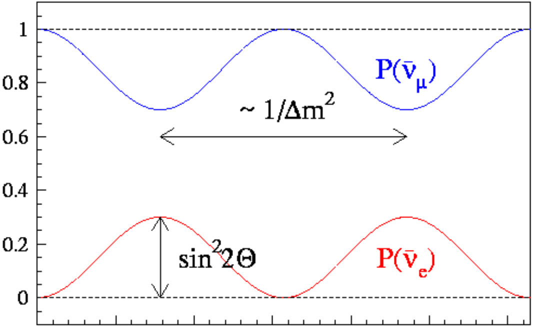
By conservation of probability $1 = \Pr(\nu_\mu \to \nu_\mu) + \Pr(\nu_\mu \to \nu_e)$, so the $\nu_\mu \to \nu_e$ *appearance probability* is

$$\Pr(\nu_\mu \to \nu_e) = 1 - \Pr(\nu_\mu \to \nu_\mu) = \sin^2(2\theta)\sin^2\left[\frac{\Delta m^2_{21}L}{4E}\right]$$

Understanding Neutrino Oscillation Plots

$$\Pr(\nu_\mu \to \nu_\mu) = \underbrace{1}_{\text{unitarity}} - \underbrace{\sin^2(2\theta)}_{\text{minimum of dip}} \underbrace{\sin^2\left[\frac{\Delta m^2_{21} L}{4E}\right]}_{\text{spacing between beats}}$$

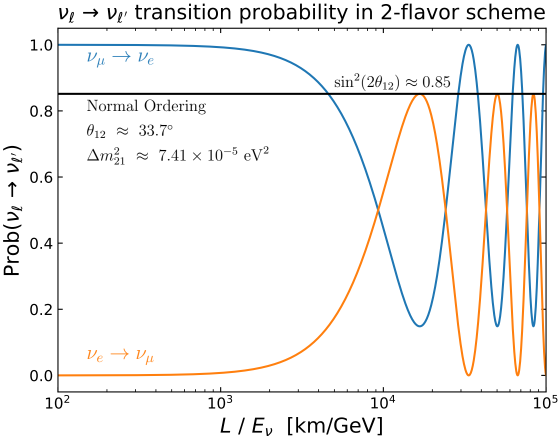
$$\Pr(\nu_\mu \to \nu_e) = \underbrace{\sin^2(2\theta)}_{\text{maximum of peak}}$$



With updated inputs:

gitlab.cern.ch/riruiz/public-projects/-/tree/master/NuPhysSandbox

$$\Pr(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left[\frac{\Delta m^2_{21} L}{4E}\right] \qquad \Pr(\nu_\mu \to \nu_e) = \sin^2(2\theta)$$



The 3×3 Paradigm

To date, all oscillation data can be described within the 3×3 Paradigm

- 3 ν_ℓ (flavor states) $\implies$ 3 mixing angles
- 3 ν_k (mass states) $\implies$ 2 mass splittings **one may be massless!**
- 1 CP phase (if Dirac); +2 CP phases (if Majorana)

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{\mathrm{CP}}} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}\,,\ \ c_{ij} = \cos\theta_{ij}$

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NuFIT 5.3 (2024)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
	$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
	$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
	$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
	$\delta_{CP}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ eV^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ eV^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
	$\theta_{12}/^\circ$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
	$\sin^2\theta_{23}$	$0.454^{+0.019}_{-0.016}$	$0.411 \to 0.606$	$0.568^{+0.016}_{-0.021}$	$0.412 \to 0.611$
	$\theta_{23}/^\circ$	$42.3^{+1.1}_{-0.9}$	$39.9 \to 51.1$	$48.9^{+0.9}_{-1.2}$	$39.9 \to 51.4$
	$\sin^2\theta_{13}$	$0.02224^{+0.00056}_{-0.00057}$	$0.02047 \to 0.02397$	$0.02222^{+0.00069}_{-0.00057}$	$0.02049 \to 0.02420$
	$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.13}_{-0.11}$	$8.23 \to 8.95$
	$\delta_{CP}/^\circ$	232^{+39}_{-25}	$139 \to 350$	273^{+24}_{-26}	$195 \to 342$
	$\frac{\Delta m^2_{21}}{10^{-5}\ eV^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ eV^2}$	$+2.505^{+0.024}_{-0.026}$	$+2.426 \to +2.586$	$-2.487^{+0.027}_{-0.024}$	$-2.566 \to -2.407$

Success of the ν oscillation paradigm opens many questions:

- Are there other, heavier ν's?

Would manifest as non-unitarity of 3×3 $U_{\ell m}$

- How much CP violation is in the lepton sector? $\delta_{\rm CP} \sim 270^\circ$ ('24), $\eta_1, \eta_2 \sim$???

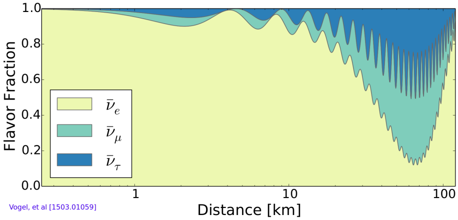
- What drives the CKM matrix "diagonal" but the PMNS matrix "non-diagonal"? part of the "flavor" problem

- Are neutrinos Majorana fermions?

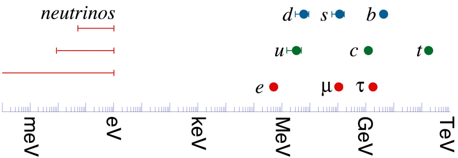
Would manifest **violation of lepton number symmetry**

- $m_\nu \neq 0$ breaks SM gauge symmetry.

What generates? the SM Higgs or another Higgs?



Vogel, et al [1503.01059]



coffee time!