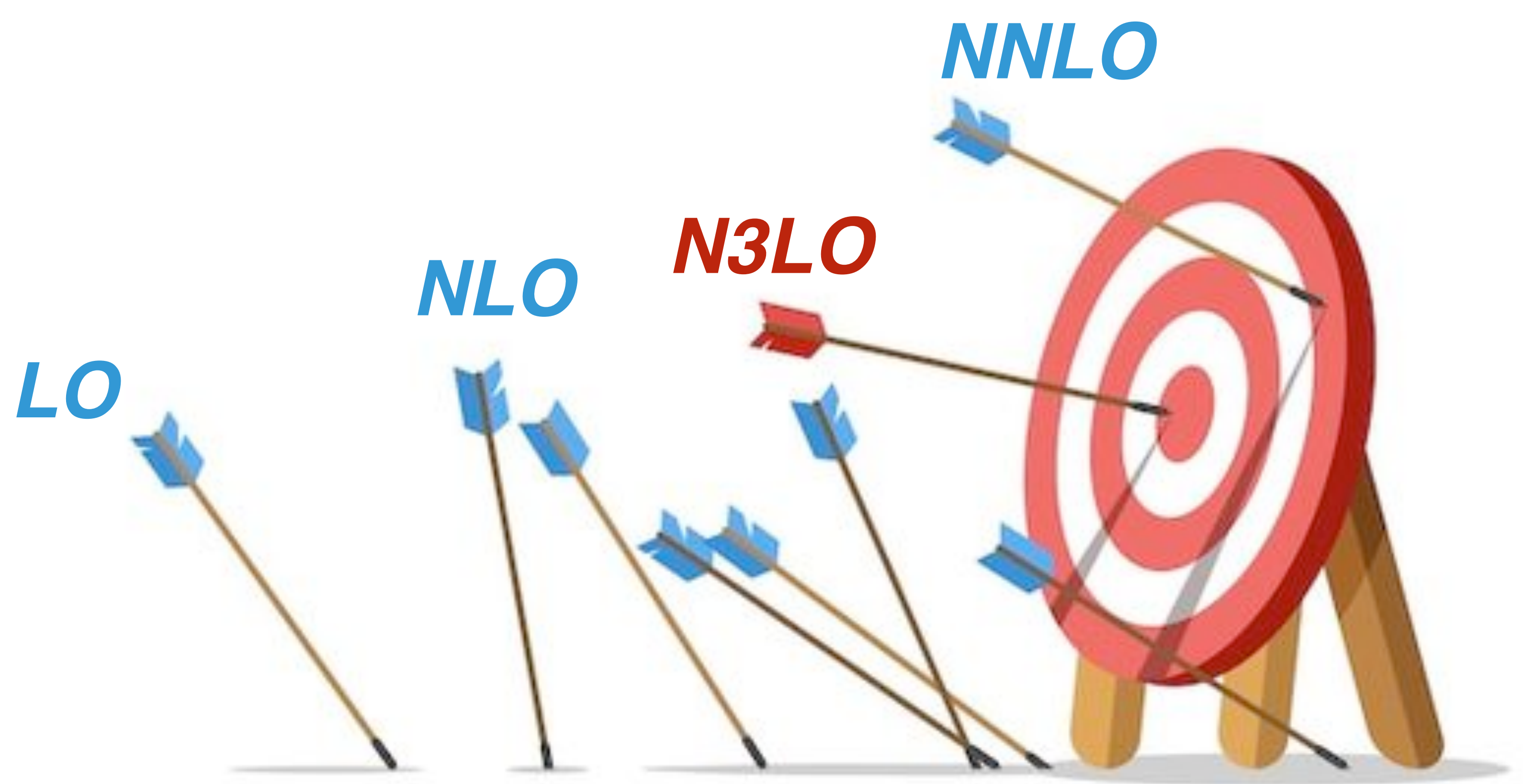
QCD and Monte Carlo event generators (Lecture 1 — Fixed-order calculations)

Marius Wiesemann

Max-Planck-Institut für Physik



BND summer school 2024

Blankenberge (Belgium), September 2-12th, 2024

QUIZ: Getting to know the room

★ Please raise your hands!

QUIZ: Getting to know the room

★ Please raise your hands!

➙ Who is working on collider/LHC physics?

QUIZ: Getting to know the room

★ Please raise your hands!

→ Who is working on collider/LHC physics?

→ Who is working on cosmology/astroparticle physics?

QUIZ: Getting to know the room

★ Please raise your hands!

→ Who is working on collider/LHC physics?

→ Who is working on cosmology/astroparticle physics?

→ Who is in a different field?

QUIZ: Getting to know the room

★ Please raise your hands!

QUIZ: Getting to know the room

★ Please raise your hands!

→ Who is currently a PhD student?

QUIZ: Getting to know the room

★ Please raise your hands!

➙ Who is currently a PhD student?

➙ Who already has a PhD?

QUIZ: Getting to know the room

★ Please raise your hands!

→ Who is currently a PhD student?

→ Who already has a PhD?

→ Who has already finished a PostDoc?

QUIZ: Getting to know the room

★ Please raise your hands!

→ Who is currently a PhD student?

→ Who already has a PhD?

→ Who has already finished a PostDoc?

→ Who is staff member?

QUIZ: Getting to know the room

★ Let's divide the room…!

→ **Who is a theorist?**

QUIZ: Getting to know the room

★ Let's divide the room…!

→ **Who is a theorist?**

→ **Who is an experimentalist?**

QUIZ: Getting to know the room

★ Let's divide the room…!

→ **Who is a theorist?**

→ **Who is an experimentalist?**

→ **Who is non-binary?**

QUIZ: Getting to know the room

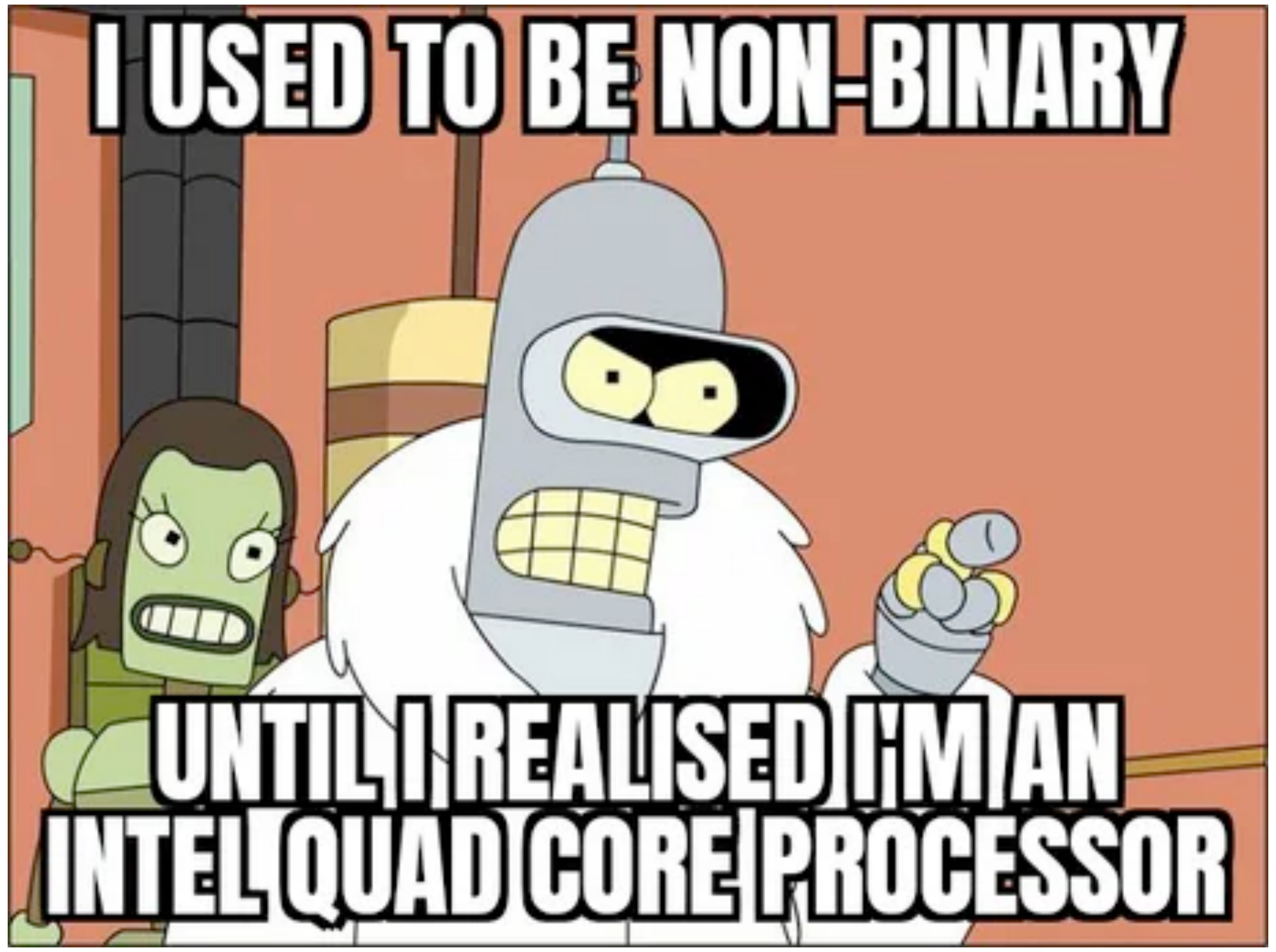
★ Let's divide the room…!

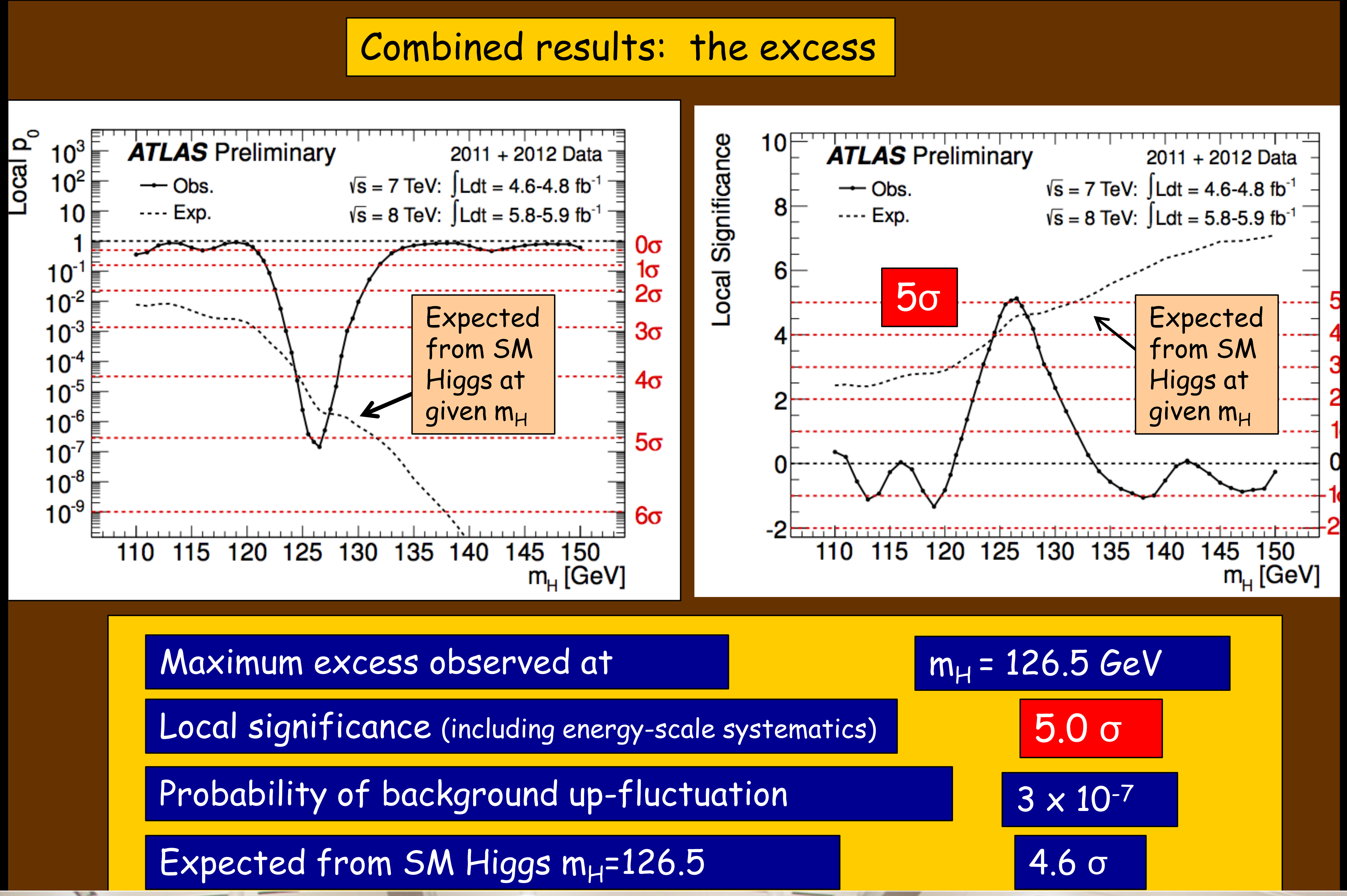
→ **Who is a theorist?**

→ **Who is an experimentalist?**

→ **Who is ~~non-binary~~?**

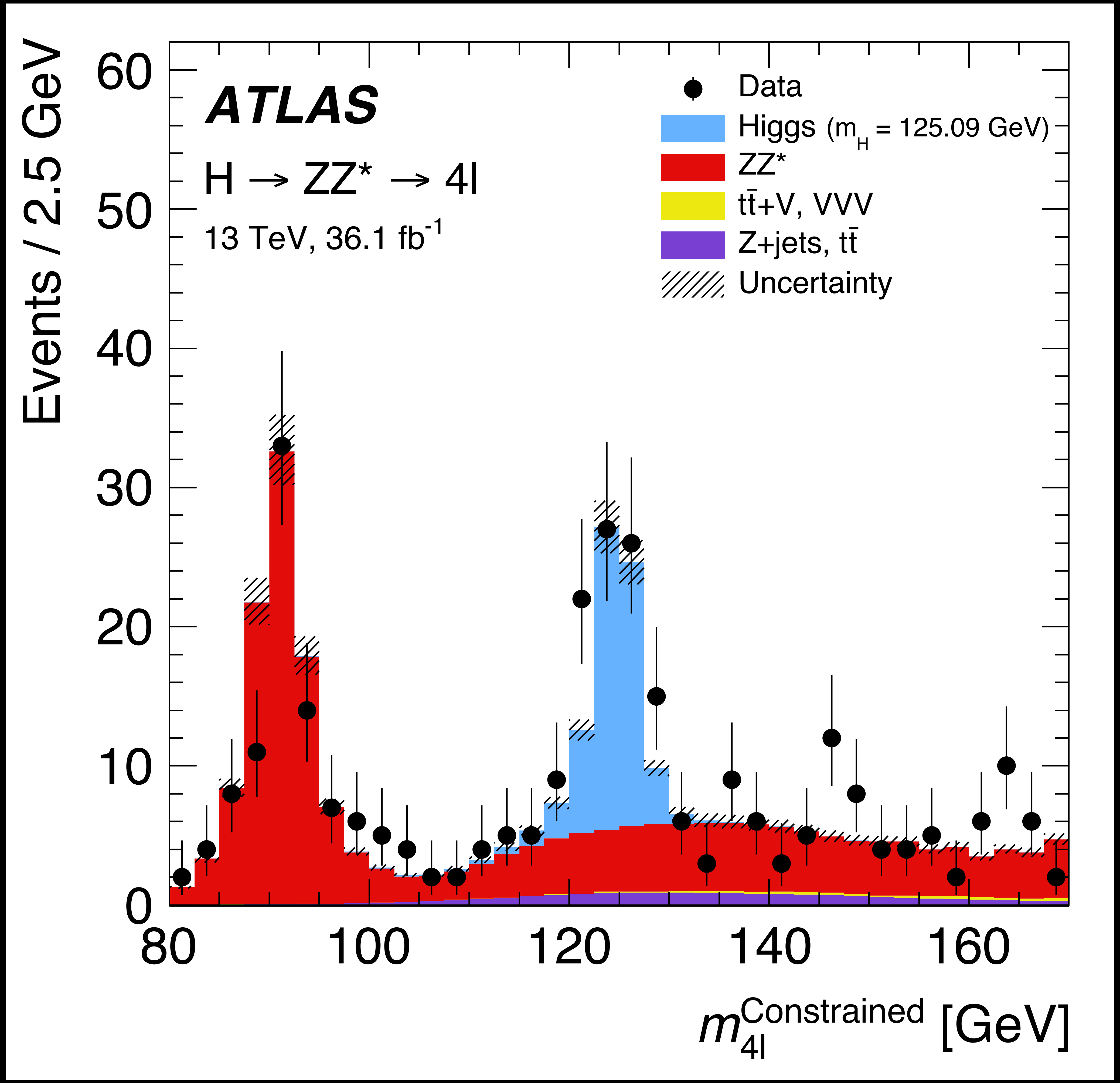
phenomenologist

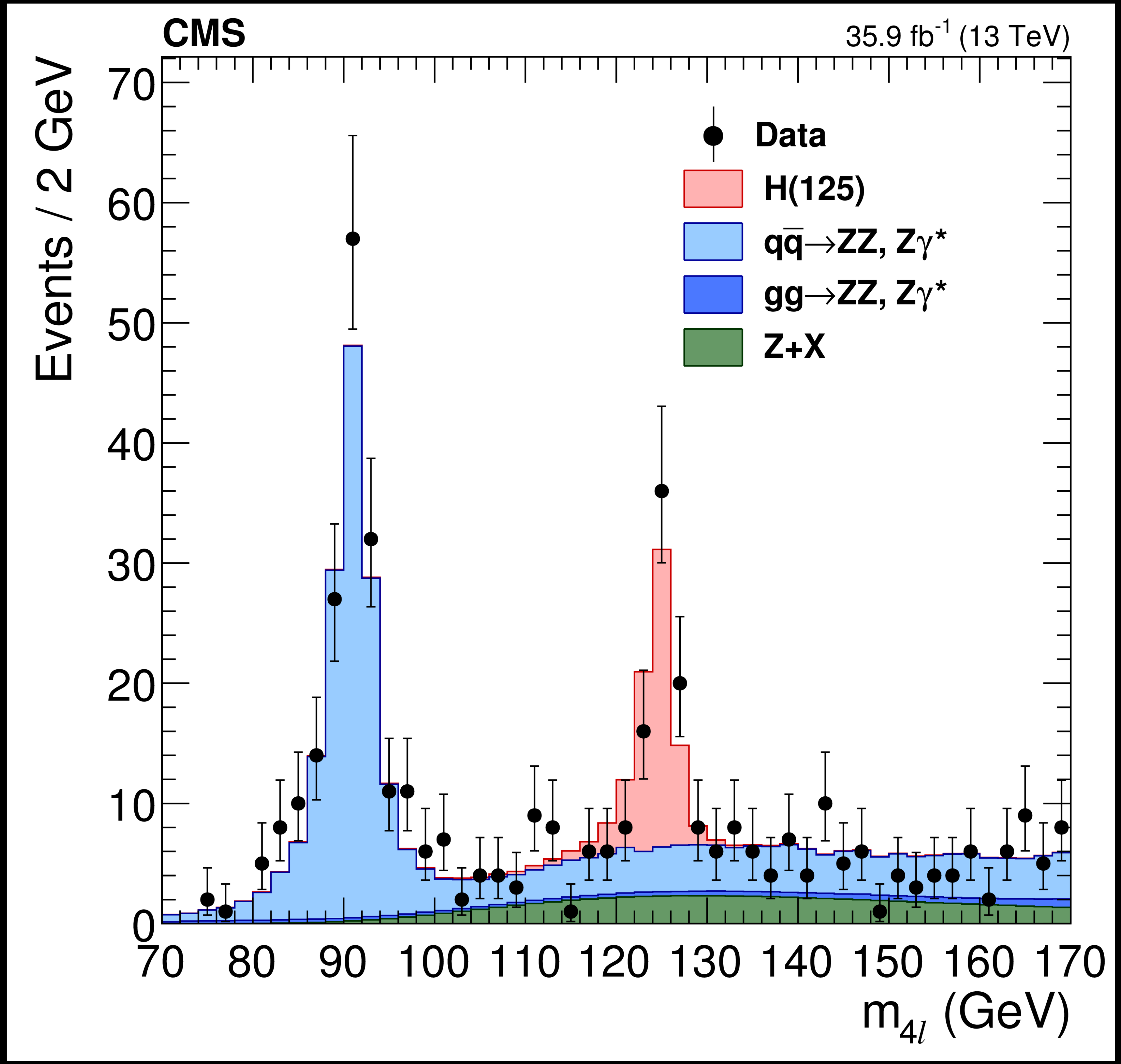




4th July 2012

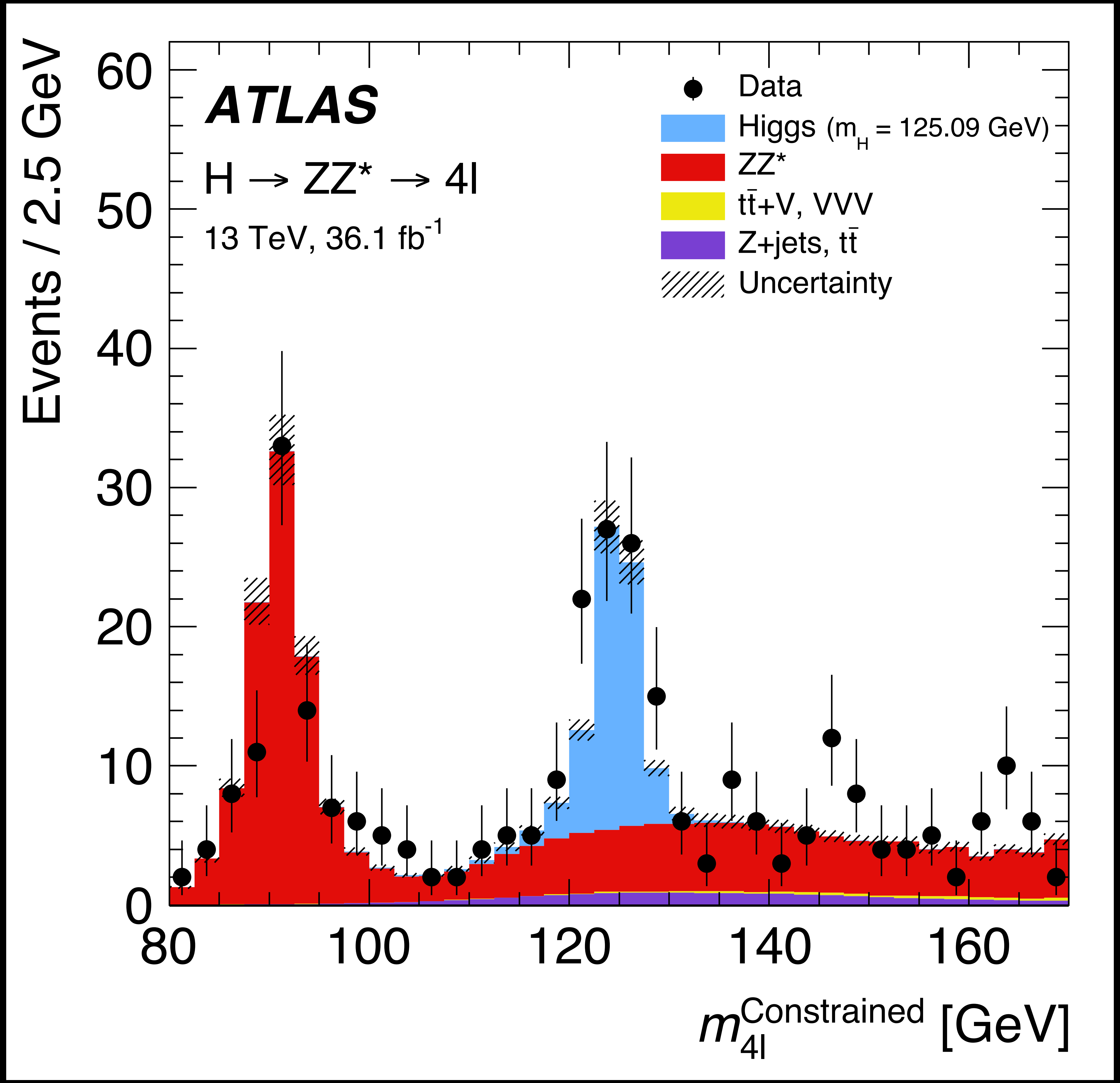
Did we need theory to observe the Higgs resonance?

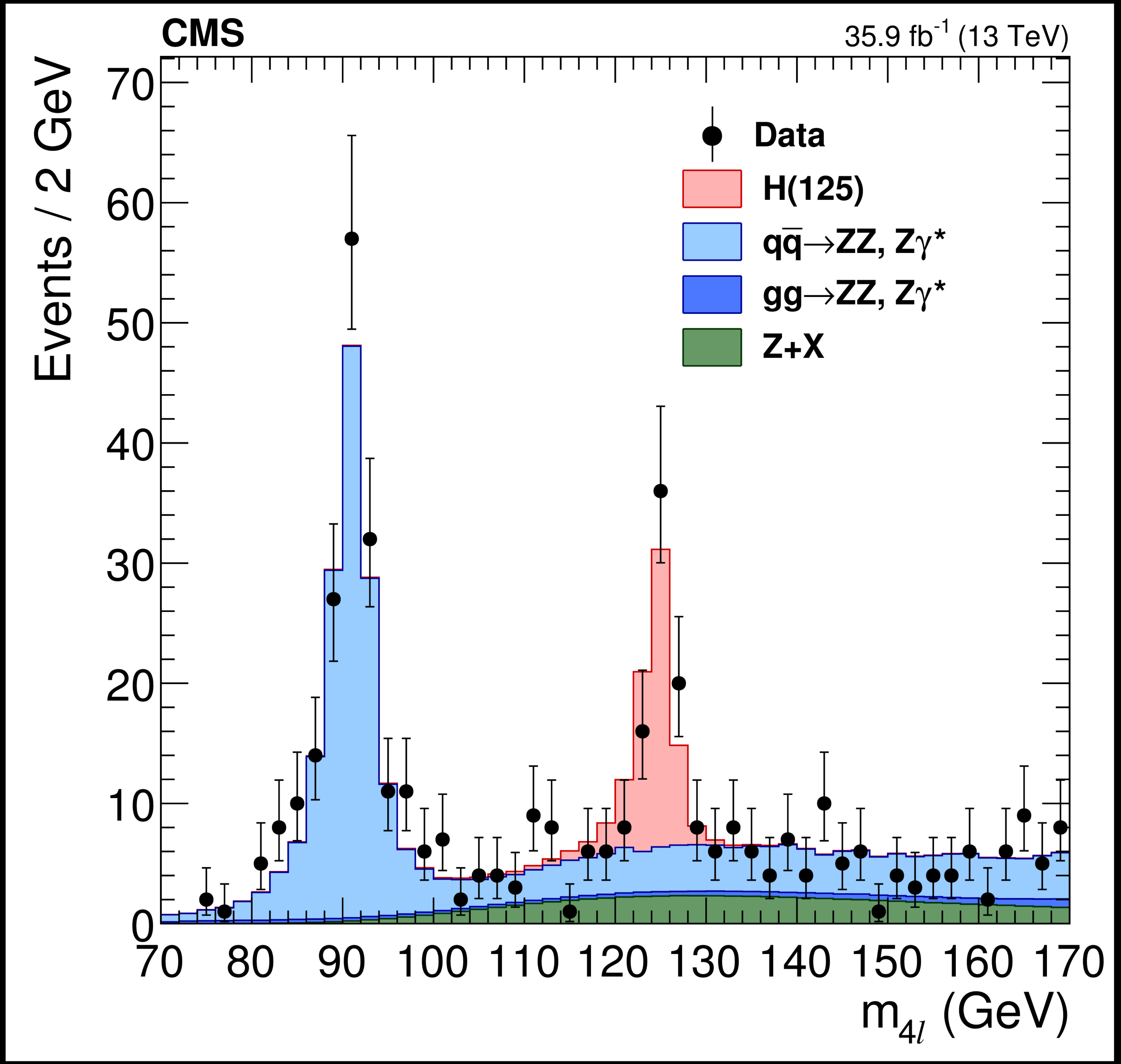




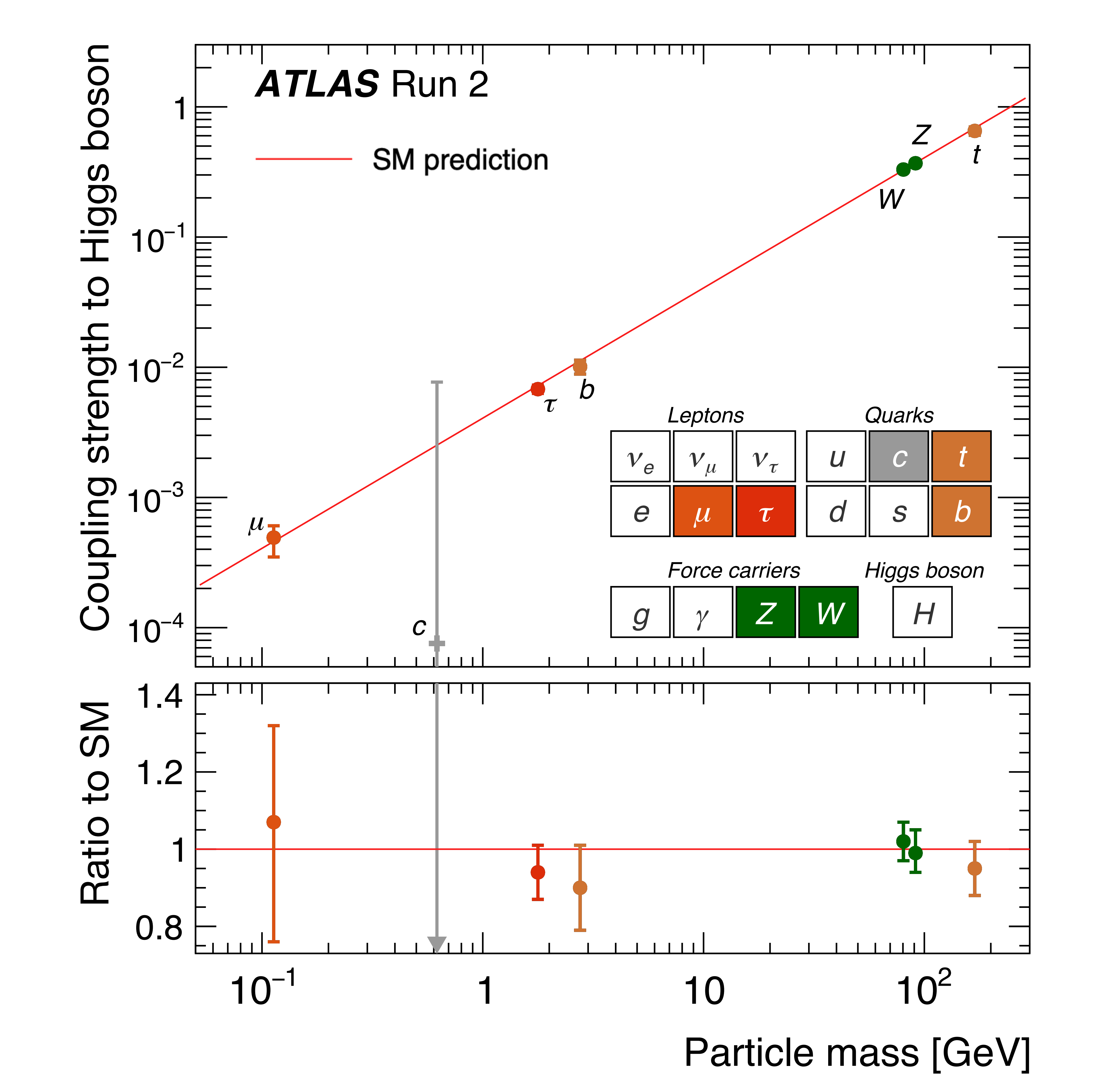
Did we need theory to observe the Higgs resonance?

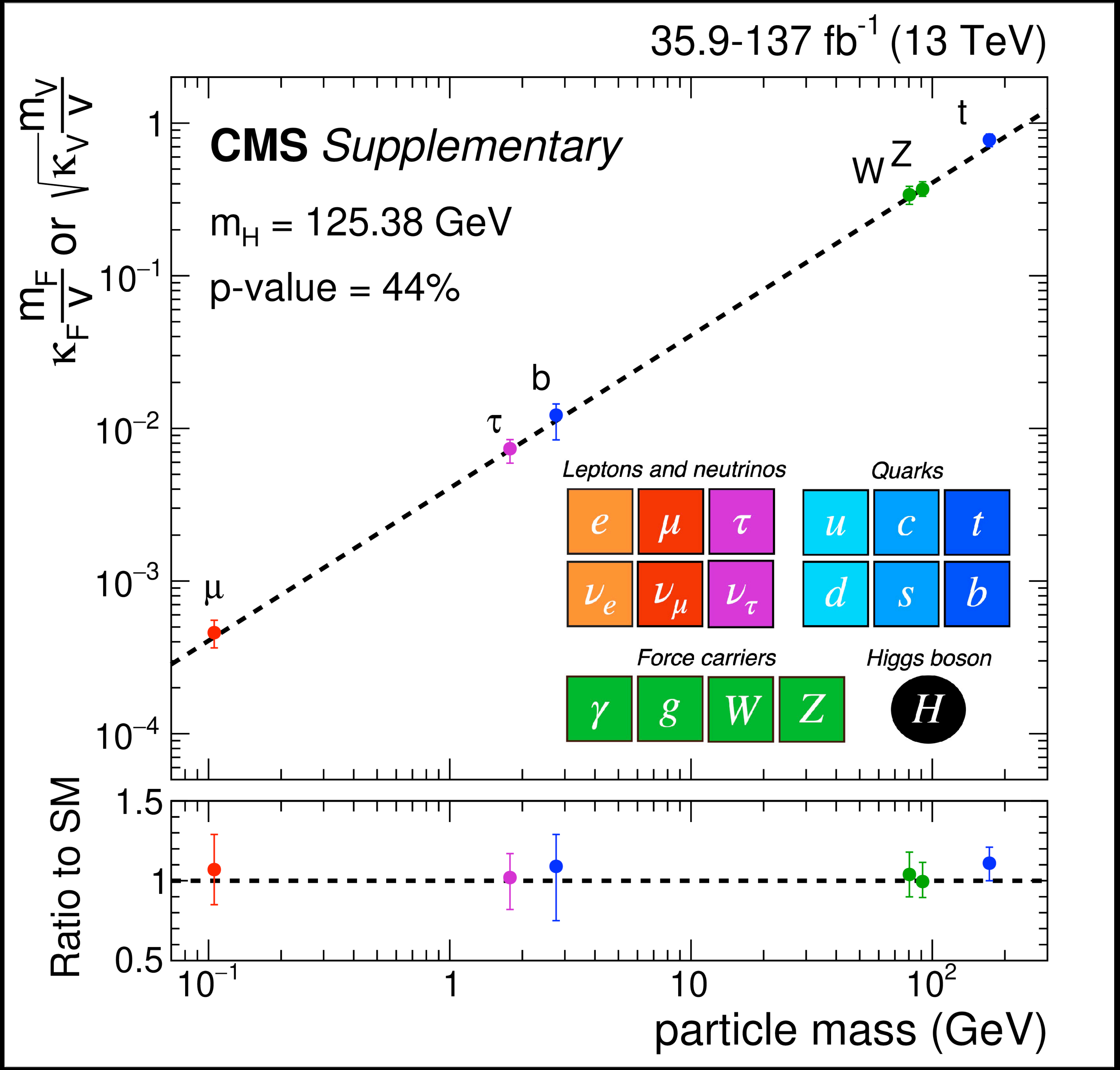
...no! (not really)





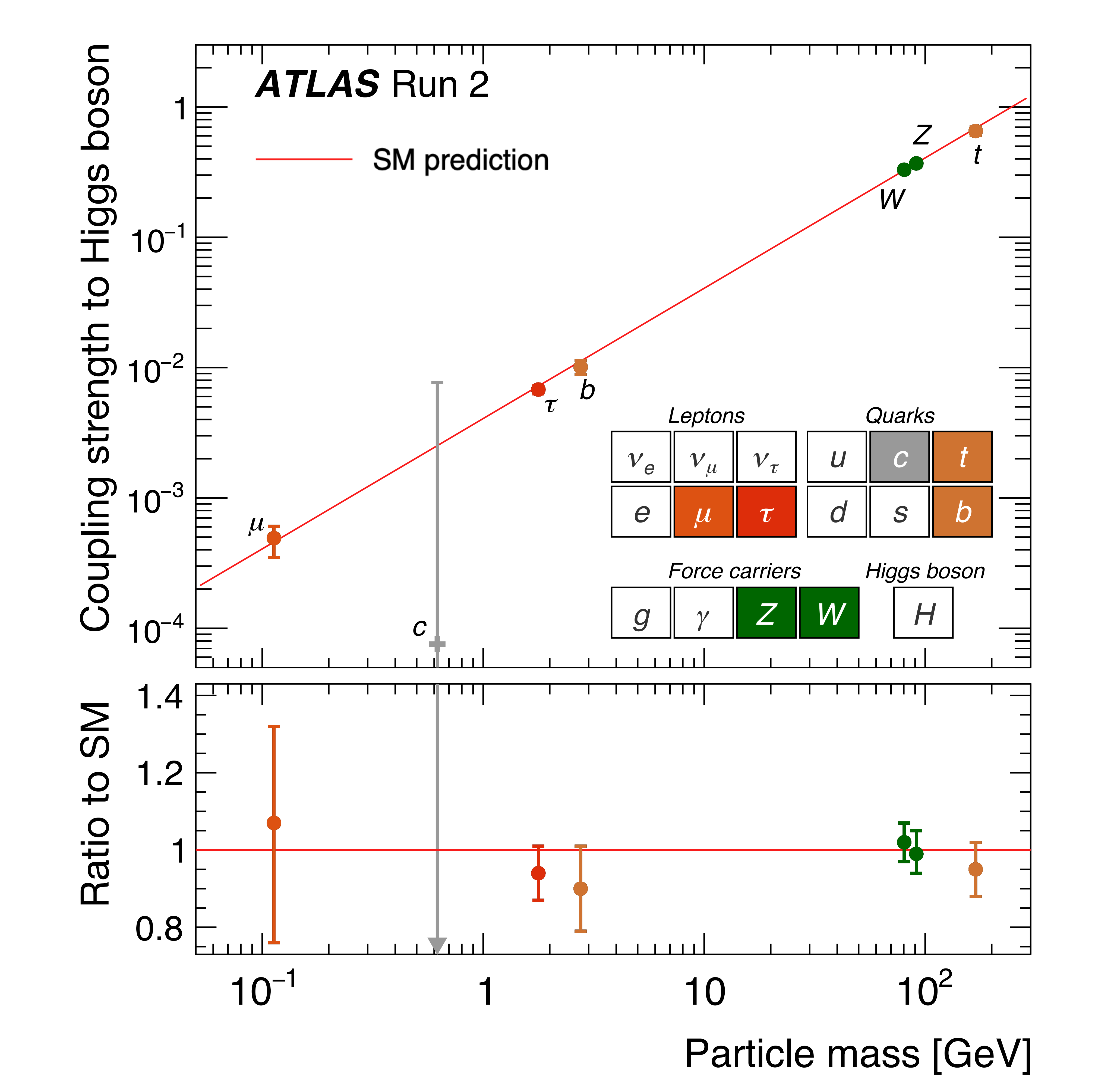
Do we need theory to measure Higgs couplings?

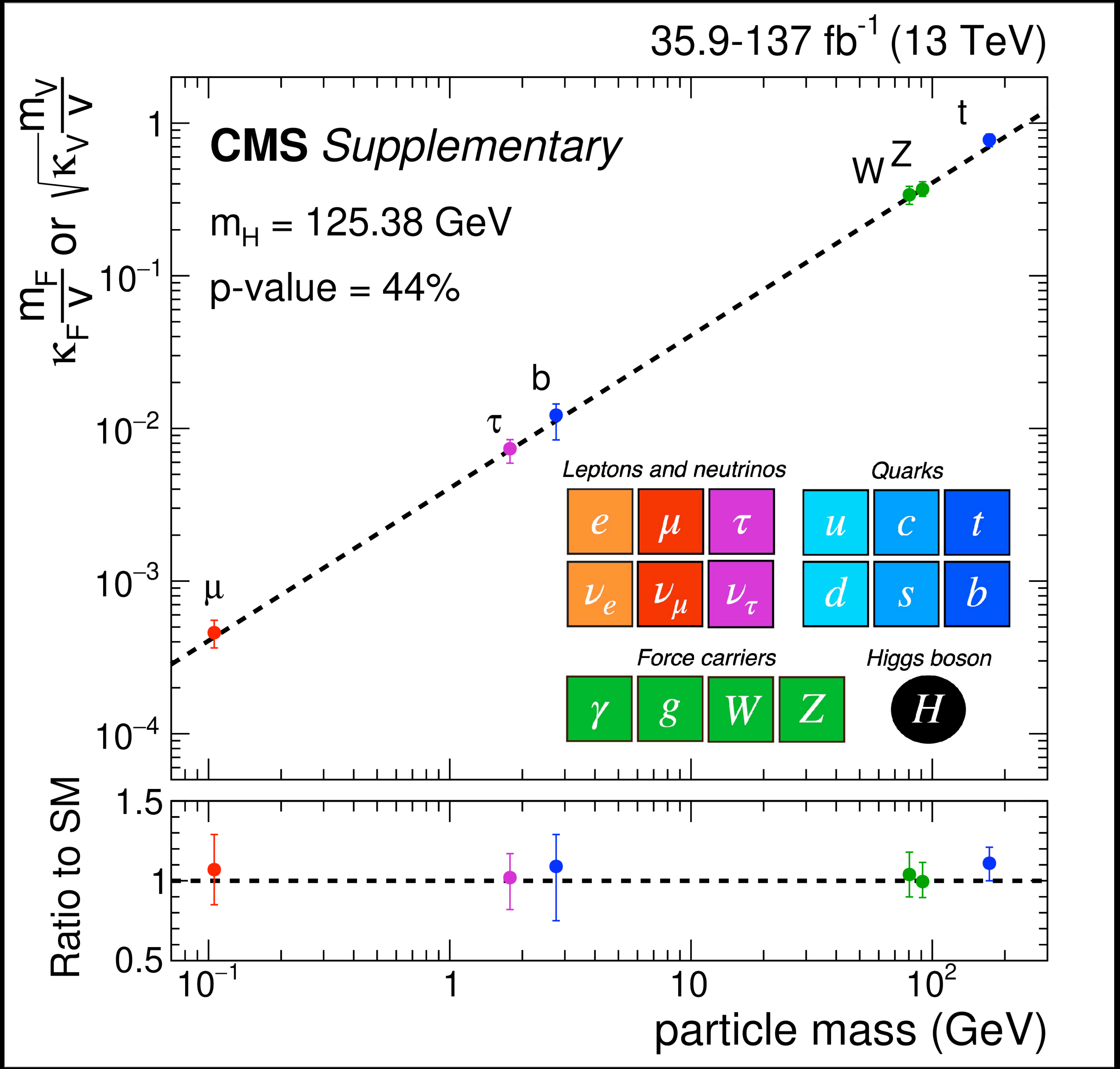




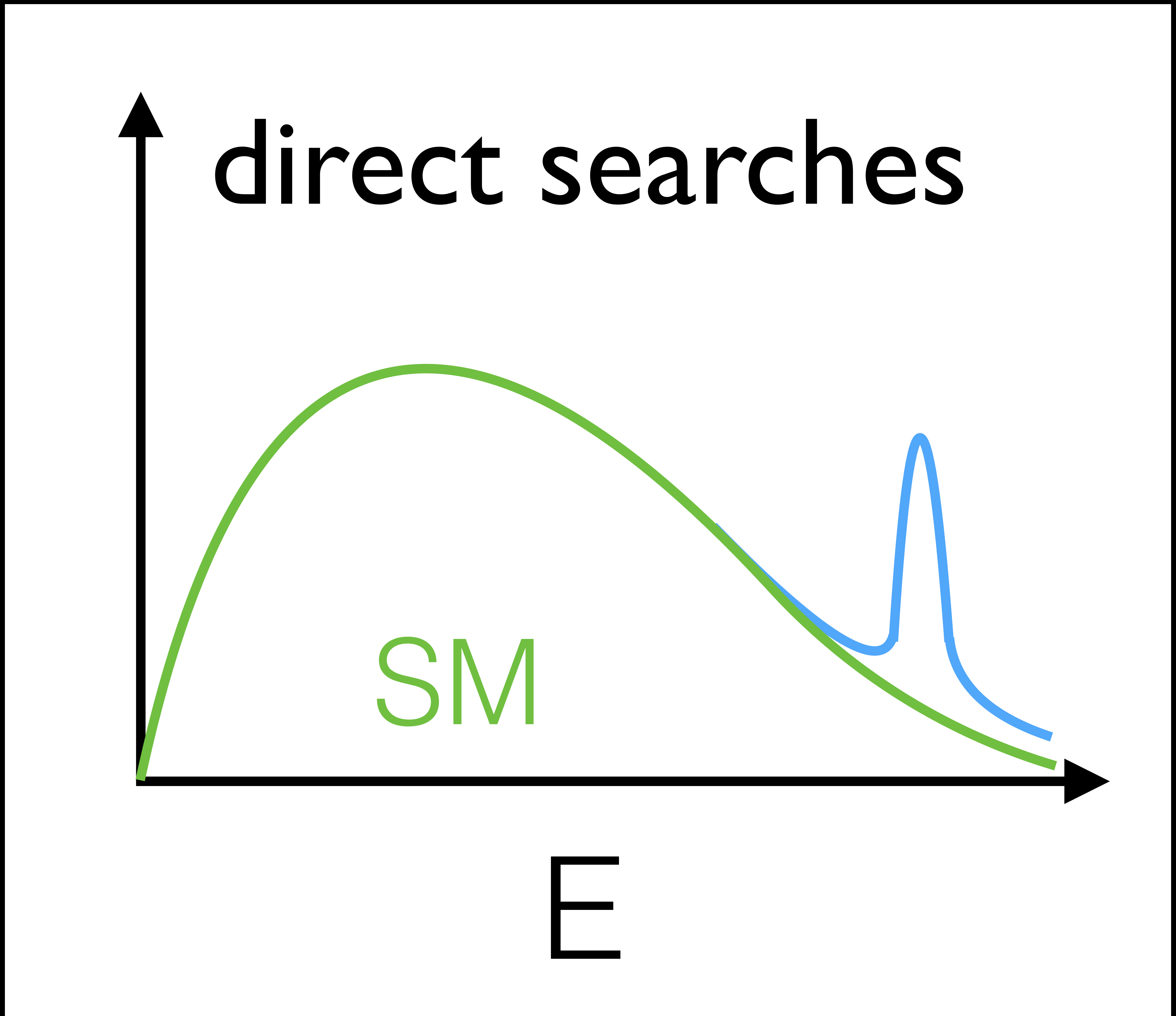
Do we need theory to measure Higgs couplings?

Yes, absolutely!



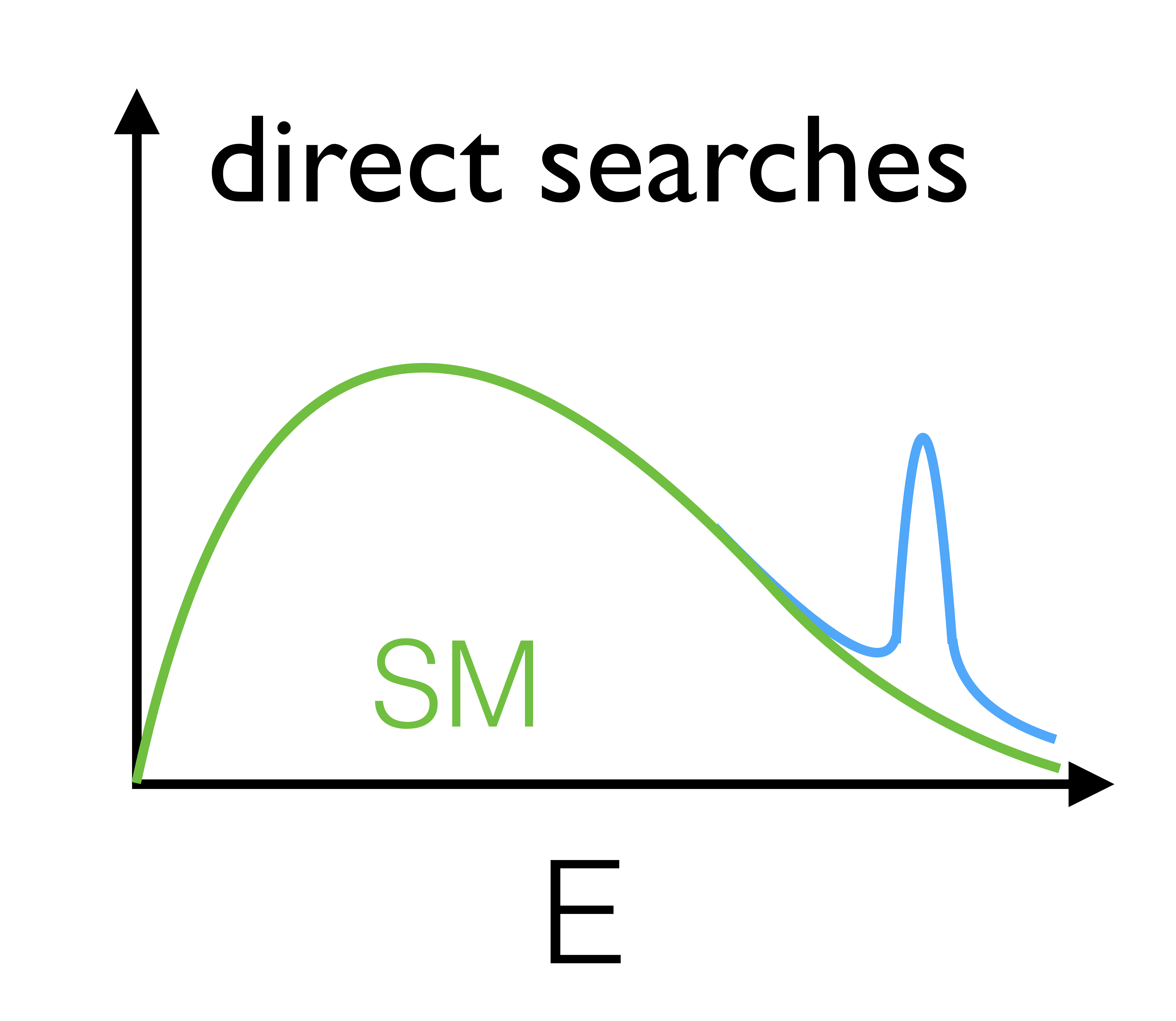


Do we need theory to find a New-Physics resonance?



Do we need theory to find a New-Physics resonance?

No!



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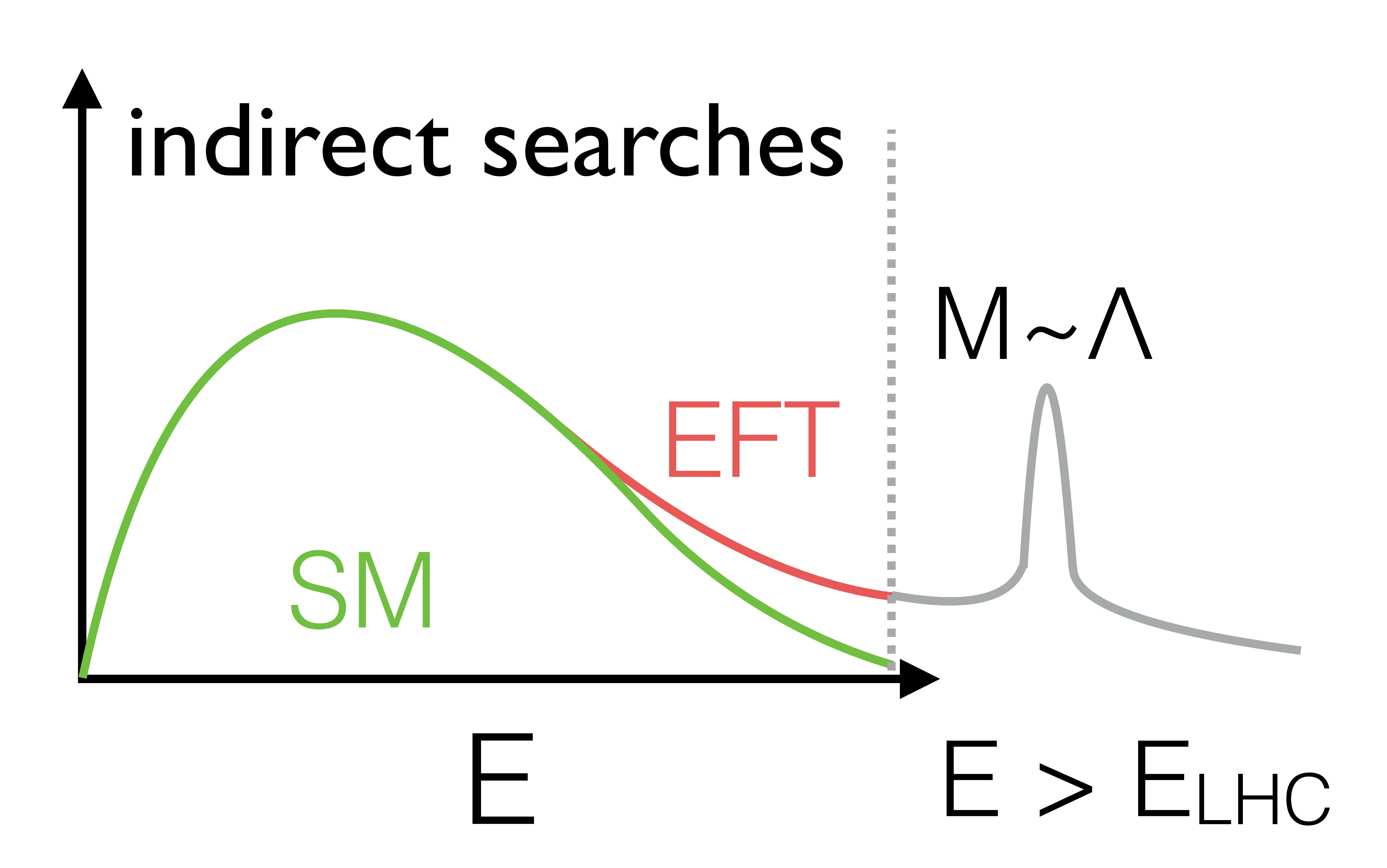

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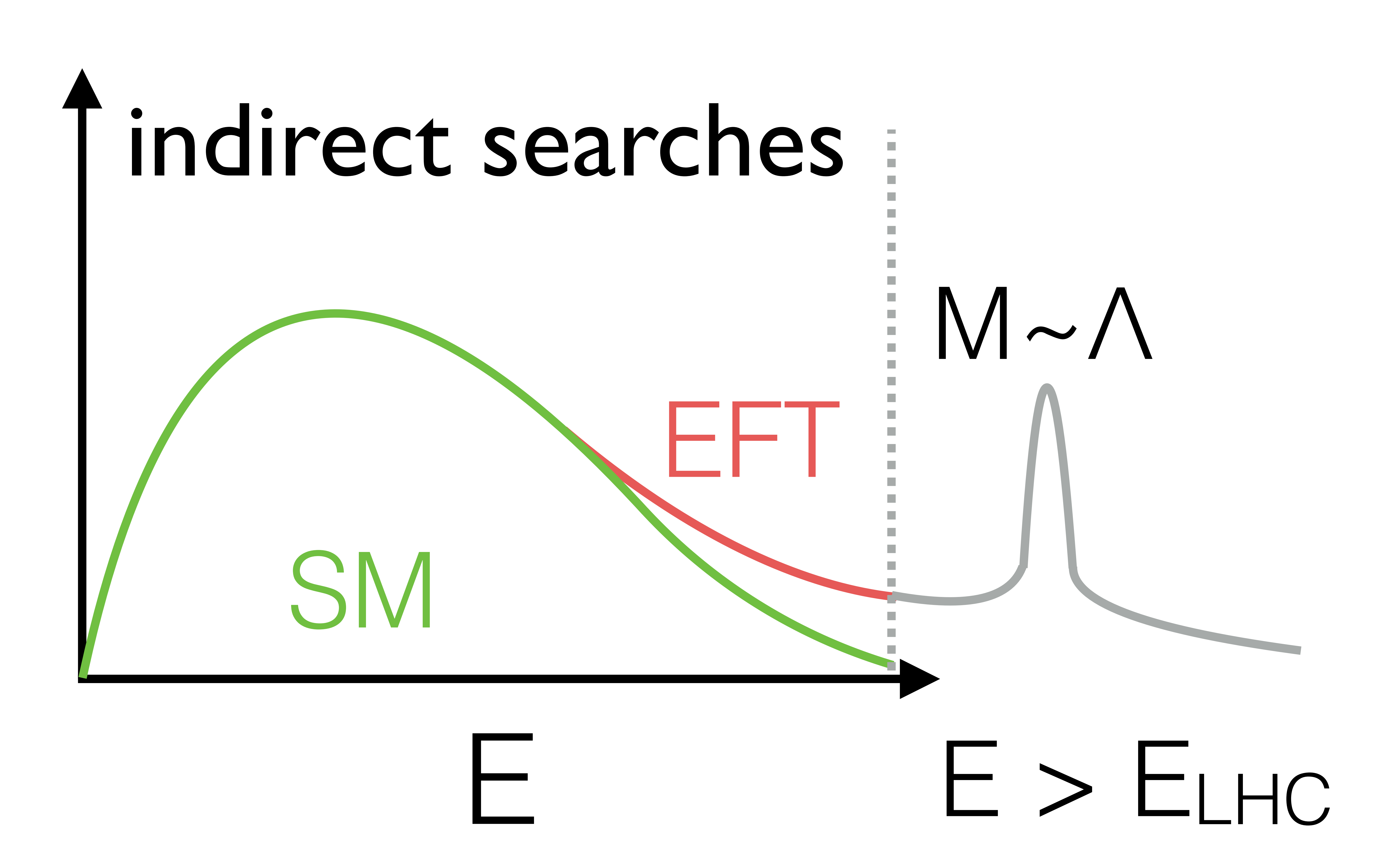
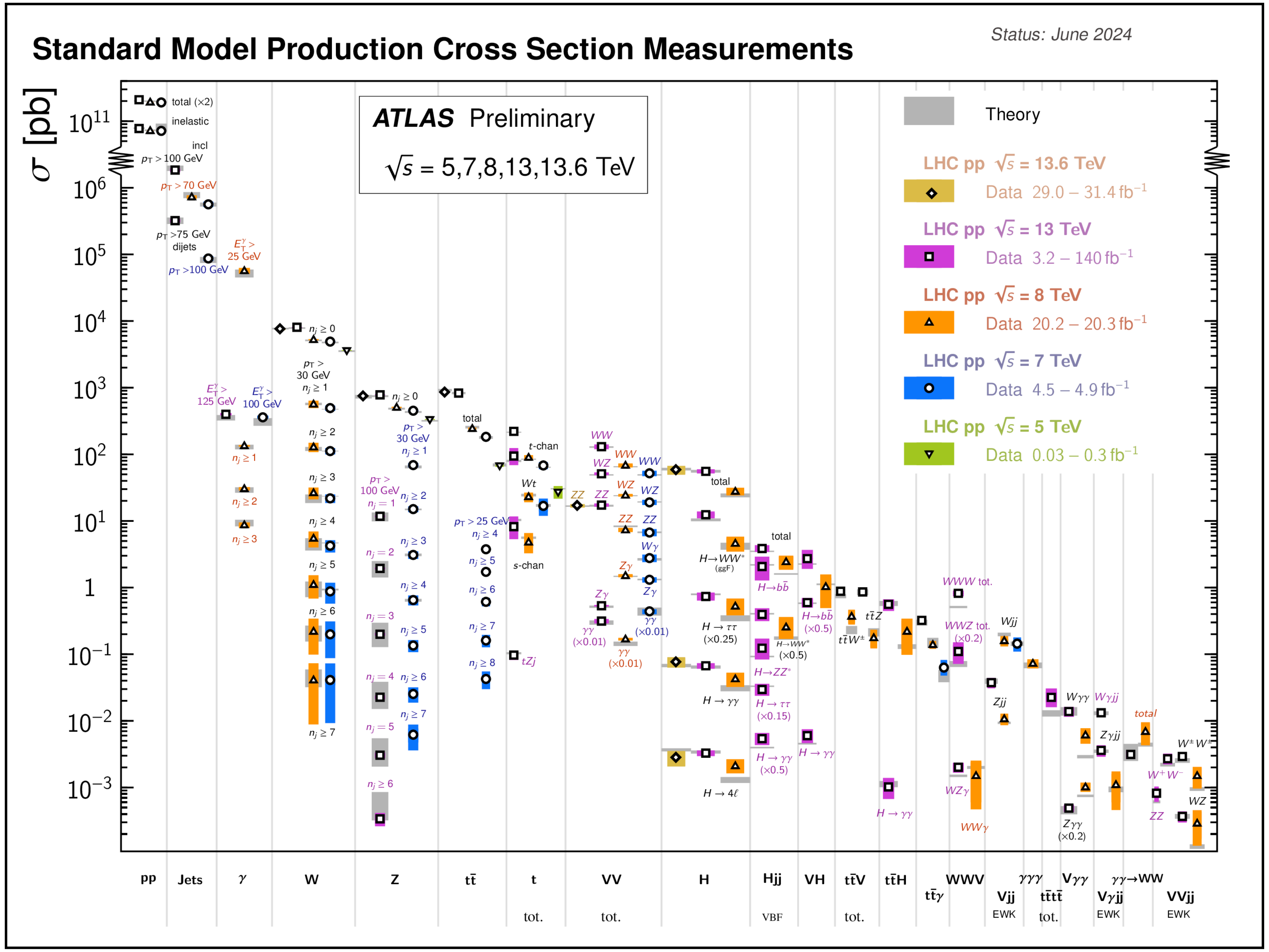
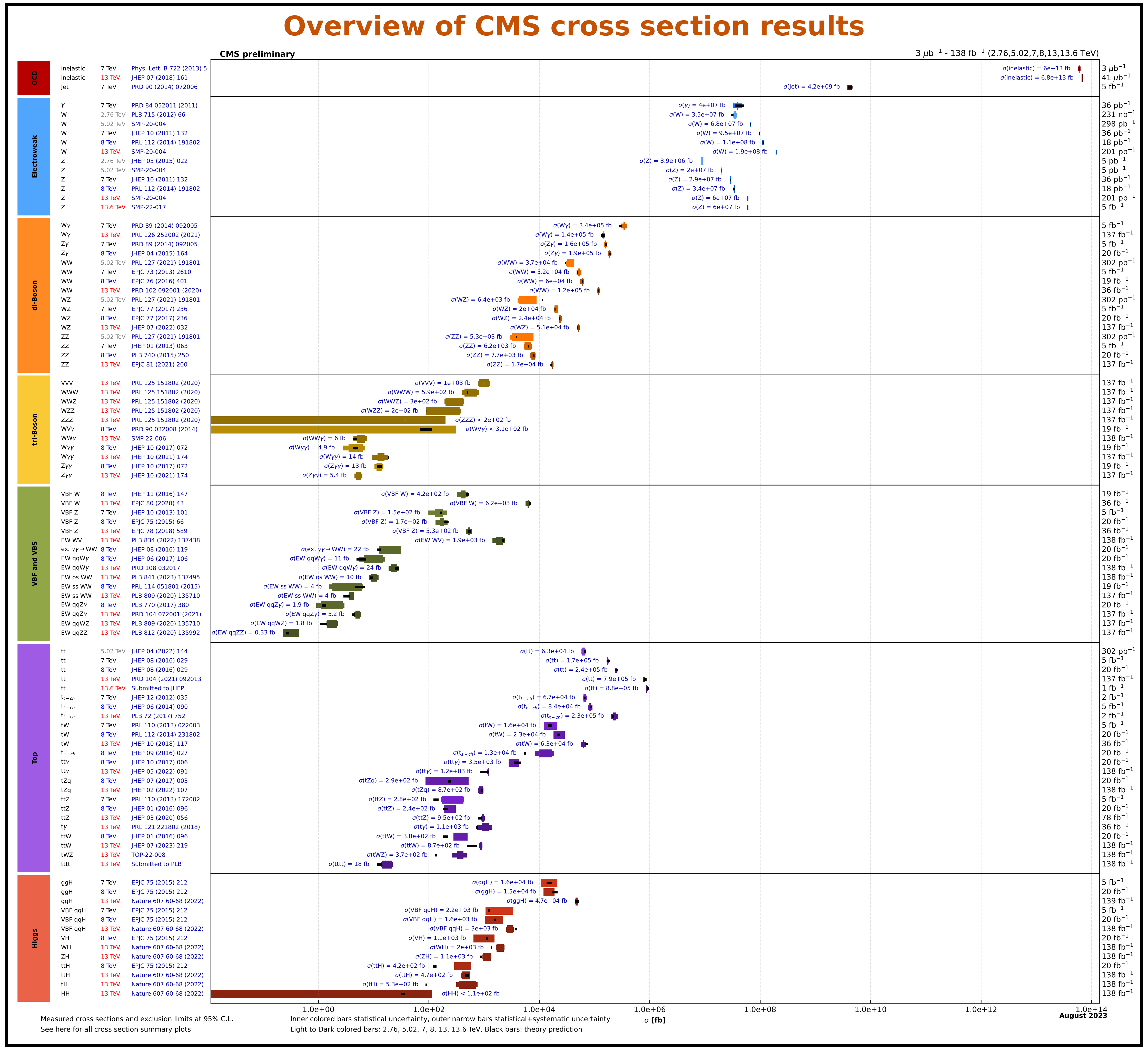


How do we get here?





Outline

Lecture 1

★ Fixed-order calculations

- QCD basics (Lagrangian, Feynman rules, strong coupling)
- LHC Factorization/Master Formula (PDFs, partonic cross section)
- NLO QCD (methods, slicing vs. subtraction vs. analytic)
- NNLO QCD (methods, timeline)
- EW corrections (NLO, Sudakov logarithms, mixed QCD-EW corrections?)

Lecture 2: Hands-on session on MATRIX

Lecture 3

★ Monte Carlo Event Generation & Resummation

- Resummation
- Parton Shower Generators (formalism, hadronization, MPI)
- NLO+PS Matching (MC@NLO, Powheg, merging)
- NNLO+PS Matching (MiNNLO, Geneva)

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Useful literature

★ Introductory level (QCD lecture notes from CERN schools)

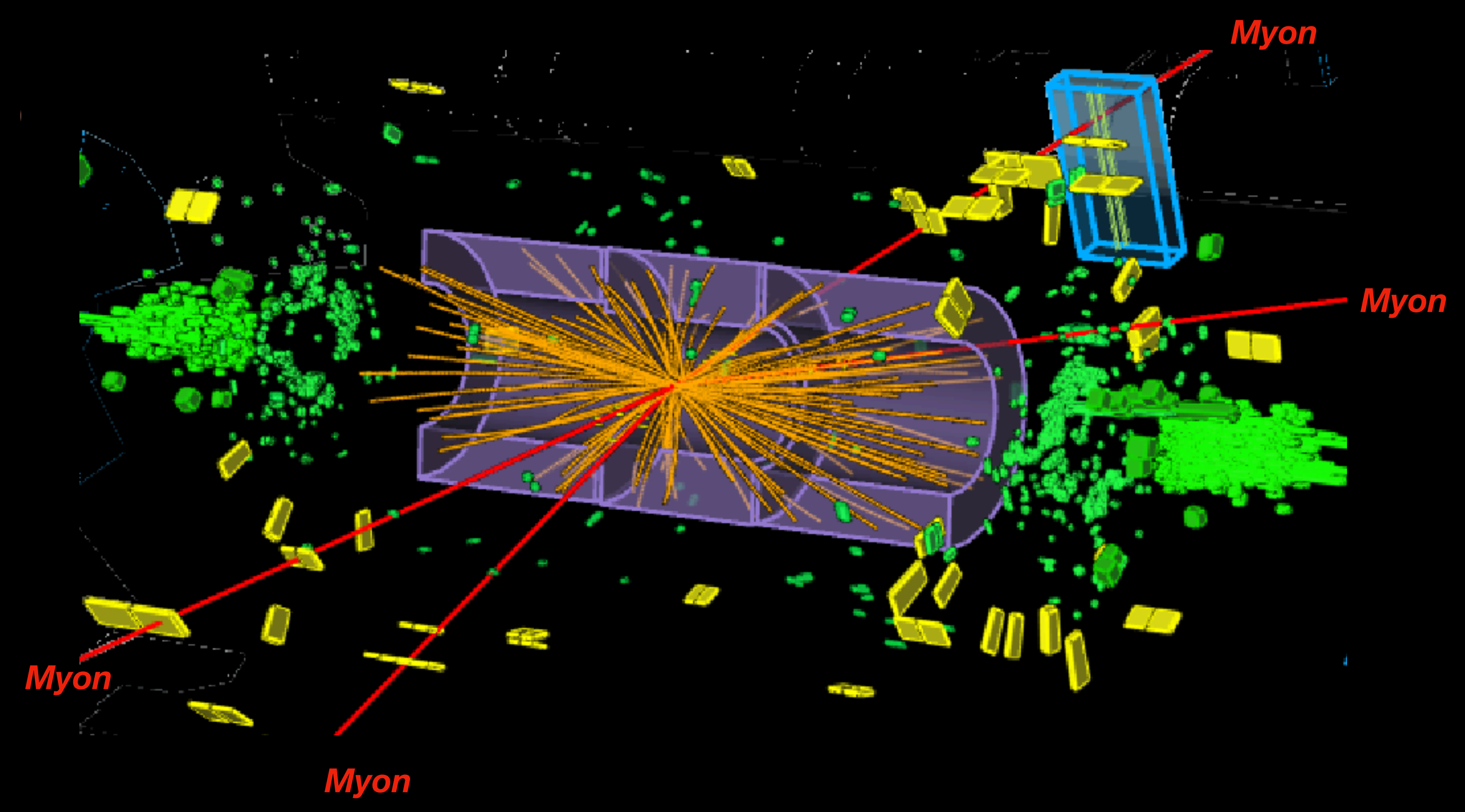
- Peter Skands, arXiv:1207.2389
- Gavin Sakam, arXiv:1011.5131

★ Books on QCD

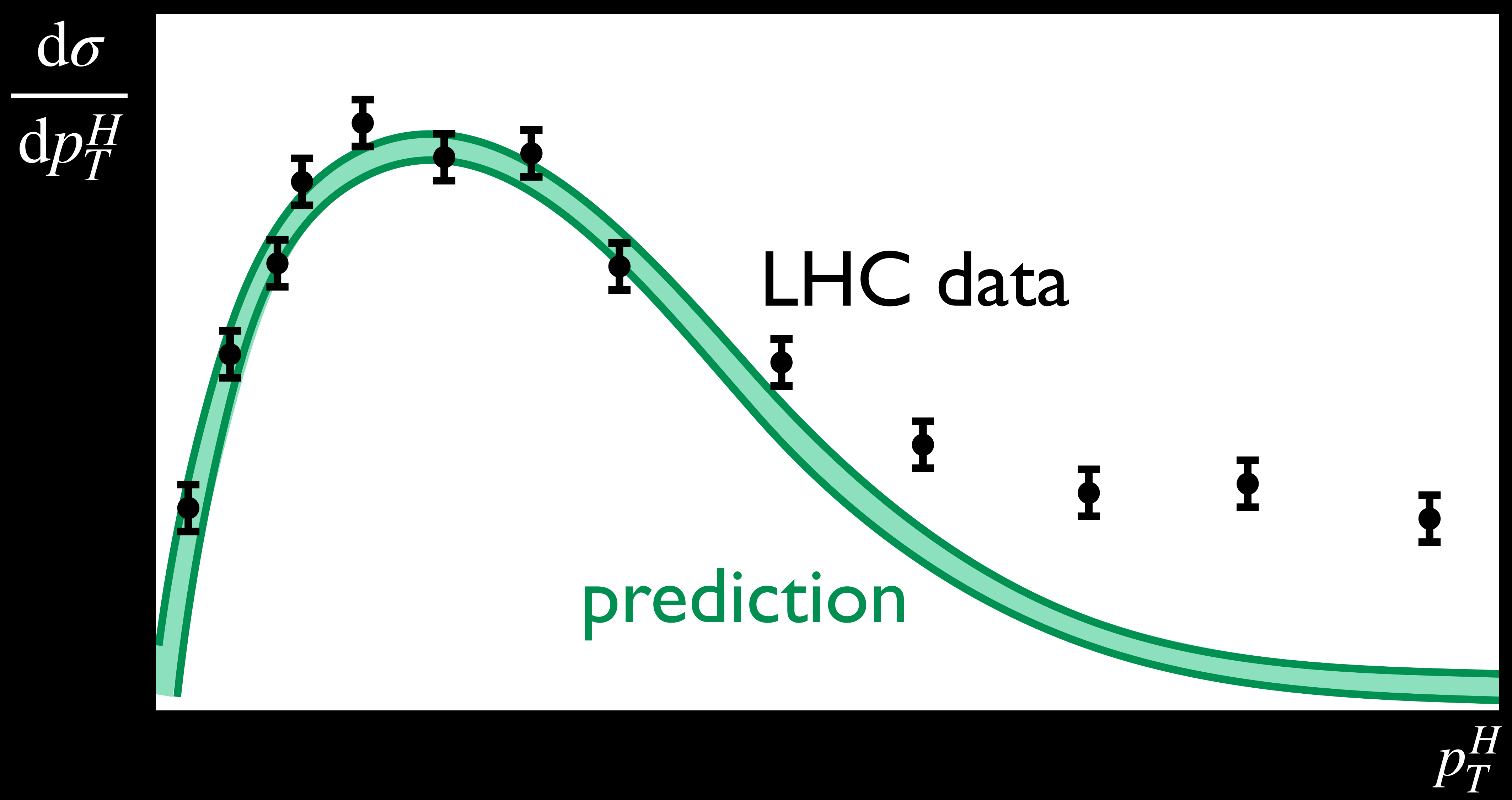
- "QCD and collider Physics", R.K. Ellis, W.J. Stirling, B.R. Webber, Cambridge, 1996
- "The Black Book of Quantum Chromodynamics: A Primer for the LHC Era", J. Campbell, J. Houston, F. Krauss, Oxford, 2018

Imagine…

…LHC records enough statistics…



…to observe an excess in a Higgs distribution



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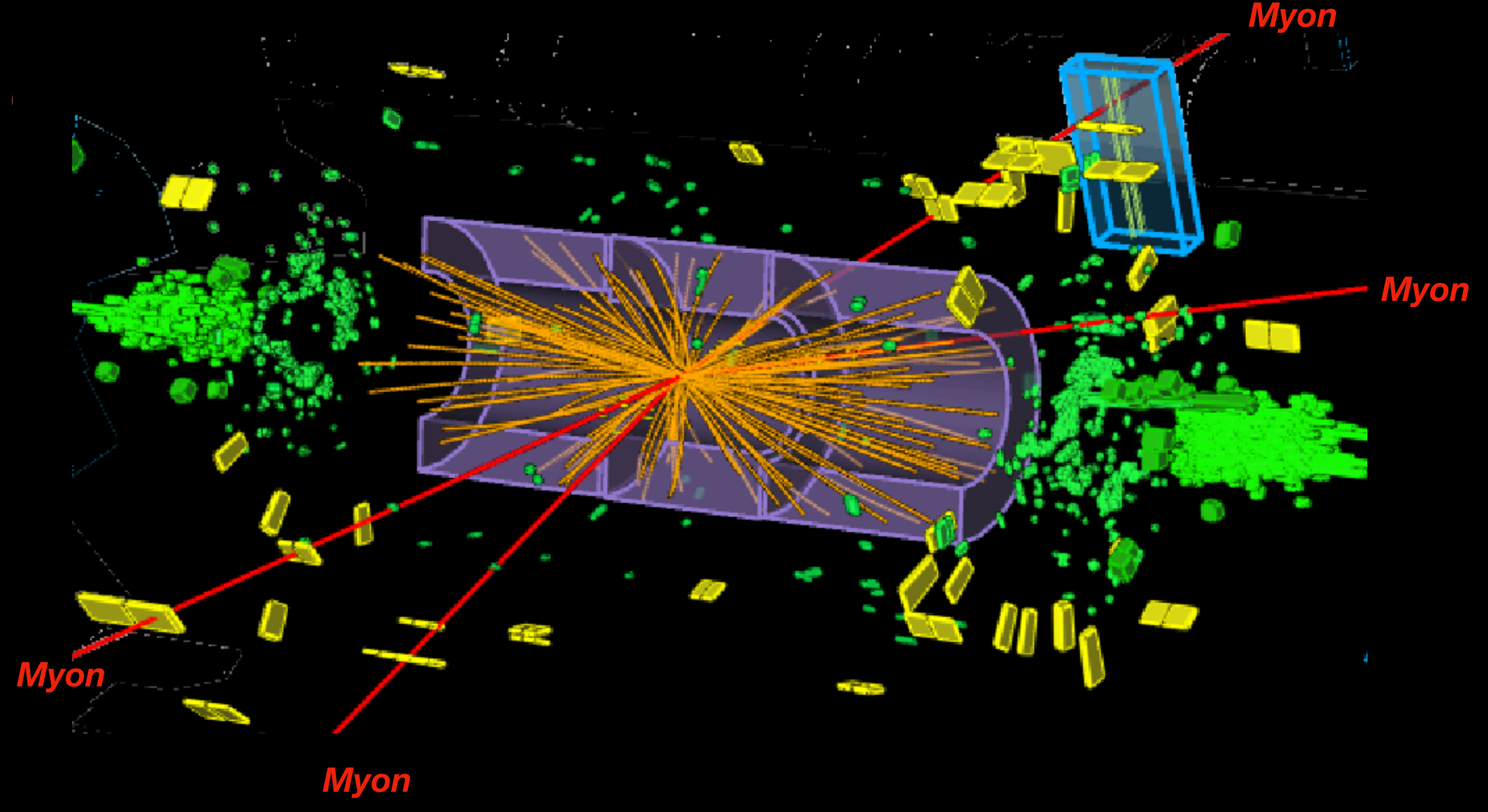

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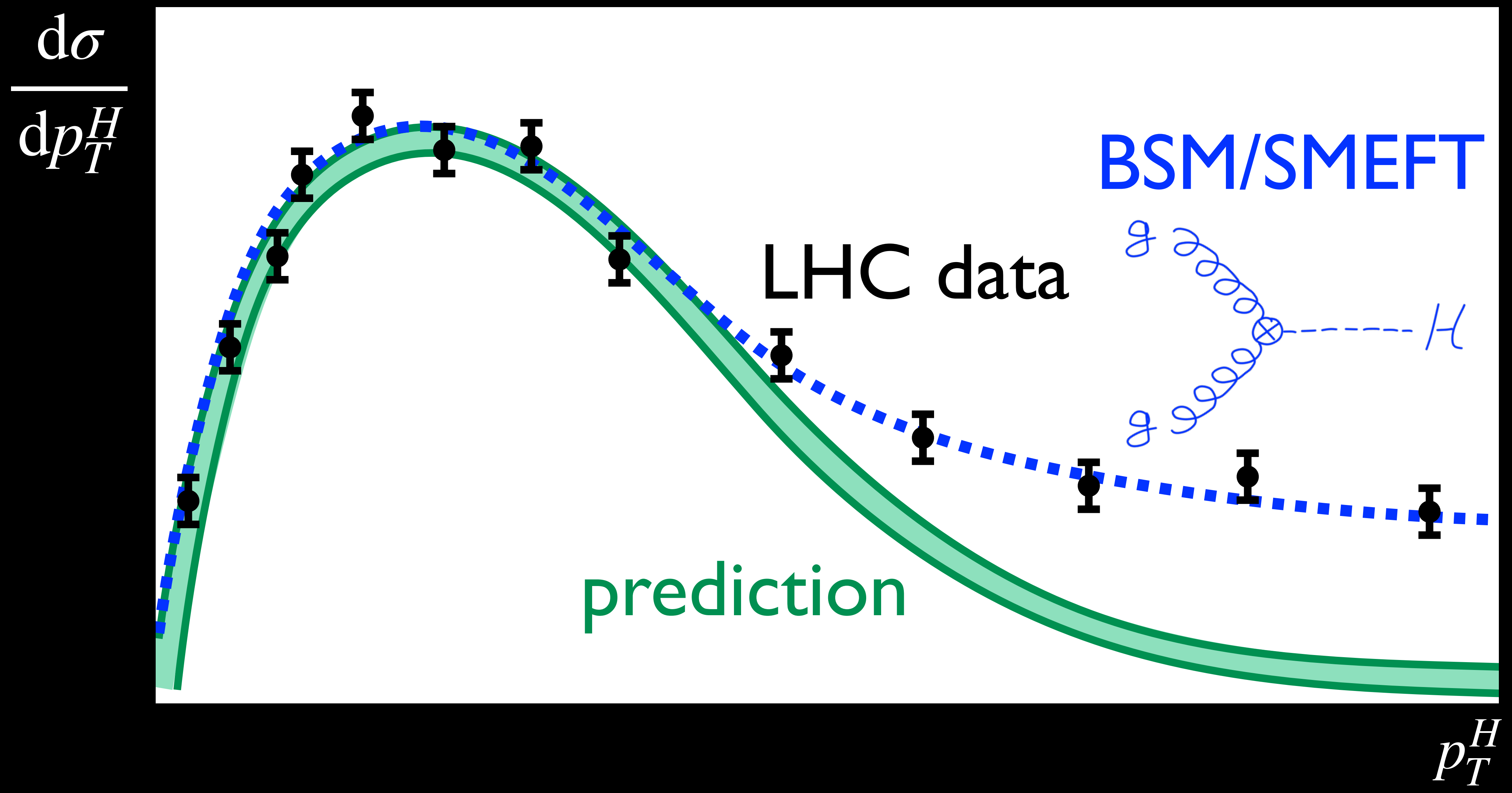
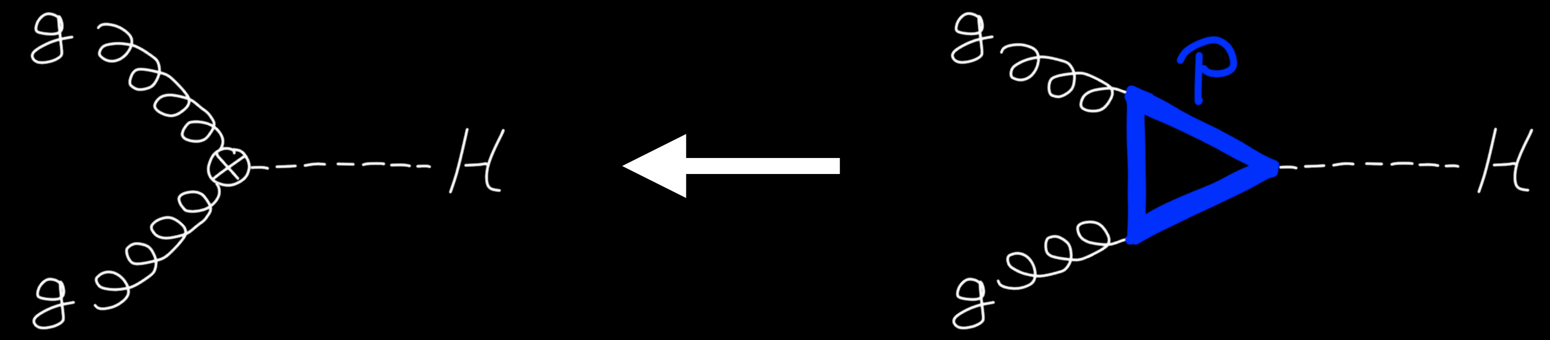


New Physics discovered!

→ point-like Higgs-gluon interaction
see e.g. [Grazzini, Ilnicka, Spira, **MW** '16]



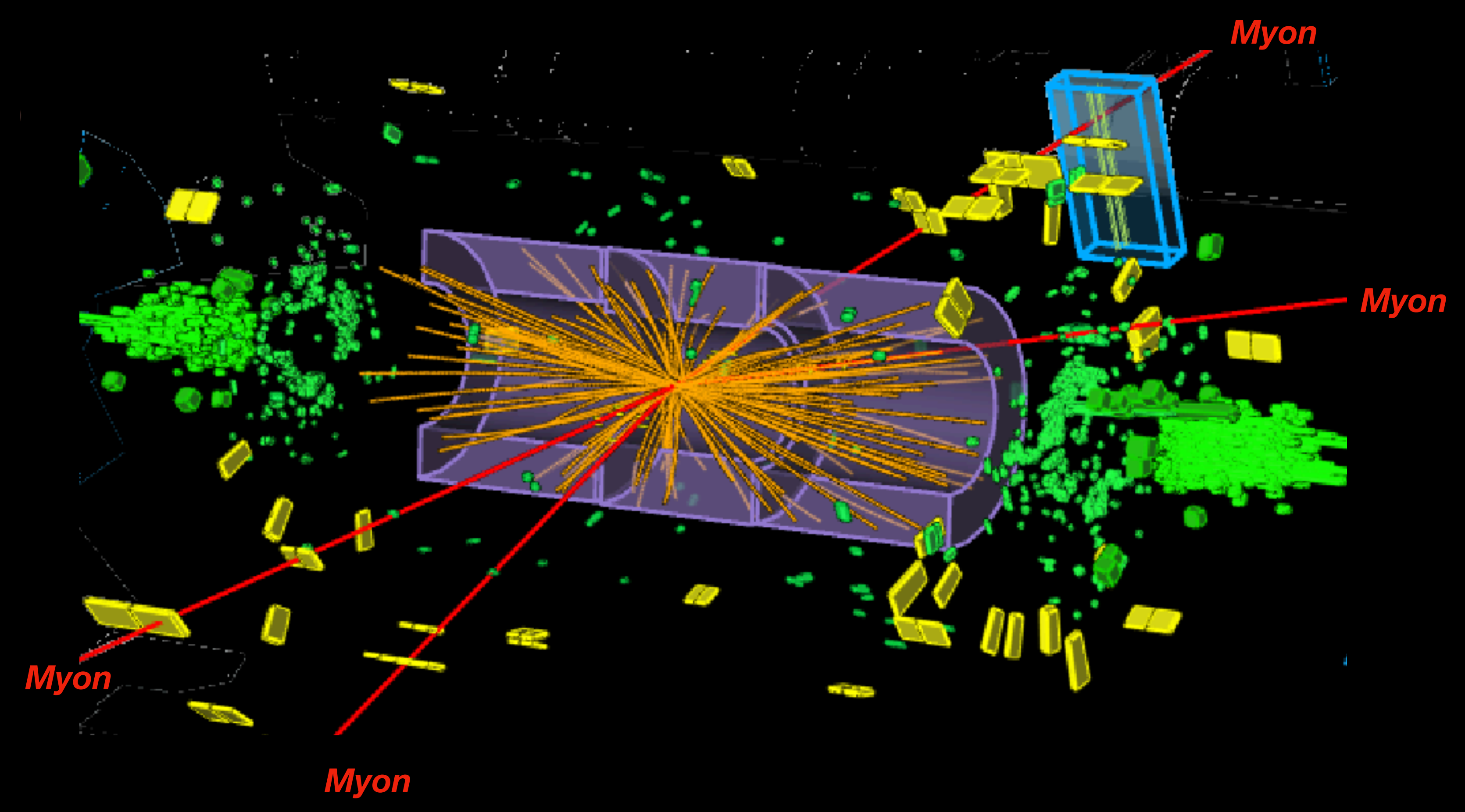
→ new **heavy particle** running in loop


Sunday, July 4, 2025

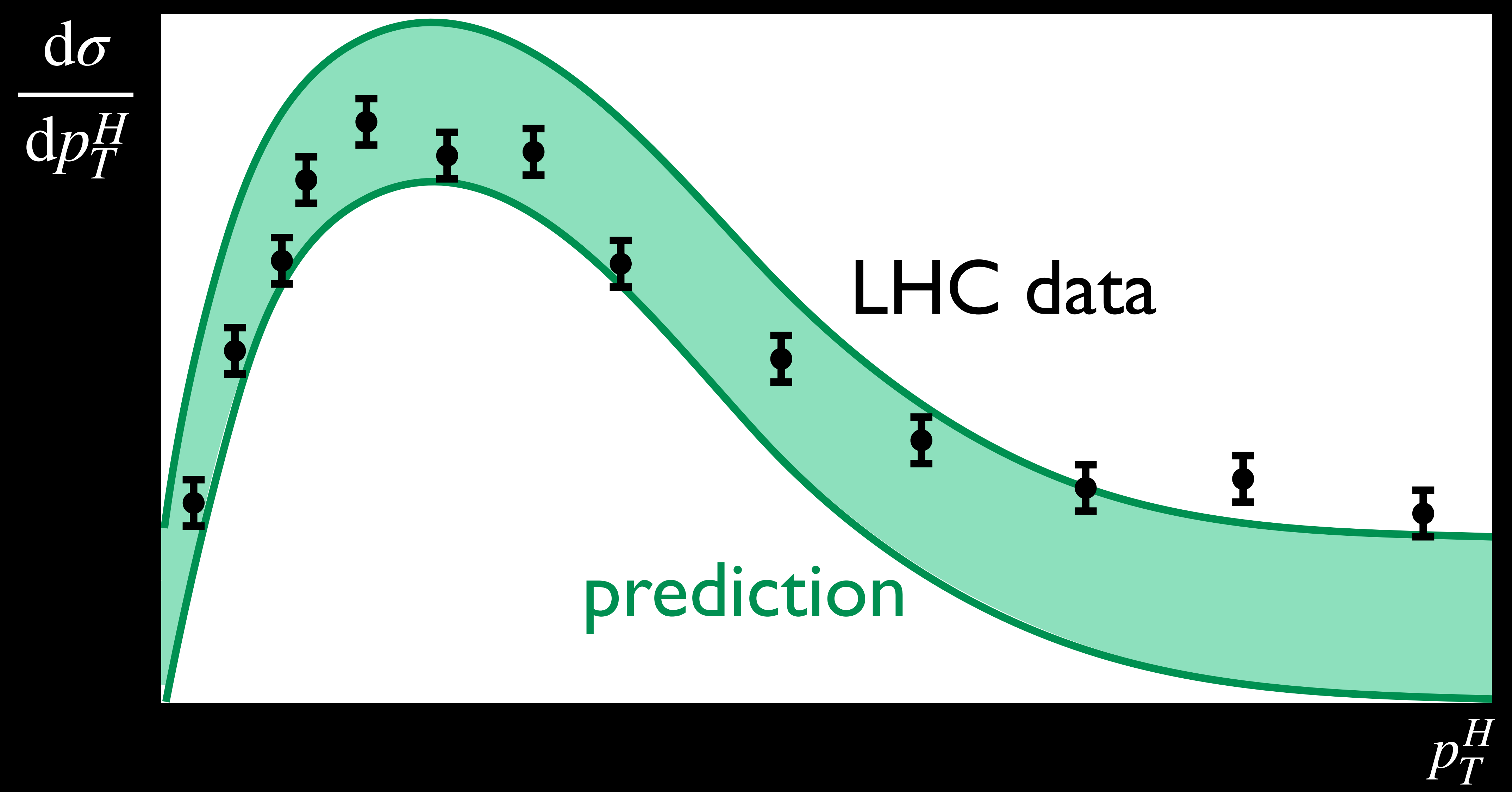
LHC discovers new particle

Likely another Nobel prize in particle physics

Now Imagine…



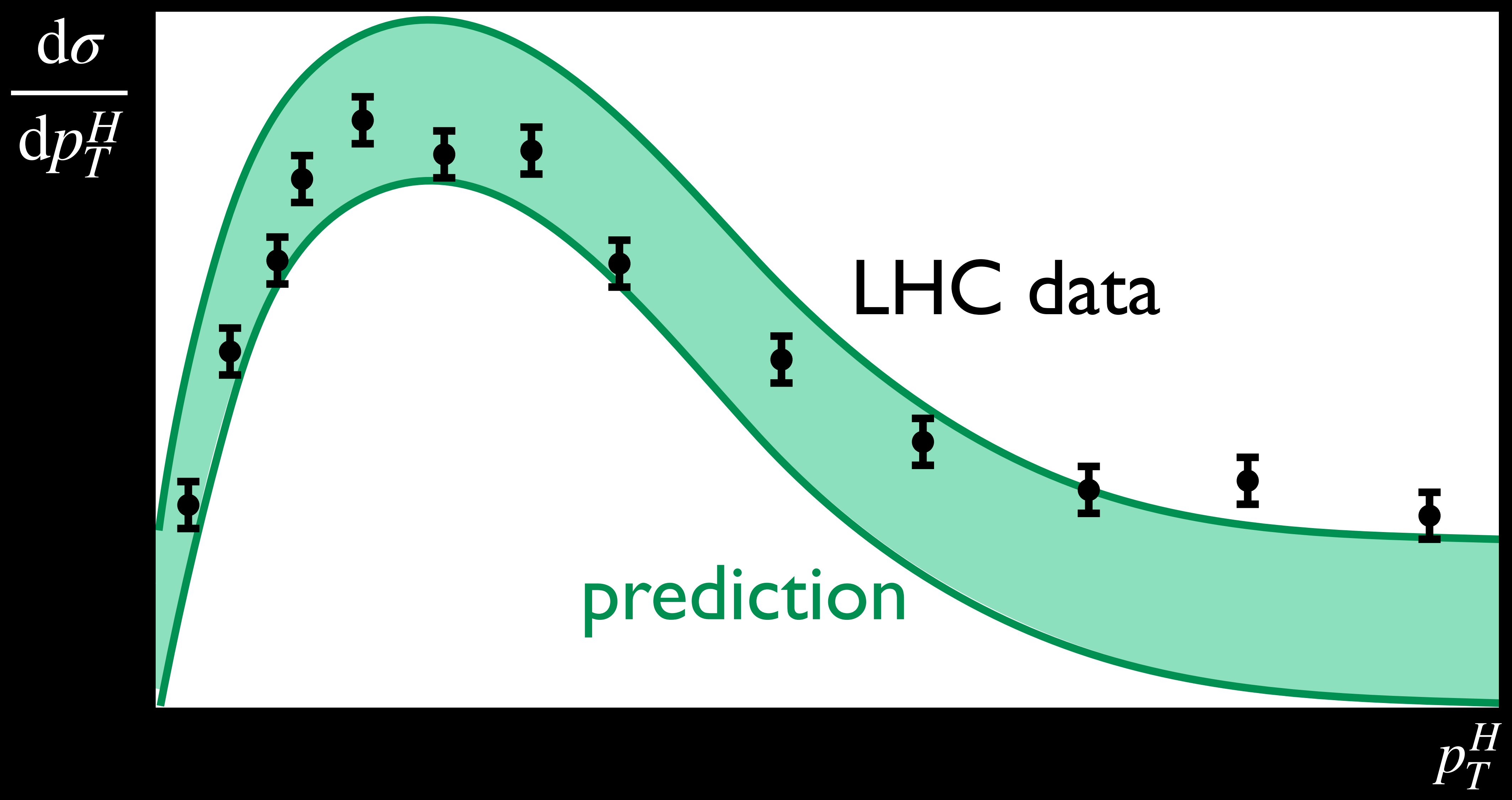
…the theory error was five times larger

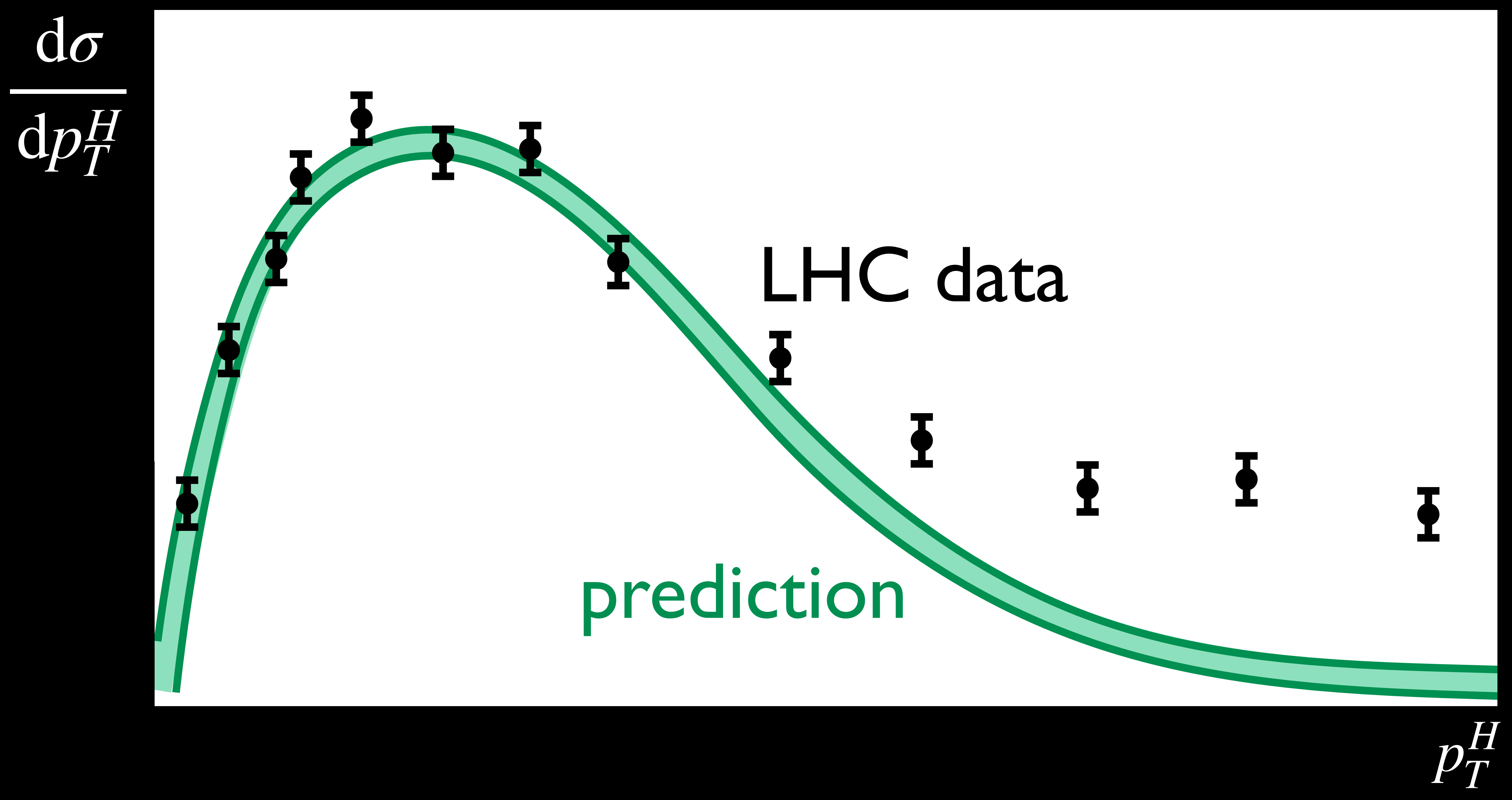


WE MISSED DISCOVERING NEW PHYSICS

make sure there are only two LHC scenarios:

1. establish SM for accessible energy scales at LHC
2. find deviation pattern that hints to BSM Physics





➙ more precise predictions translate into higher discovery reach almost "for free"

The QED Lagrangian

$$\mathscr{L}_{\text{QED}} = \mathscr{L}_{\text{Dirac}} + \mathscr{L}_{\text{Maxwell}} + \mathscr{L}_{\text{int}}$$

$$= \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e\bar{\psi}\gamma^{\mu} A_{\mu} \psi$$

ψ dirac fermion fields with mass m

A_{μ}^{a} electromagnetic photon gauge fields

$F_{\mu\nu}^{a}$ photon field strength tensor $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$

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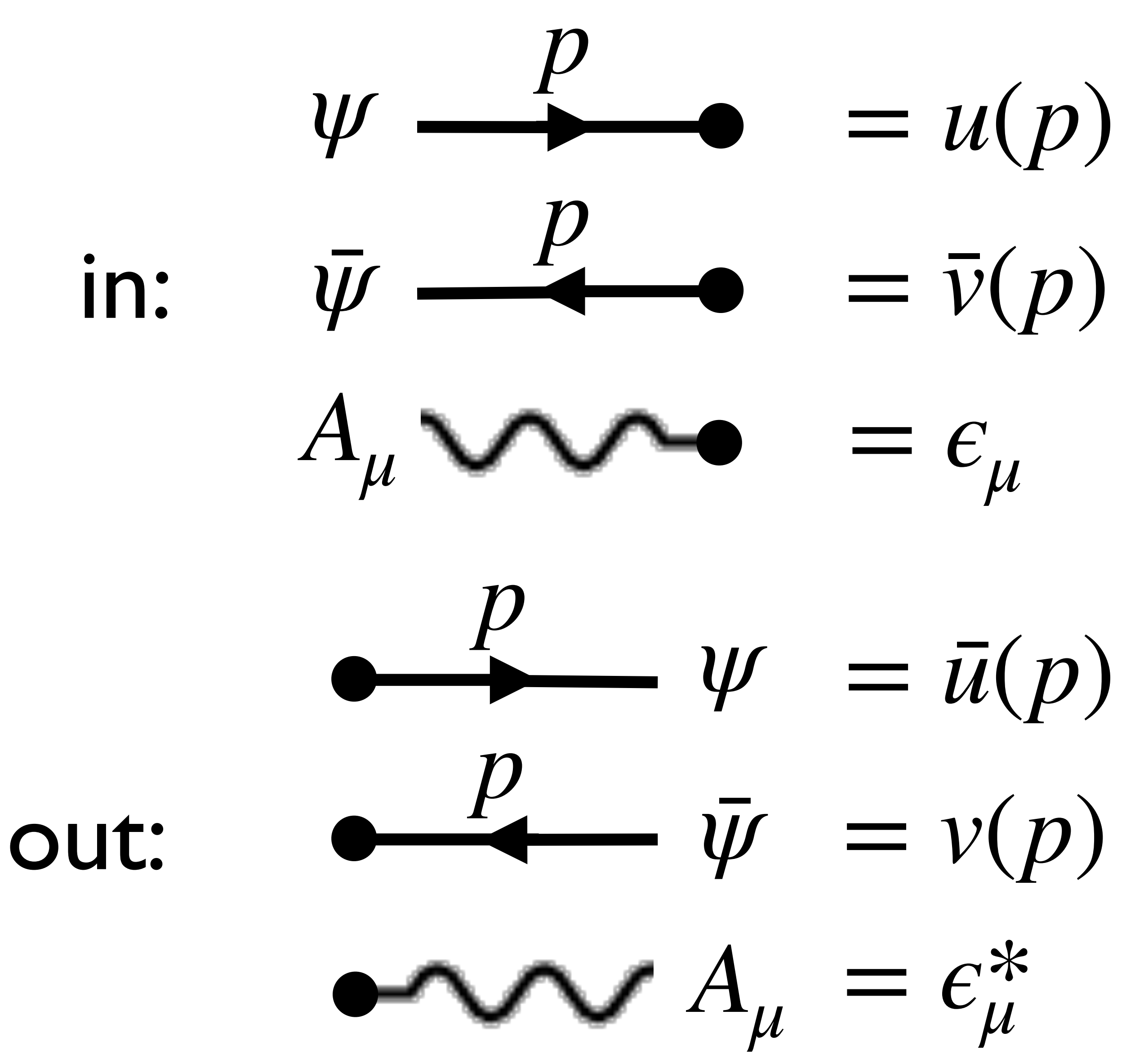
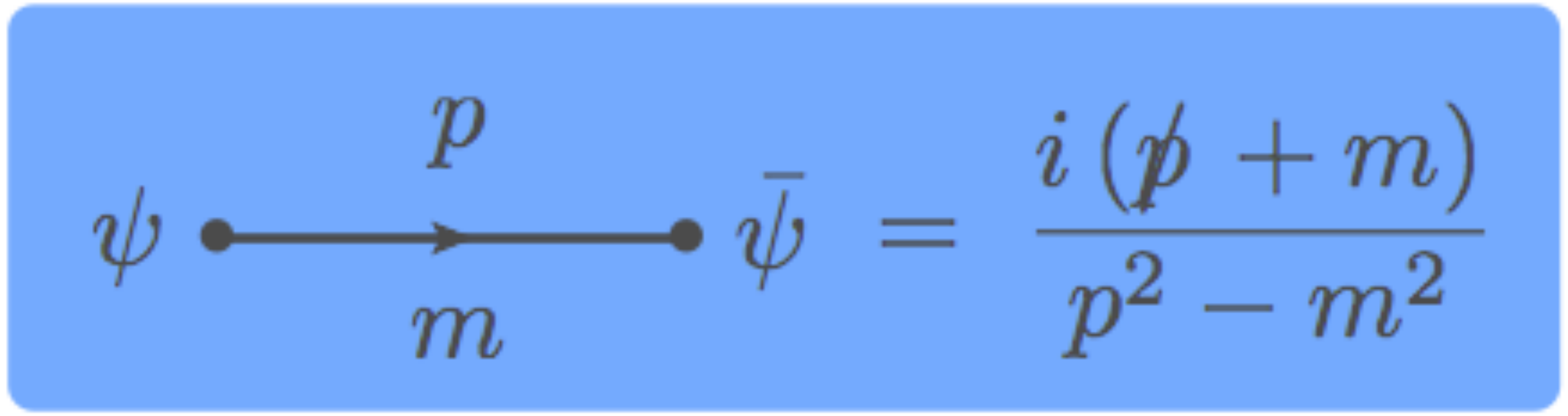
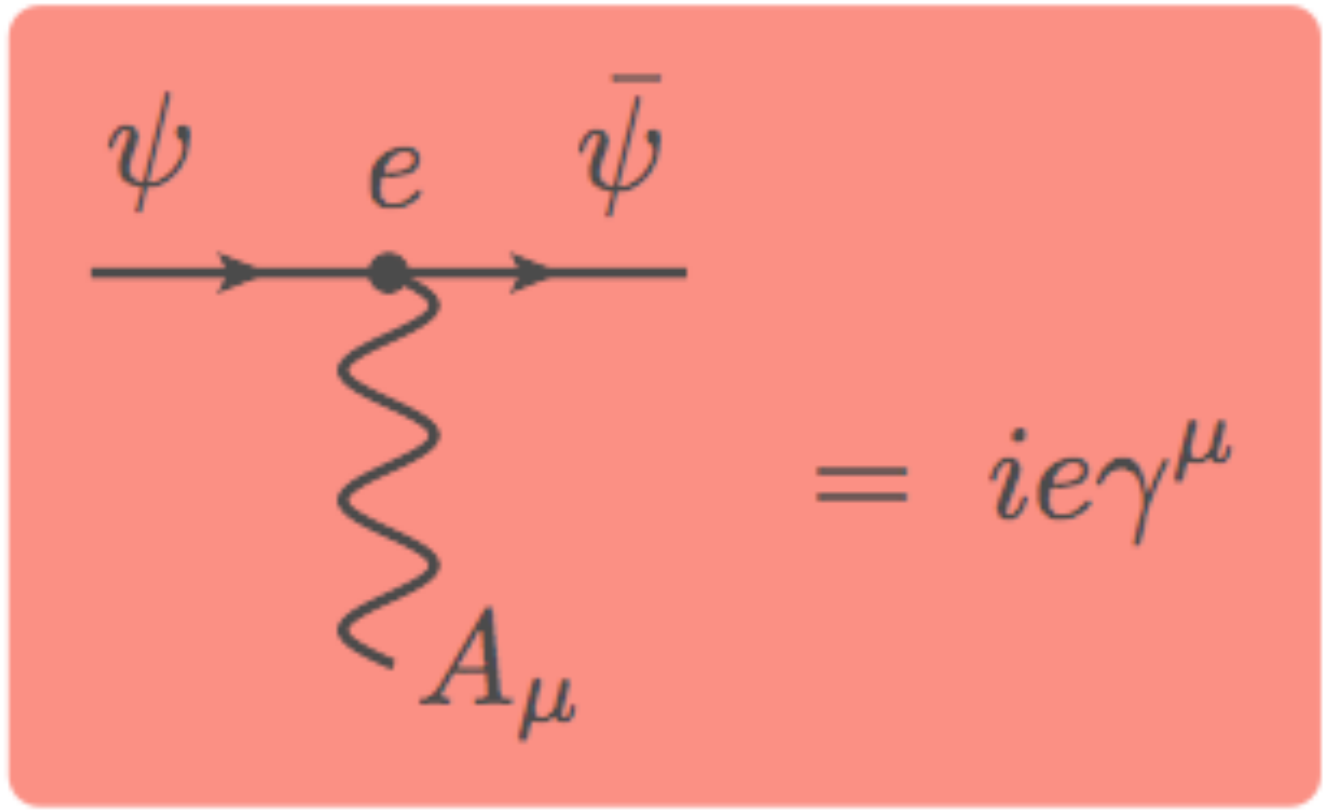
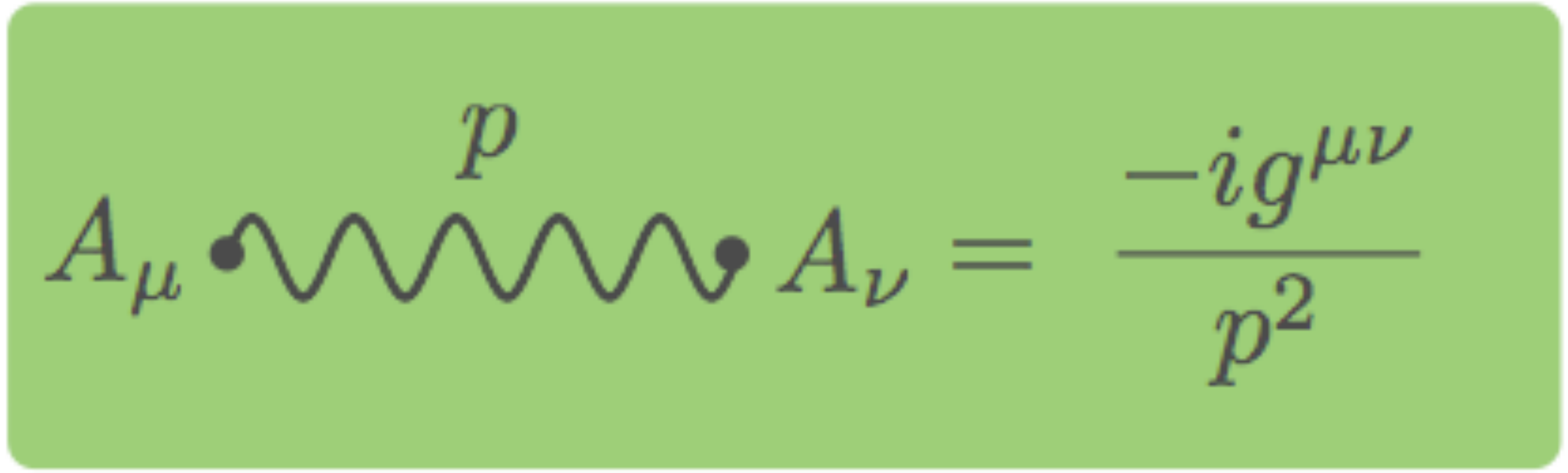
Feynman rules:









The QCD Lagrangian

$$\mathscr{L}_{\text{QCD}} = \mathscr{L}_{\text{Dirac}} + \mathscr{L}_{\text{Yang–Mills}} + \mathscr{L}_{\text{int}}$$

$$= \bar{\psi}_i (i\partial\!\!\!/ - m)\, \psi_i - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - g_s \bar{\psi}_i \gamma^\mu A^a_\mu t^a_{ij} \psi_j$$

- ψ_i quark fields with colour charge index i and mass m ➙ quarks come in 3 colours $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$
- A^a_μ gluon gauge fields $a = 1,\ldots,8$
- $F^a_{\mu\nu}$ gluon field strength tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s\, f_{abc}\, A^b_\mu\, A^c_\nu$ with SU(3) structure constants f_{abc}
- t^a_{ij} SU(3) colour matrices (genarators of the SU(3) gauge group; representation: Gell-Mann matrices)

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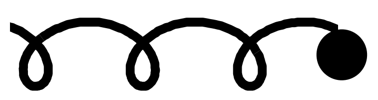
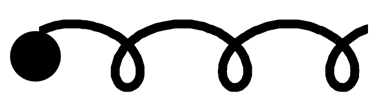
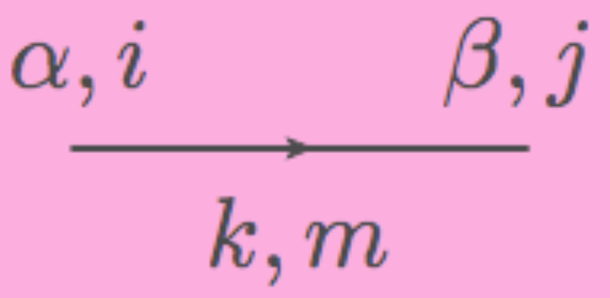
$$= \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)\left(\partial^\mu A^\nu_a - \partial^\nu A^\mu_a\right) + g_s f_{abc}(\cdots) + g_s^2 f_{abc} f_{ade}(\cdots)$$

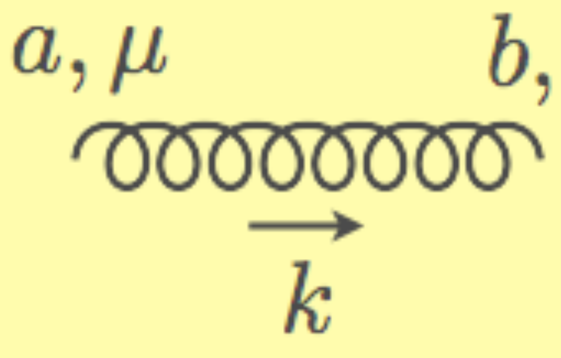
Feynman rules:

in: A^a_μ  $= \epsilon_\mu$

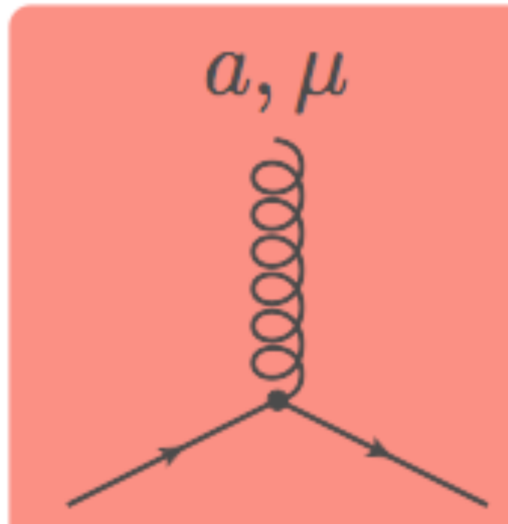
out:  A^a_μ $= \epsilon^*_\mu$



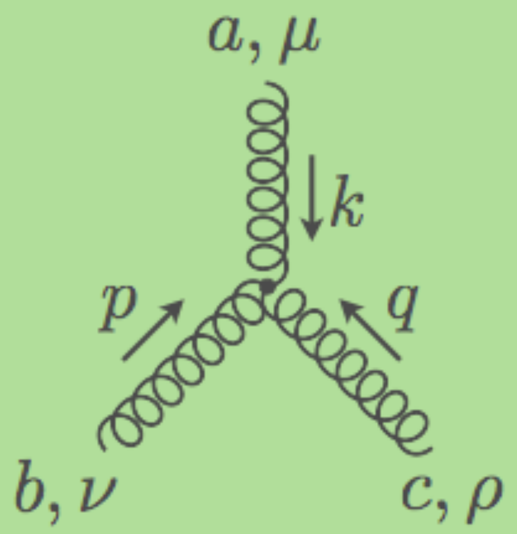
$$= \left(\frac{i}{k\!\!\!/ - m}\right)_{\alpha\beta} \delta_{ij}$$



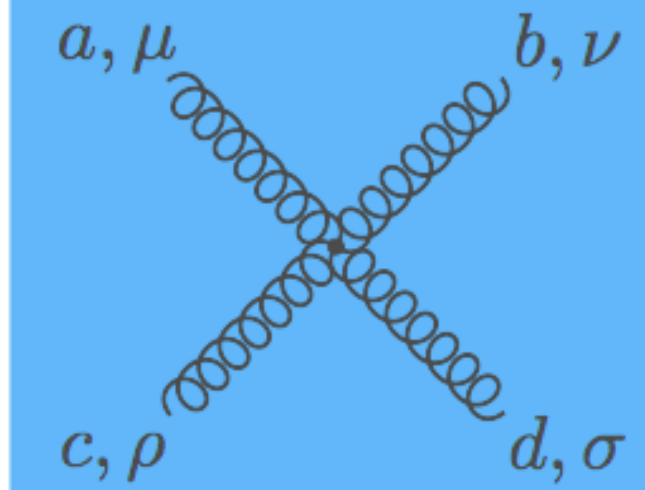
$$= \left(\frac{-i g_{\mu\nu}}{k^2}\right) \delta^{ab}$$



$$= i g_s \gamma^\mu t^a$$



$$= g_s f^{abc} \Big[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu \Big]$$



$$= -i g_s^2 \Big[f^{abe} f^{cde} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}\right) + f^{ace} f^{bde} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}\right) + f^{ade} f^{bce} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}\right) \Big]$$

The strong coupling constant

★ The SM is a renormalizable gauge theory

➙ couplings (and masses) need to be renormalized (because of UV divergences)

➙ theory does not predict value of α, but the dependence on scale

Renormalization group equation (RGE):

$$\frac{\mathrm{d}\alpha(\mu^2)}{\mathrm{d}\ln(\mu^2)} = \beta(\alpha(\mu^2)) = \beta_0\,\alpha^2 + \beta_1\,\alpha^3 + \beta_2\,\alpha^4 + \cdots$$

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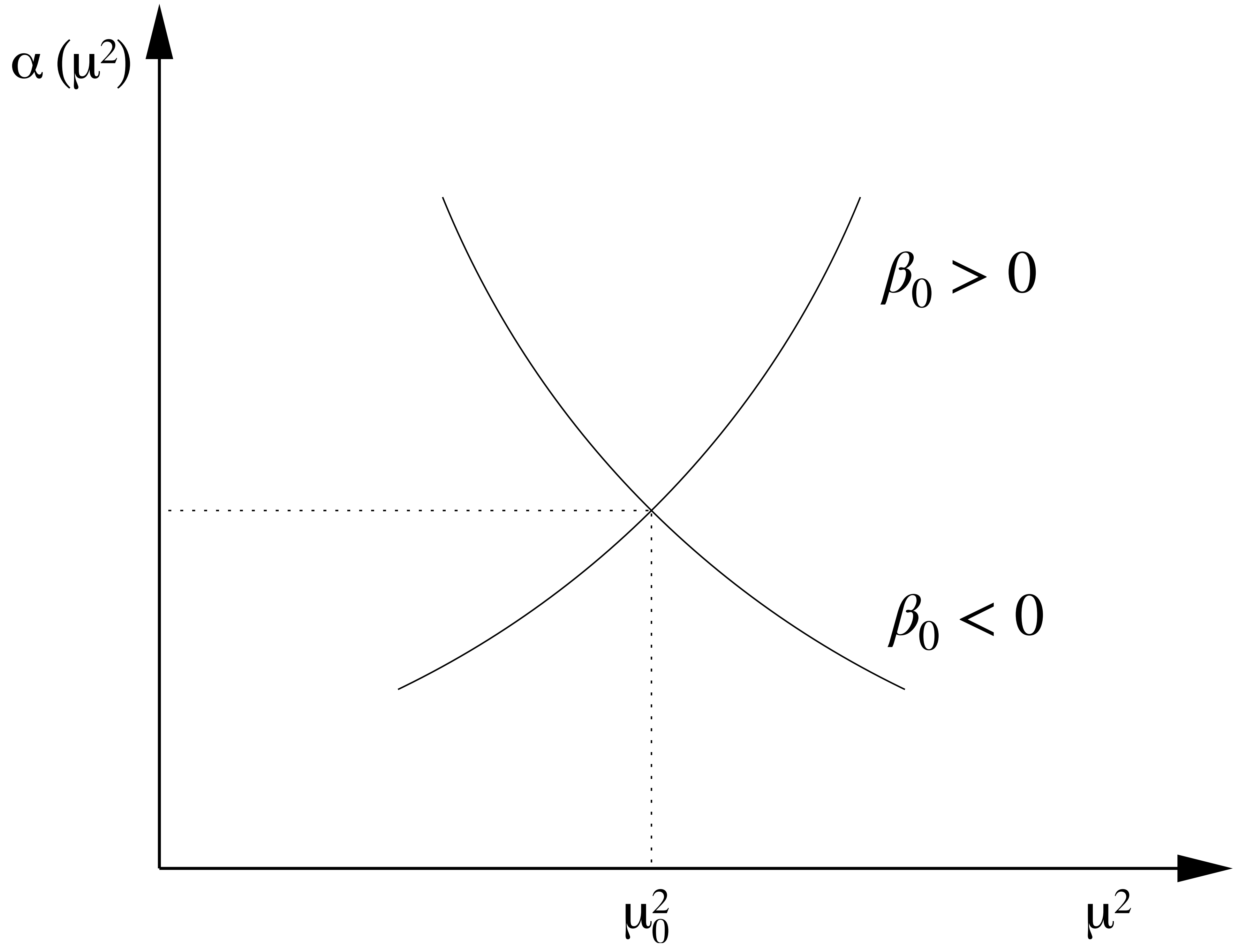
perturbative ($\alpha \ll 1$) solution at one-loop:

$$\alpha(\mu^2) = \frac{\alpha(\mu_0^2)}{1 - \alpha(\mu_0^2)\,\beta_0\ln(\mu^2/\mu_2^2)}$$



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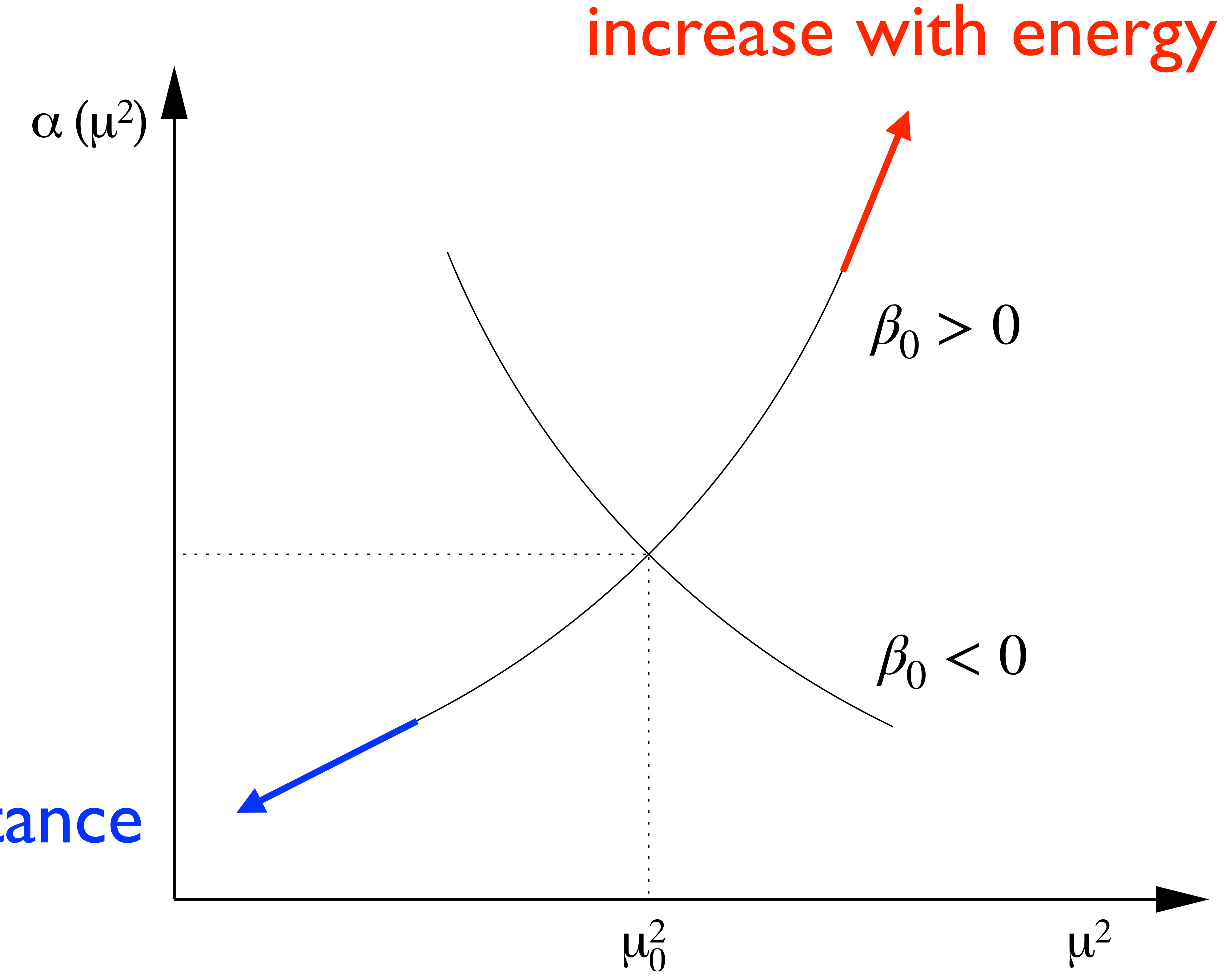
QED: $\beta_0 > 0$



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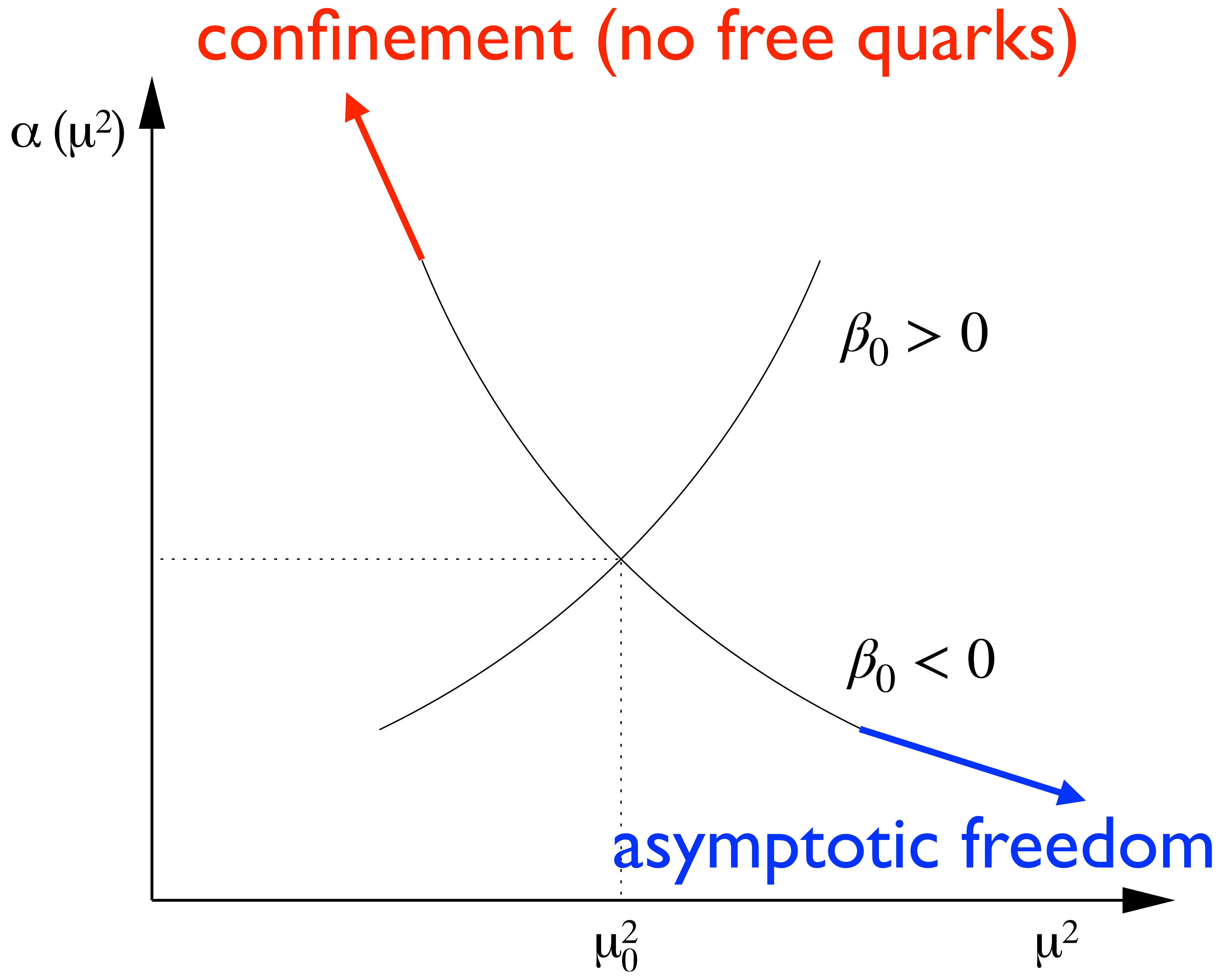
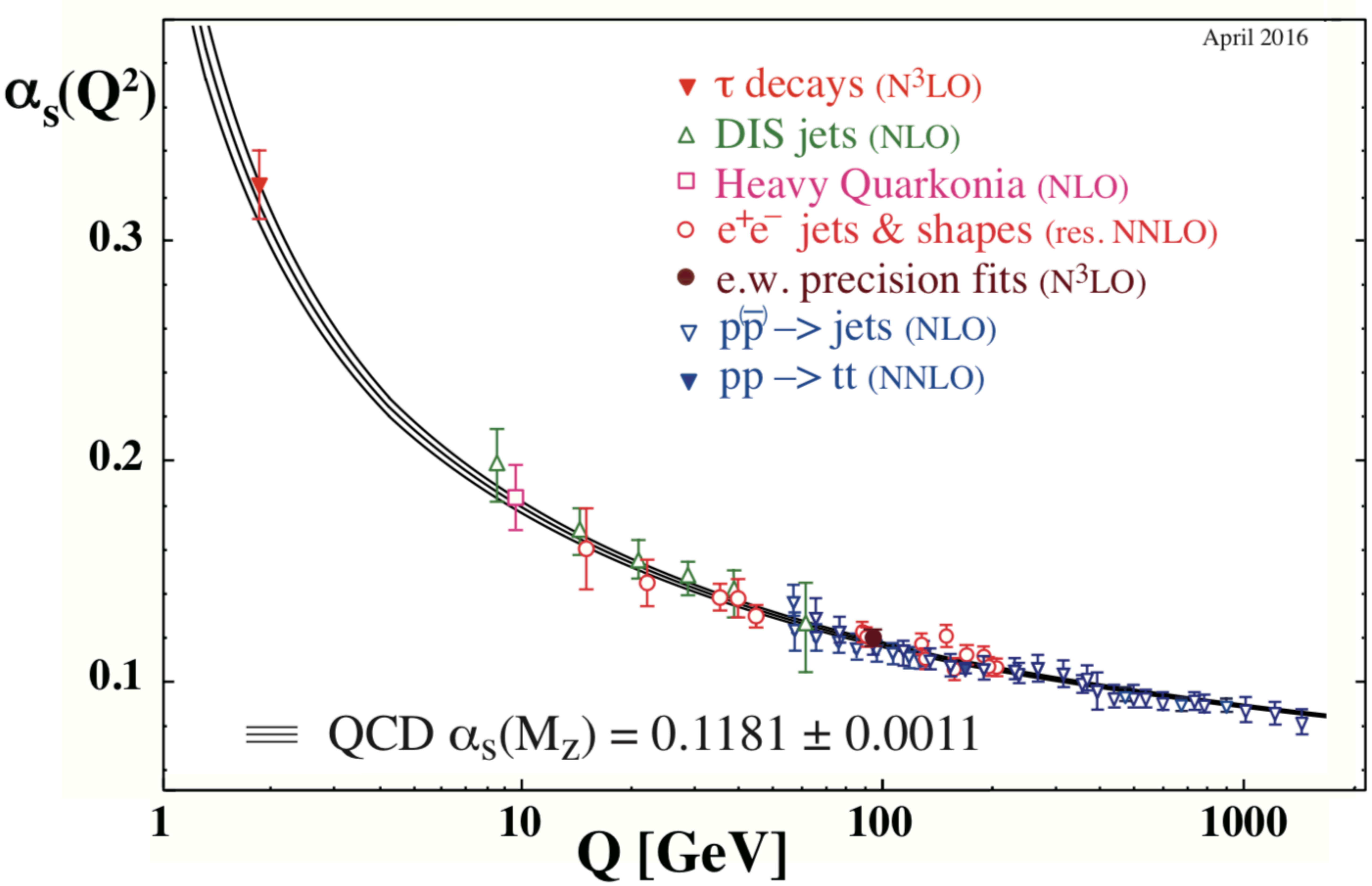
QCD: $\beta_0 < 0$ (due to gluon self interaction)



We can actually measure asymptotic freedom



Nobel prize in 2004
Gross, Pollitzer, Wilczek

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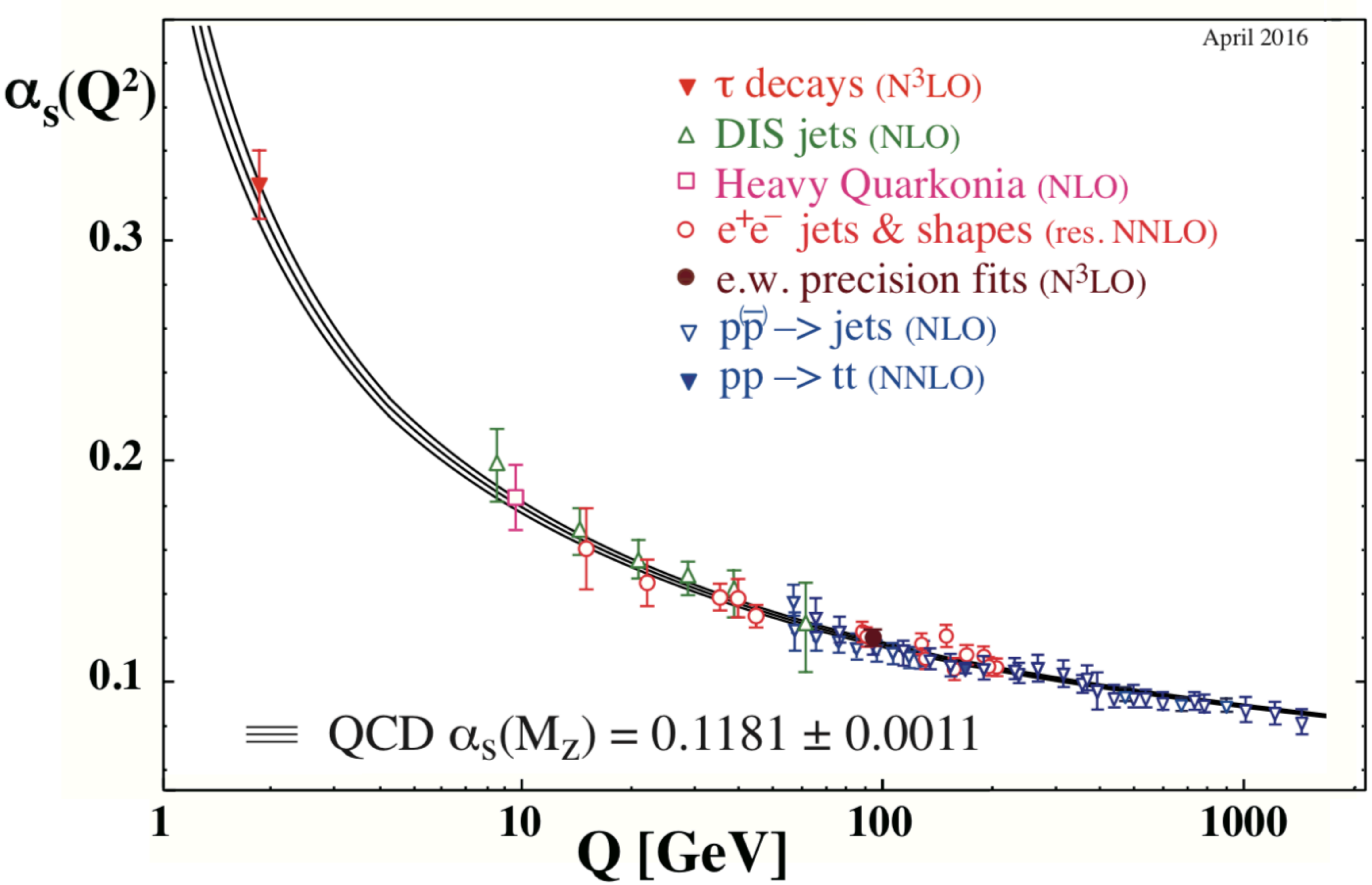

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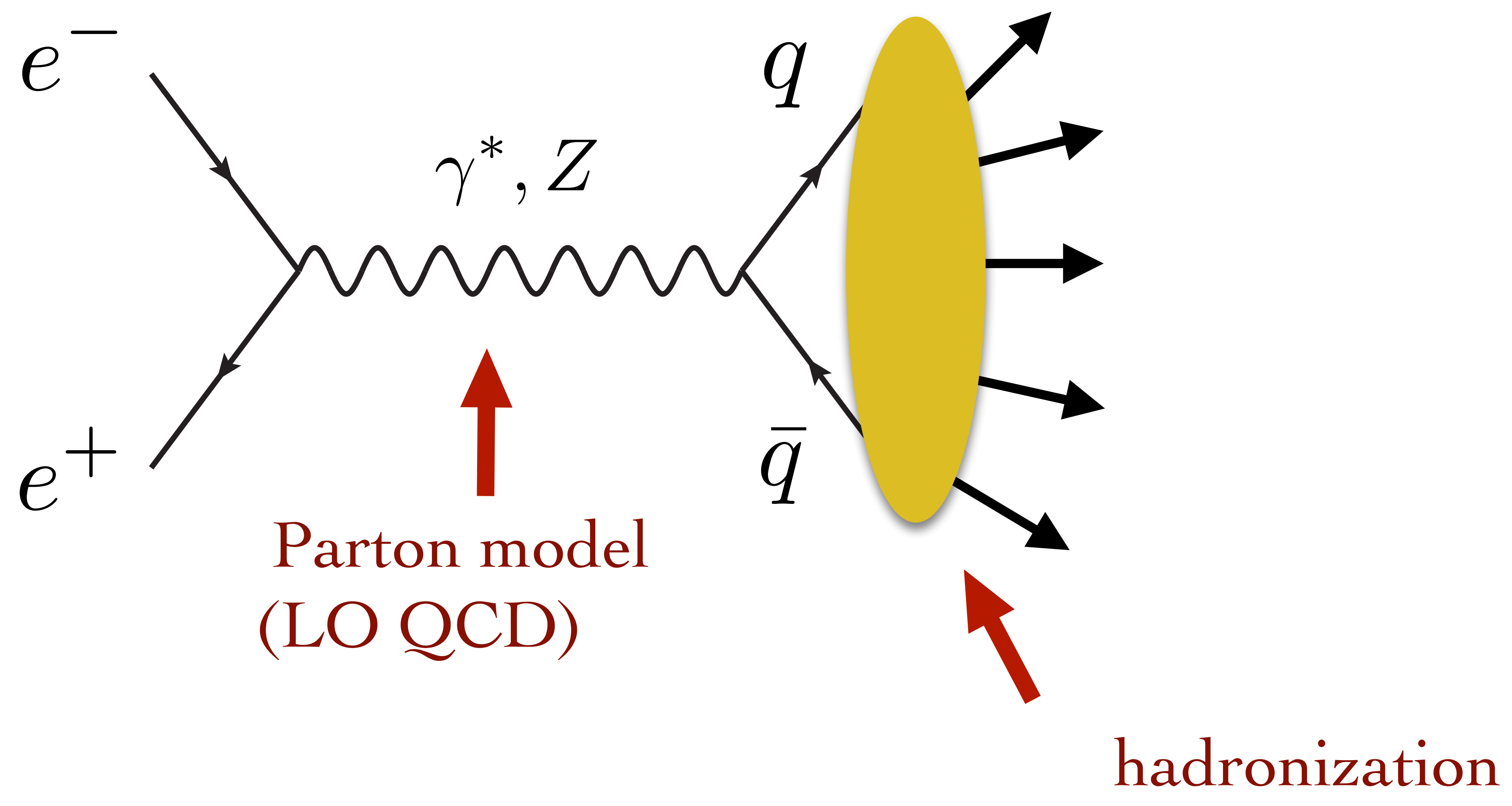
➙ perturbation theory valid for high-energy collisions ($Q \gg \Lambda_{QCD} \simeq 0.2\,\text{GeV}$)

Parton model

asymptotic freedom → at large momentum-transfer hadrons behave as collection of free (weakly interacting) partons



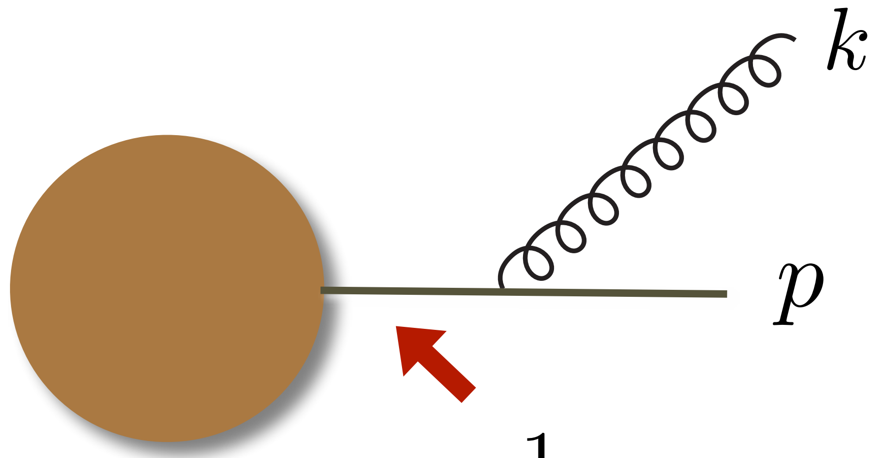
quark-hadron duality:

due to large time separation between hard scattering and hadronization there is no quantum interference and the hard momentum flow is not altered "significantly" ➞ if we are not interested in the hadron dynamics (sufficiently inclusive observables) the parton picture is valid

Infrared singularities

two kinds of infrared (IR) singularities appear in theories with massless particles:

- ◉ soft ➙ vanishing parton (gluon) momentum
- ◉ collinear ➙ two partons become collinear

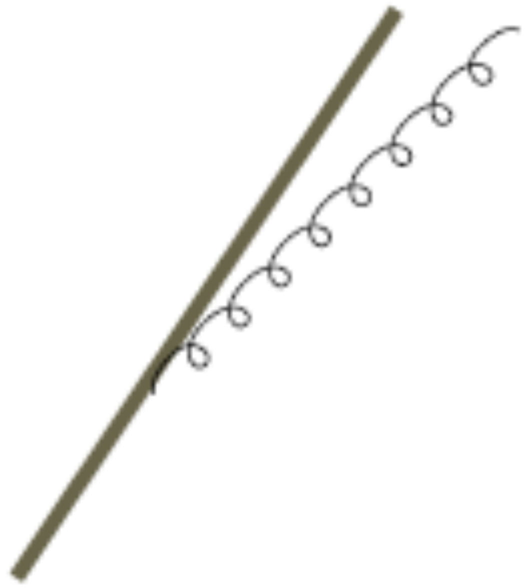


$$\frac{1}{(p+k)^2} = \frac{1}{2E_p E_k (1-\cos\theta)}$$

soft (E_k) collinear ($1-\cos\theta$)

hard parton

hard parton
+ soft gluon



two collinear partons

➙ physically indistinguishable (degenerate states), IR divergencies are a manifestation of factorization of short-distance from long-distance effects (not existent in hadron picture)

Does parton model survive with radiative corrections?

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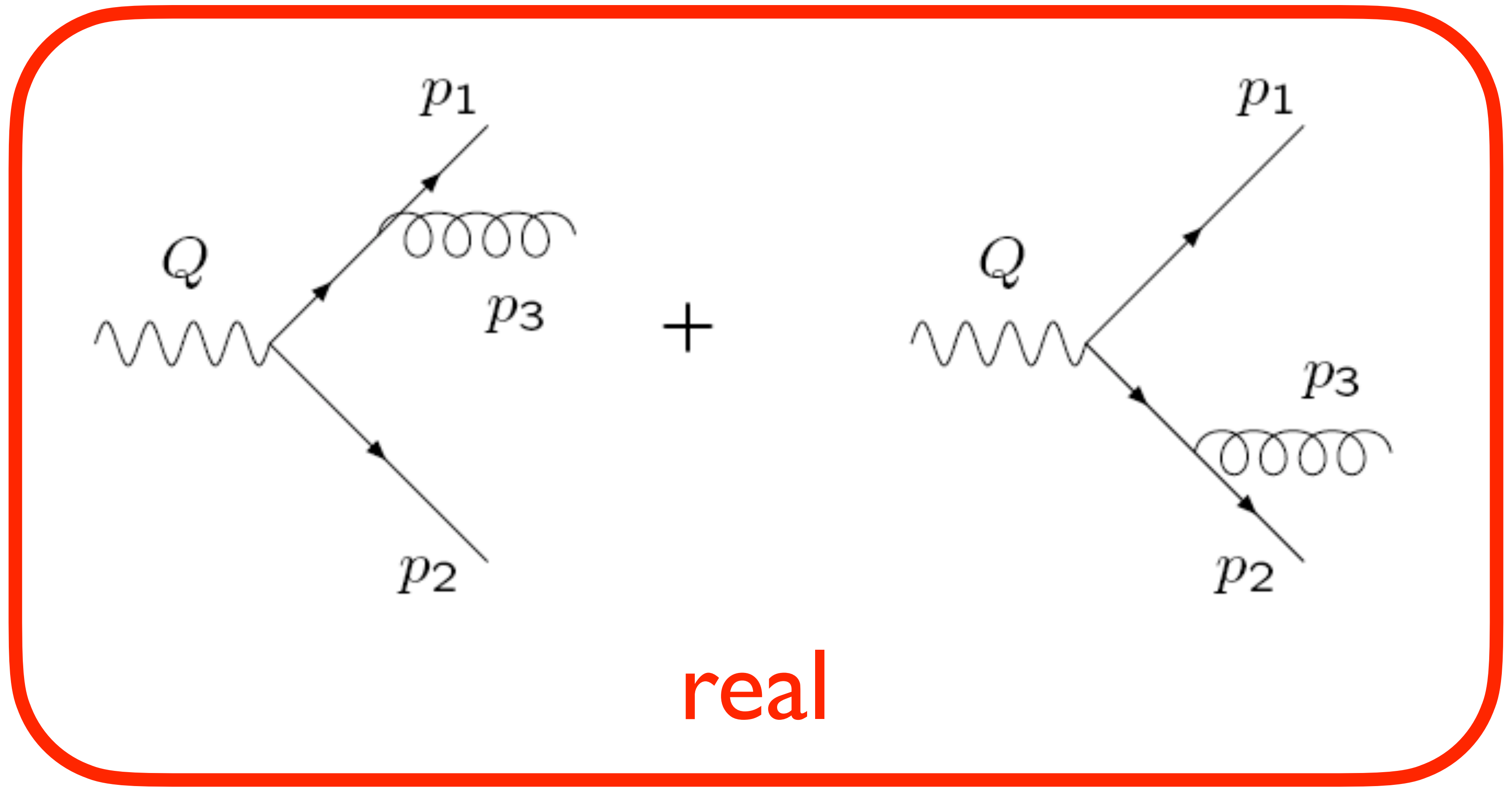
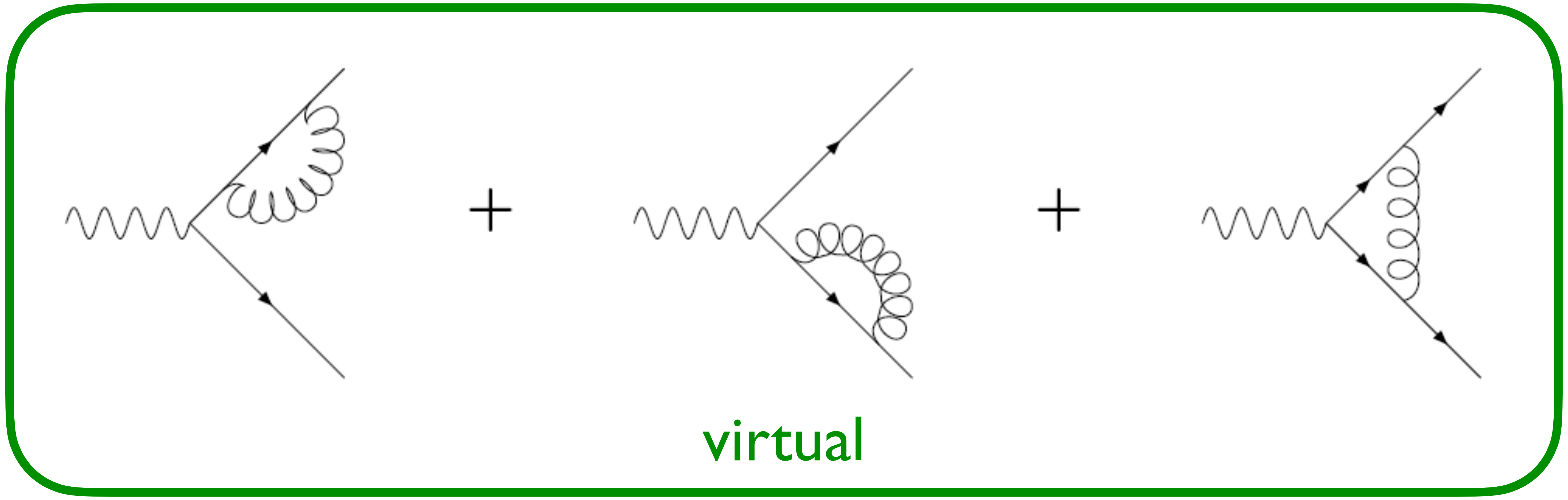
consider $\mathcal{O}(\alpha_s)$ corrections to $e^+e^- \to q\bar{q}$:





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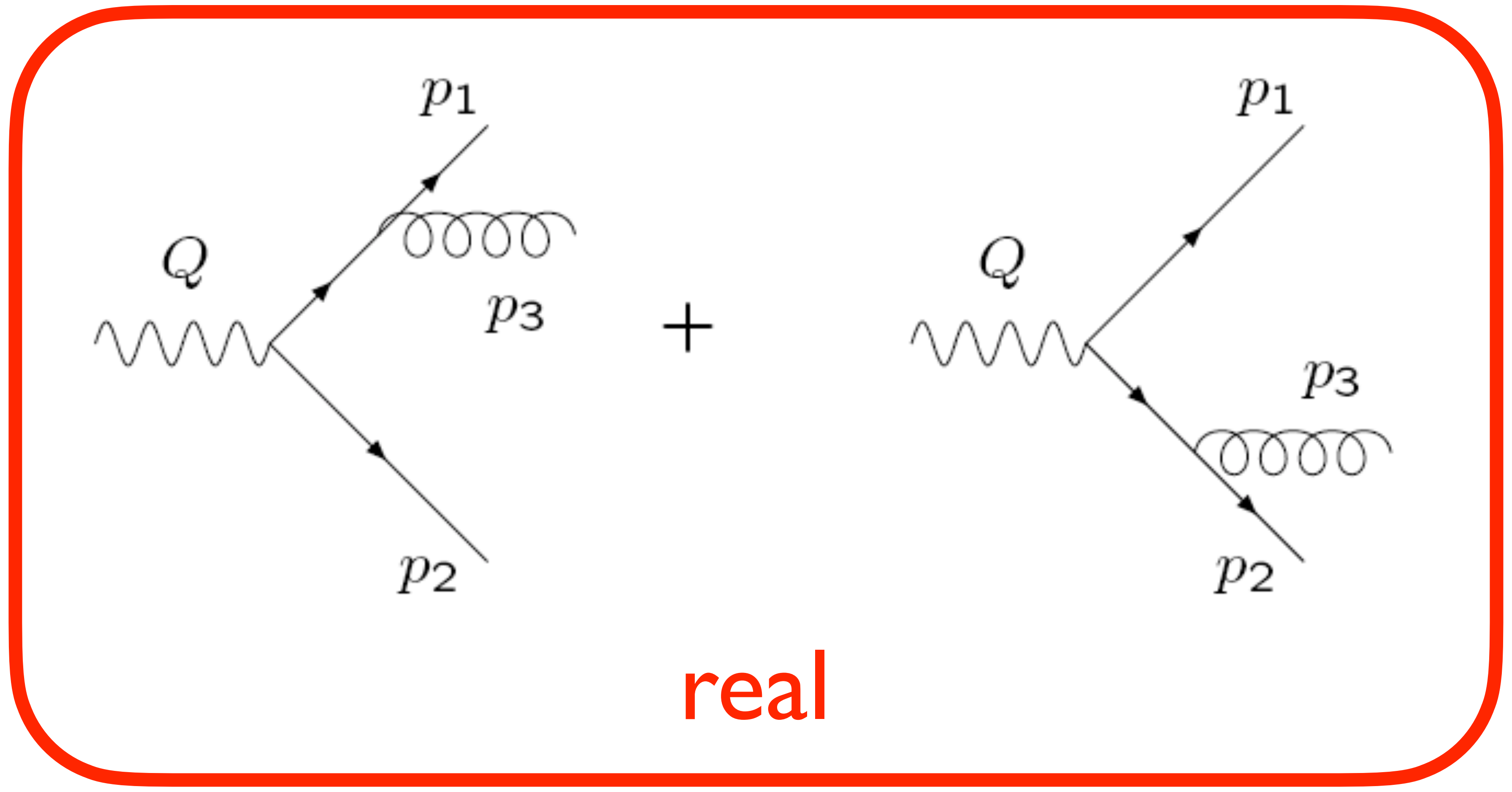

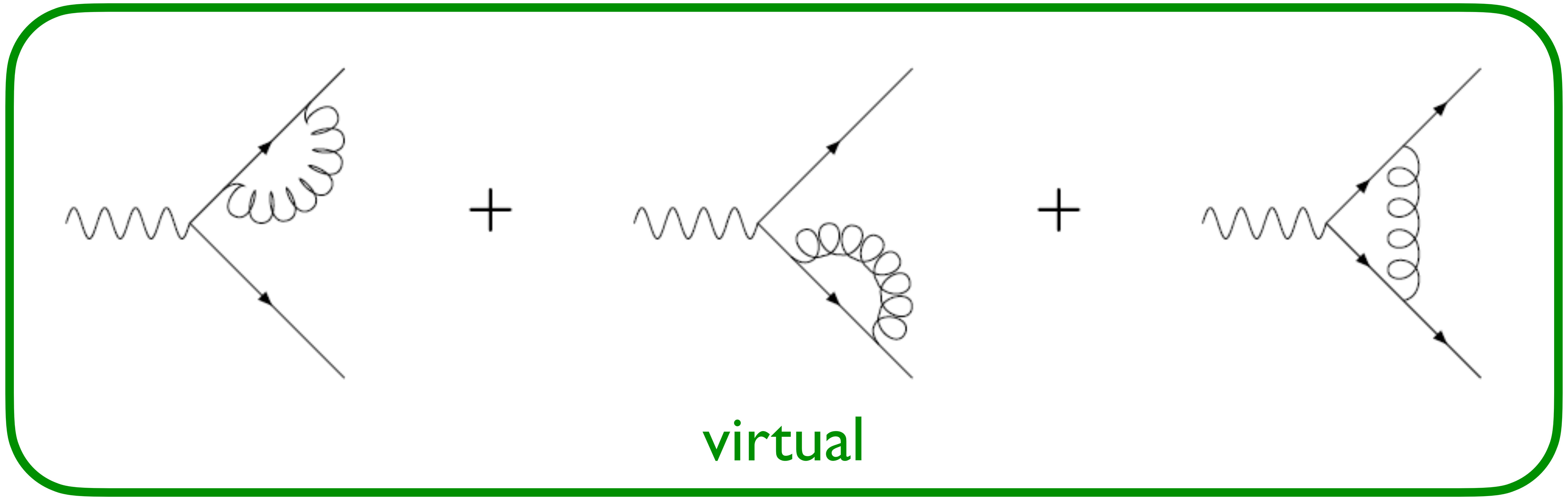

➙ degenerate states: soft/collinear real radiation cannot be distinguished from virtual correction

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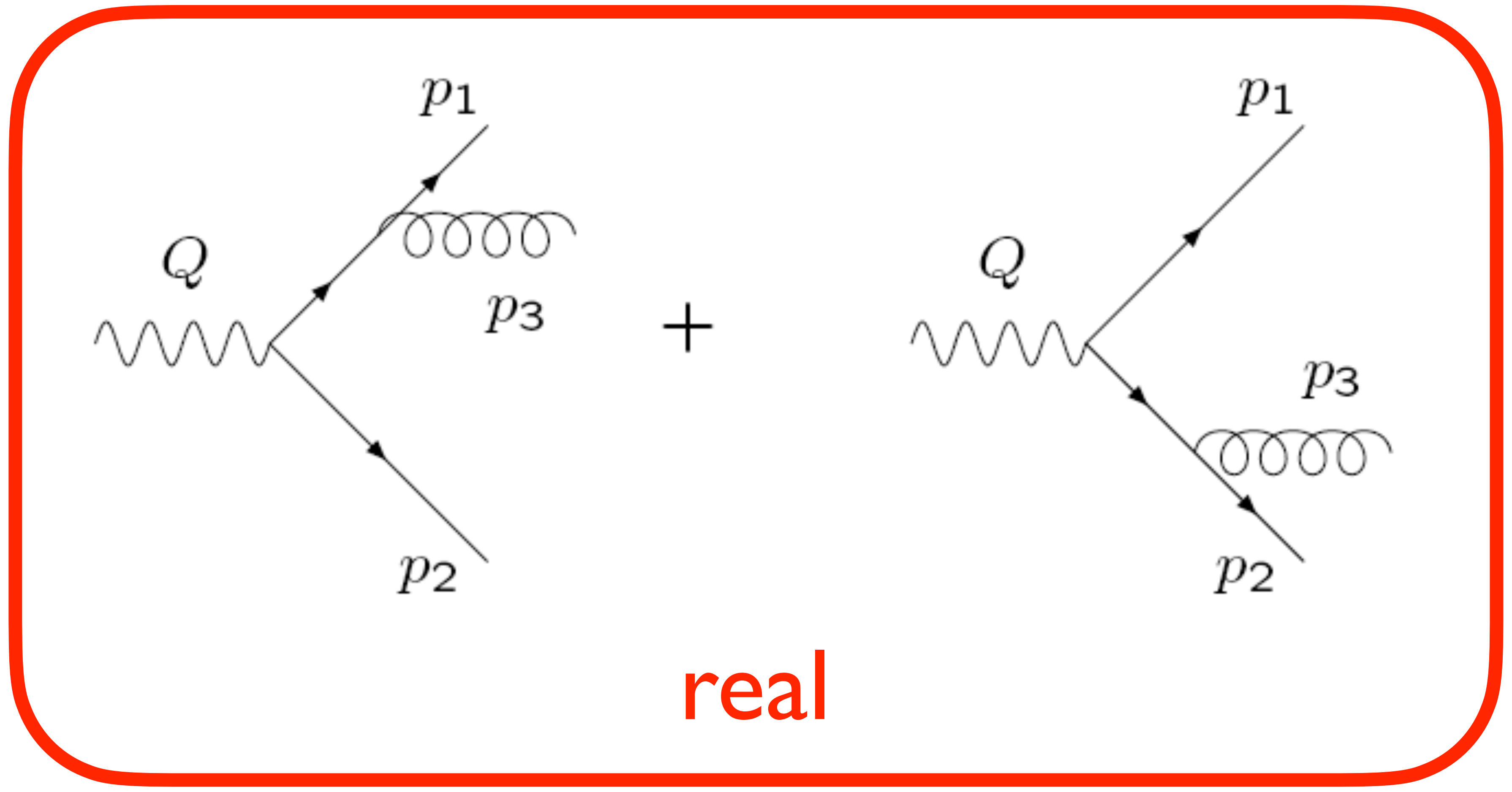
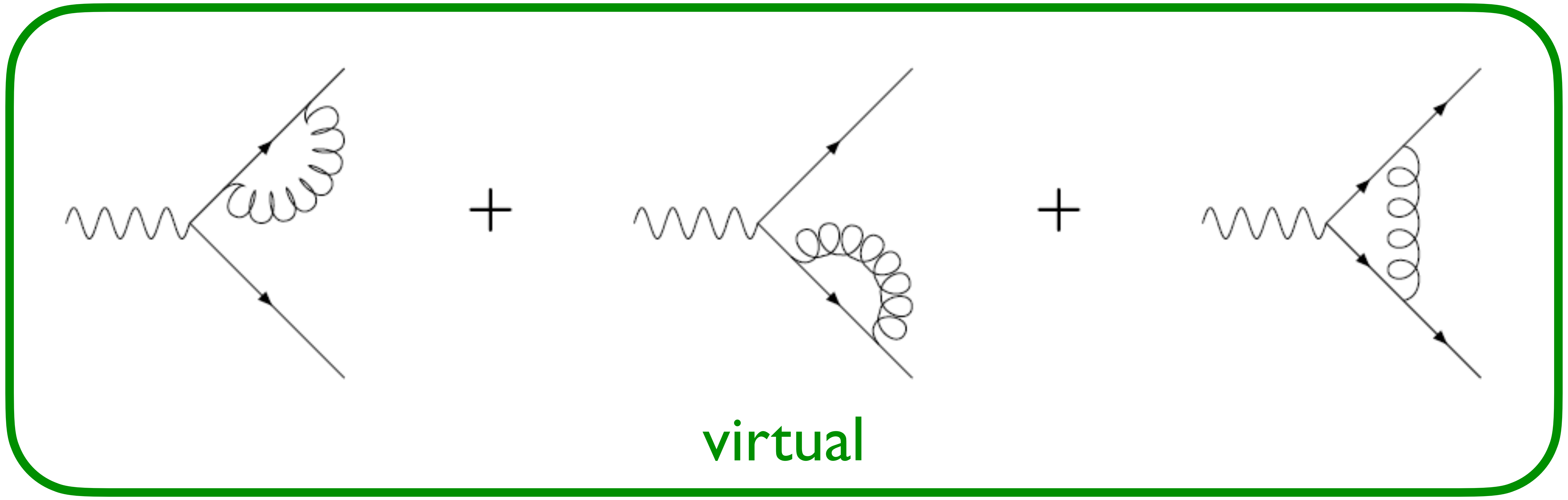




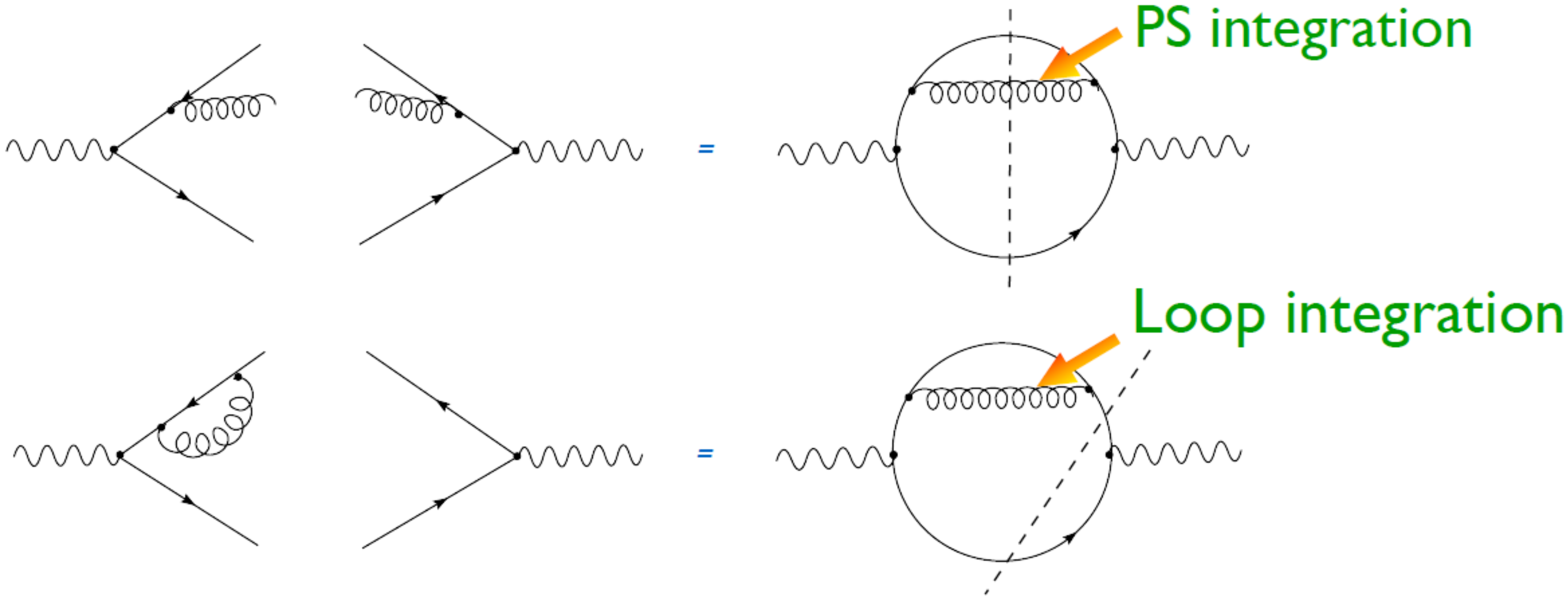
➙ degenerate states: soft/collinear real radiation cannot be distinguished from virtual correction

Kinoshita-Lee-Naumberg (KLN) theorem:

When summing over all degenerate states (inital & final-state + soft & collinear configurations) in sufficiently inclusive observables IR singularities cancel out.

Infrared safety

The cancellation of IR singularities is not a miracle, but a direct consequence from unitarity:



➙ in the IR region real and virtual amplitudes are kinematically equivalent up to a different sign

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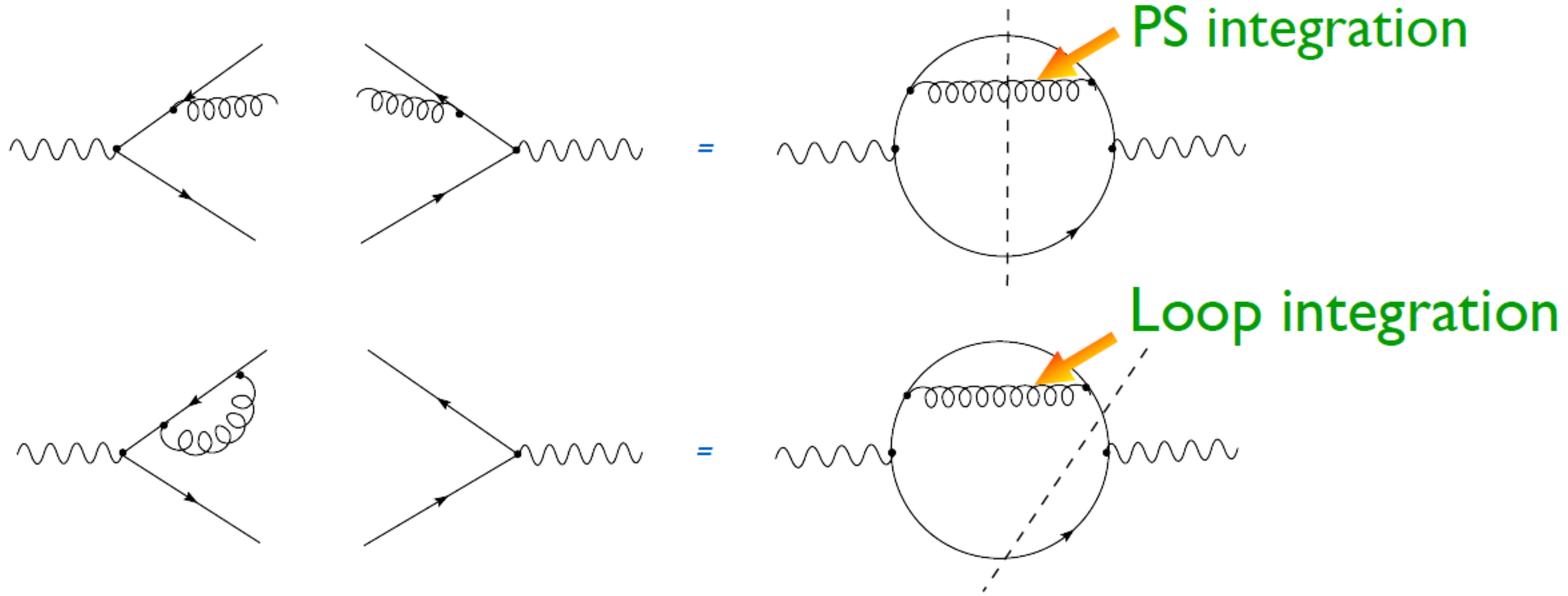


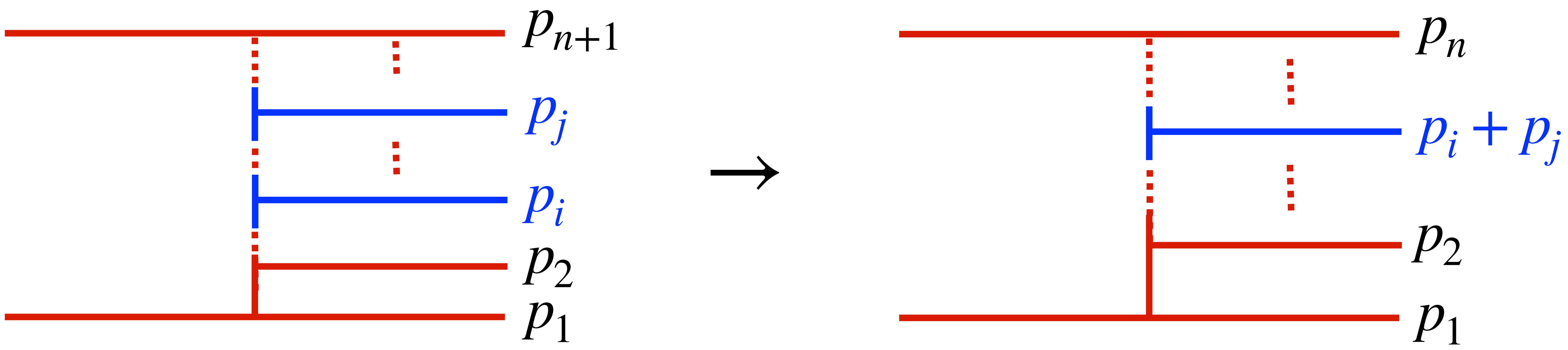
➙ in the IR region real and virtual amplitudes are kinematically equivalent up to a different sign

This cancellation happens for sufficiently inclusive (i.e. IR-safe) observable, but was does this mean?

Infrared safety

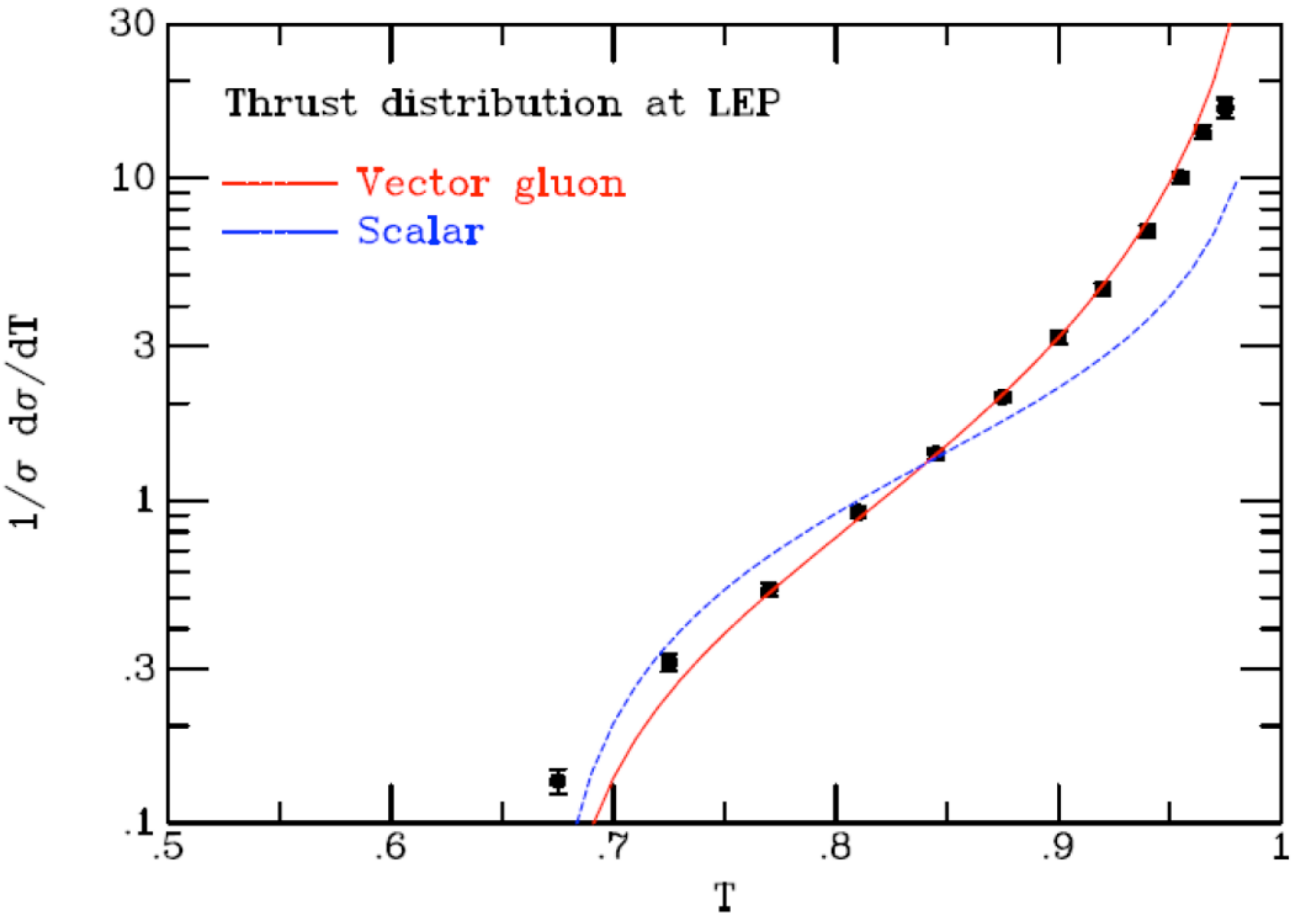
An observable $\mathcal{O}$ is infrared and collinear safe if

$$\mathcal{O}_{n+1}(p_1, \ldots, p_i, \ldots, p_j, \ldots, p_{n+1}) \to \mathcal{O}_n(p_1, \ldots, p_i + p_j, \ldots, p_n) \quad \text{if } p_i \parallel p_j \text{ or } p_j \to 0$$

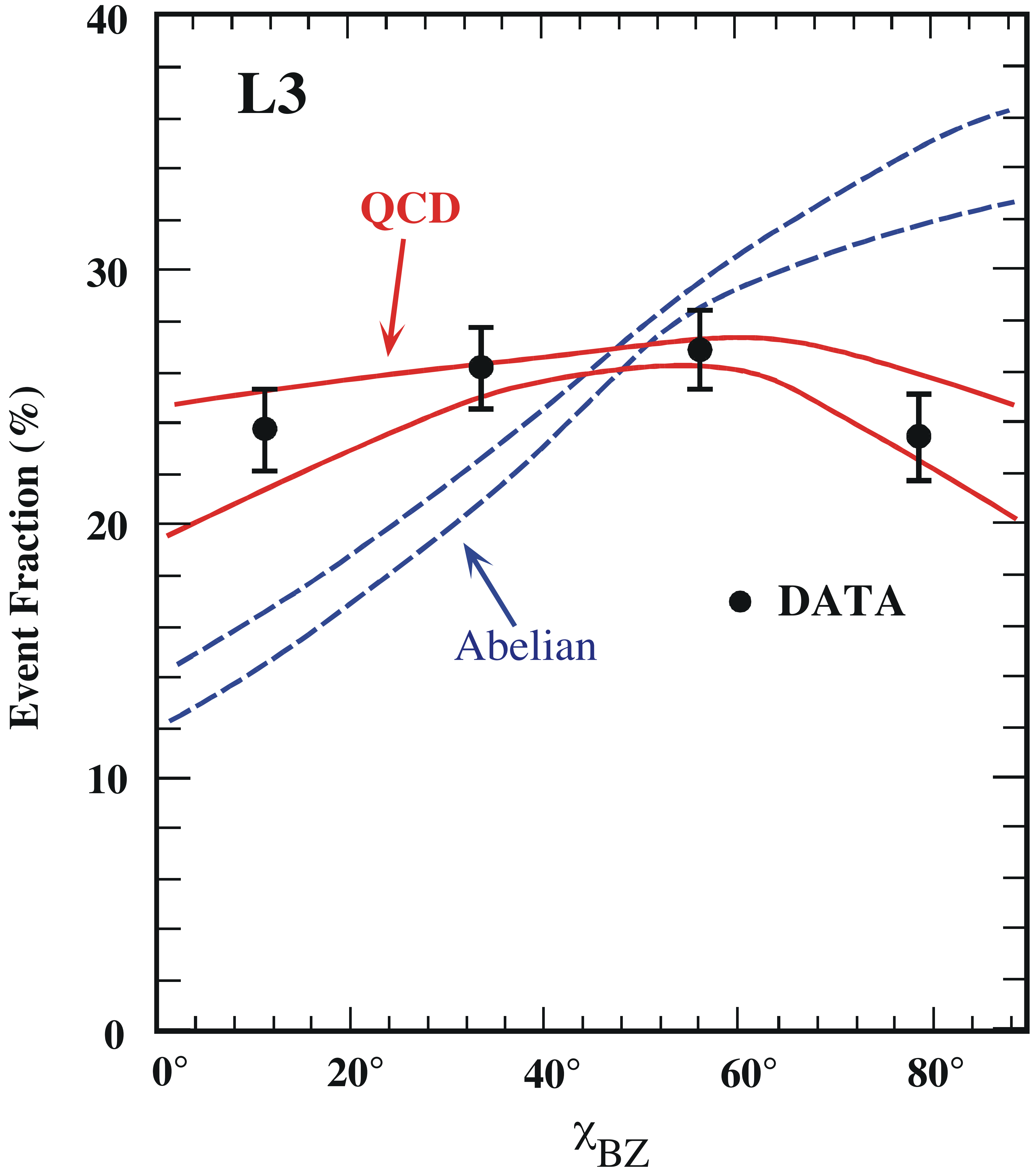


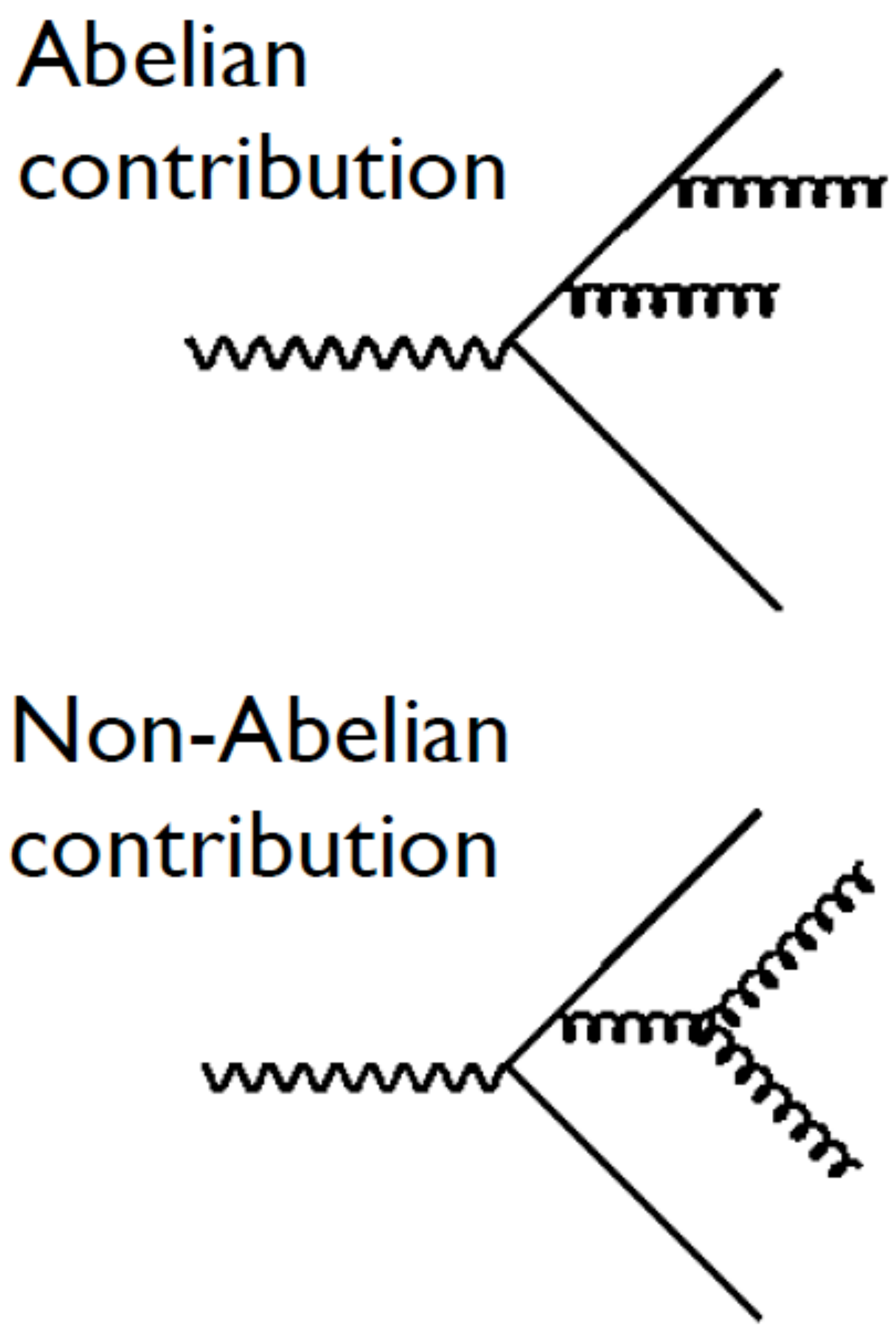
i.e. the observable is not sensitive to soft or collinear emissions

QCD measurements: spin of gluon

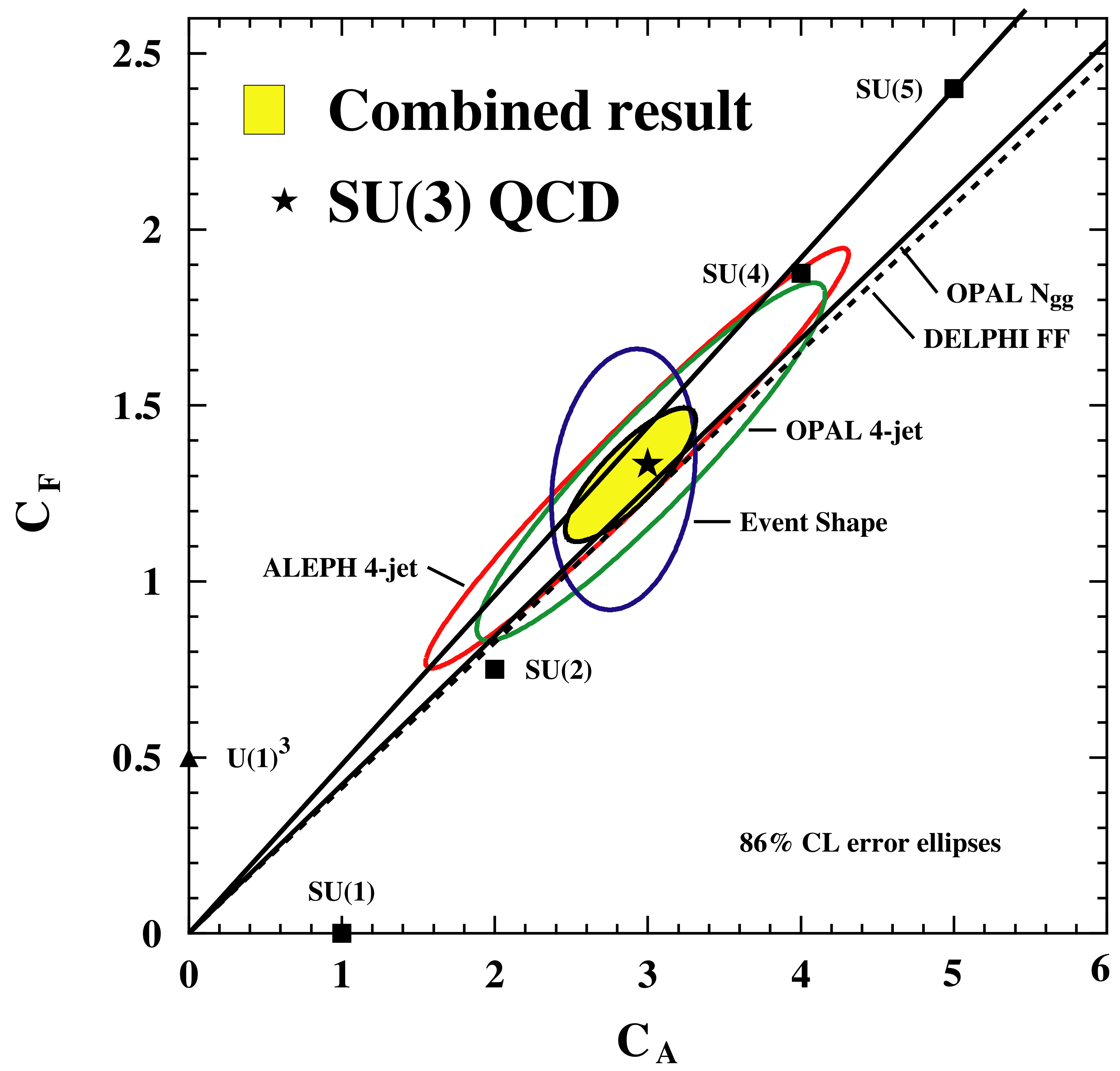


QCD measurements: non-abelian nature





QCD measurements: colour factors



Fits of colour factors from 4-jet rates and event shapes

$$C_A = 2.89 \pm 0.21$$
$$C_F = 1.30 \pm 0.09$$

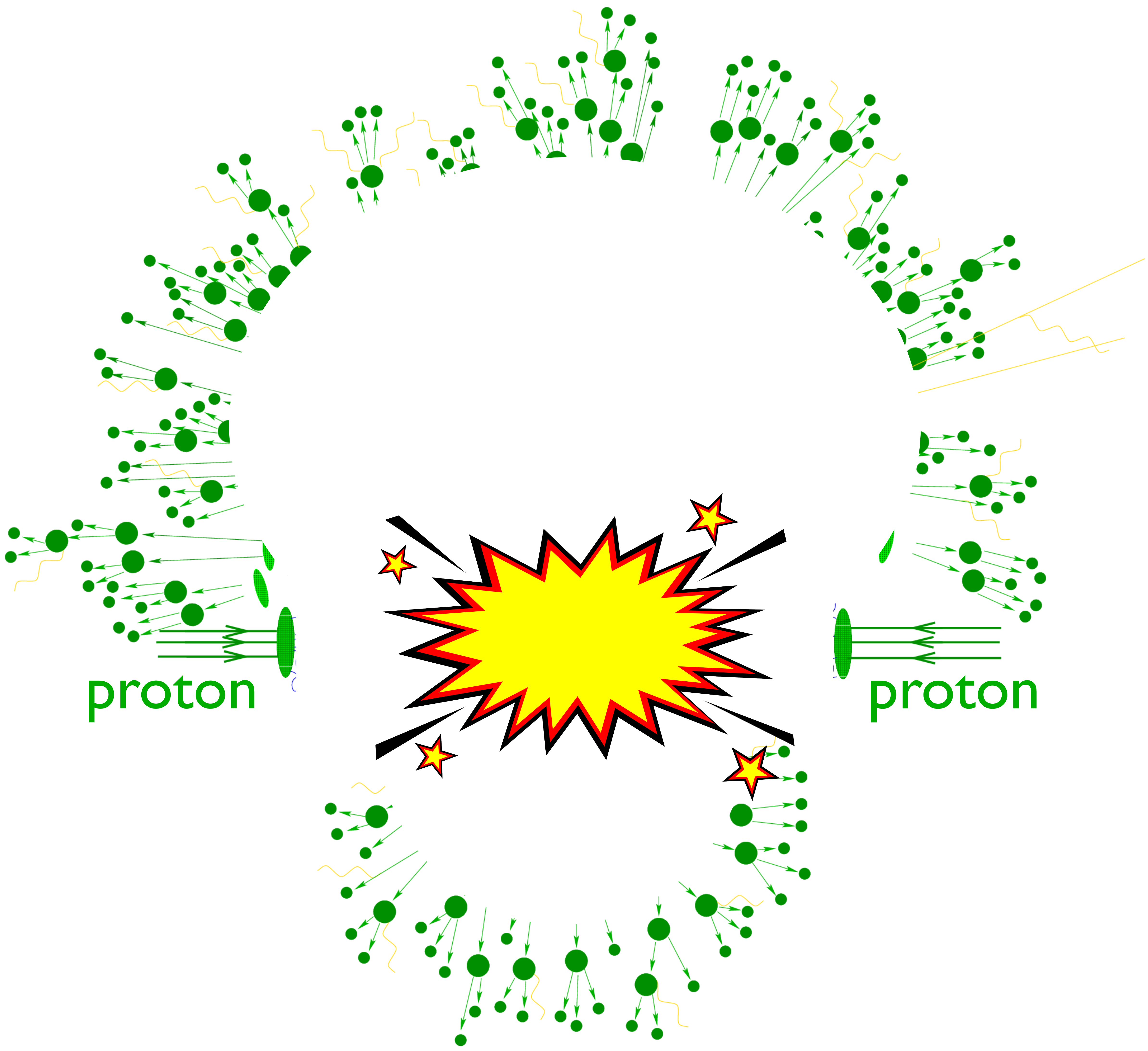
Well compatible with QCD:

$$C_A = 3$$
$$C_F = \frac{4}{3}$$

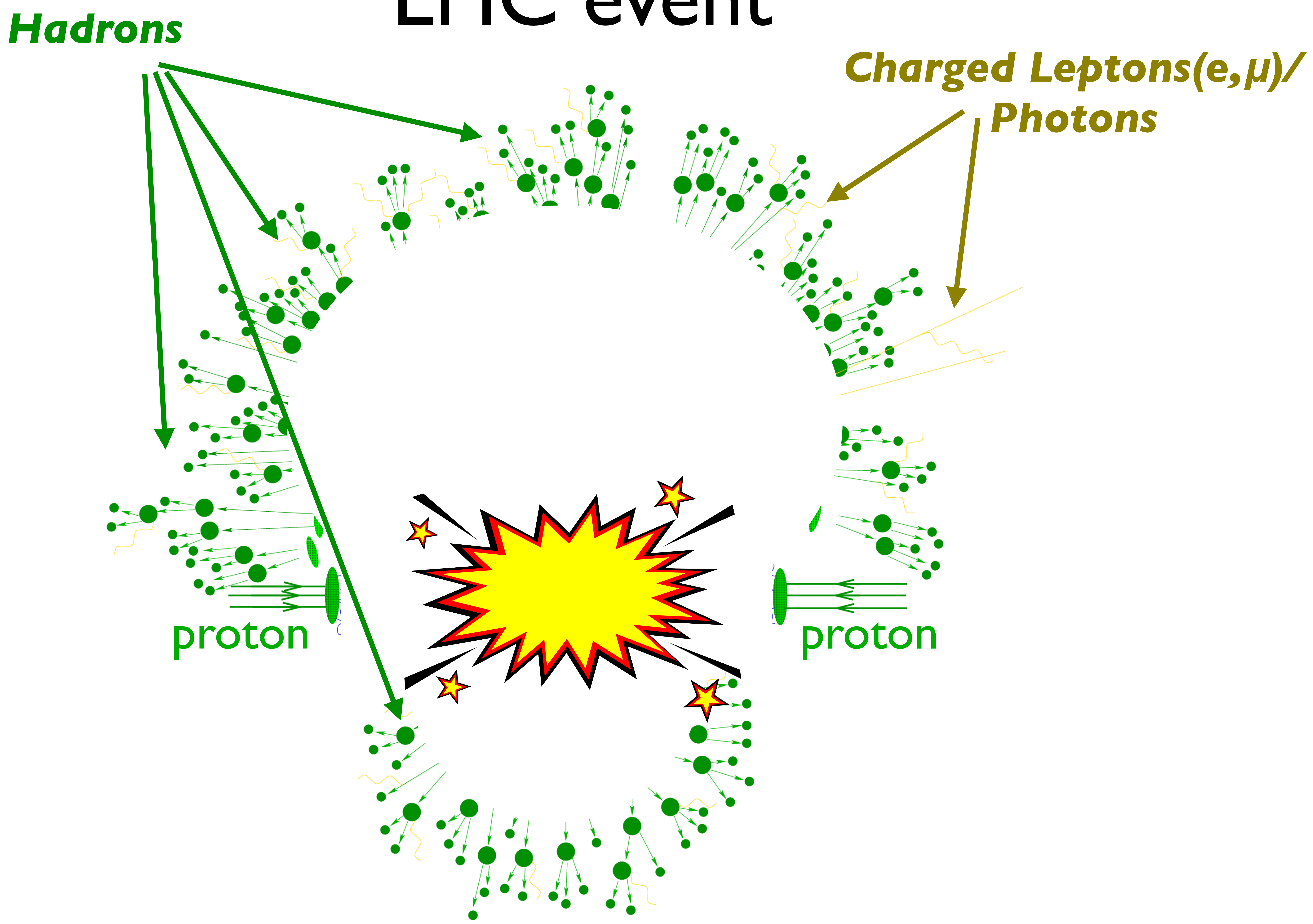
Questions?

How to make predictions for proton-proton collisions

LHC event



LHC event



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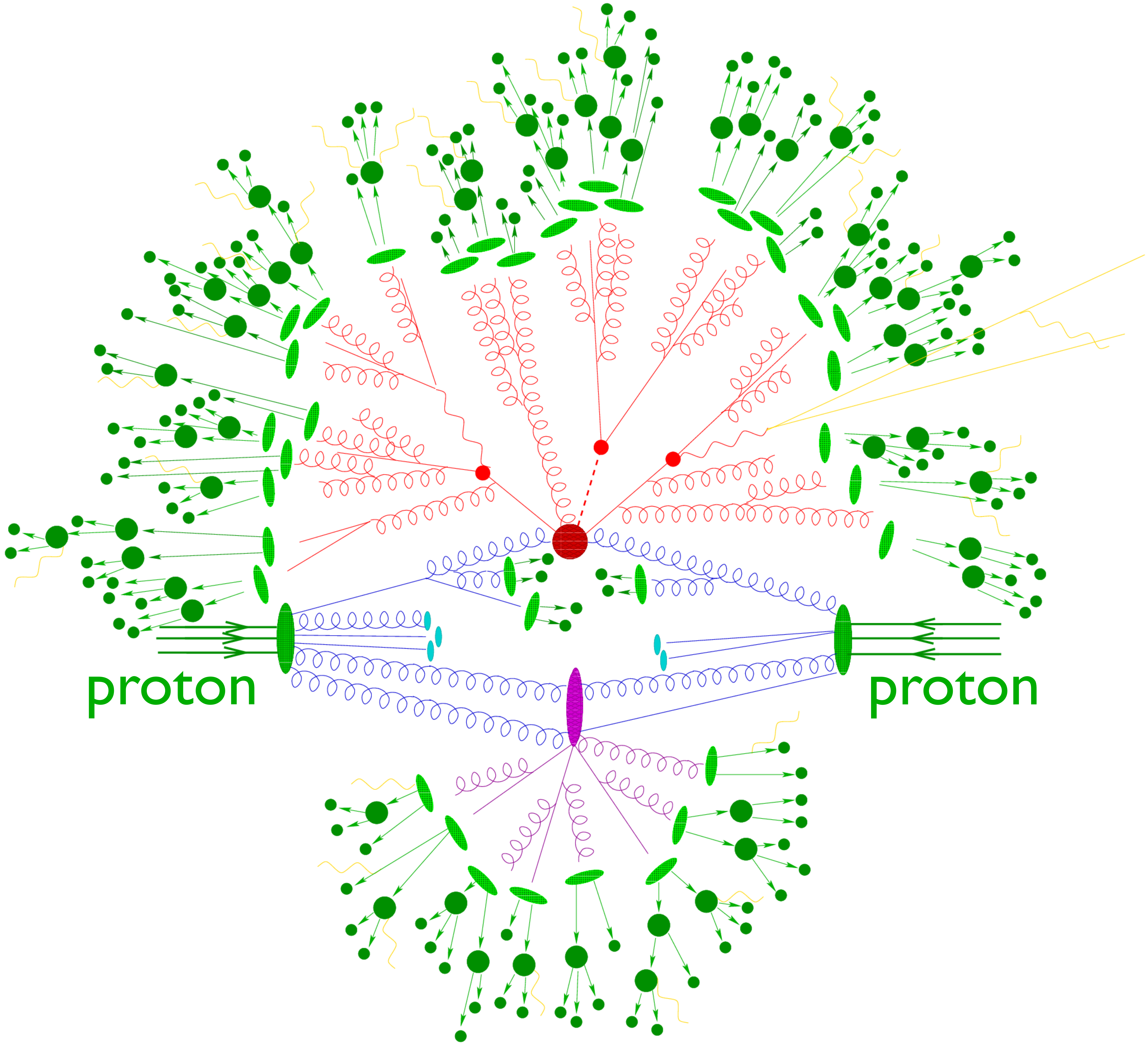

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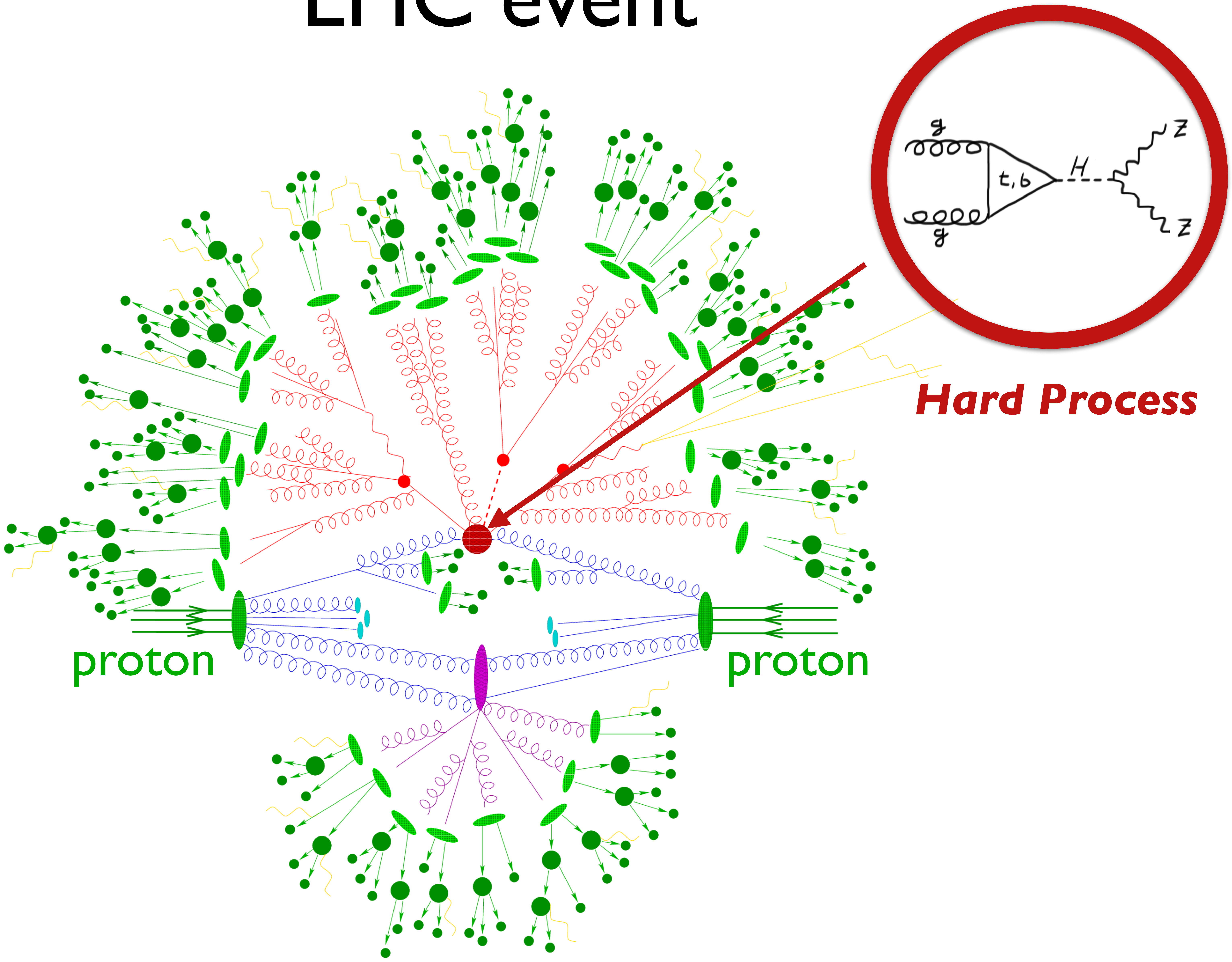

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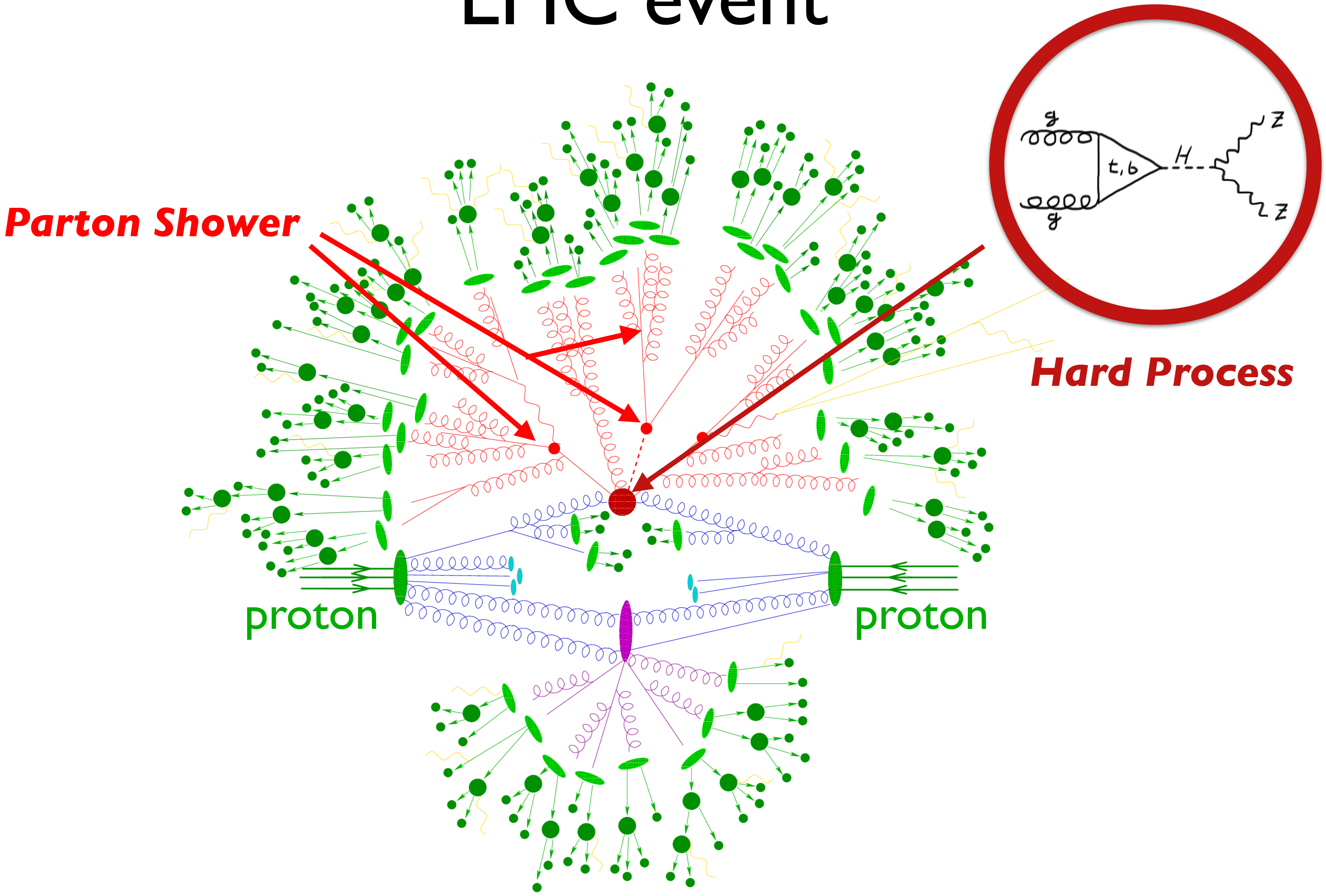

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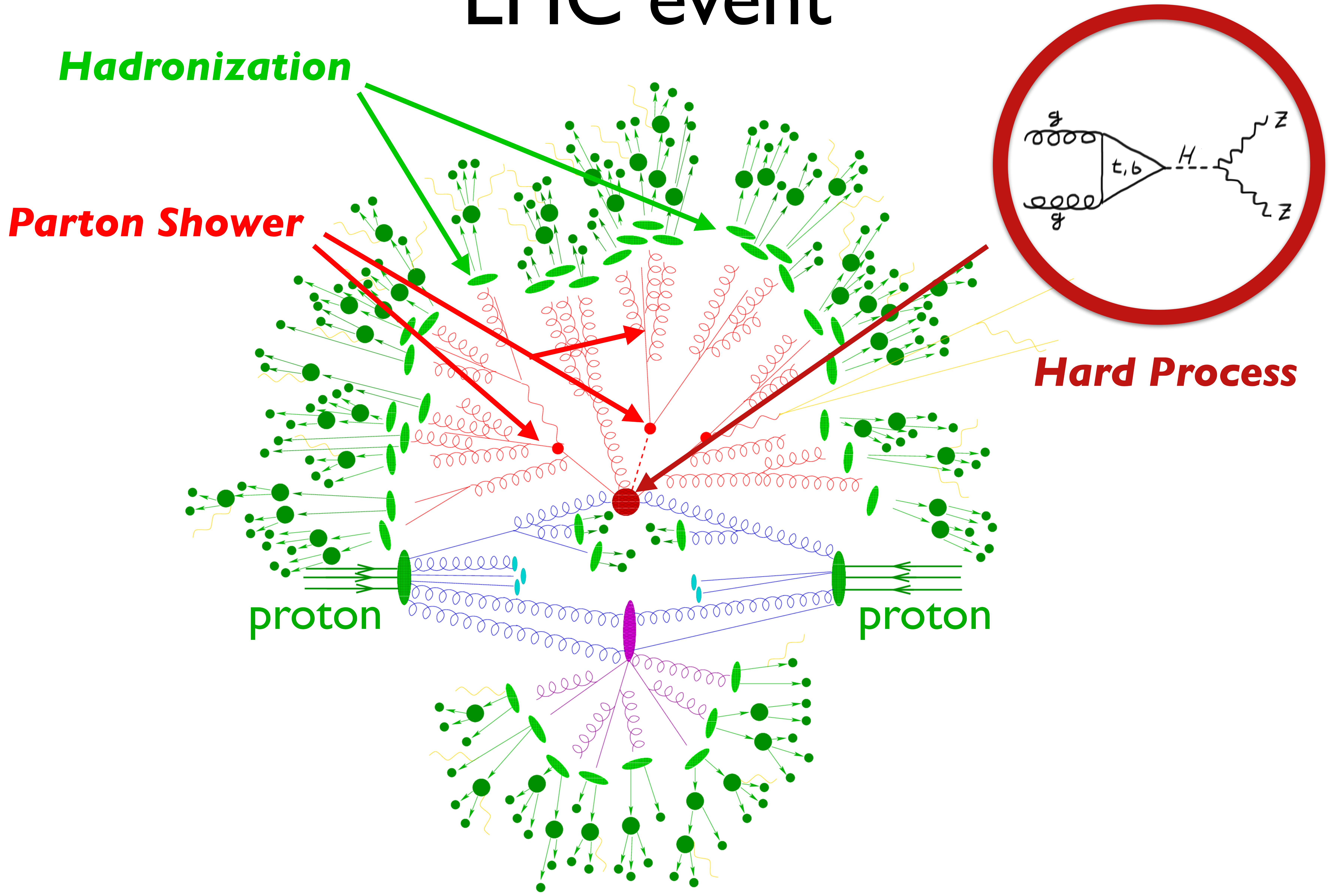
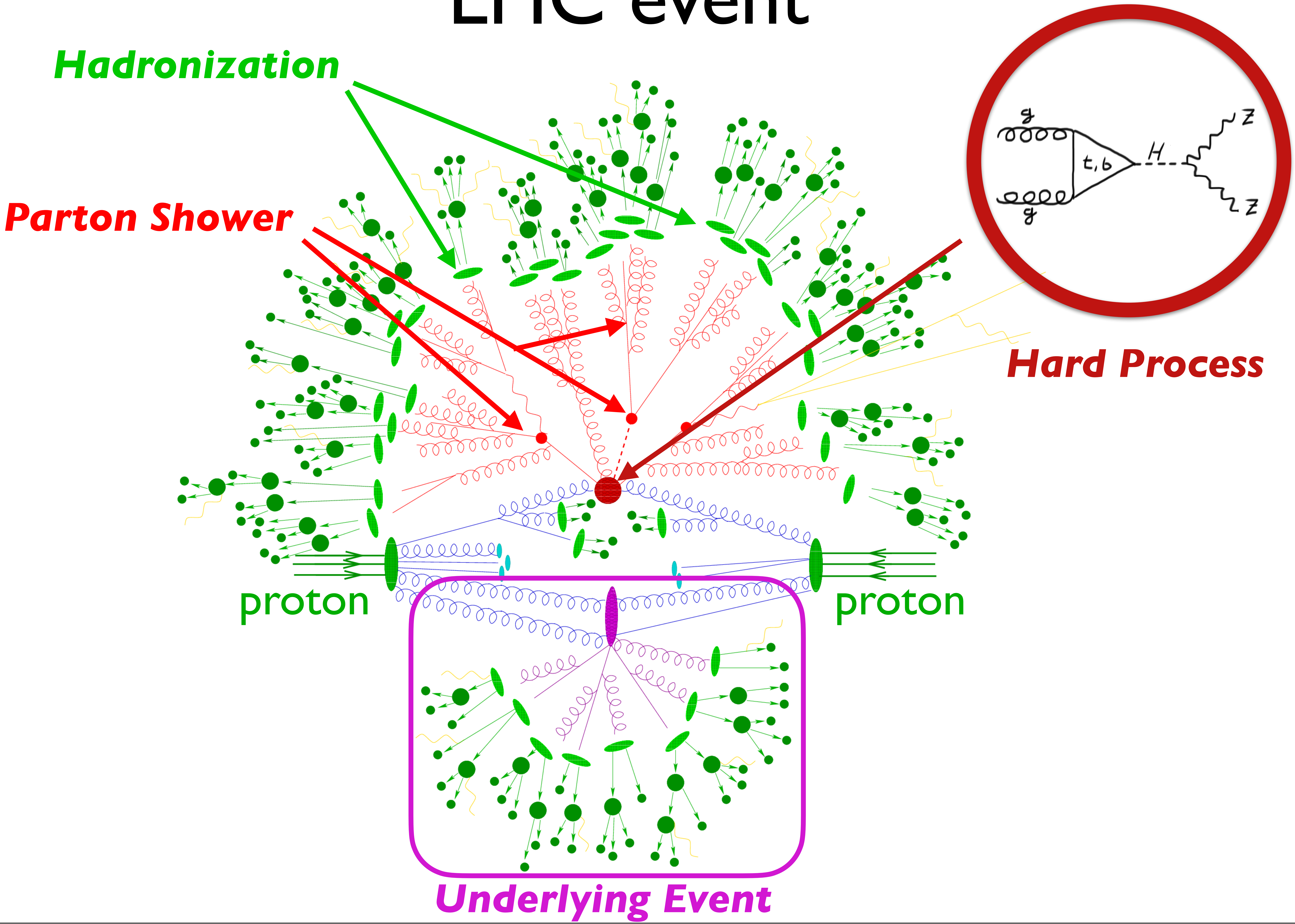

Hadronization
Parton Shower
Hard Process
g
g
t, b
H
Z
Z
proton
proton

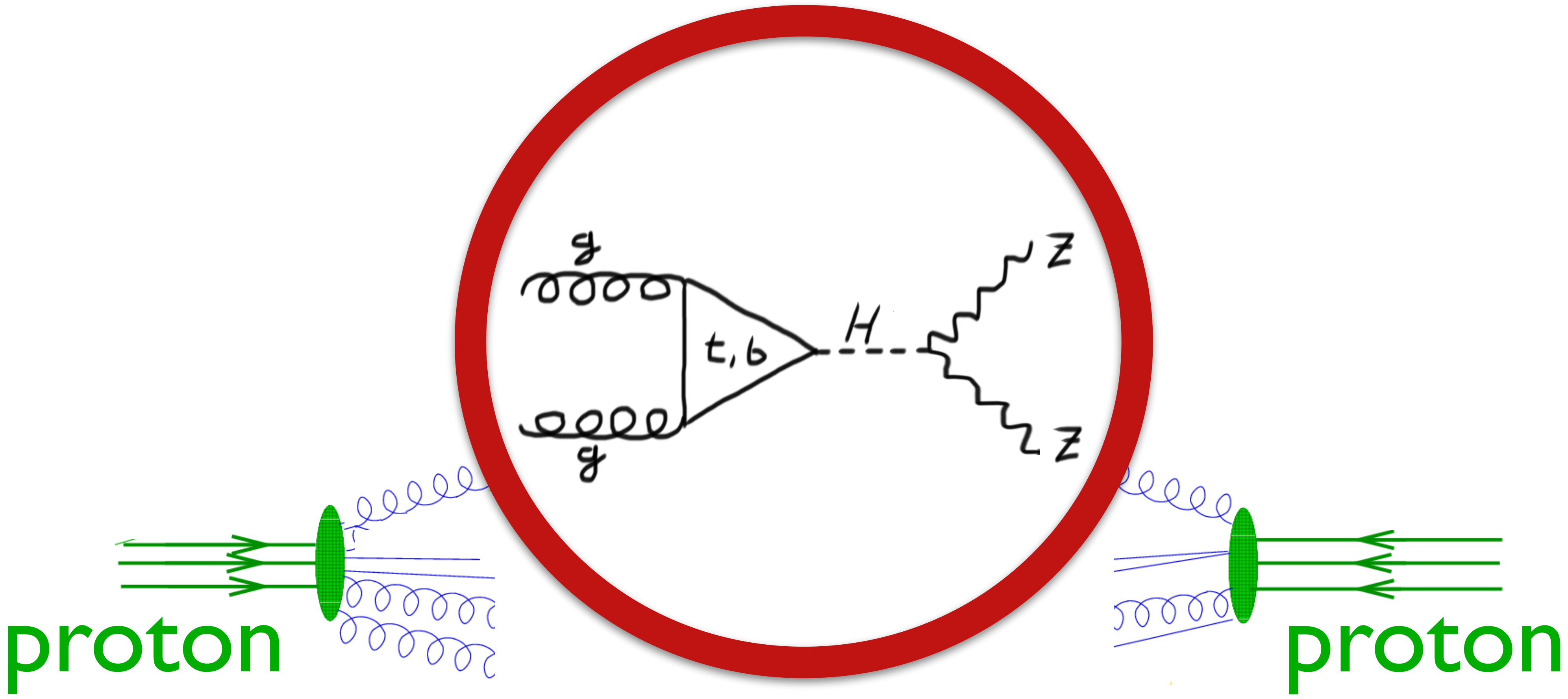
LHC event



Hadronization
Parton Shower
Hard Process
g
g
t, b
H
Z
Z
proton
proton
Underlying Event

LHC Master Formula

$$\sigma_{\text{had}} =$$



Hard Process

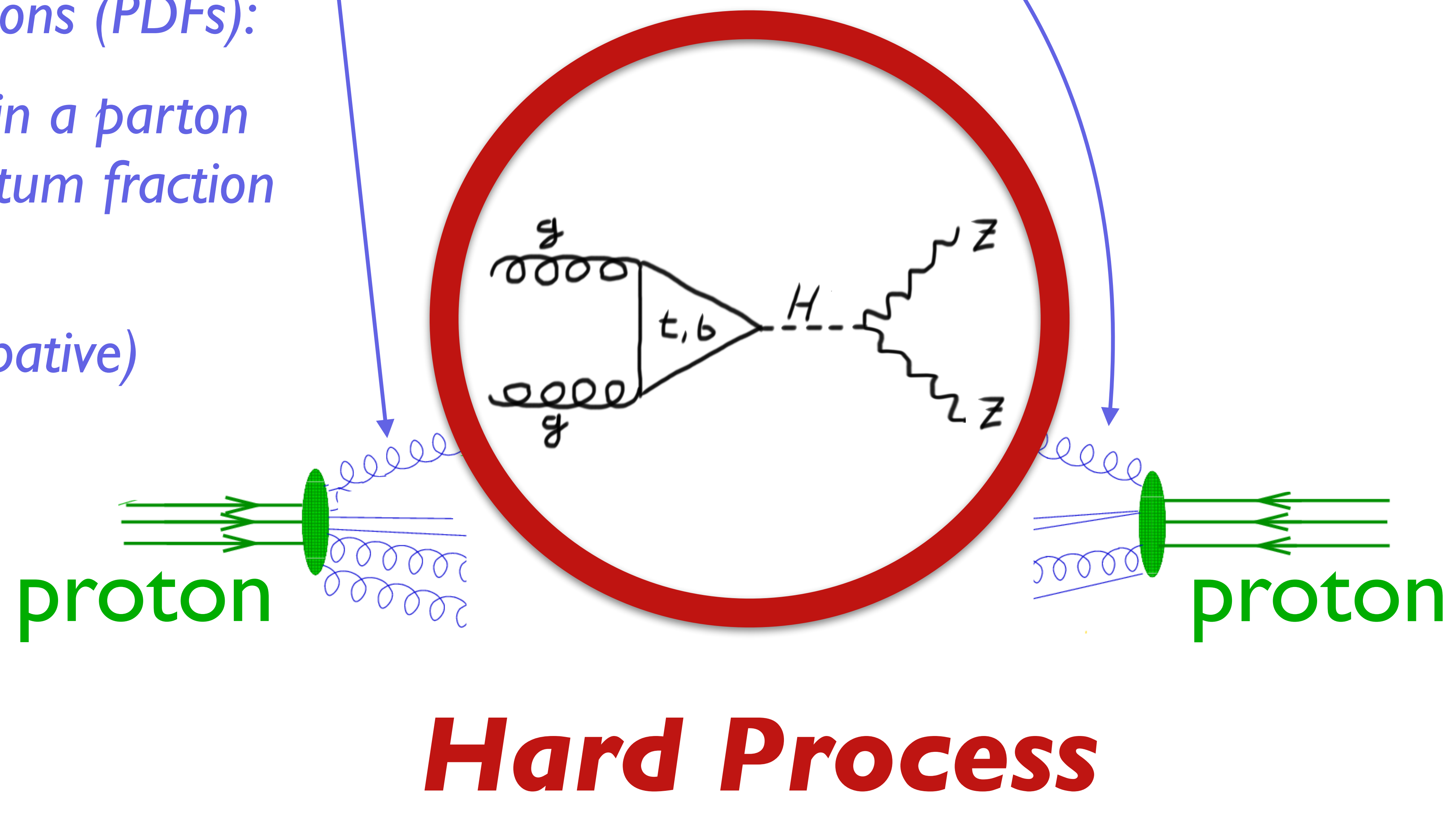
LHC Master Formula

$$\sigma_{\text{had}} = \qquad f_i(x_1, \mu_F)\, f_j(x_2, \mu_F)$$

Parton Distribution Functions (PDFs):

probability to find a certain a parton (here: gluon) with momentum fraction x_i inside the proton

long distance (non-perturbative)



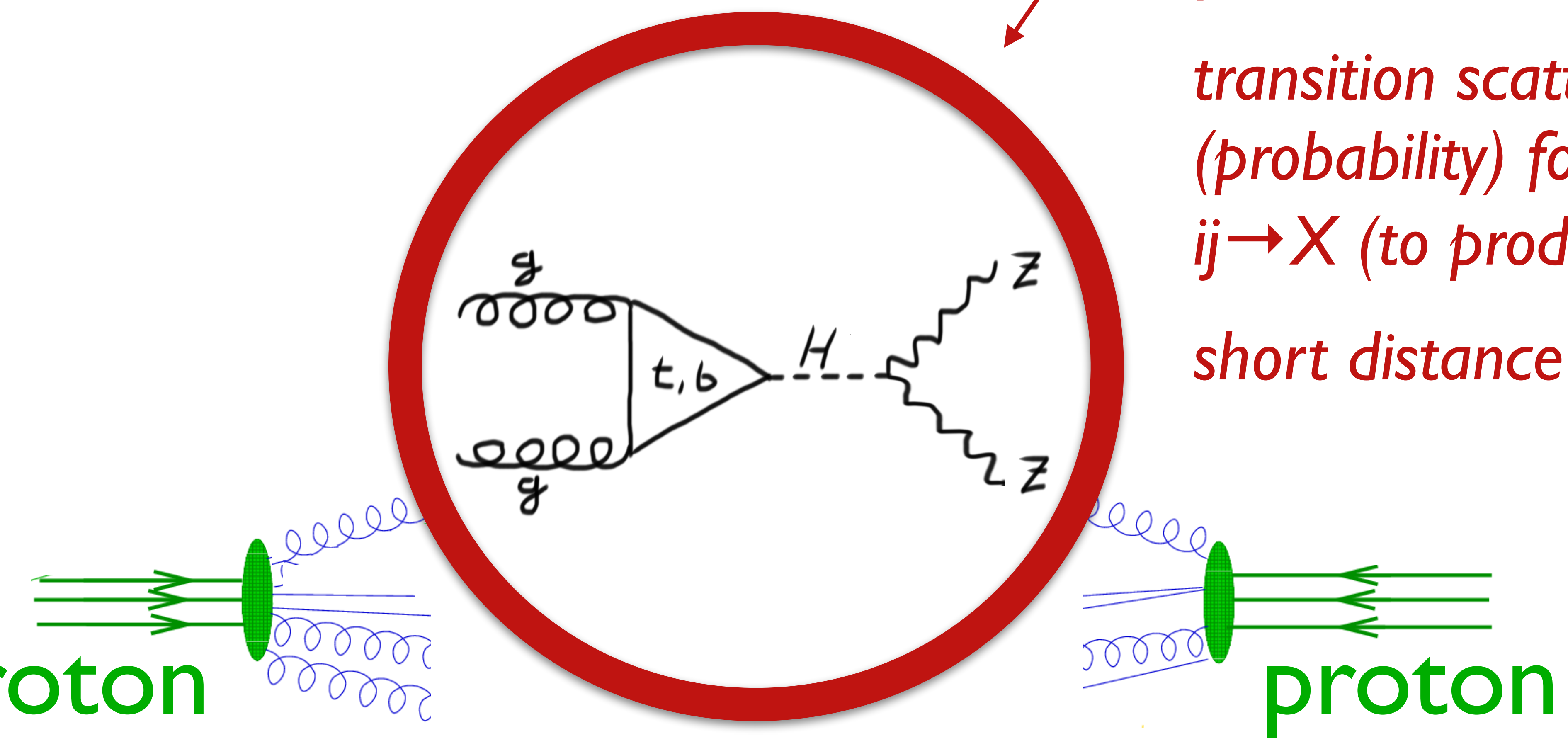
LHC Master Formula

$$\sigma_{\text{had}} = f_i(x_1,\mu_{\text{F}})\, f_j(x_2,\mu_{\text{F}}) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F)$$

partonic cross section:

transition scattering amplitude (probability) for the partonic process ij→X (to produce some final state X)

short distance (perturbative)



Hard Process

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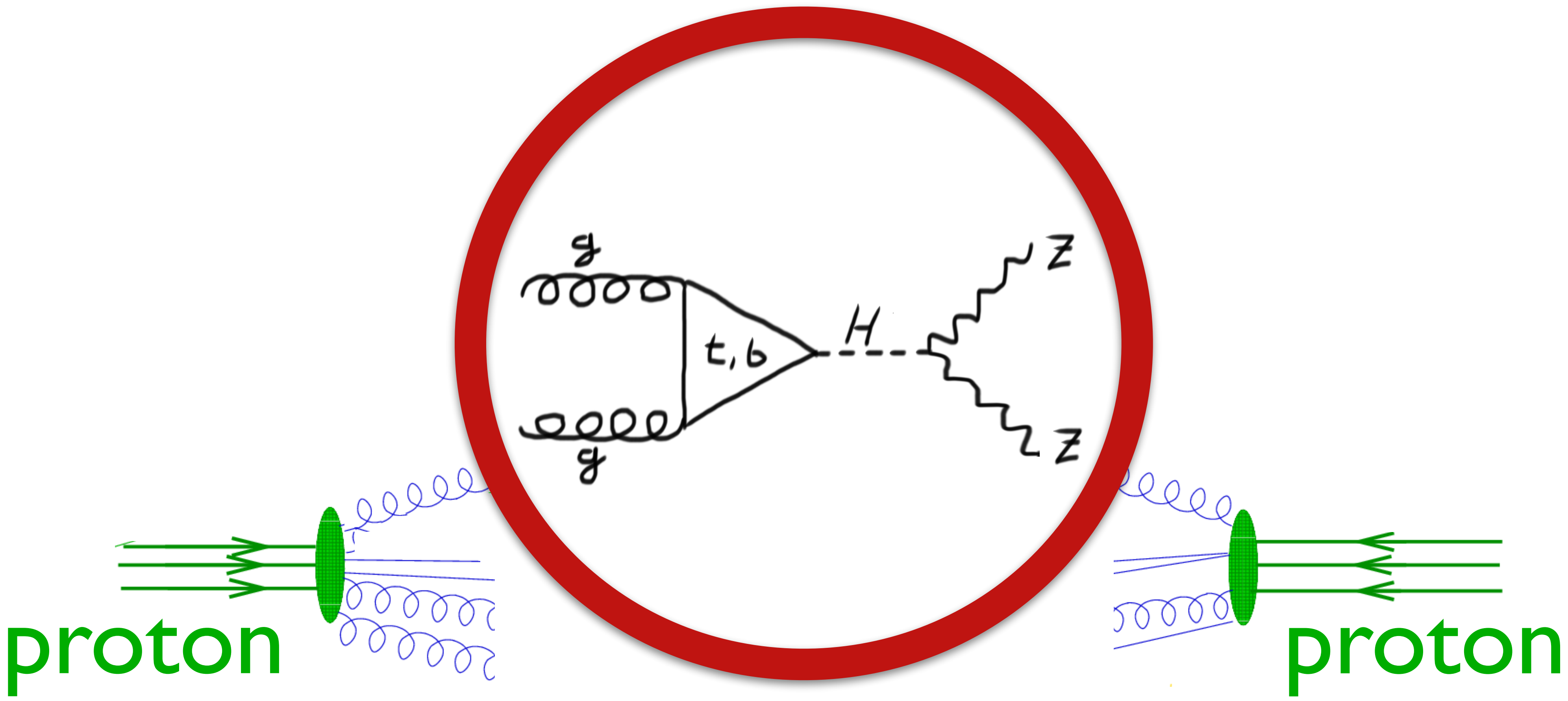
$$\sigma_{\text{had}} = \sum_{ij} \int \mathrm{d}x_1\, \mathrm{d}x_2\, f_i(x_1, \mu_{\mathrm{F}})\, f_j(x_2, \mu_{\mathrm{F}}) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F)$$



Hard Process

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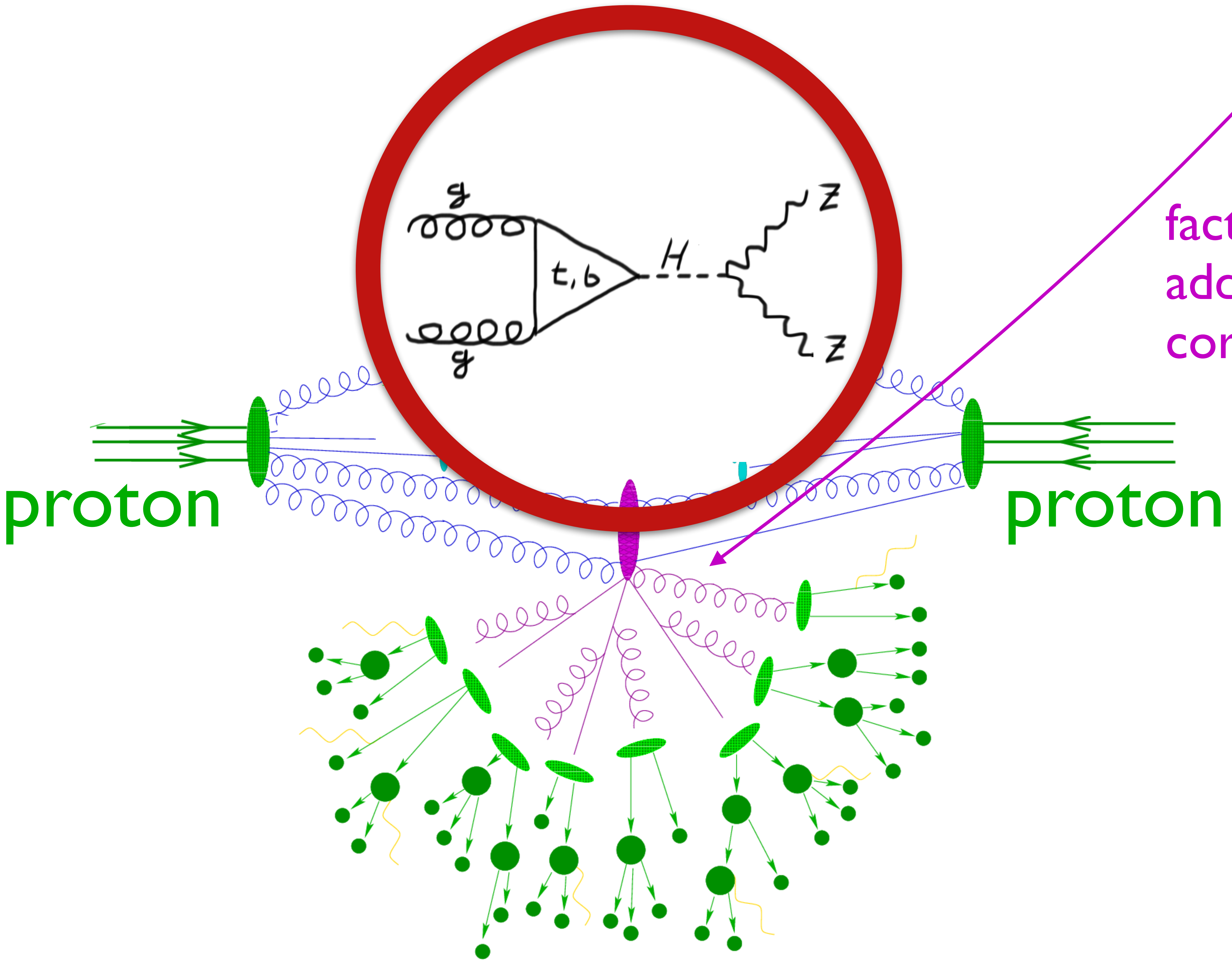
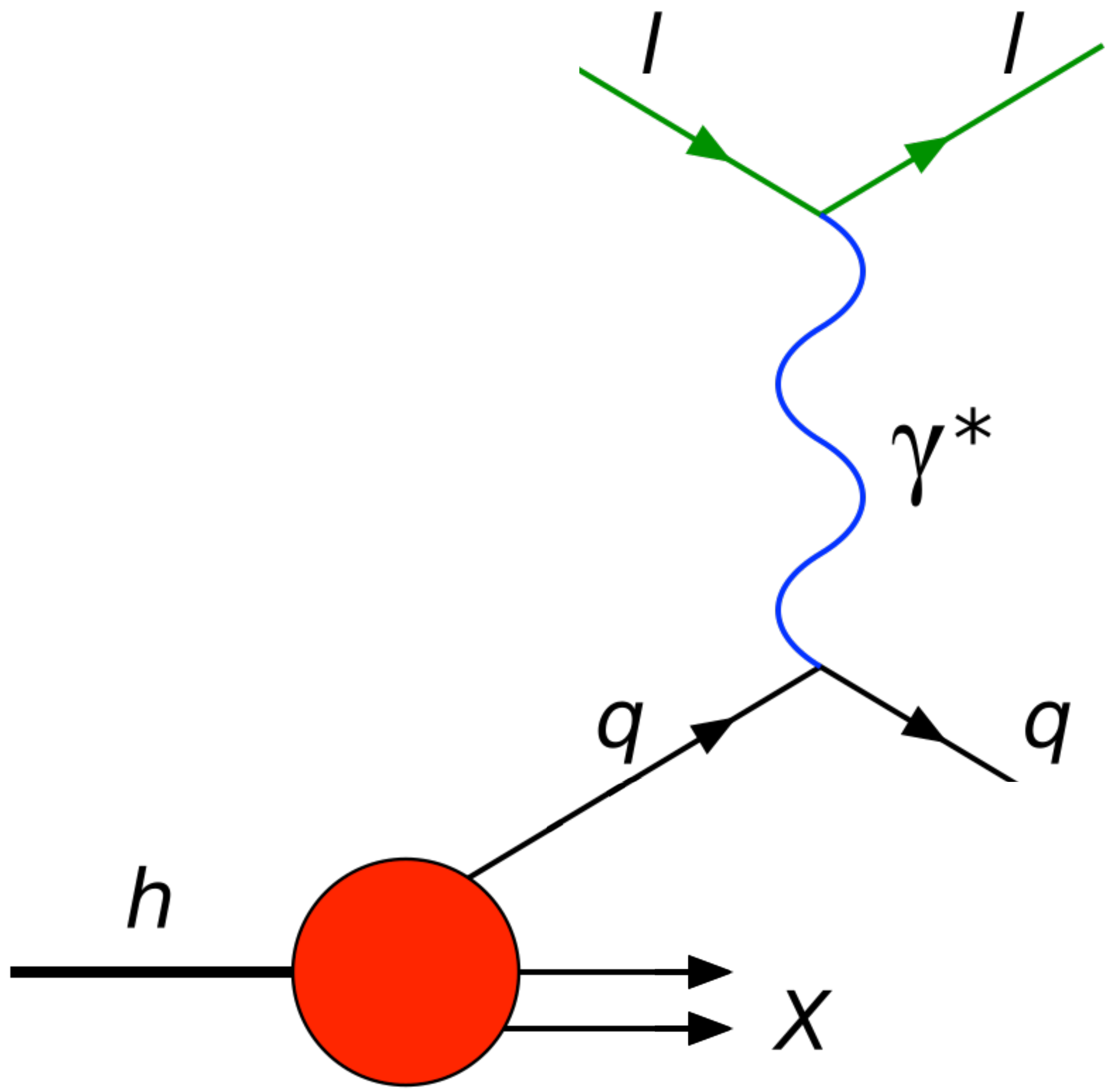
factorization holds up to additional non-perturbative corrections (higher "twist")



Factorization: a few comments

consider deep inelastic scattering (DIS, just one hadron) for simplicity:

$$\sigma_{\rm had}^{\rm DIS} = f^{\rm bare} \otimes \sigma_p$$



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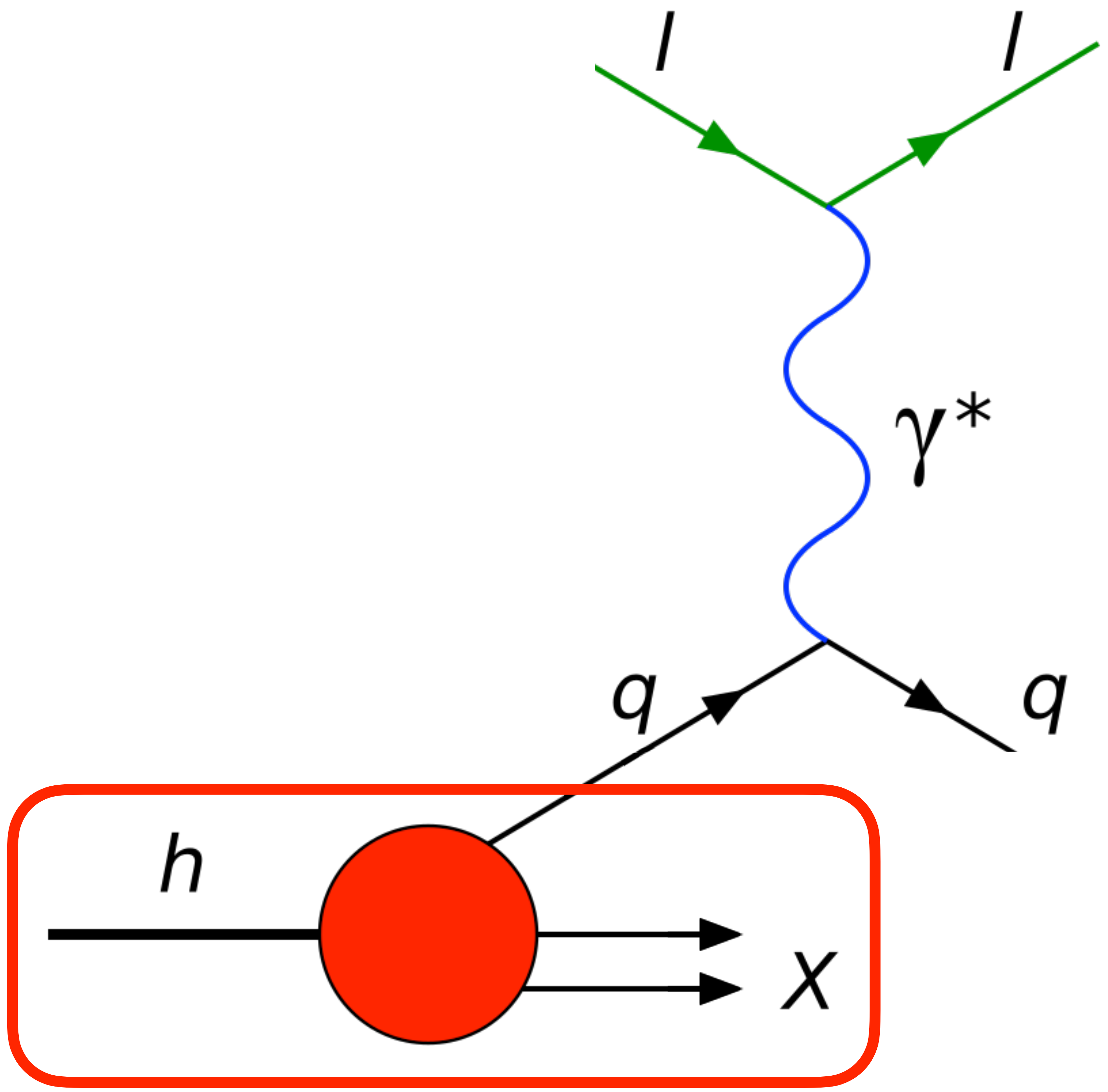
bare PDF
(UV divergent)



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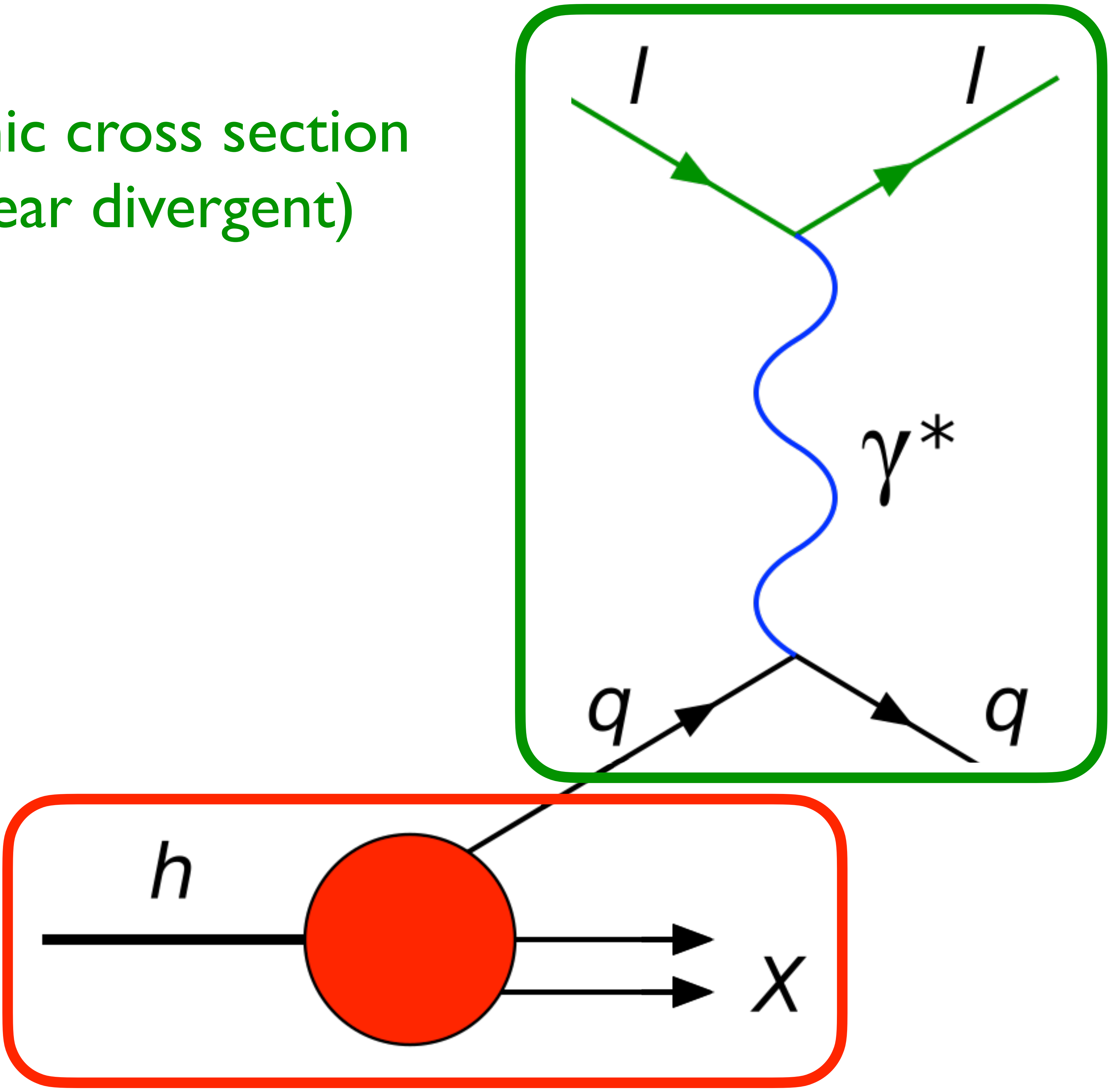
bare PDF
(UV divergent)

partonic cross section
(collinear divergent)



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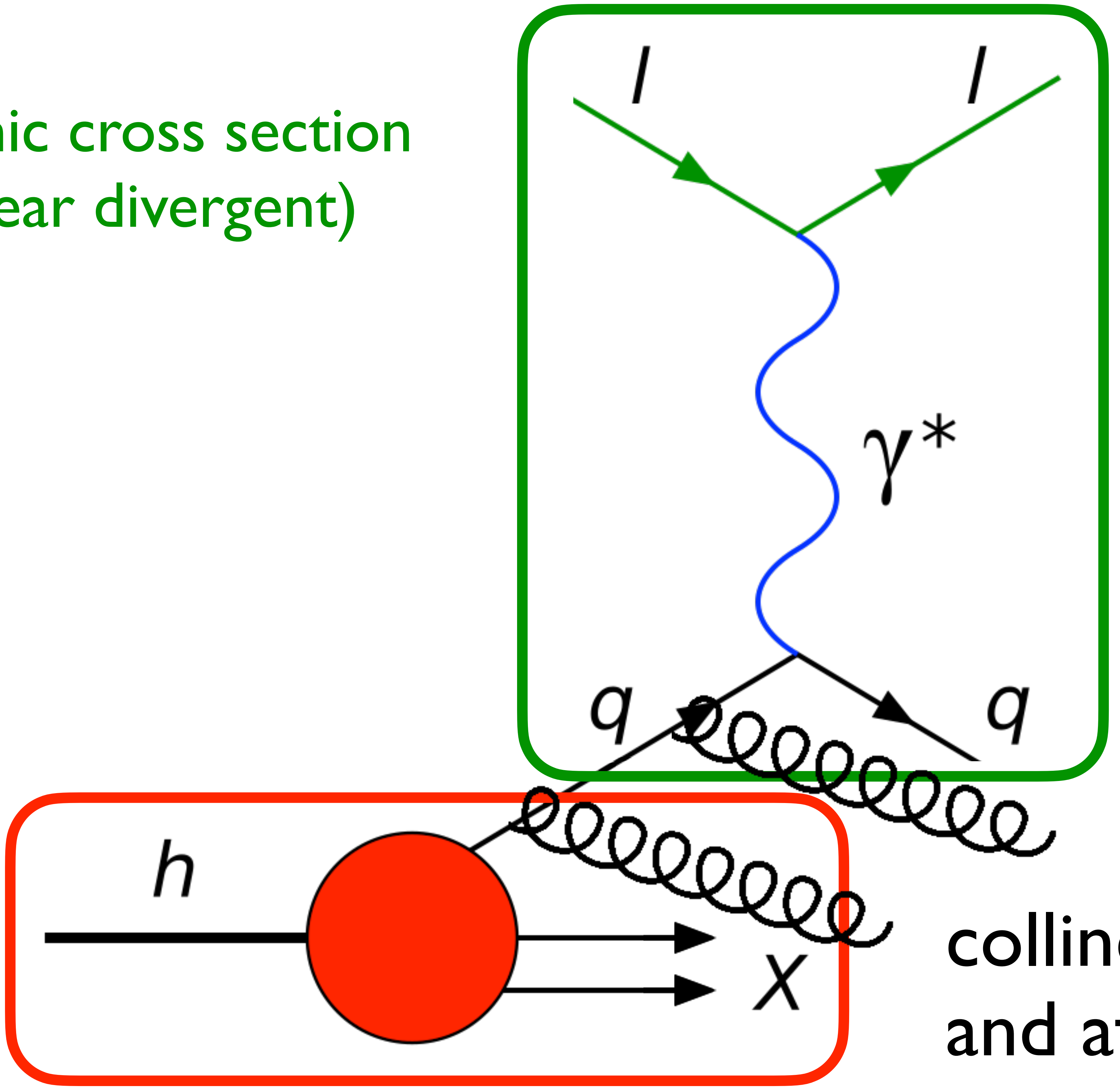
bare PDF (UV divergent) | partonic cross section (collinear divergent)



collinear radiation can appear in either regime and affects both the PDFs and the partonic cross section

Factorization: a few comments

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$$\sigma_{\text{had}}^{\text{DIS}} = f^{\text{bare}} \otimes \sigma_p = f^{\text{bare}} \otimes \underbrace{Z_{\text{UV}}(\mu_F) \otimes Z_{\text{IR}}^{-1}(\mu_F)}_{\text{multiply by one}} \otimes \sigma_p$$

$$Z_{\text{UV}} = Z_{\text{IR}}$$

UV renormalization of bare PDFs cancels collinear singularities of partonic cross section

Factorization: a few comments

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UV renormalized PDFs (finite)

$$Z_{\text{UV}} = Z_{\text{IR}}$$

UV renormalization of bare PDFs cancels collinear singularities of partonic cross section

Factorization: a few comments

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UV renormalized PDFs (finite)

partonic cross section +collinear counterterm (finite)

$$Z_{\rm UV} = Z_{\rm IR}$$

UV renormalization of bare PDFs cancels collinear singularities of partonic cross section

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factorization introduces factorization scale μ_F , consider the simple example:

$$O = F(\mu_F) \cdot S(\mu_F) \quad \Rightarrow \quad \mu_F \frac{\mathrm{d}O}{\mathrm{d}\mu_F} = 0 \quad \Rightarrow \quad \mu_F \frac{\mathrm{d}\ln F(\mu_F)}{\mathrm{d}\mu_F} = \gamma(\mu_F) = -\mu_F \frac{\mathrm{d}\ln S(\mu_F)}{\mathrm{d}\mu_F}$$

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factorization → evolution

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factorization → evolution

back to DIS:

$$\sigma_{\text{had}}^{\text{DIS}}(m, Q) = f(m, \mu_F) \otimes \hat{\sigma}_p(Q, \mu_F)$$

$$E(m, Q) \sim \exp\left(\int_m^Q \frac{\mathrm{d}\mu}{\mu} \gamma(\mu) \right)$$

Factorization: a few comments

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factorization → evolution

back to DIS:

$$\begin{aligned}\sigma_{\text{had}}^{\text{DIS}}(m, Q) &= f(m, \mu_F) \otimes \hat{\sigma}_p(Q, \mu_F) \\ &= f(m, \mu_0) \otimes E(\mu_0, \mu_F) \otimes \hat{\sigma}_p(Q, \mu_F)\end{aligned}$$

Factorization: a few comments

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factorization → evolution → resummation

back to DIS:

$$\begin{aligned} \sigma_{\text{had}}^{\text{DIS}}(m, Q) &= f(m, \mu_F) \otimes \hat{\sigma}_p(Q, \mu_F) \\ &= f(m, \mu_0) \otimes E(\mu_0, \mu_F) \otimes \hat{\sigma}_p(Q, \mu_F) \\ \mu_0 \simeq m, \mu_F \simeq Q \quad &= f(m, m) \otimes E(m, Q) \otimes \hat{\sigma}_p(Q, Q) \end{aligned}$$

$$E(m, Q) \sim \exp\left(\int_m^Q \frac{\mathrm{d}\mu}{\mu} \gamma(\mu) \right)$$

Evolution of PDFs: DGLAP equation

in practice much more complicated:

- ✦ convolution (use Mellin space for exponentiation/resummation)
- ✦ scale dependence through strong coupling
- ✦ coupled differential equation mixing all PDF sets
- ✦ …

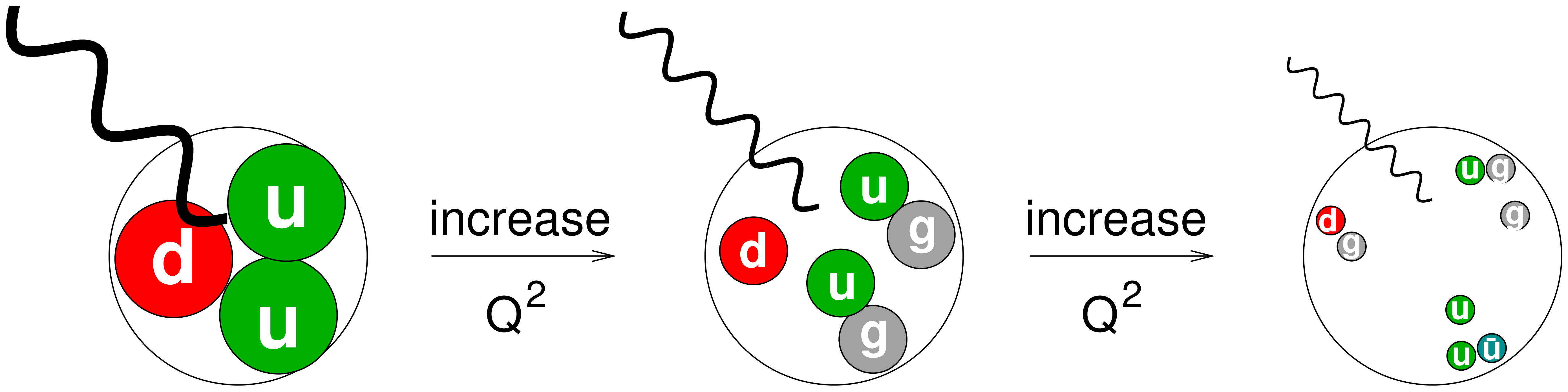
convolution:

$$\big(a \otimes b\big)(x) = \int_x^1 \frac{\mathrm{d}x'}{x'} a(x')\, b(x/x')$$

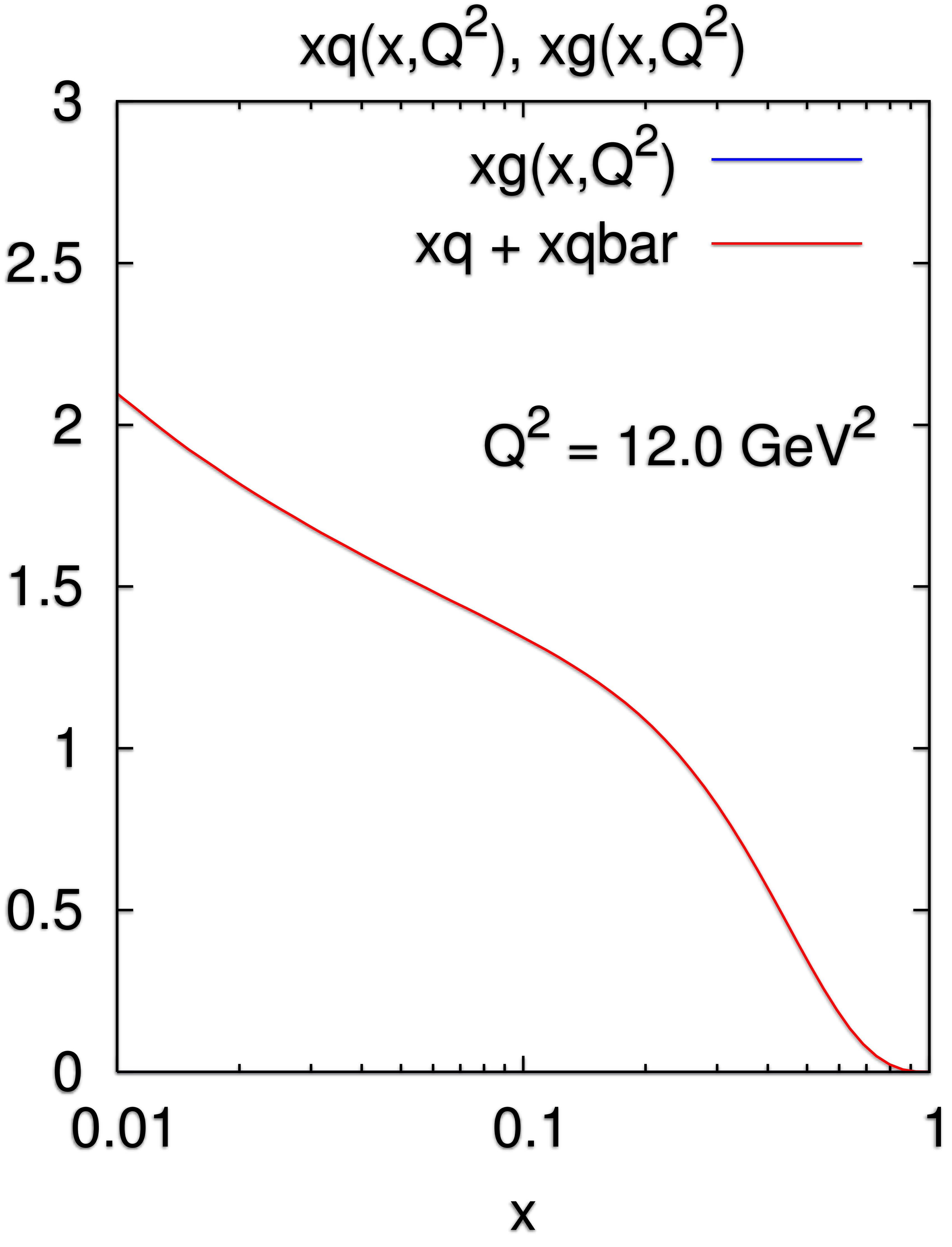
$$\frac{\partial}{\partial \ln\mu^2} f(x,\mu^2) = \sum_j \frac{\alpha_s(\mu)}{2\pi} \Big(f_j(\mu^2) \otimes P_{ij}(\alpha_s(\mu)) \Big)(x)$$

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (DGLAP)

physical meaning:



Evolution of PDFs: Example #1



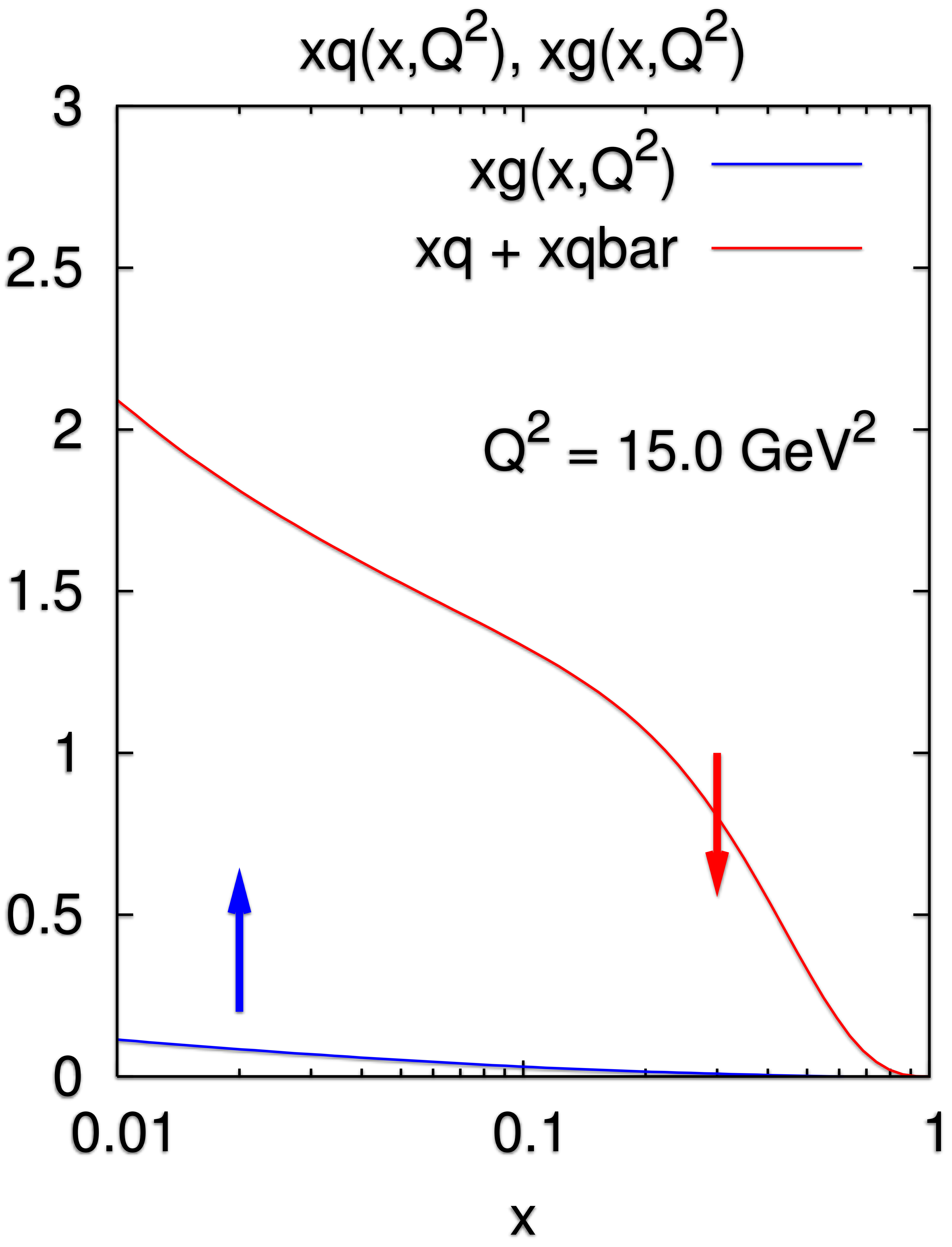
Take example evolution starting with just quarks:

$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

...slide borrowed from Gavin Salam

Evolution of PDFs: Example #1



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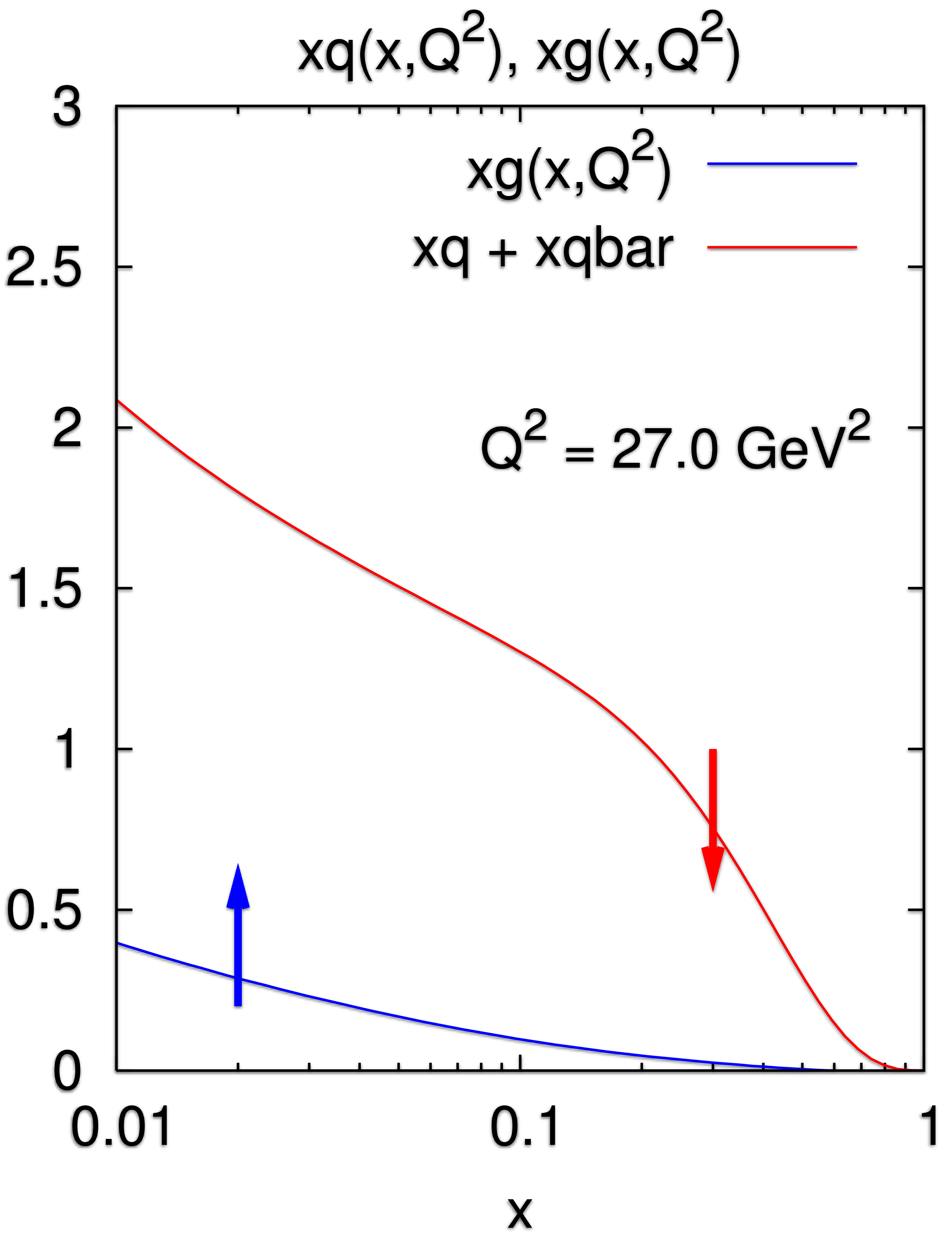

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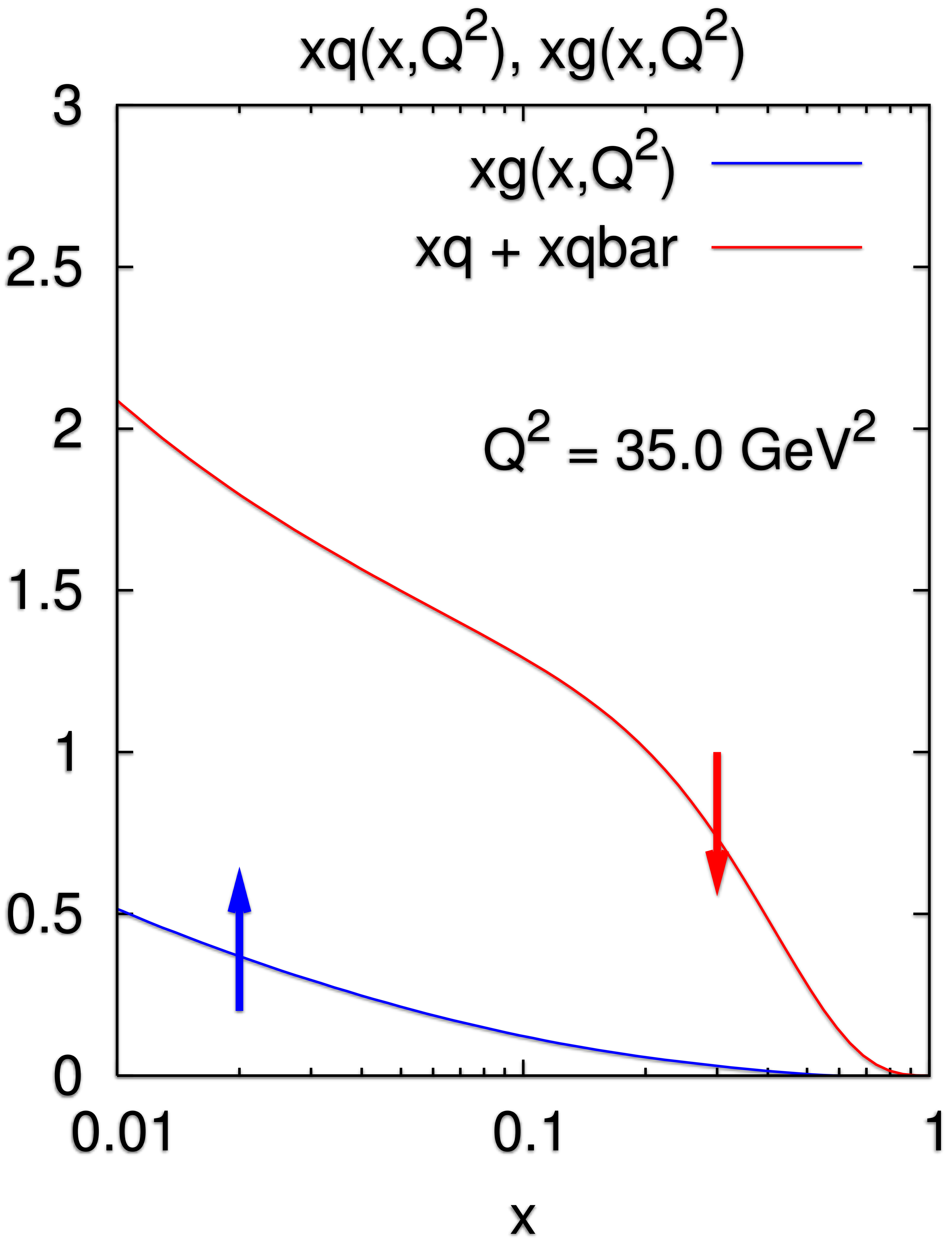

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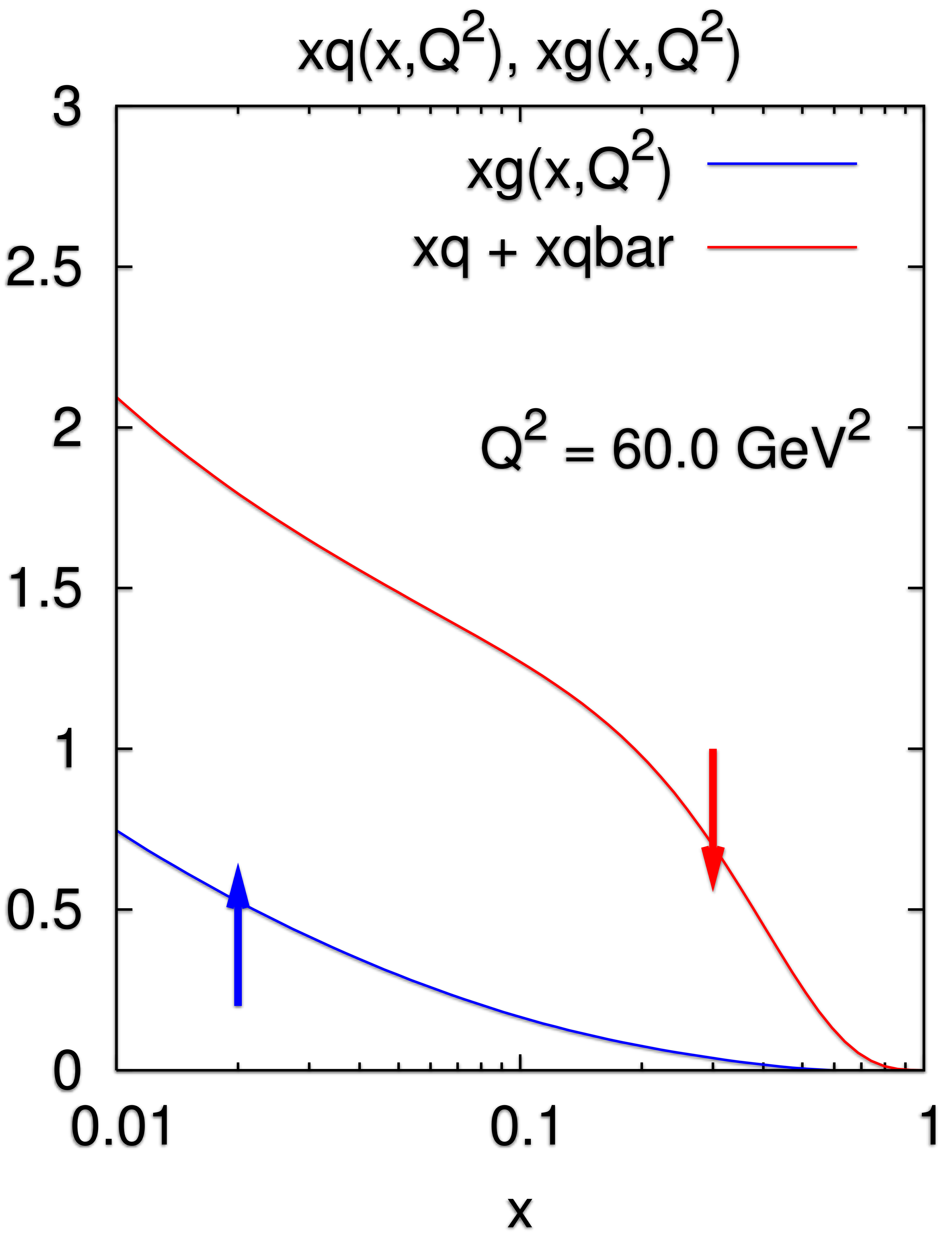
$$\partial_{\ln Q^2} q = P_{q\leftarrow q} \otimes q$$
$$\partial_{\ln Q^2} g = P_{g\leftarrow q} \otimes q$$

- quark is depleted at large x
- gluon grows at small x

...slide borrowed from Gavin Salam

Evolution of PDFs: Example #1



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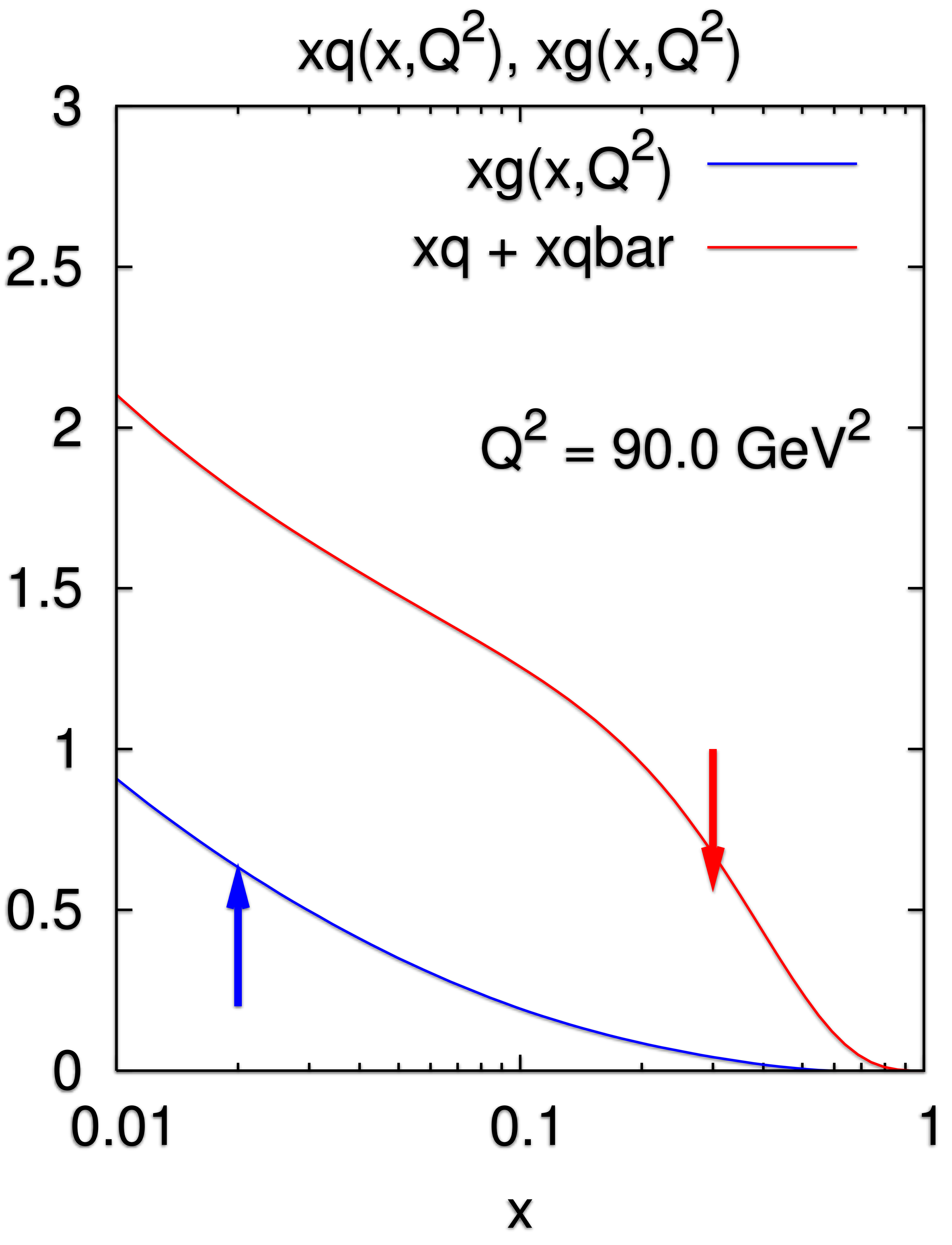

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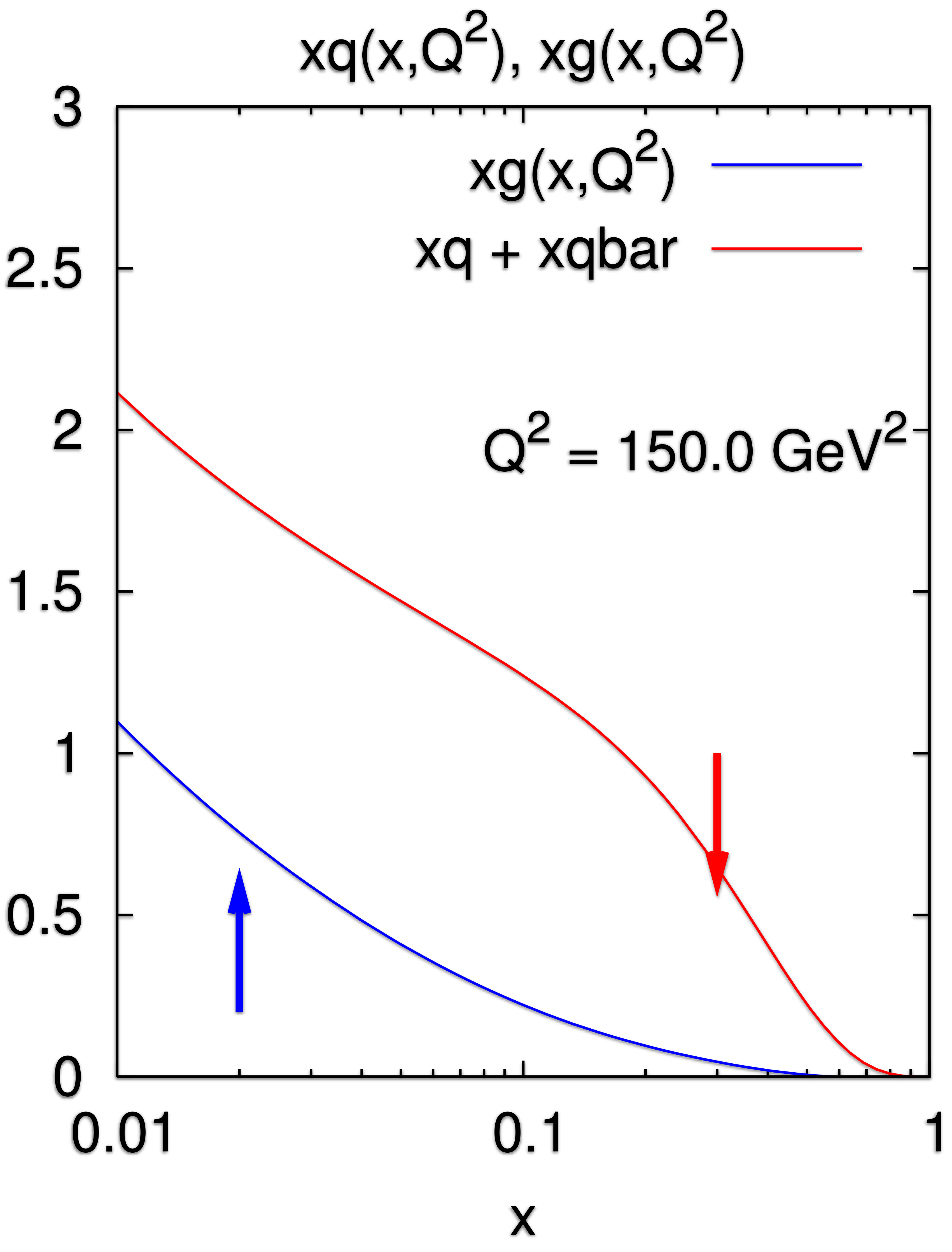

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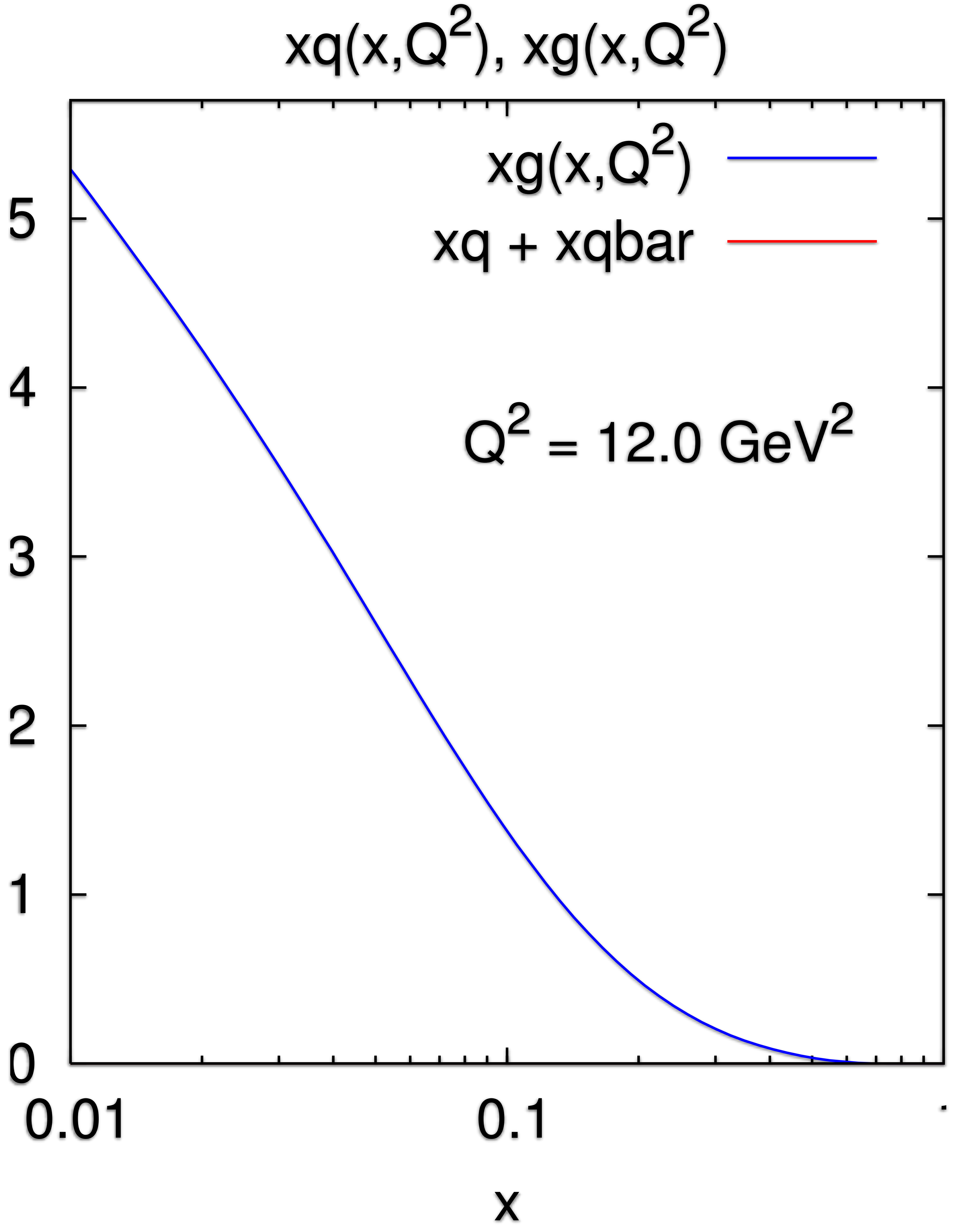
Evolution of PDFs: Example #2



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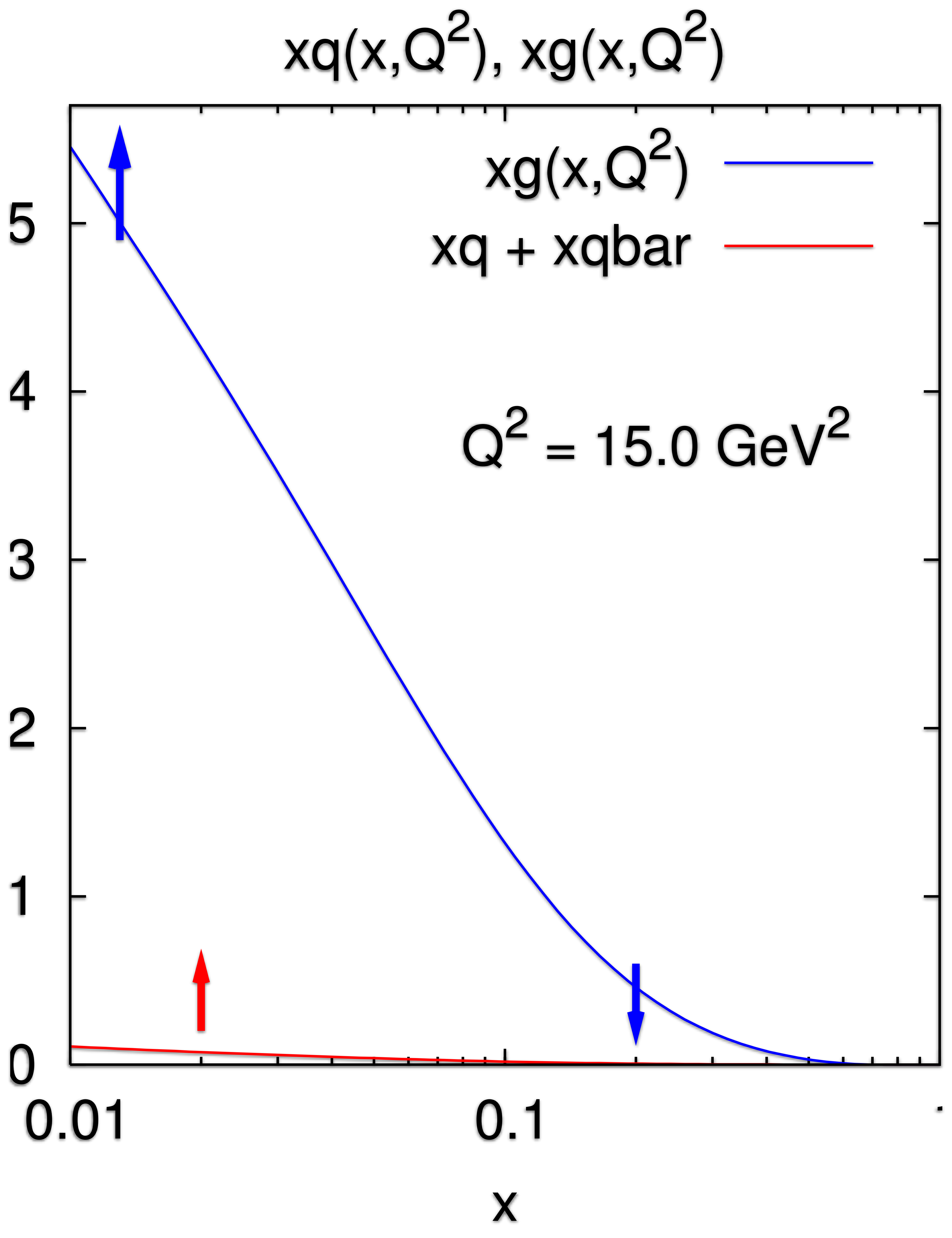

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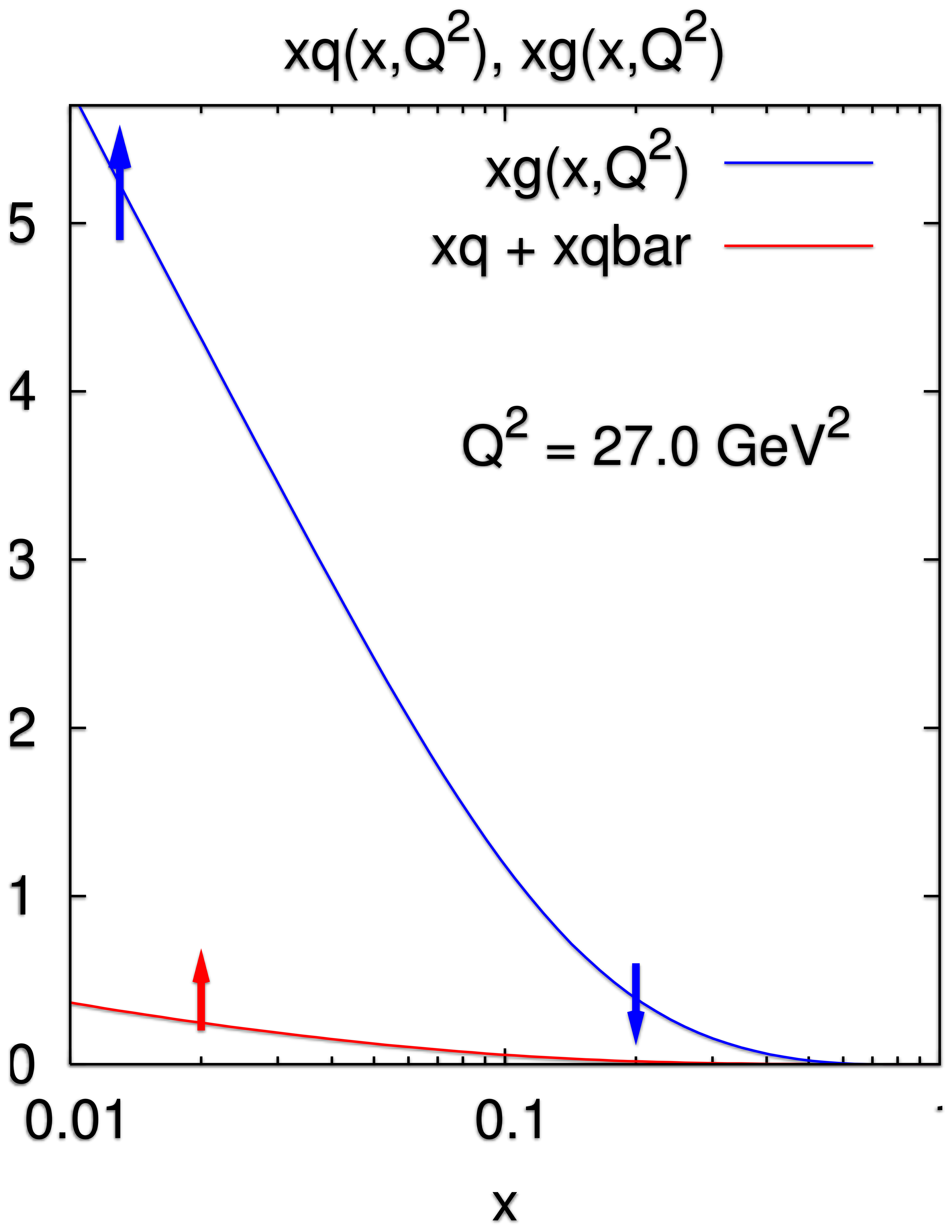

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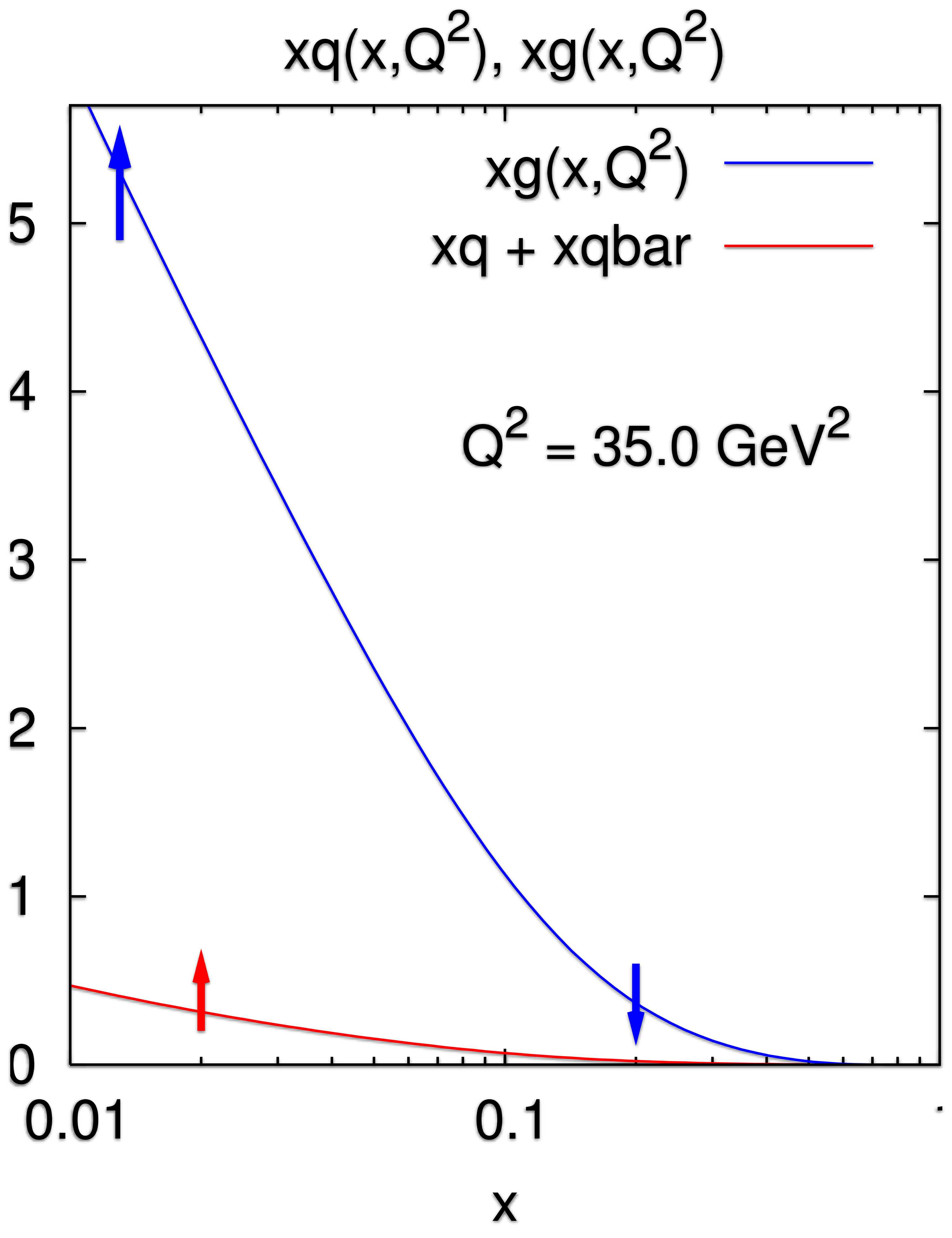

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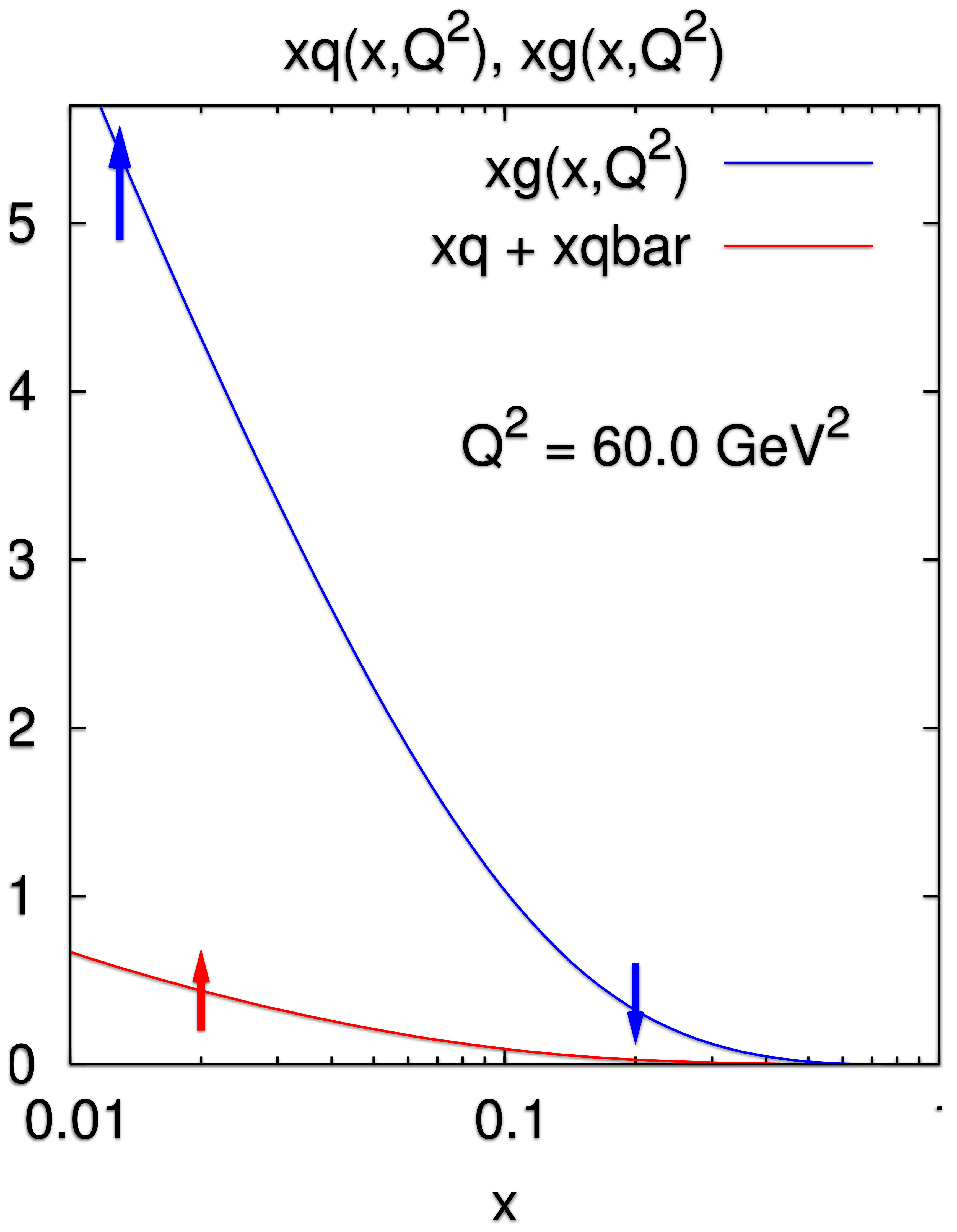

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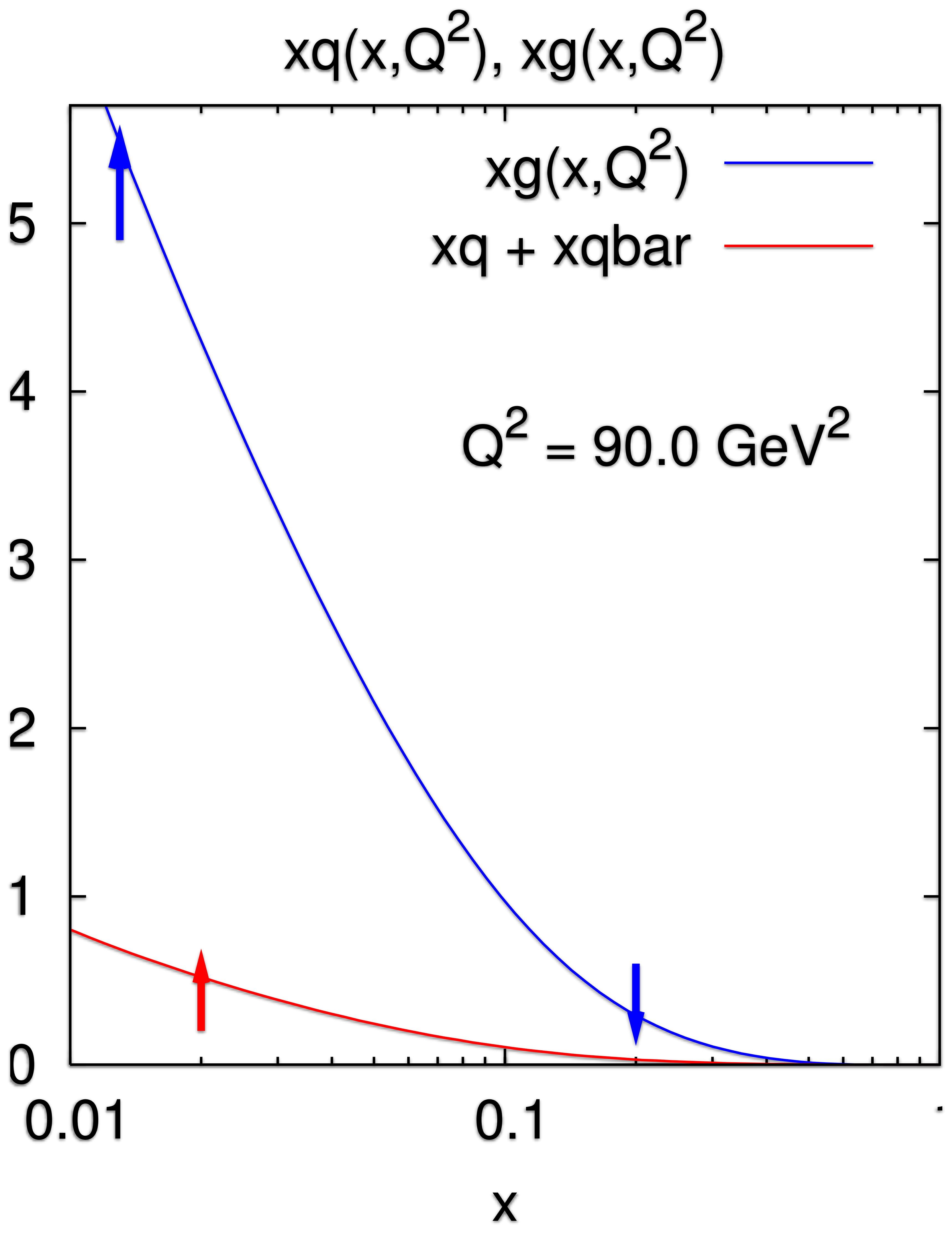

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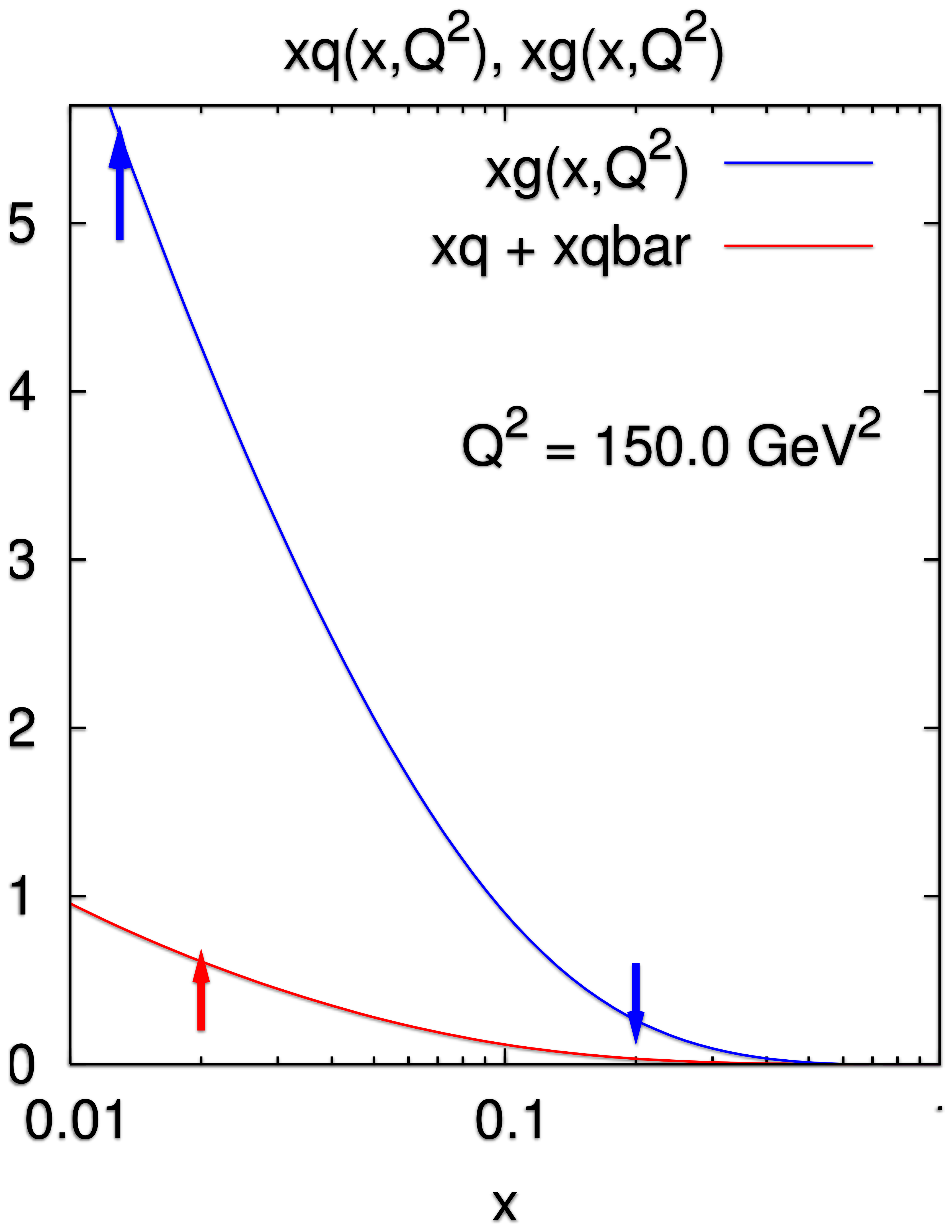

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DGLAP evolution:

- ➤ partons lose momentum and shift towards smaller x
- ➤ high-x partons drive growth of low-x gluon

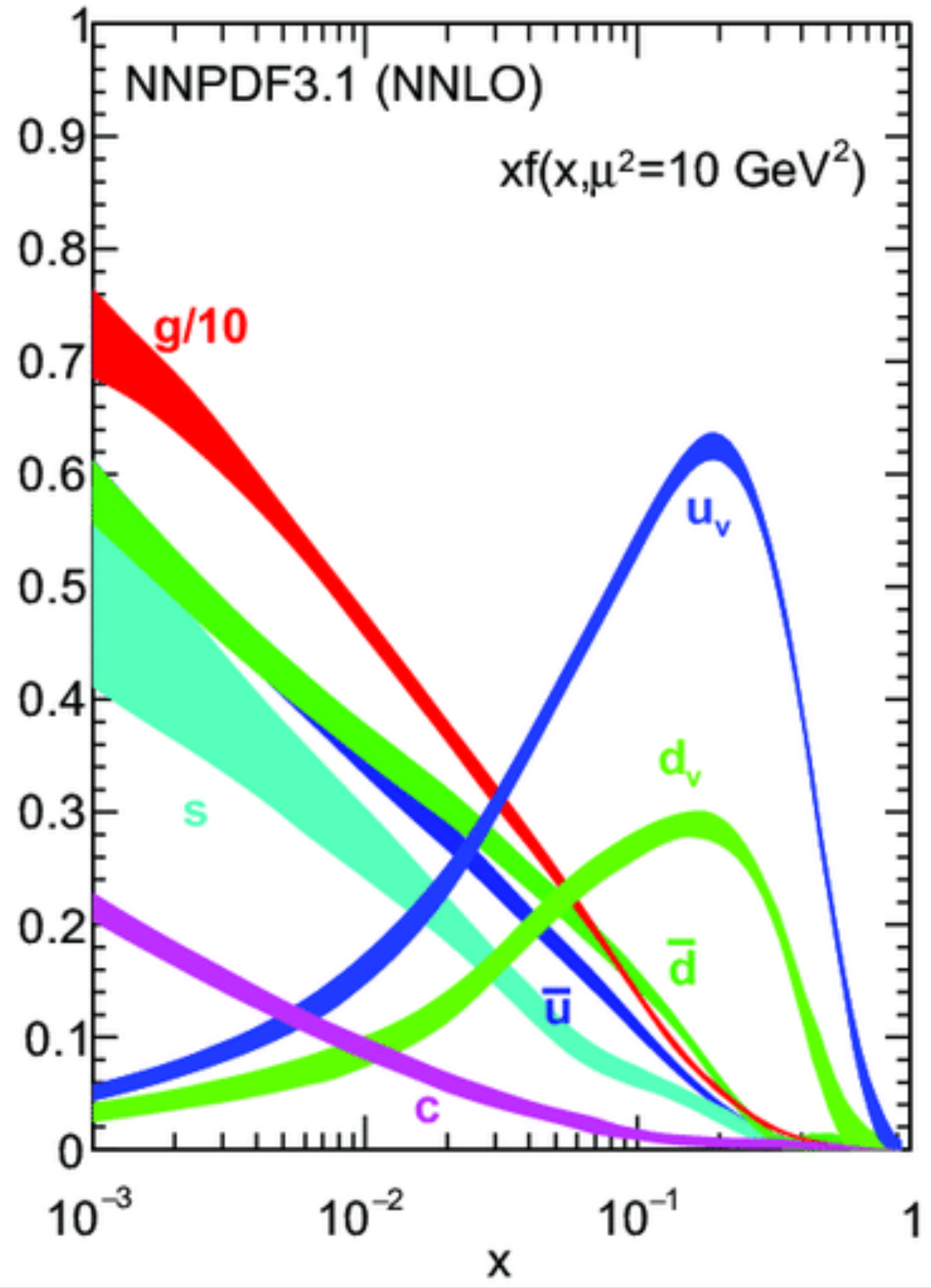
Parton Distribution Functions (PDFs)

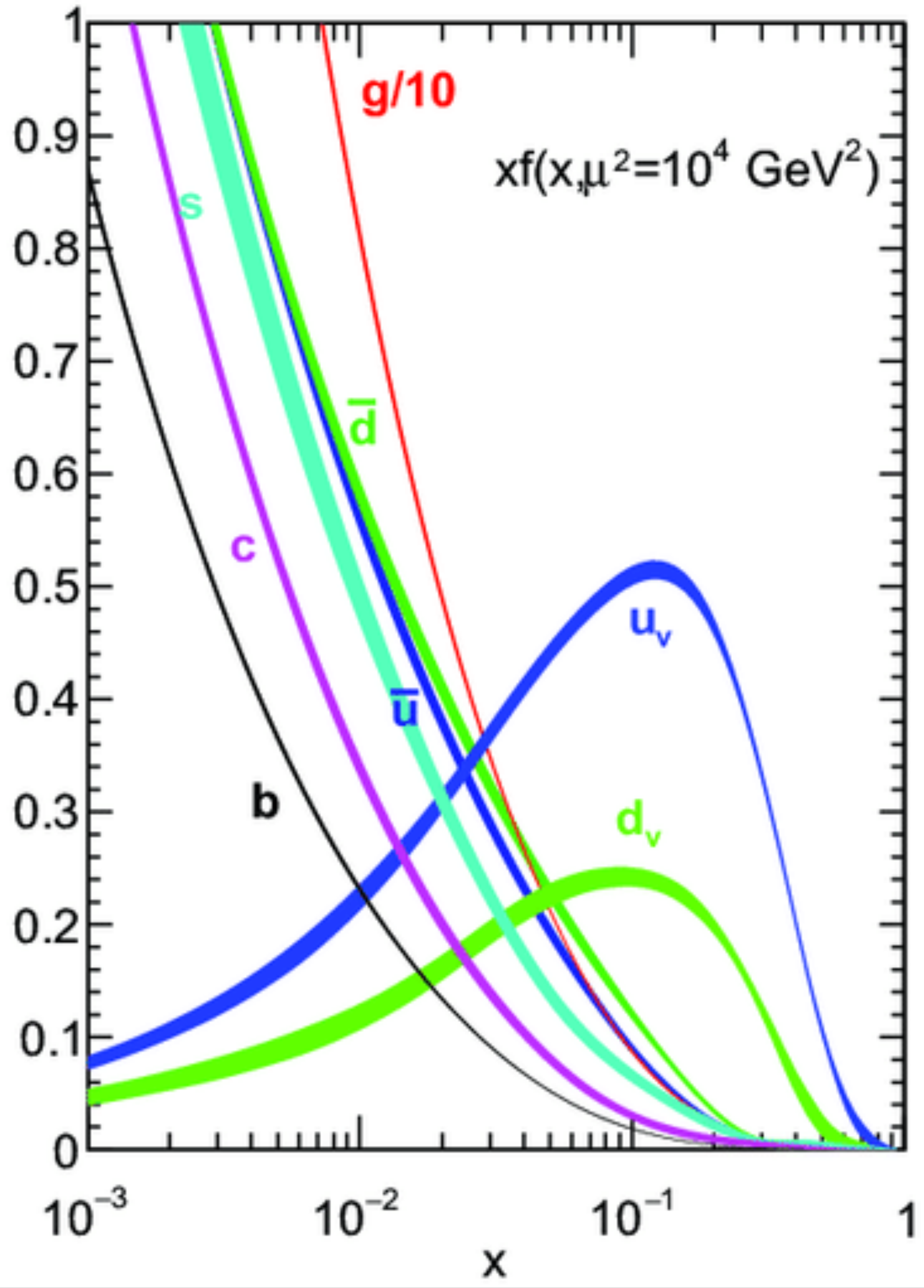
$$\sigma_{\text{had}} = \sum_{ij} \int \mathrm{d}x_1\, \mathrm{d}x_2\, f_i(x_1, \mu_\mathrm{F})\, f_j(x_2, \mu_\mathrm{F}) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}(\Lambda^2/Q^2)$$

- *universal distributions containing long-distance structure of hadrons*
- *scale dependence via DGLAP evolution (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi):*

$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(x, \mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \sum_j \int_x^1 \frac{dy}{y} f_j(y, \mu^2) P_{ij}(x/y, \alpha_S(\mu^2))$$

- $f_i(x, \mu_0^2)$ *determined from:*
  - *lattice QCD (in principle)*
  - *fits to data (in practice)*
    e.g. *MSTW, MMHT, CTEQ, HERA, ABM, NNPDF, ...*
  - *photon PDF calculated*
    [Manohar, Nason, Salam, Zanderighi, '17]





Sum rules & indirect (gluon) PDF determination

Momentum sum rule: conservation of incoming total momentum

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

u_v	0,267
d_v	0,111
u_s	0,066
d_s	0,053
s_s	0,033
c_c	0,016
total	0,546

➟ *half of the longitudinal momentum carried by gluons*

...slide borrowed from Giulia Zanderighi

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Momentum sum rule: conservation of incoming total momentum

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

Conservation of flavour: e.g. for a proton

$$\int_0^1 dx \left(f_u^{(p)}(x) - f_{\bar{u}}^{(p)}(x) \right) = 2$$

$$\int_0^1 dx \left(f_d^{(p)}(x) - f_{\bar{d}}^{(p)}(x) \right) = 1$$

$$\int_0^1 dx \left(f_s^{(p)}(x) - f_{\bar{s}}^{(p)}(x) \right) = 0$$

In the proton: u, d valence quarks, all other quarks are called sea-quarks

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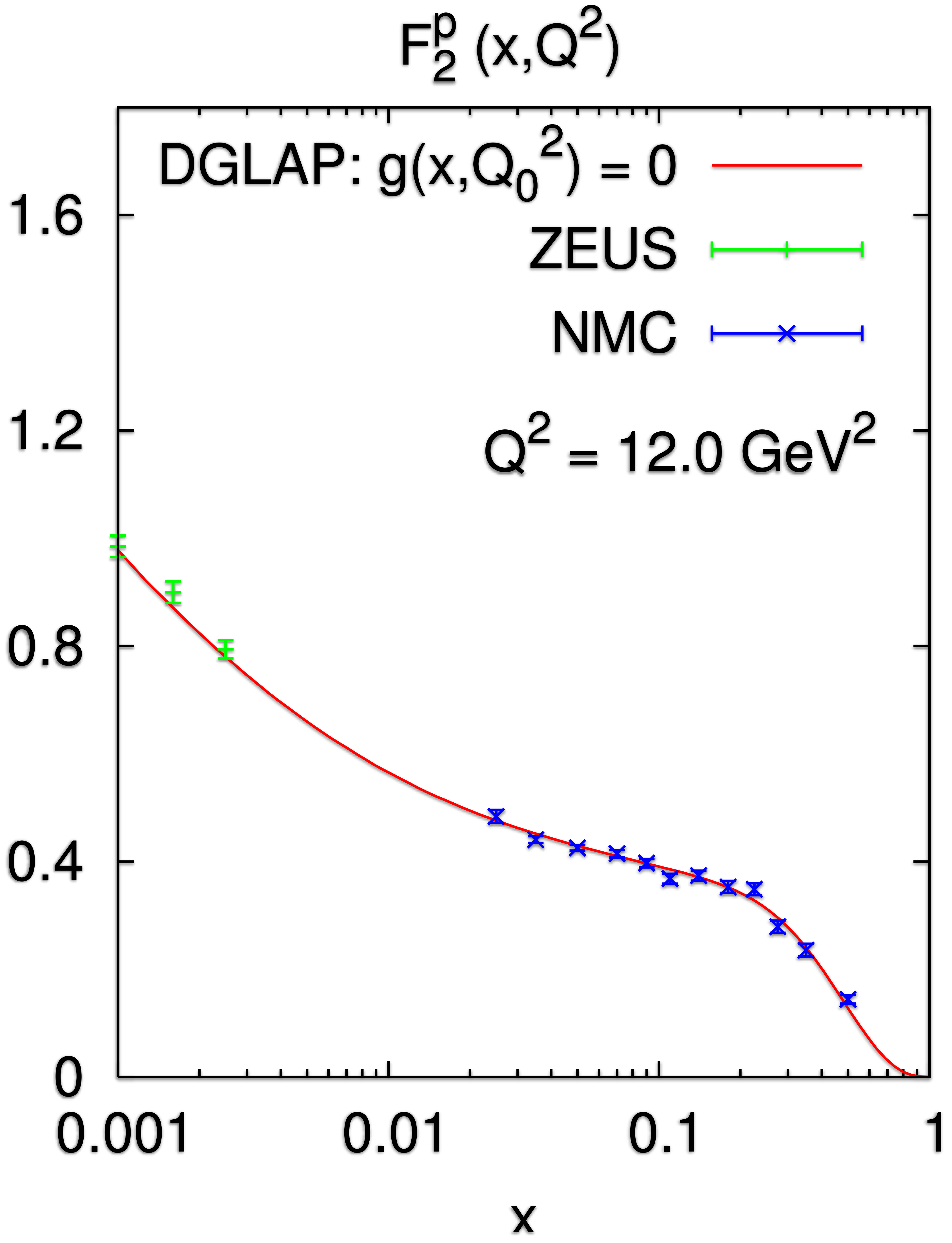

Fit quark distributions to $F_2(x, Q_0^2)$, at *initial scale* $Q_0^2 = 12$ GeV^2.

NB: Q_0 often chosen lower

Assume there is no gluon at Q_0^2:

$$g(x, Q_0^2) = 0$$

Use DGLAP equations to evolve to higher Q^2; compare with data.

...slide borrowed from Gavin Salam

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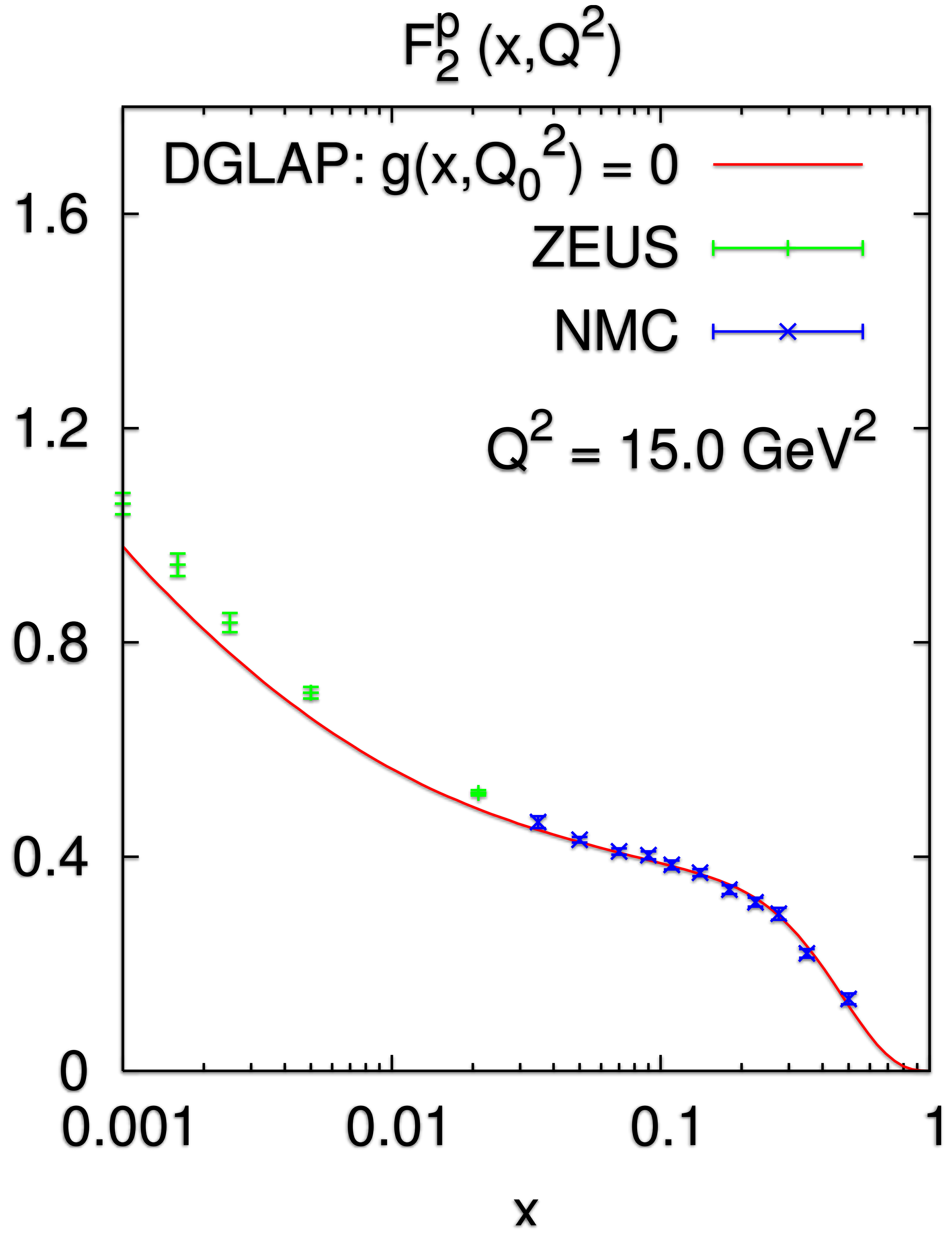

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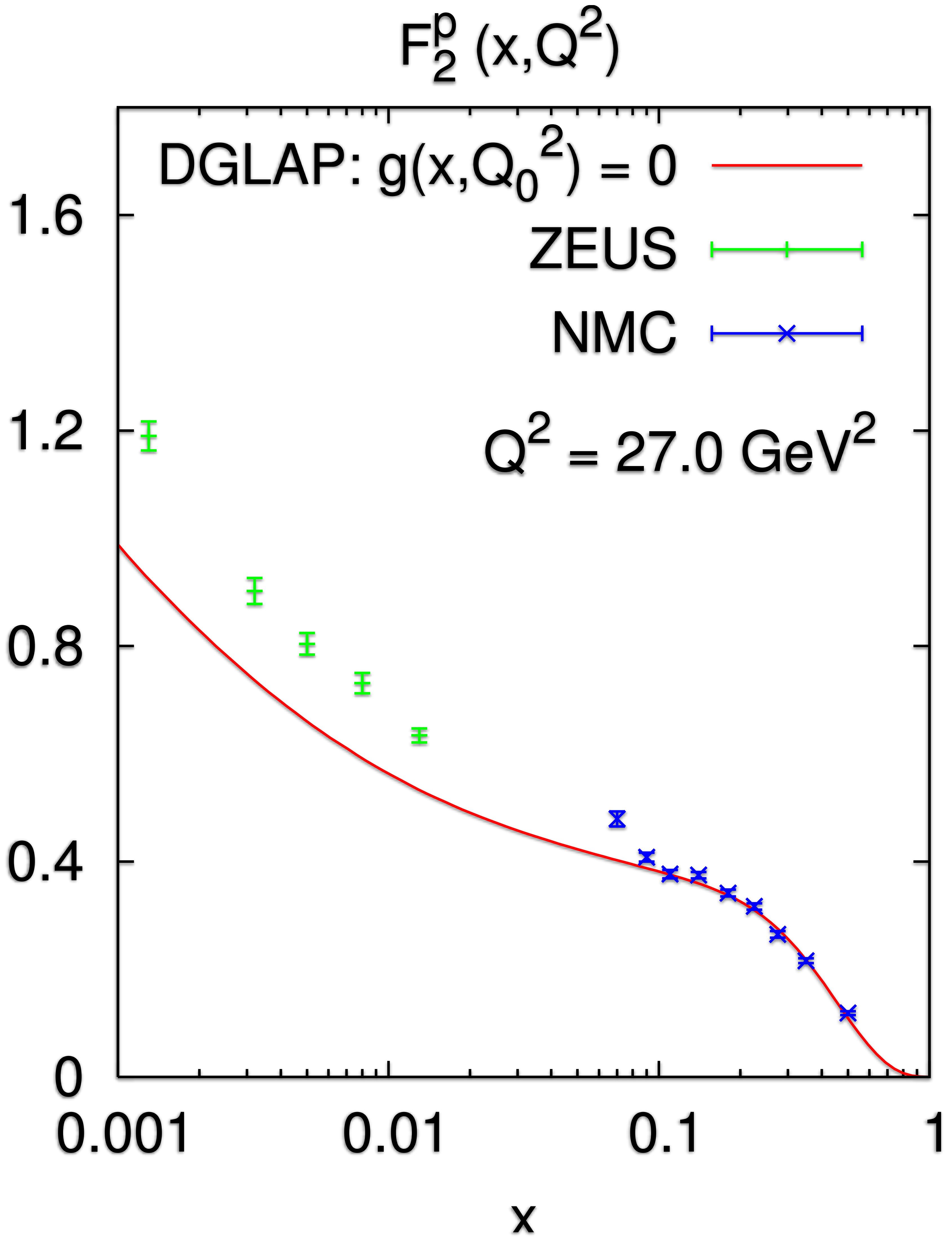

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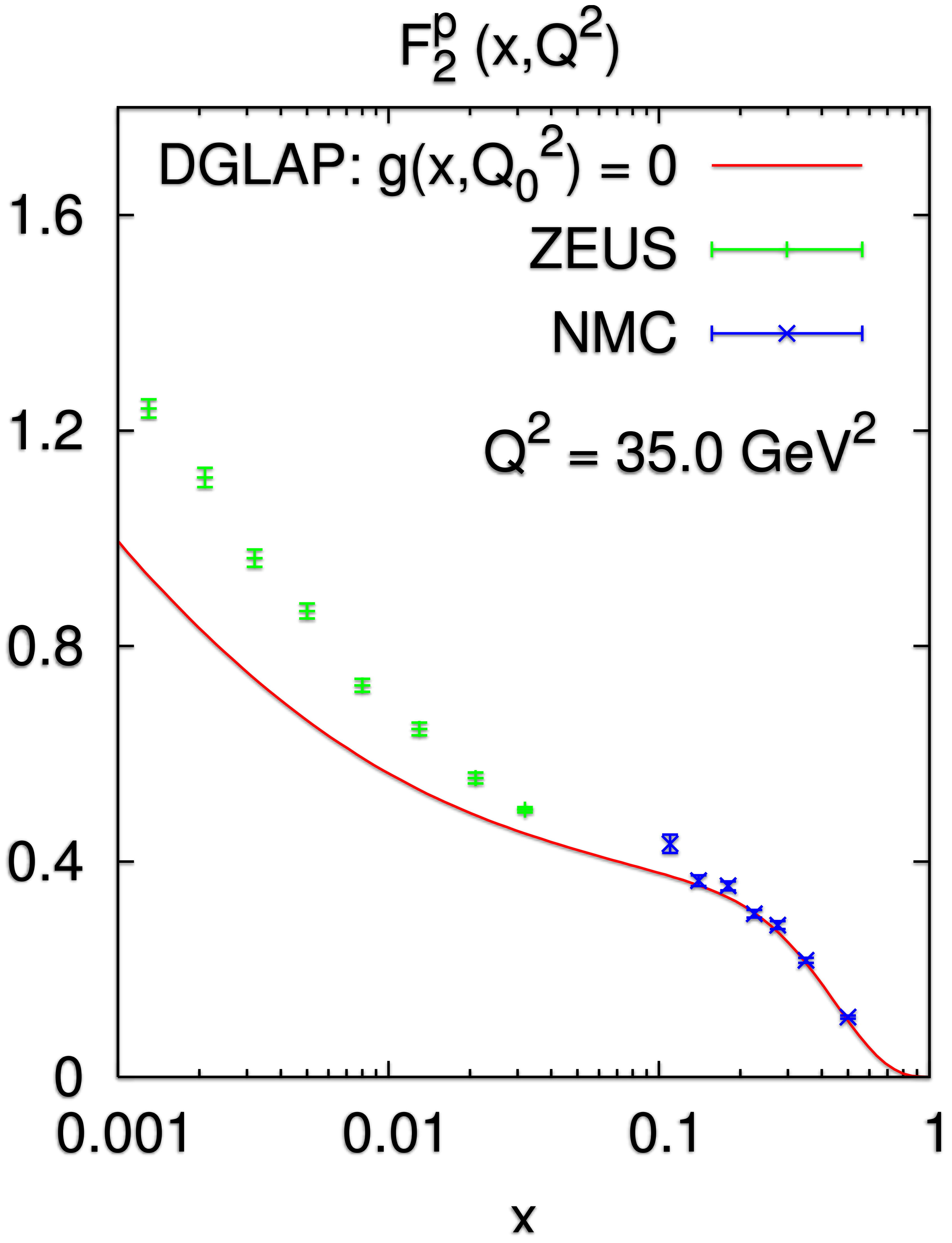

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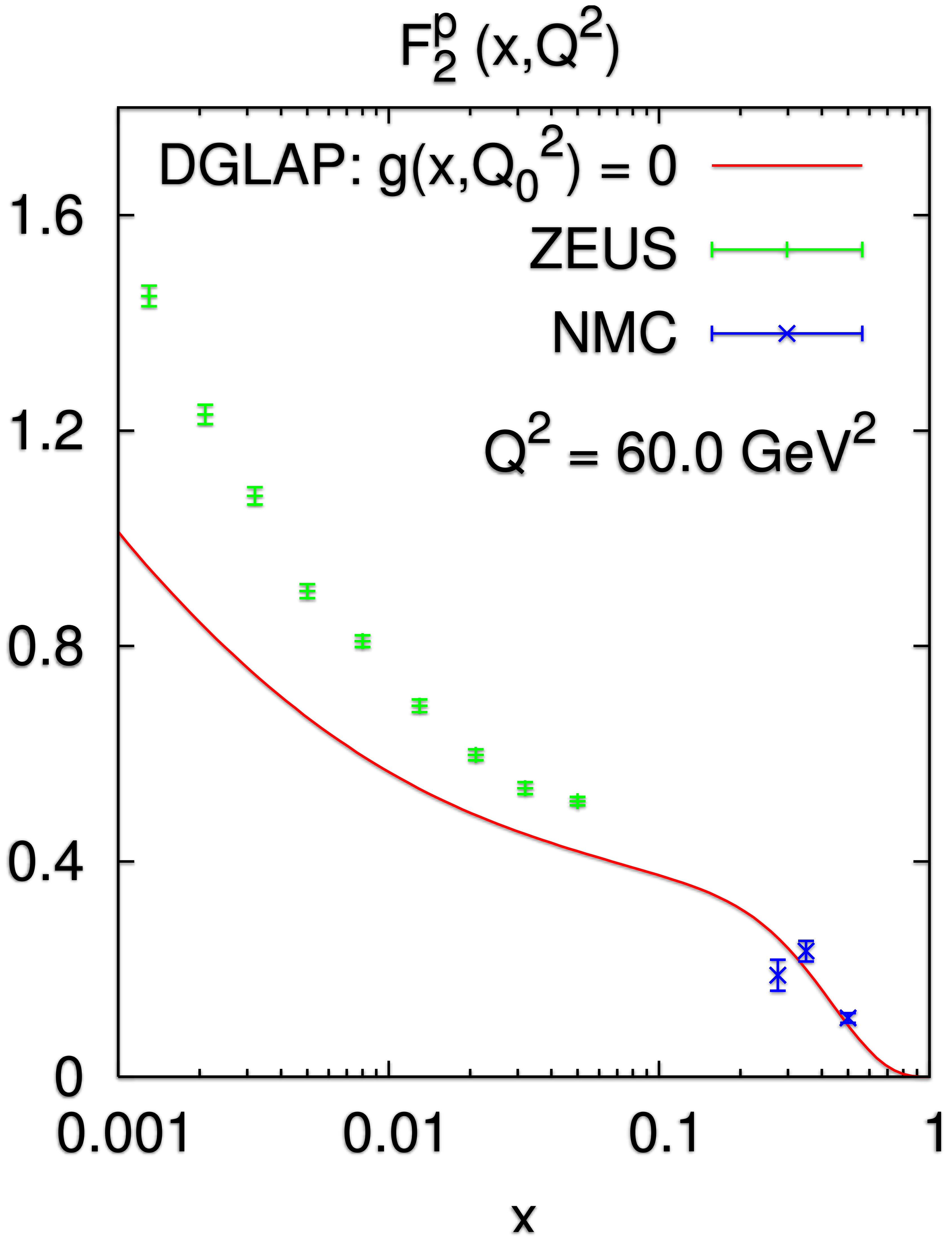

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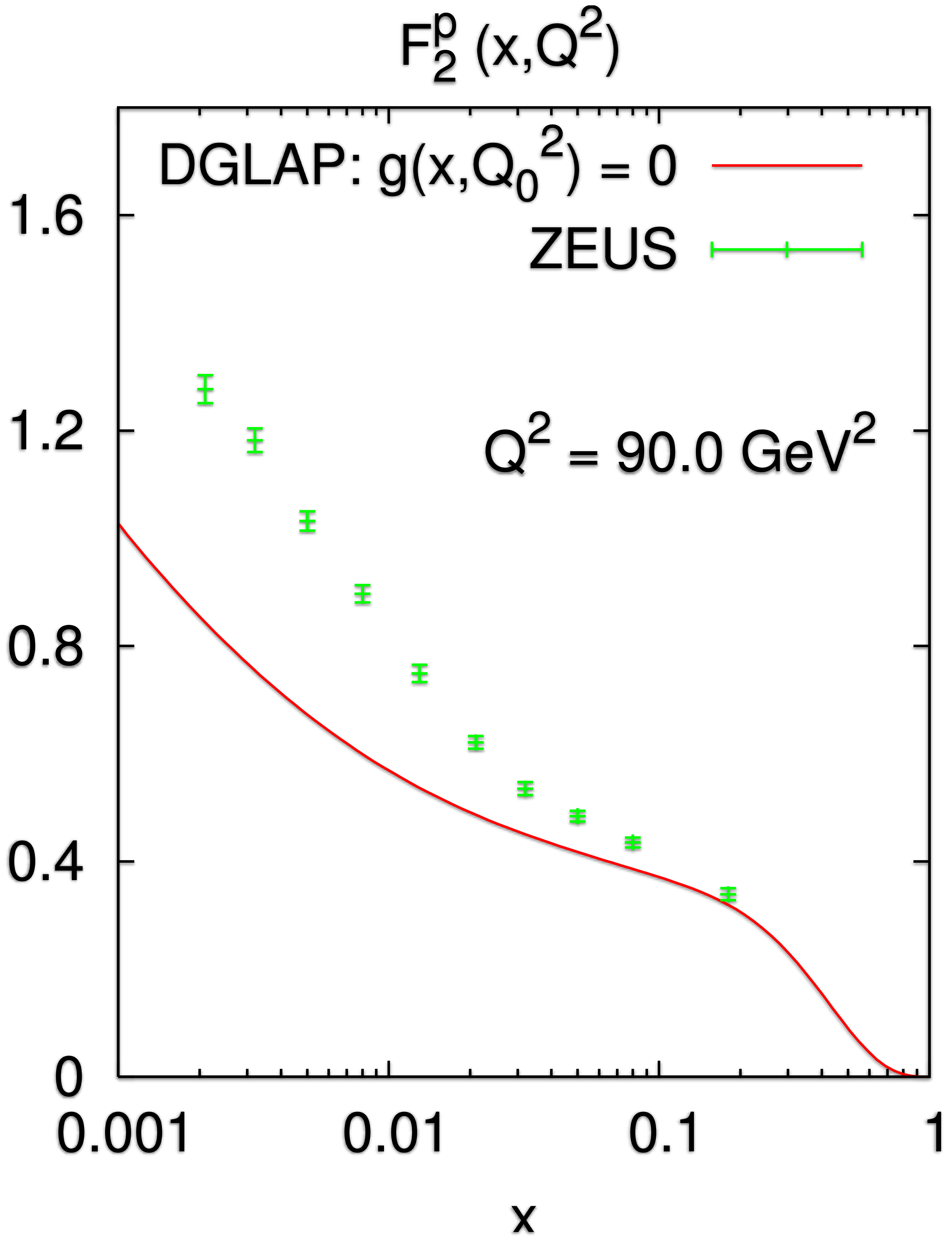

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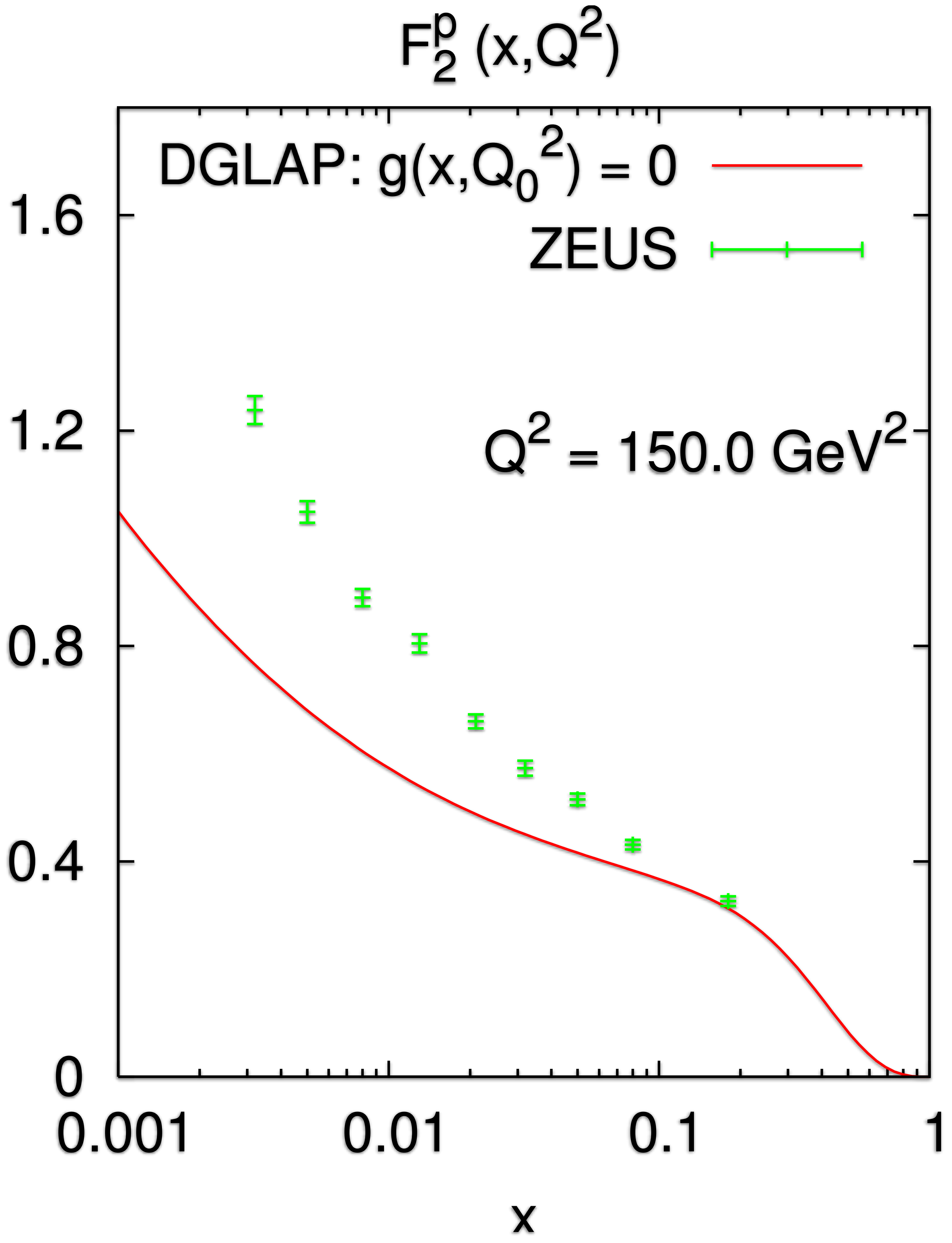

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COMPLETE FAILURE
to reproduce data evolution

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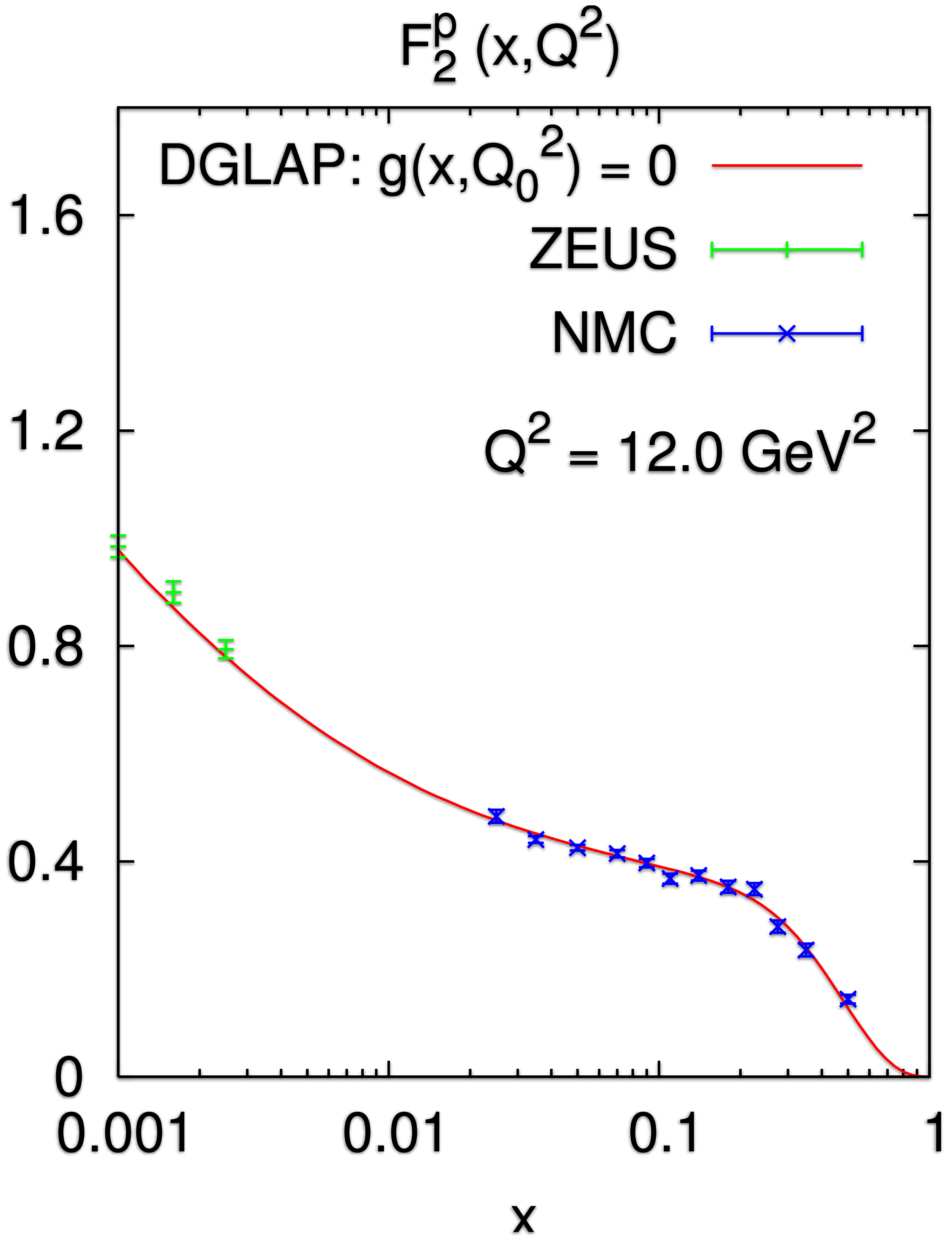


If gluon $\neq 0$, splitting

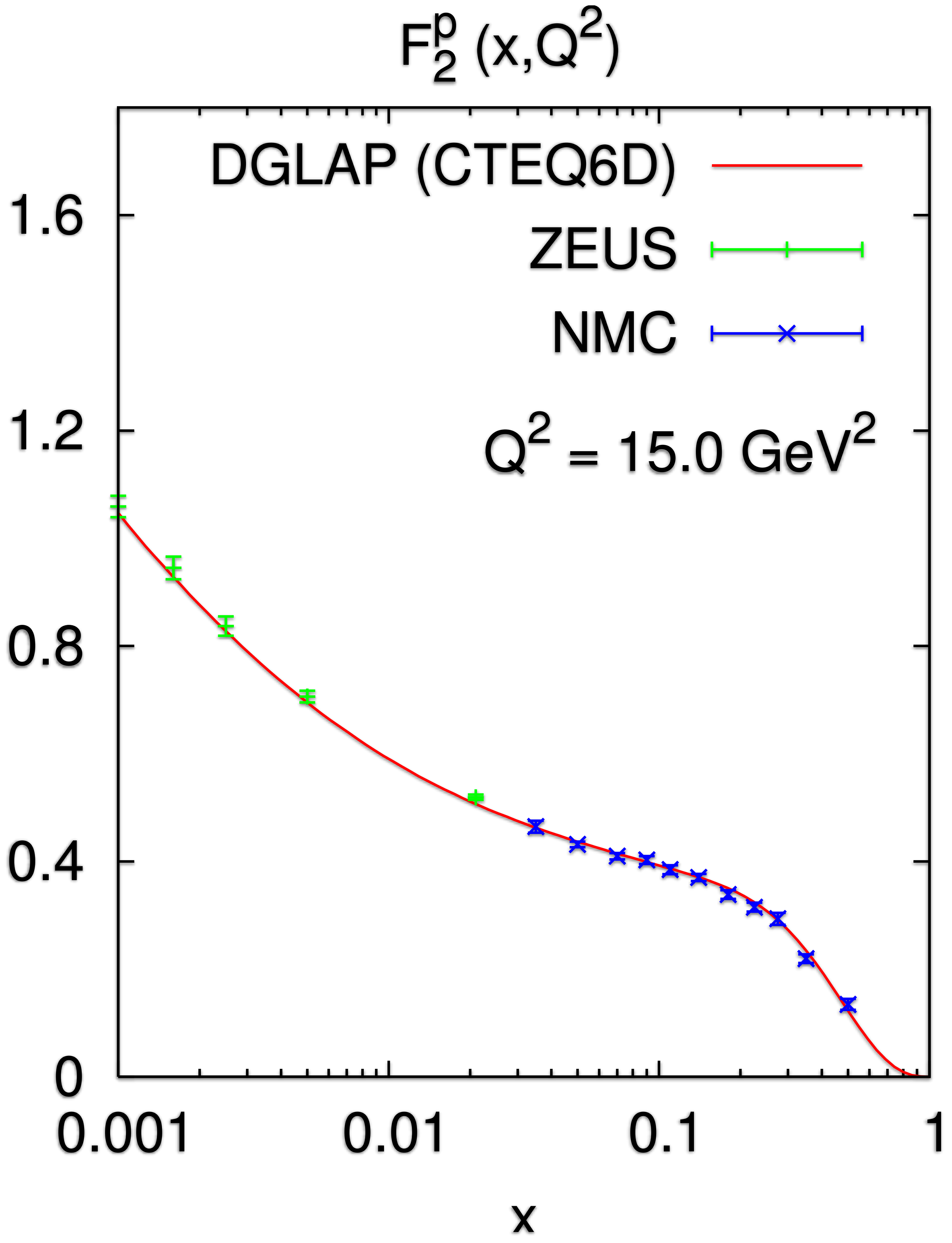
$$g \to q\bar{q}$$

generates extra quarks at large Q2 ➠ faster rise of F2

Global PDF fits (CT, MMHT, NNPDF, etc.) choose gluon distribution that leads to the correct Q2 evolution.

...slide borrowed from Gavin Salam

Sum rules & indirect (gluon) PDF determination



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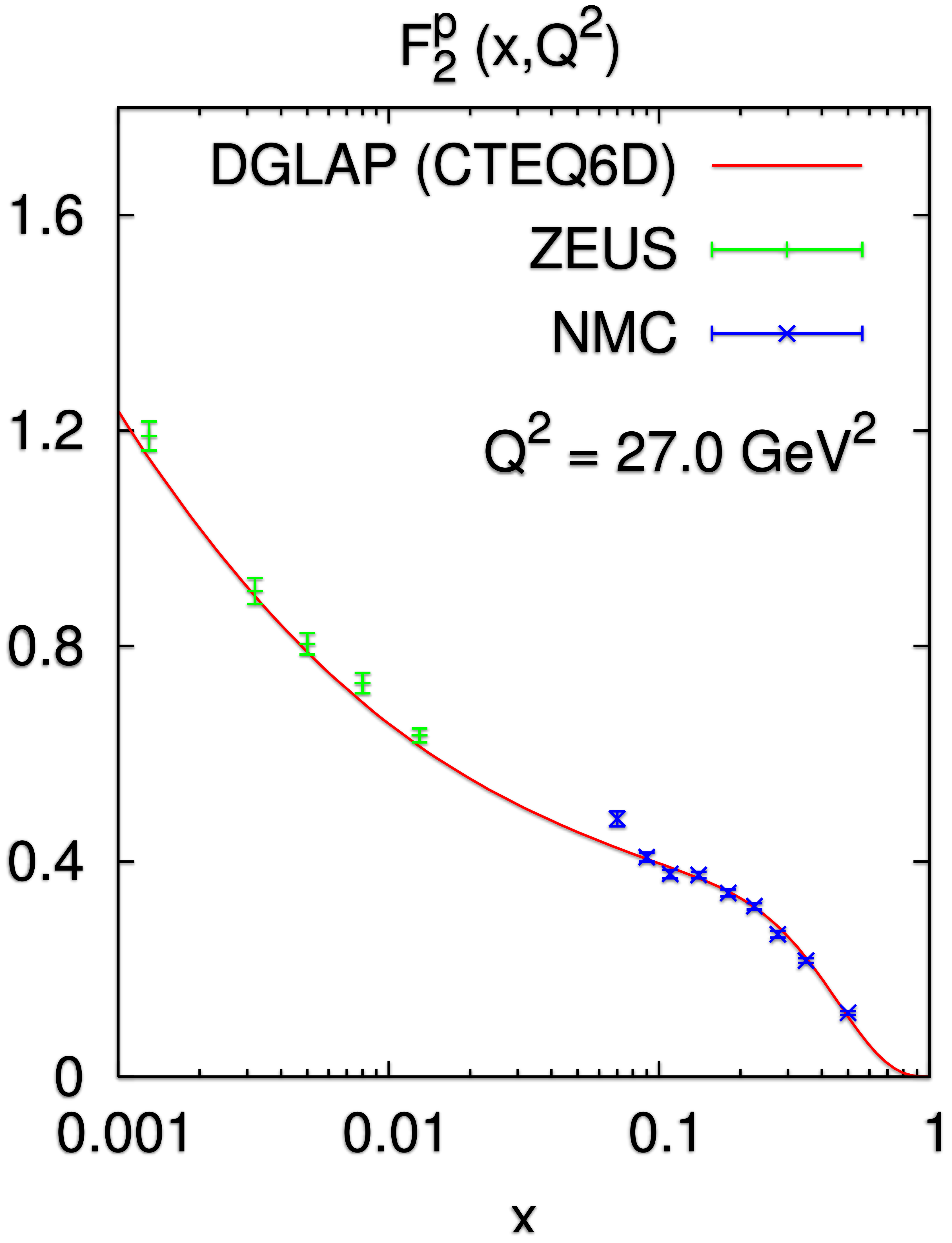

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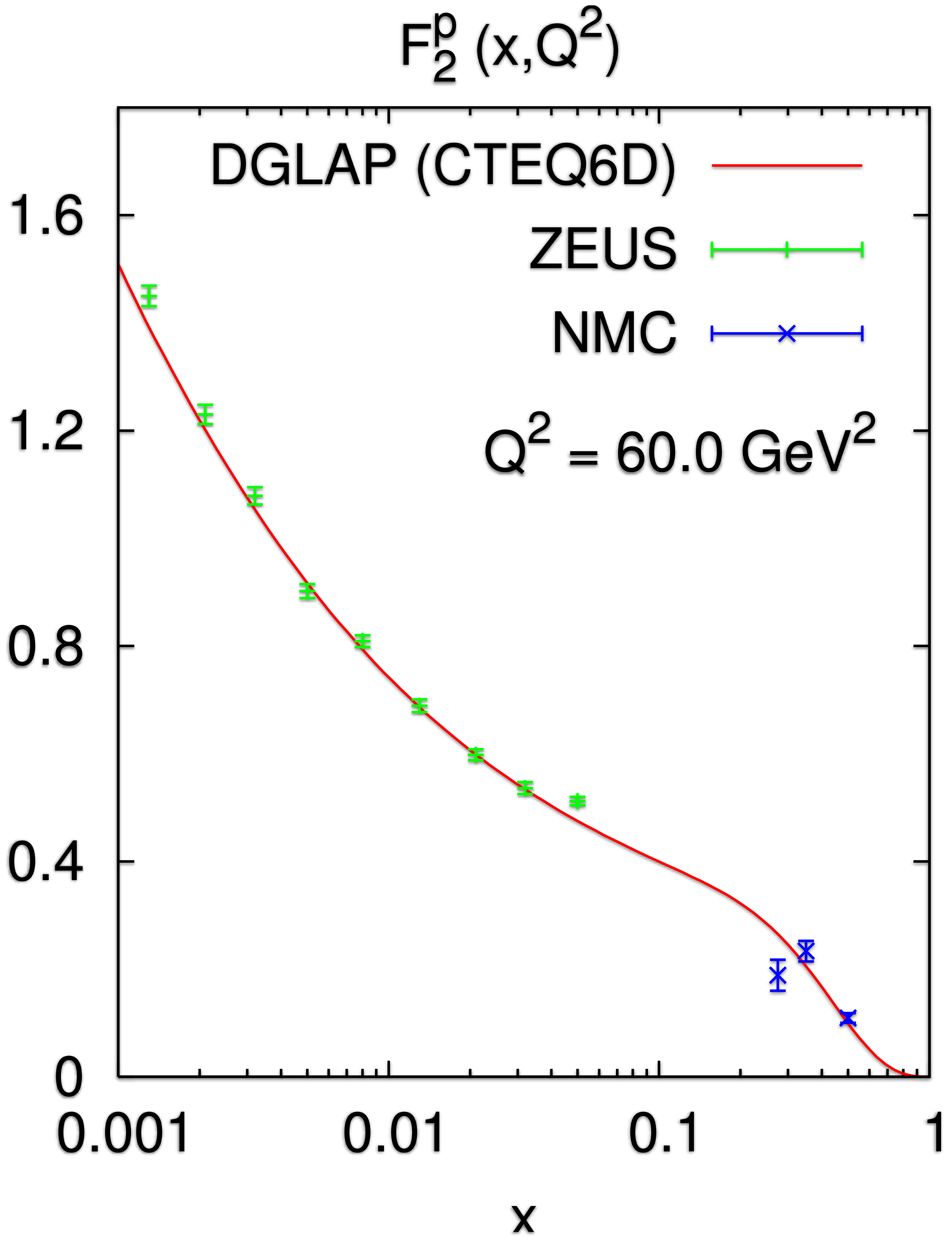

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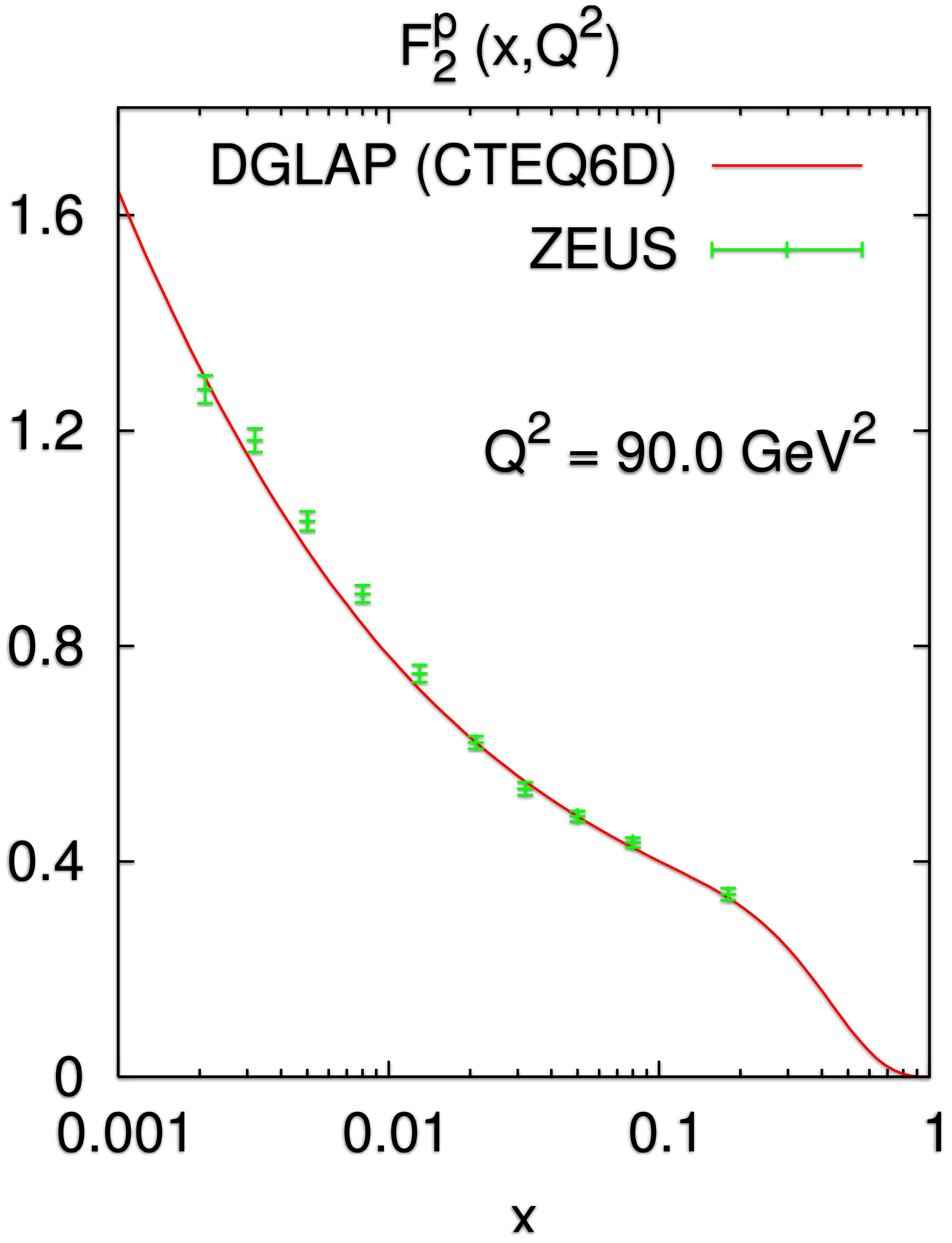

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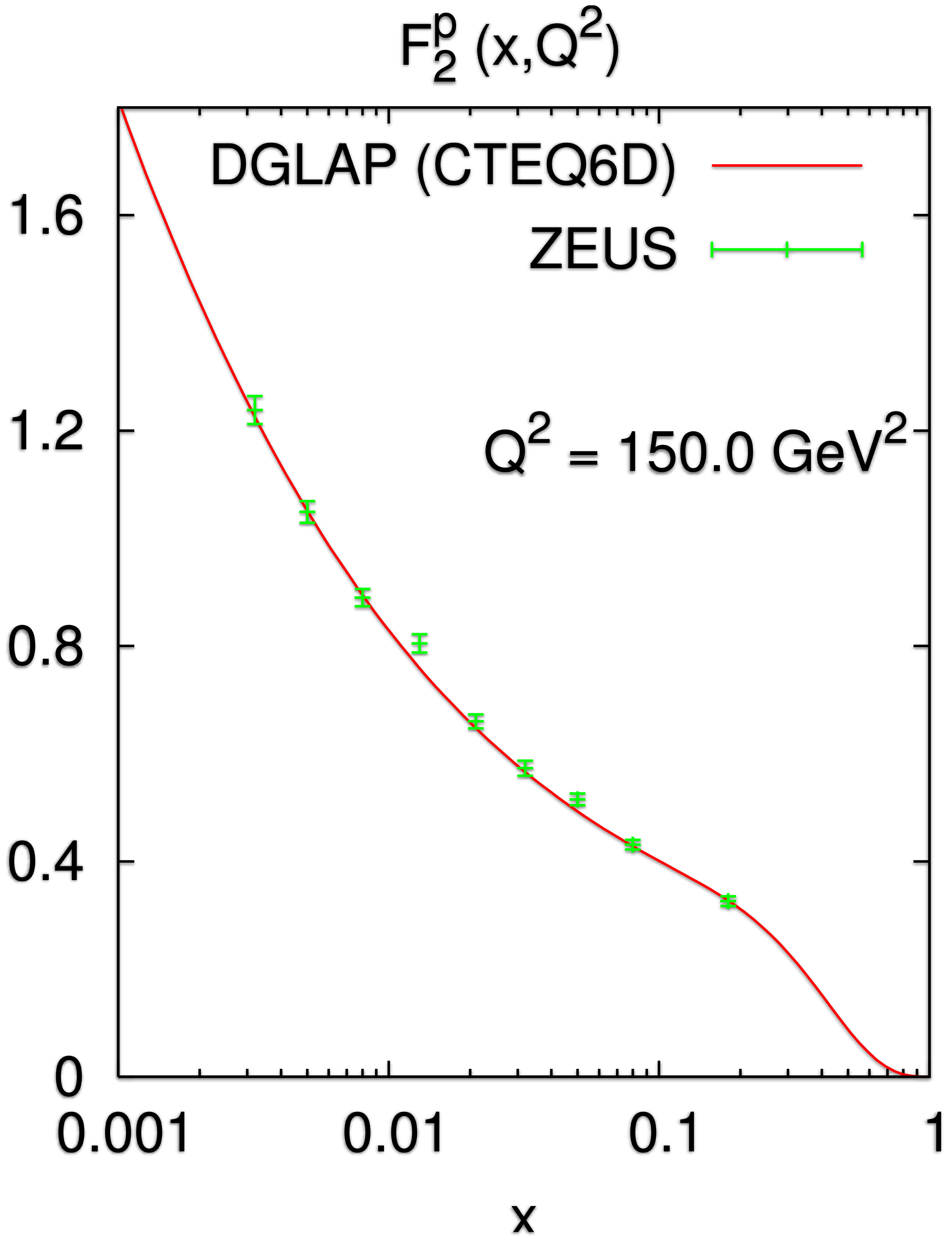

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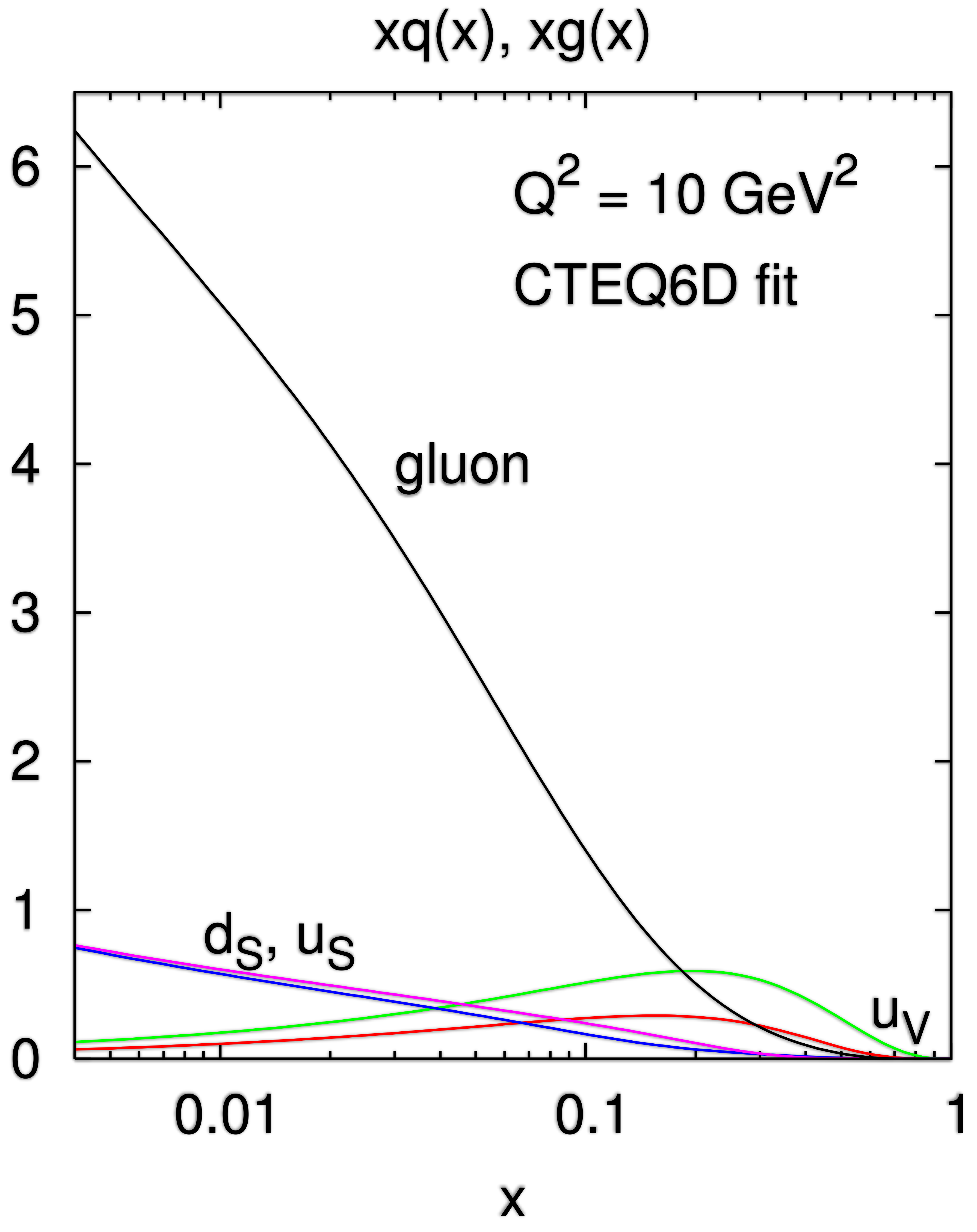
SUCCESS

Sum rules & indirect (gluon) PDF determination



Resulting gluon distribution is **HUGE!**

Carries 47% of proton's momentum
(at scale of 100 GeV)

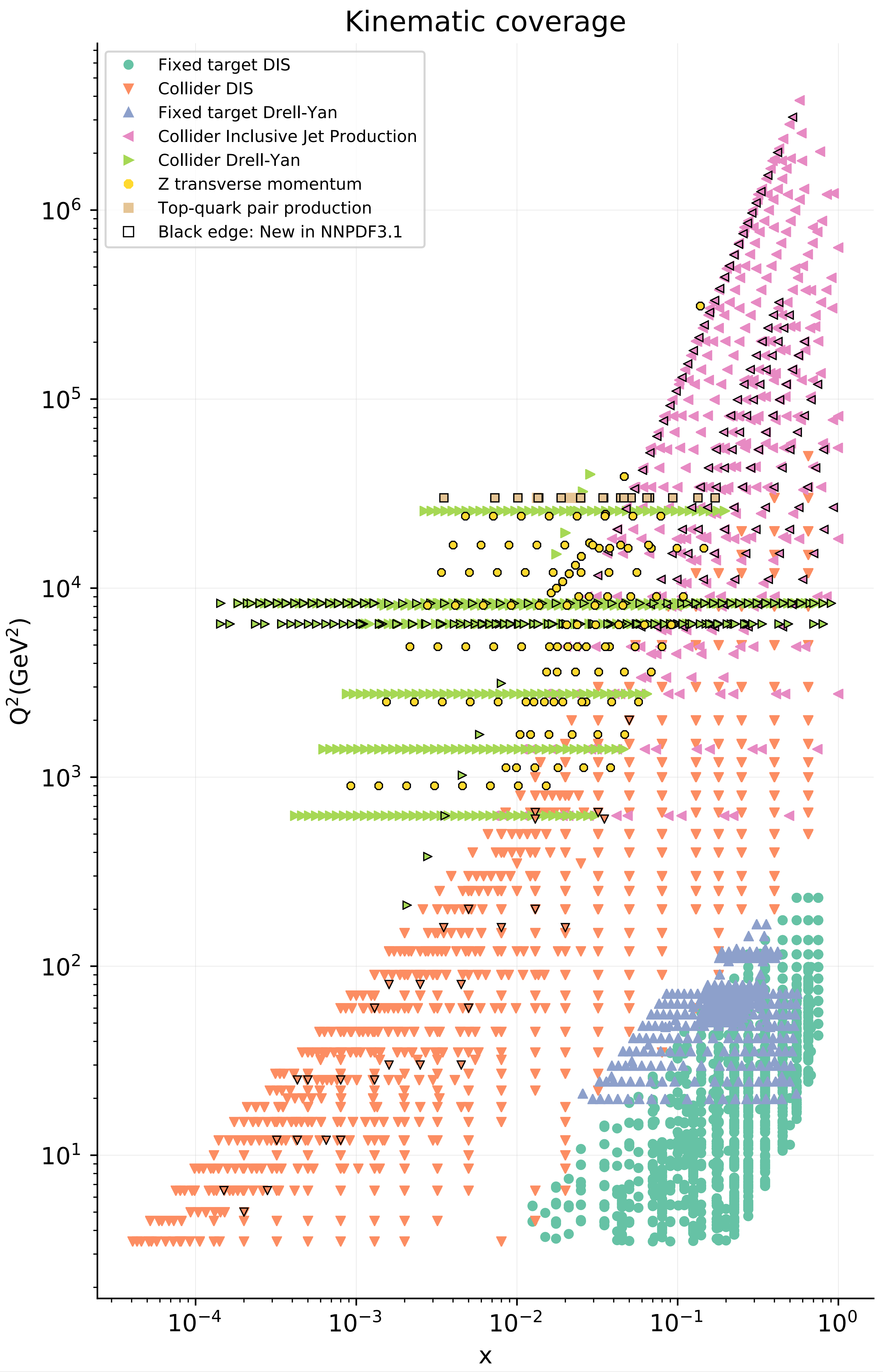
Crucial in order to satisfy momentum sum rule.

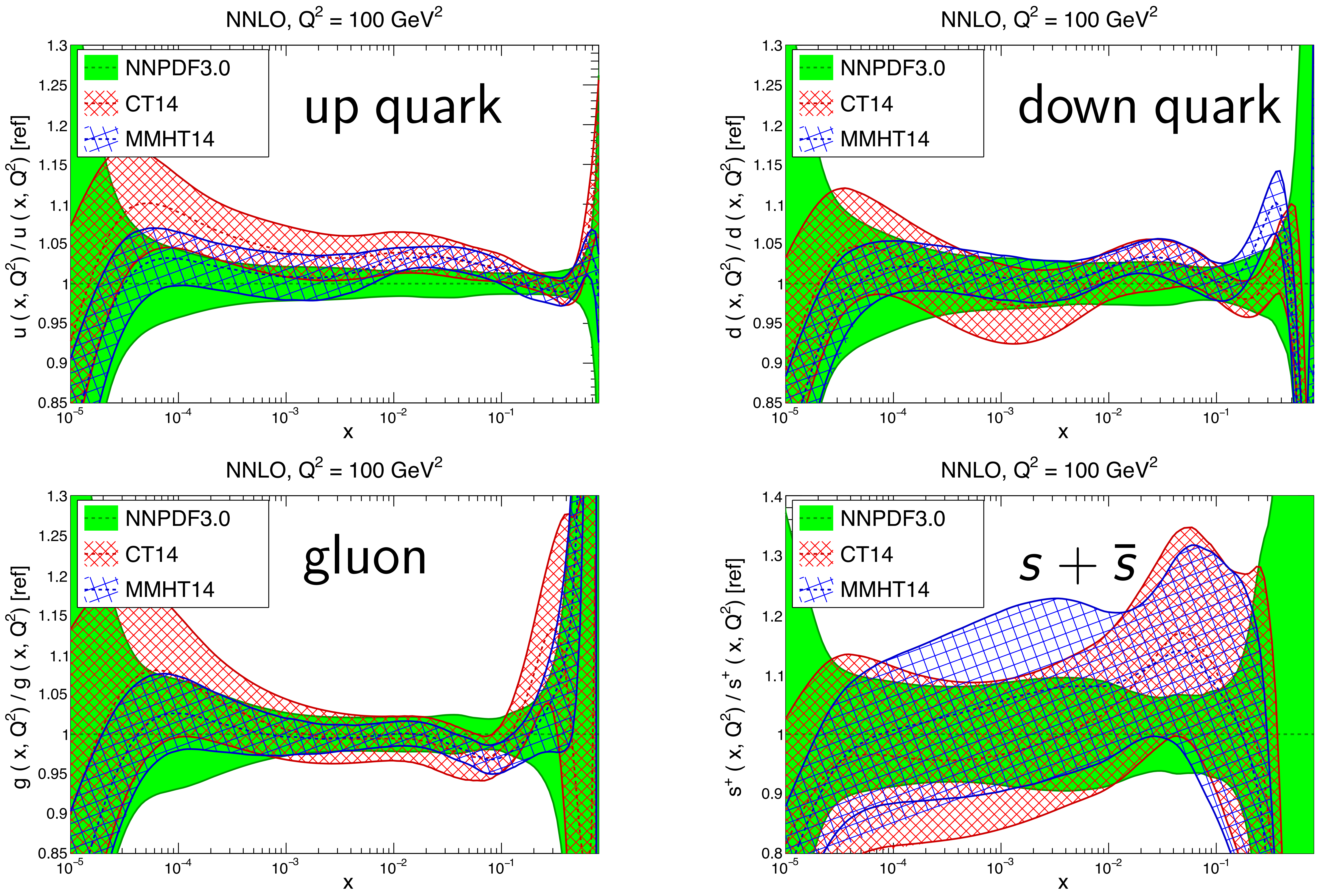
Large value of gluon has big impact on phenomenology

...slide borrowed from Gavin Salam

NNPDF3.1 dataset

PDF fits today





Questions?

Partonic Cross Section

$$\sigma_{\mathrm{had}} = \sum_{ij} \int \mathrm{d}x_1\, \mathrm{d}x_2\, f_i(x_1, \mu_\mathrm{F})\, f_j(x_2, \mu_\mathrm{F}) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}(\Lambda^2/Q^2)$$

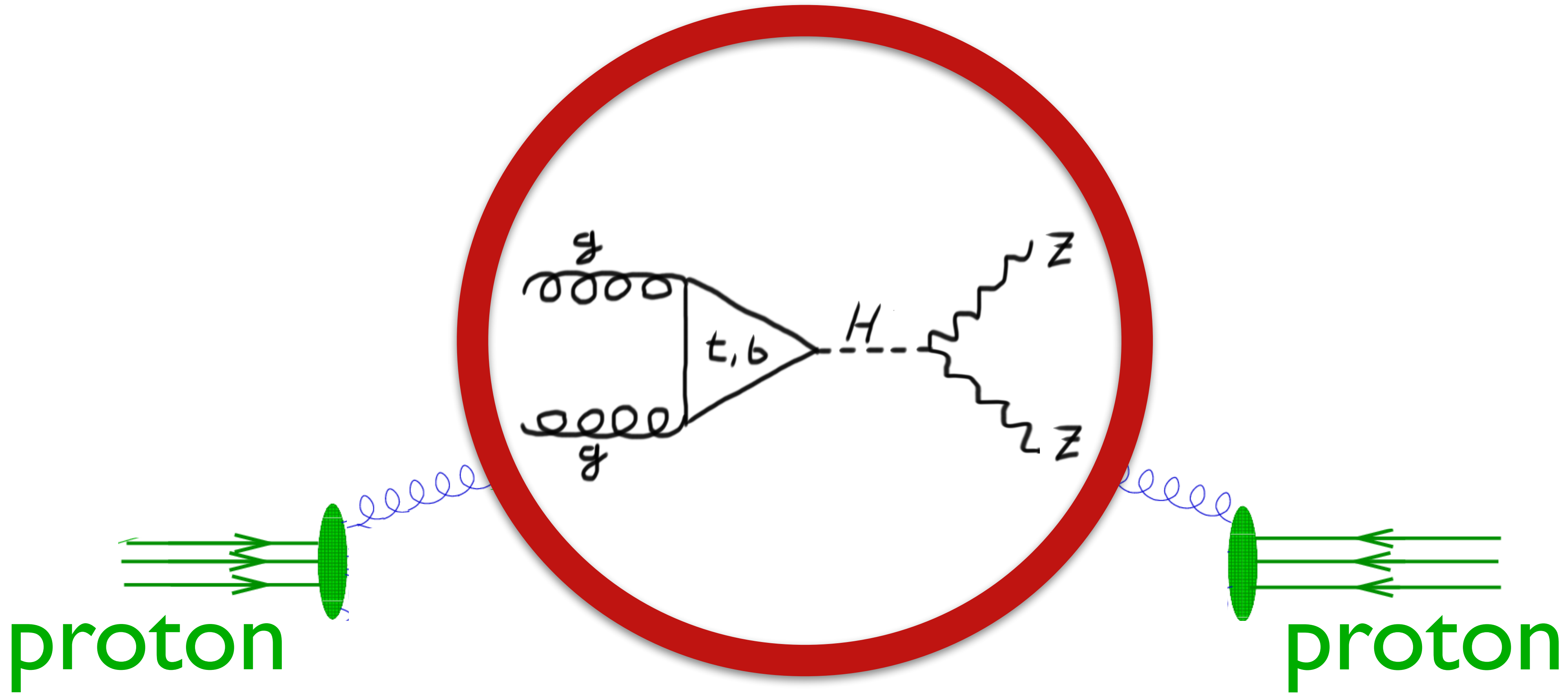
$$\sigma_{ij} \sim \underbrace{\underbrace{\sigma_{\mathrm{LO}} \cdot (1 + \alpha}_{\mathrm{NLO}} + \alpha^2}_{\mathrm{NNLO}} + \ldots)$$

Uncertainties: (α ~ 0.118)

LO ~ $\mathcal{O}$(100%)

NLO ~ $\mathcal{O}$(10%)

NNLO ~ $\mathcal{O}$(1%)



Hard Process

Partonic Cross Section

$$\sigma_{\mathrm{had}} = \sum_{ij} \int \mathrm{d}x_1\, \mathrm{d}x_2\, f_i(x_1, \mu_{\mathrm{F}})\, f_j(x_2, \mu_{\mathrm{F}}) \times \boxed{\sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F)} + \mathcal{O}(\Lambda^2/Q^2)$$

$$\sigma_{ij} = \frac{1}{2s} \underbrace{\int \left[\prod_{i=1}^{n} \frac{\mathrm{d}^3\vec{q}_i}{(2\pi)^3 2E_i}\right] \left[(2\pi)^4 \delta^4 \left(\sum_{i=1}^{n} q_i^\mu - (p_1 + p_2)^\mu\right)\right]} \overline{|\mathcal{M}_{ij}(p_1, p_2, q_i)|^2}$$

[flux factor] [phase-space integral - Φ_n] [squared matrix element]

Partonic Cross Section

$$\sigma_{\text{had}} = \sum_{ij} \int dx_1\, dx_2\, f_i(x_1, \mu_F)\, f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}(\Lambda^2/Q^2)$$

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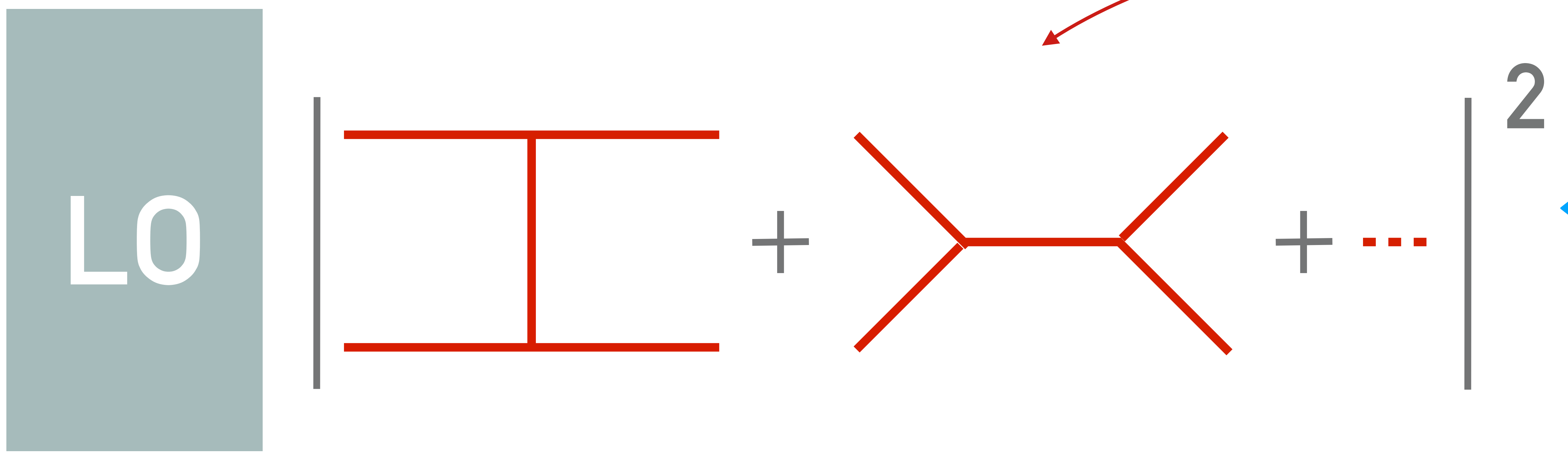
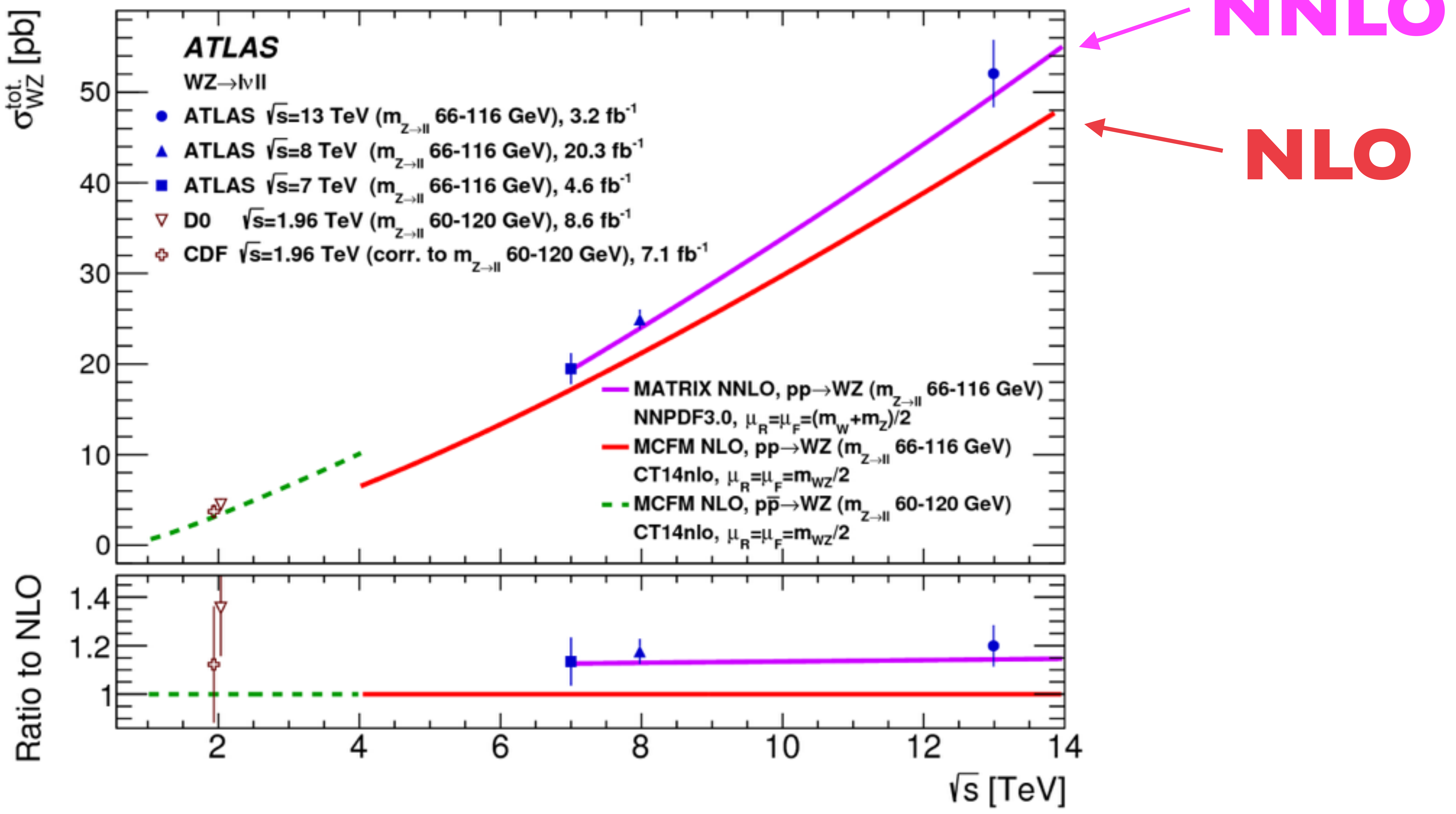


to illustrate the concepts, we don't care what the particles are — just draw lines

Importance of QCD corrections
(example WZ)

[Grazzini, Kallweit, Rathlev, MW '16]



NNLO crucial for accurate description of data

Higher-order corrections

$$\mathrm{d}\sigma = \underbrace{\underbrace{\underbrace{\mathrm{d}\sigma^{(0)}}_{\mathrm{LO}} + \alpha\,\mathrm{d}\sigma^{(1)}}_{\mathrm{NLO}} + \alpha^2\,\mathrm{d}\sigma^{(2)}}_{\mathrm{NNLO}} + \mathcal{O}(\alpha^3)$$

Two (complicated) main problems to solve:

(0. phase-space integration -- easy if finite, using numerical MC methods)

1. evaluate (loop) amplitudes

(ingredients of calculation, difficulty ~ e^{loops}, understood at 1-loop, various 2-loop results, very few 3-loop results)

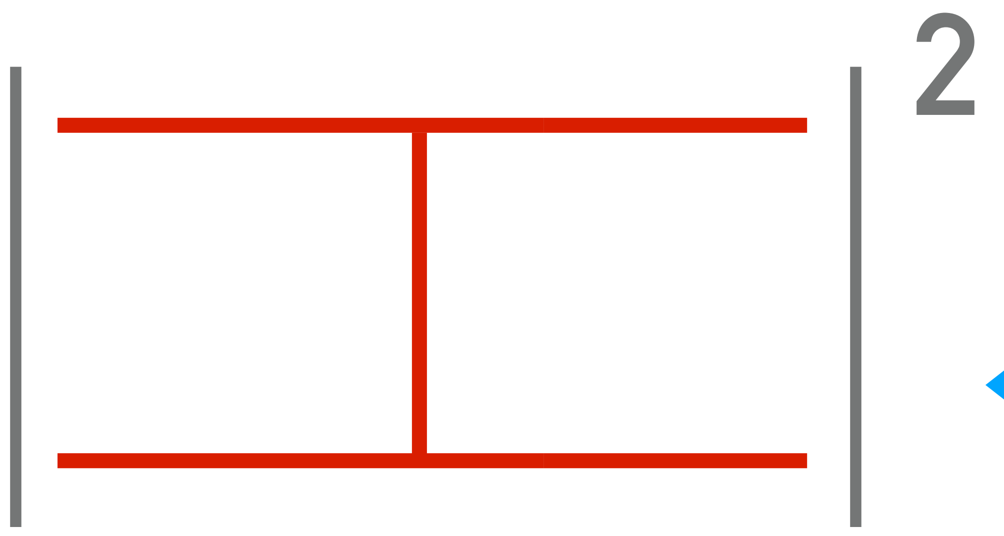
2. combination of different (singular) ingredients

(final cross section prediction, difficulty ~ e^{order}, understood up to NNLO, very few N3LO results)

INGREDIENTS FOR A CALCULATION (generic 2→2 process)

LO

Tree
2→2



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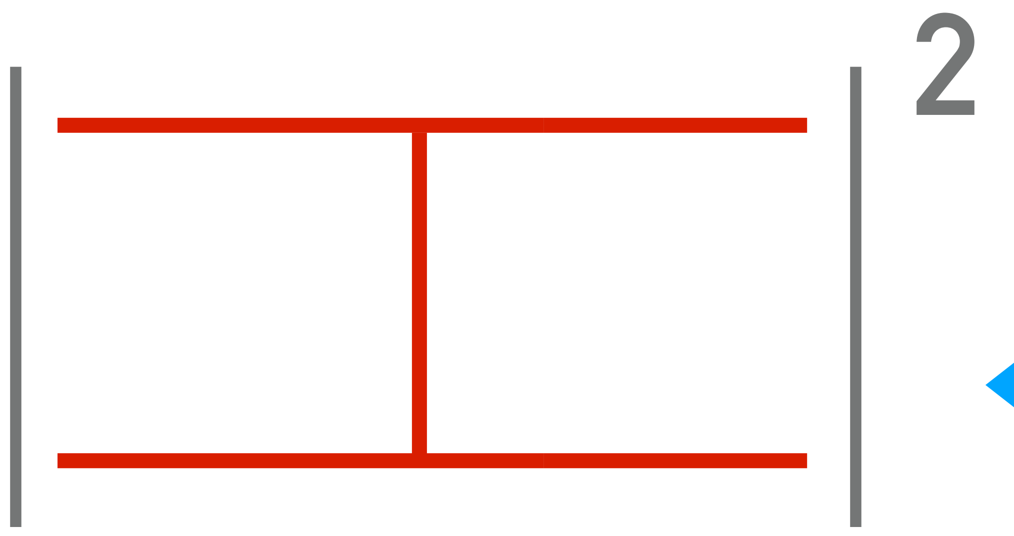

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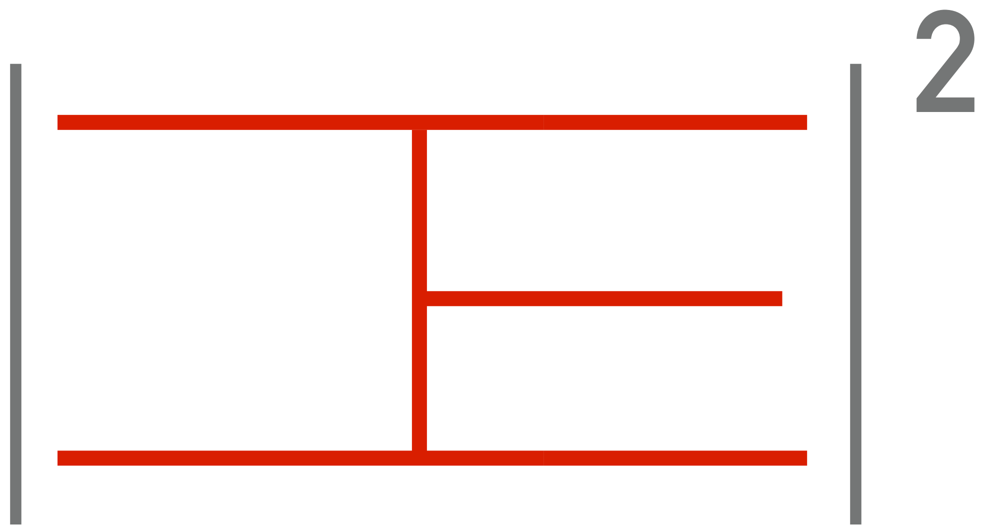
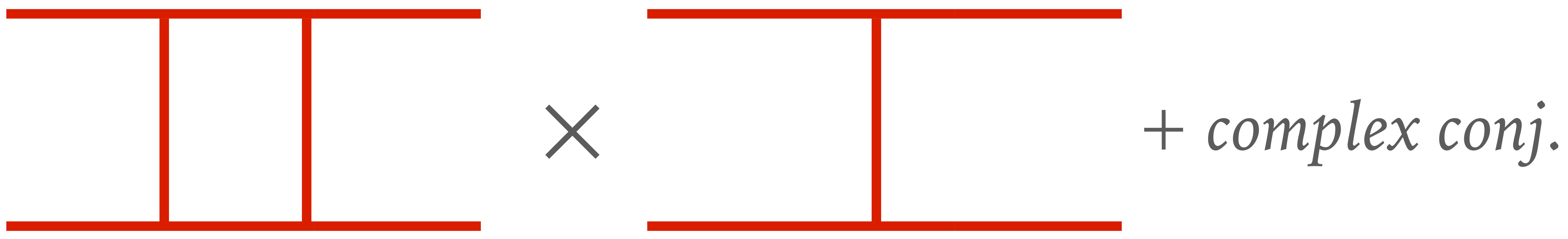
NLO

Tree
2→3



1-loop
2→2



...slide borrowed from Gavin Salam

INGREDIENTS FOR A CALCULATION (generic 2→2 process)

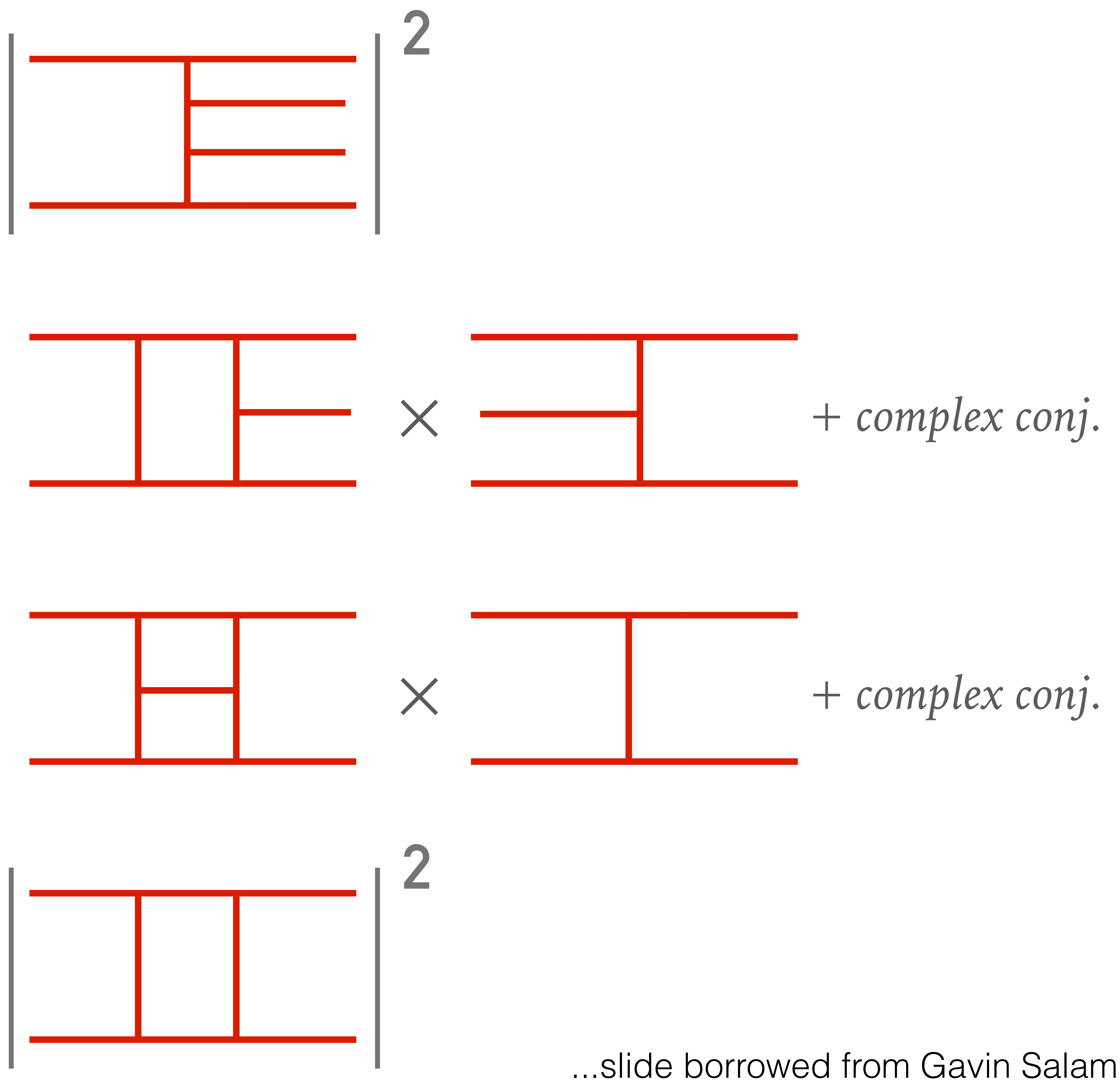
NNLO

Tree
2→4

1-loop
2→3

2-loop
2→2

1-loop
2→2



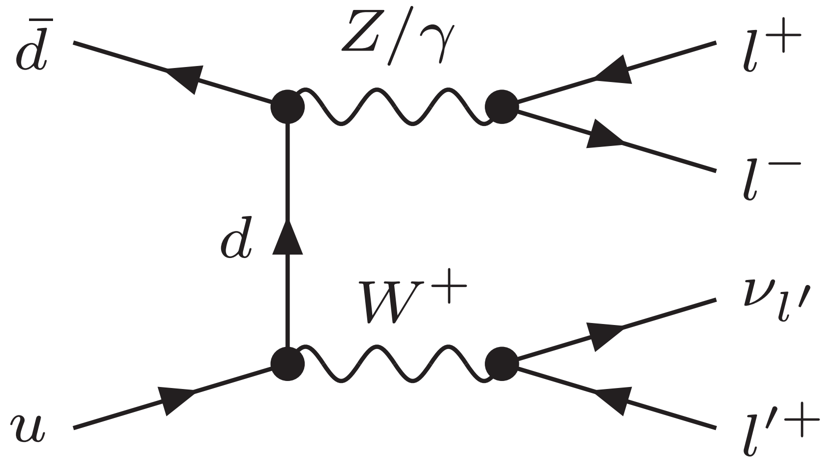
...slide borrowed from Gavin Salam

How to do a NLO calculation

NLO Calculation: The Issue

$$\sigma_{\text{LO}} = \int_{\Phi_{\text{B}}} \mathrm{d}\sigma^{\text{B}}$$

LO (pp→WZ)

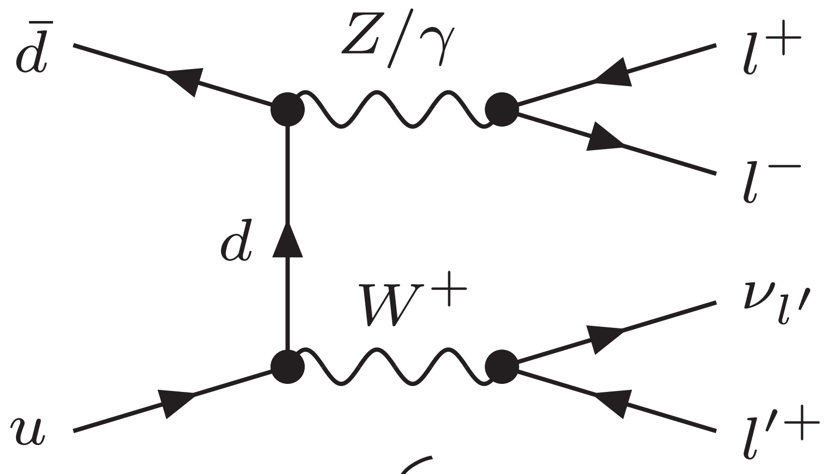


$\rightarrow \mathrm{d}\sigma^{\text{B}}$

NLO Calculation: The Issue

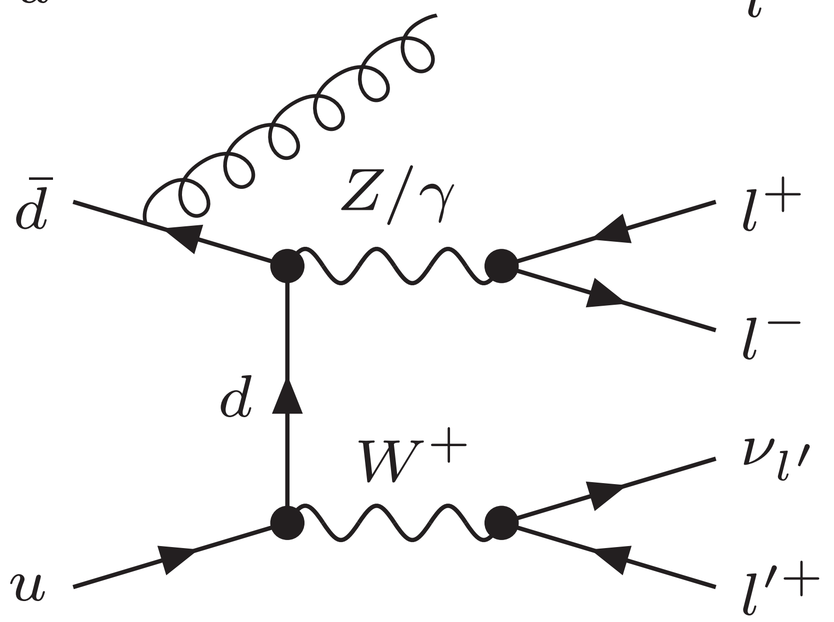
$$\sigma_{\mathrm{NLO}} = \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B}+1}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{V}}$$

LO (pp→WZ)

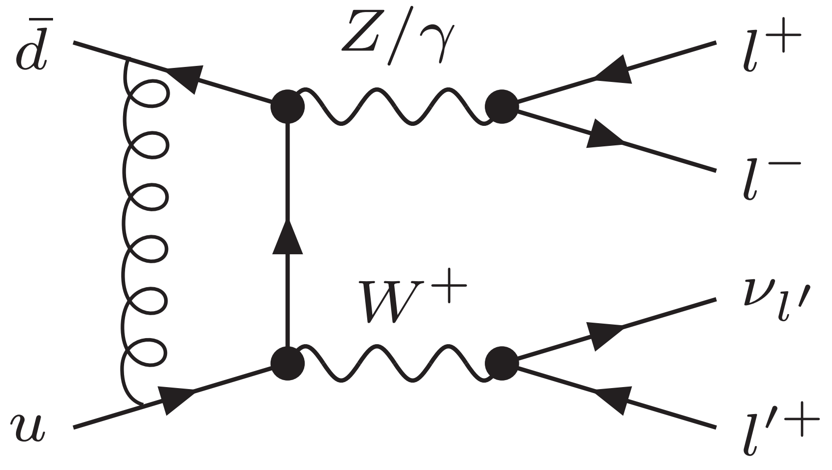


$\rightarrow \mathrm{d}\sigma^{\mathrm{B}}$

NLO (pp→WZ)



$\rightarrow \mathrm{d}\sigma^{\mathrm{R}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{V}}$

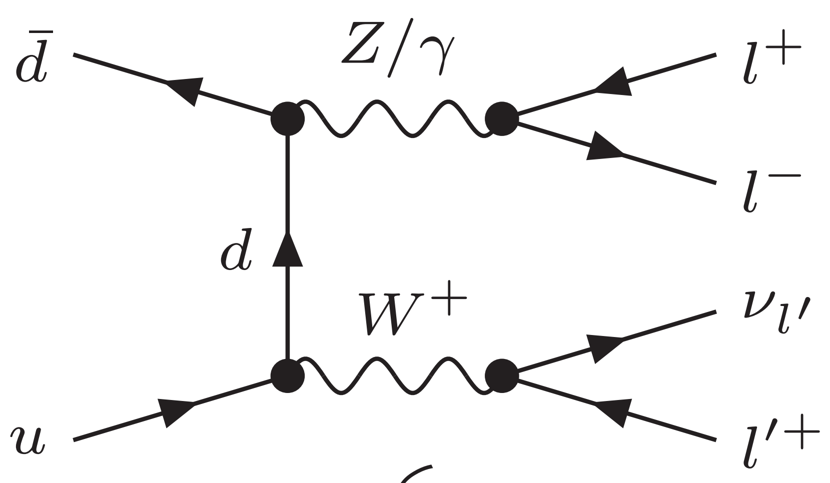
NLO Calculation: The Issue

$$\sigma_{\text{NLO}} = \int_{\Phi_{\text{B}}} \mathrm{d}\sigma^{\text{B}} + \int_{\Phi_{\text{B}+1}} \mathrm{d}\sigma^{\text{R}} + \int_{\Phi_{\text{B}}} \mathrm{d}\sigma^{\text{V}}$$

1-loop virtual amplitude:

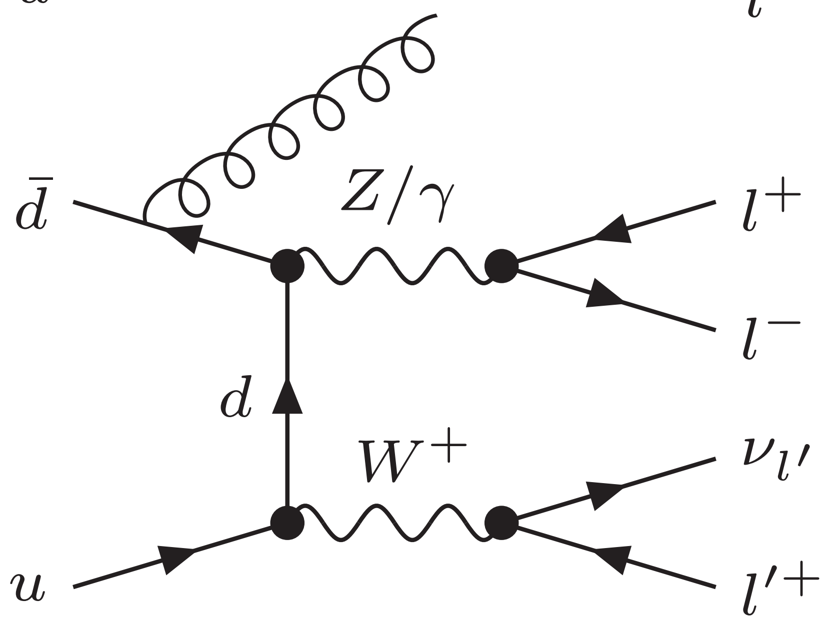
$$\mathrm{d}\sigma^{\text{V}} \sim \left(\frac{A_0}{\epsilon^2} + \frac{B_0}{\epsilon} + C_0 \right)$$

LO (pp→WZ)

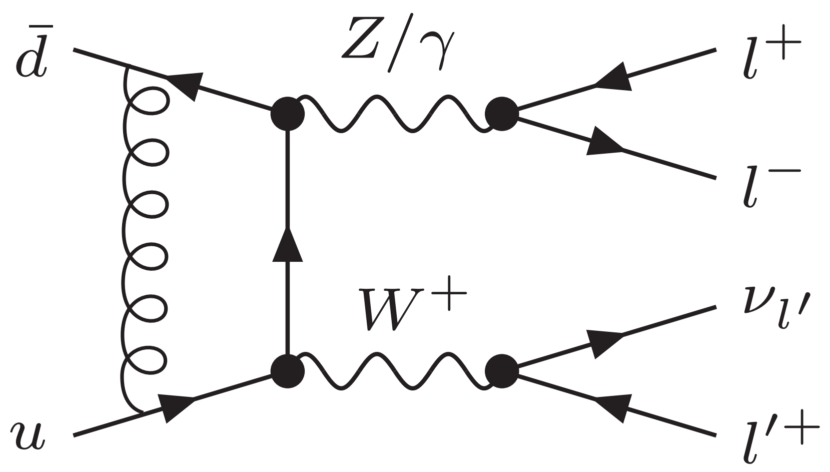


$\rightarrow \mathrm{d}\sigma^{\text{B}}$

NLO (pp→WZ)



$\rightarrow \mathrm{d}\sigma^{\text{R}}$



$\rightarrow \mathrm{d}\sigma^{\text{V}}$

NLO Calculation: The Issue

$$\sigma_{\mathrm{NLO}} = \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B}+1}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{V}}$$

real-emission amplitude finite, but integrand (propagators) become singular during phase-space integration.
After phase-space integration:

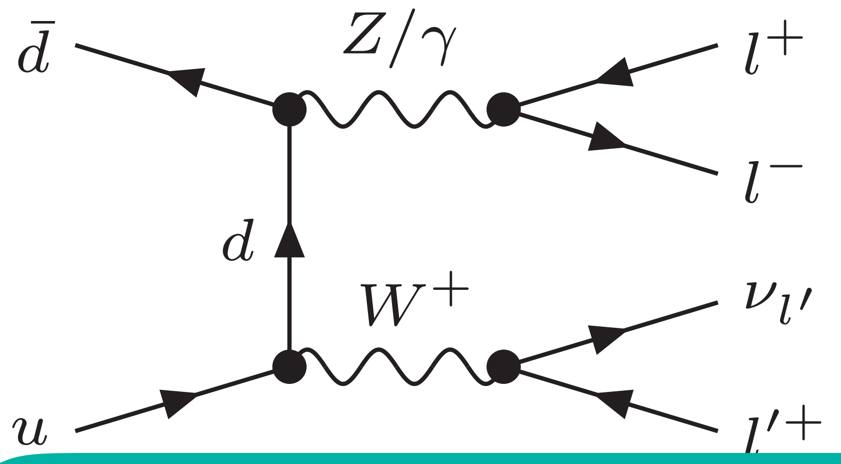
$$\int_{\Phi_{\mathrm{B+1}}} \mathrm{d}\sigma^{\mathrm{R}} \sim \left(-\frac{A}{\epsilon^2} + -\frac{B}{\epsilon} + D\right)$$

1-loop virtual amplitude:

$$\mathrm{d}\sigma^{\mathrm{V}} \sim \left(\frac{A_0}{\epsilon^2} + \frac{B_0}{\epsilon} + C_0\right)$$

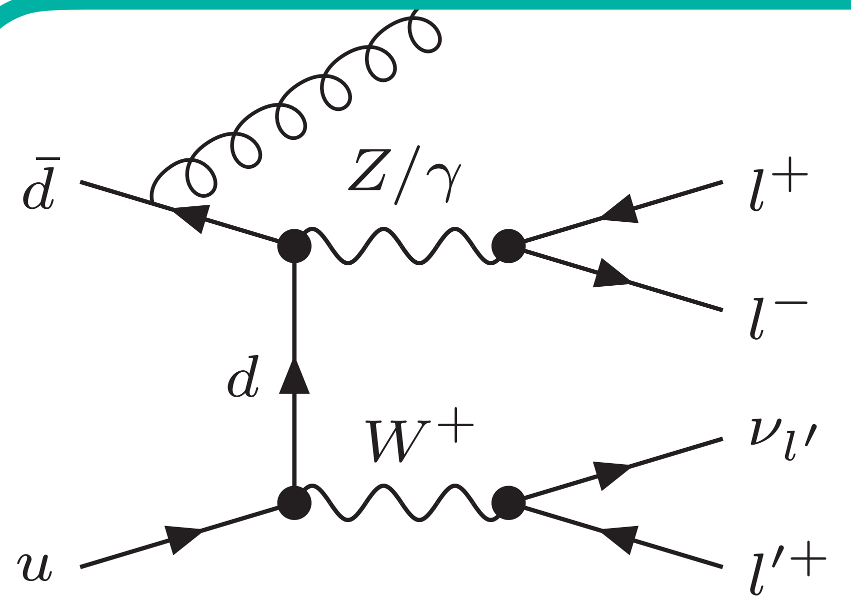
$$\int_{\Phi_B} \mathrm{d}\sigma^{\mathrm{V}} \sim \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C\right)$$

LO
(pp→WZ)

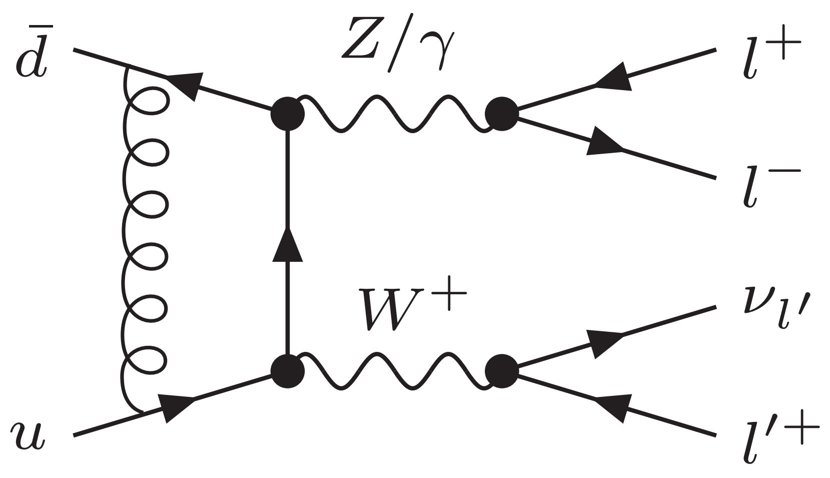


$\rightarrow \mathrm{d}\sigma^{\mathrm{B}}$

NLO
(pp→WZ)



$\rightarrow \mathrm{d}\sigma^{\mathrm{R}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{V}}$

NLO Calculation: The Issue

$$\sigma_{\text{NLO}} = \int_{\Phi_{\text{B}}} \mathrm{d}\sigma^{\text{B}} + \int_{\Phi_{\text{B}+1}} \mathrm{d}\sigma^{\text{R}} + \int_{\Phi_{\text{B}}} \mathrm{d}\sigma^{\text{V}}$$

$= C + D$

sum finite

real-emission amplitude finite, but integrand (propagators) become singular during phase-space integration.
After phase-space integration:

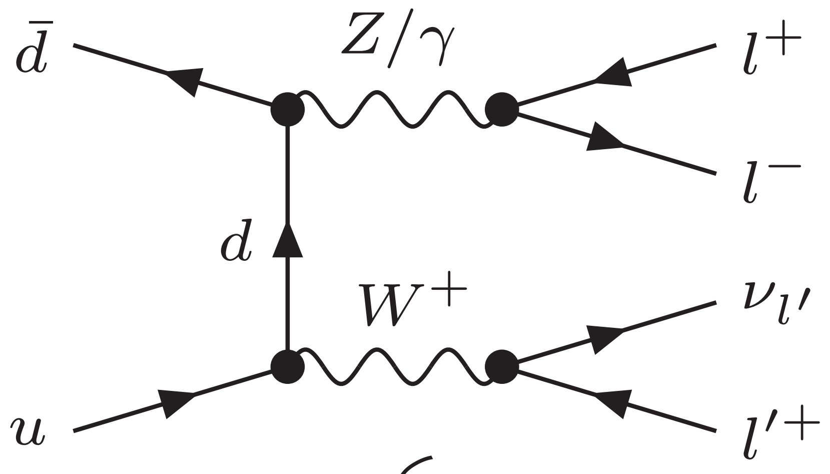
$$\int_{\Phi_{\text{B+1}}} \mathrm{d}\sigma^{\text{R}} \sim \left(-\frac{A}{\epsilon^2} + -\frac{B}{\epsilon} + D \right)$$

1-loop virtual amplitude:

$$\mathrm{d}\sigma^{\text{V}} \sim \left(\frac{A_0}{\epsilon^2} + \frac{B_0}{\epsilon} + C_0 \right)$$

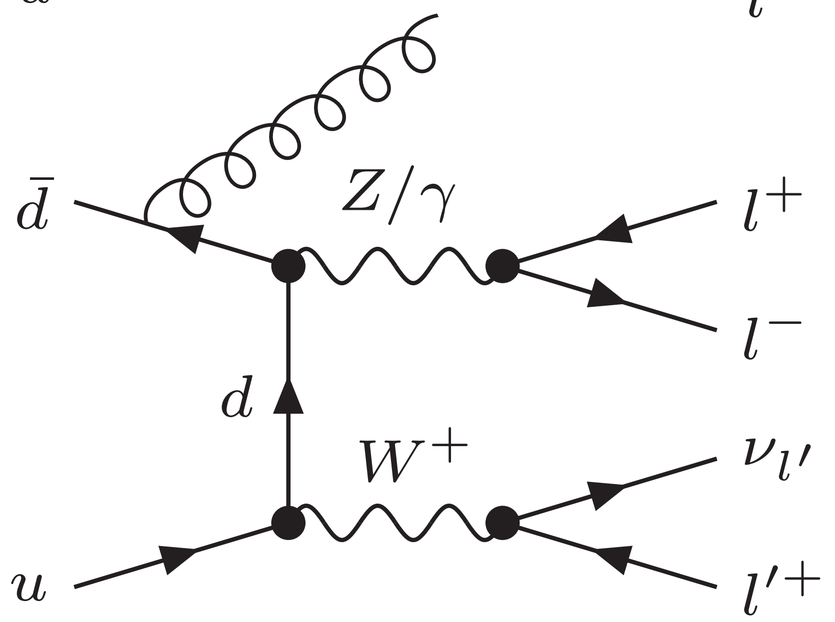
$$\int_{\Phi_B} \mathrm{d}\sigma^{\text{V}} \sim \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C \right)$$

LO (pp→WZ)

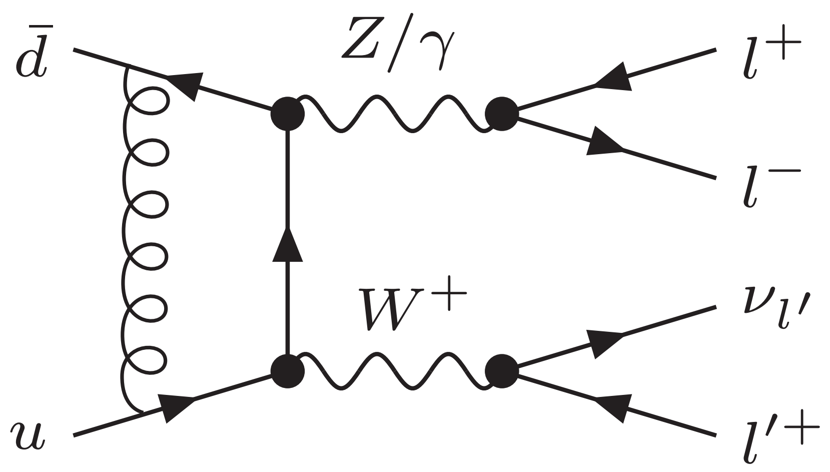


$\rightarrow \mathrm{d}\sigma^{\text{B}}$

NLO (pp→WZ)



$\rightarrow \mathrm{d}\sigma^{\text{R}}$



$\rightarrow \mathrm{d}\sigma^{\text{V}}$

$f(z)$ is some function with finite limit for $z \to 0$

LOCAL SUBTRACTION

$$\sigma = c \cdot f(0) + \int_0^1 dz \left[\frac{f(z)}{z} - \frac{f(0)}{z} \right]$$

virtual & counterterm:
may need (tough) analytic calc^n

real part:
MC integration is finite even without cut

"SLICING"

$$\sigma = \left(c - \ln \frac{1}{\text{cut}} \right) \cdot f(0) + \int_{\text{cut}}^1 dz \frac{f(z)}{z}$$

virtual & counterterm:
get from soft-collinear resummation

real part:
use MC integration (cut has to be small)

NNLO approaches

Sector decomposition
Anastasiou, Melnikov, Petriello; Binoth, Heinrich

Antenna subtraction
Kosower; Gehrmann, Gehrmann-de Ridder, Glover

Stripper
Czakon

Sector-improved residue
Boughezal, Melnikov, Petriello

CoLorFul subtraction
Del Duca, Somogyi, Troscanyi

Projection-to-Born
Cacciari, Dreyer, Karlberg, Salam, Zanderighi

qT subtraction
Catani, Grazzini

N-jettiness subtraction
Boughezal, Focke, Liu, Petriello;
Gaunt Stahlhofen, Tackmann, Walsh

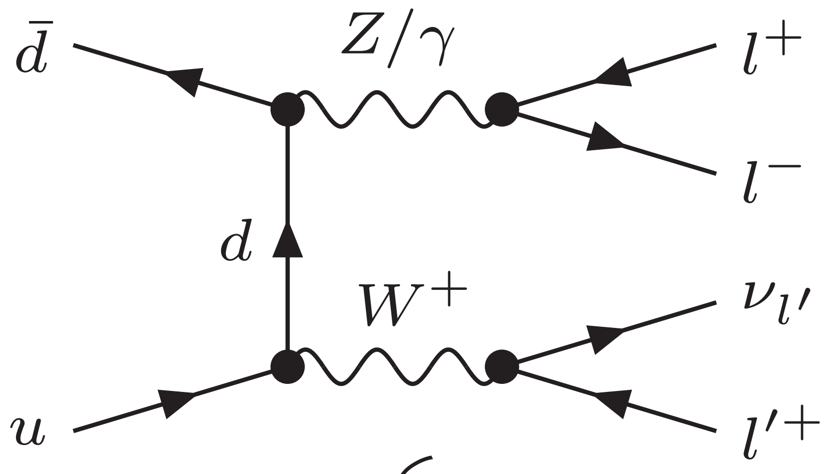
...slide borrowed from Gavin Salam

NLO through subtraction

$$\sigma_{\mathrm{NLO}} = \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \boxed{\int_{\Phi_{\mathrm{B}+1}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{V}}}$$

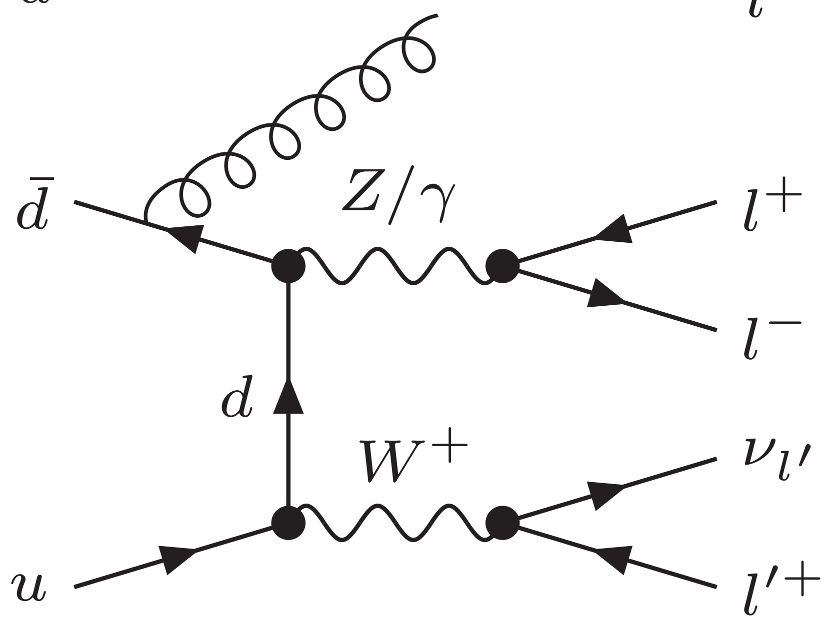
sum finite

LO (pp→WZ)

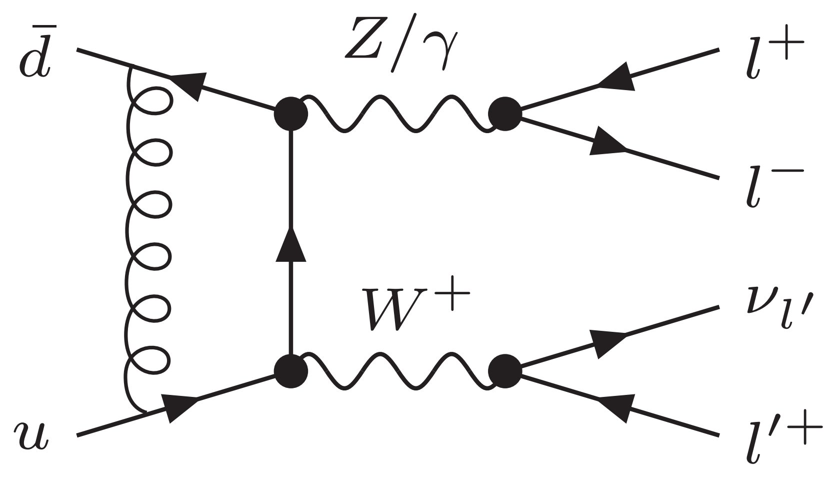


$\rightarrow \mathrm{d}\sigma^{\mathrm{B}}$

NLO (pp→WZ)



$\rightarrow \mathrm{d}\sigma^{\mathrm{R}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{V}}$

NLO through subtraction

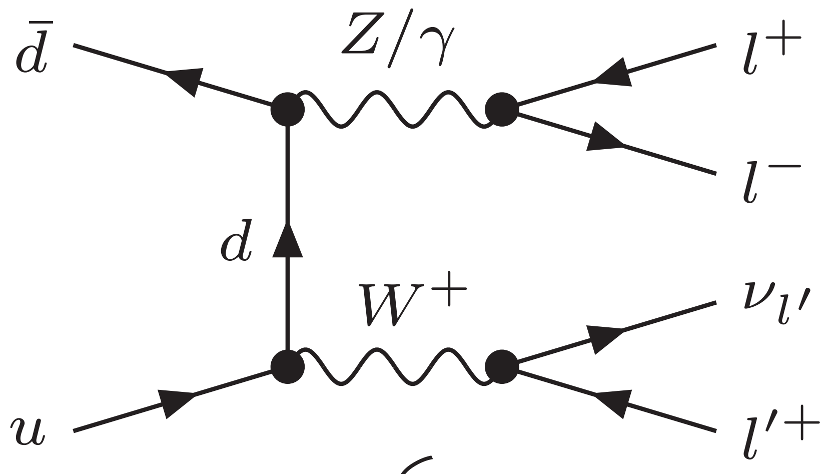
$$\sigma_{\rm NLO} = \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} {\rm d}\sigma^{\rm R} + \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm V}$$

$$= \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} \left({\rm d}\sigma^{\rm R} - {\rm d}\sigma^{\rm S}\right) + \int_{\Phi_{\rm B}} \left({\rm d}\sigma^{\rm V} + \int_1 {\rm d}\sigma^{\rm S}\right)_{\epsilon=0}$$

${\rm d}\sigma^{\rm S}$: subtraction term

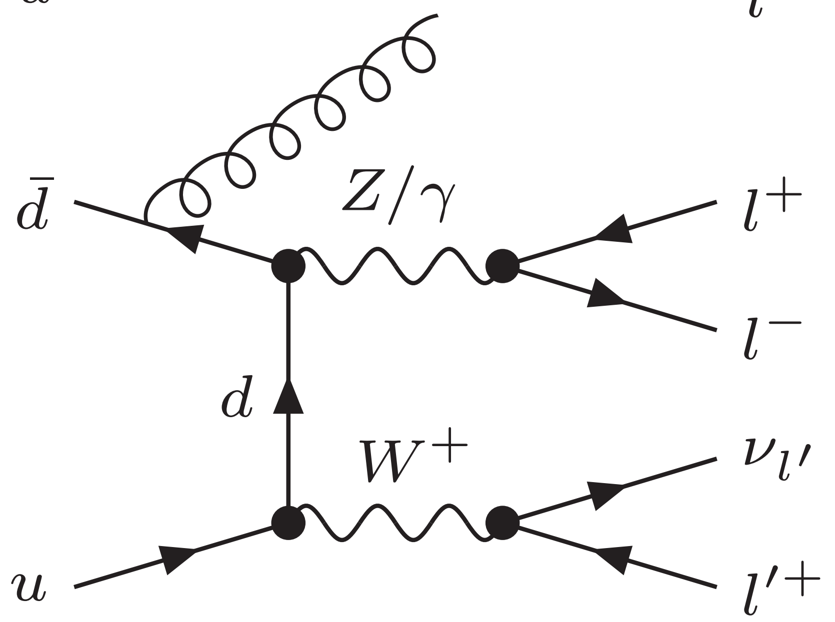
- → Dipole [Catani, Seymour '96]
- → FKS [Frixione, Kunszt, Signer '96]
- → Antenna [Gehrmann et al. '05]
- → ...

LO (pp→WZ)

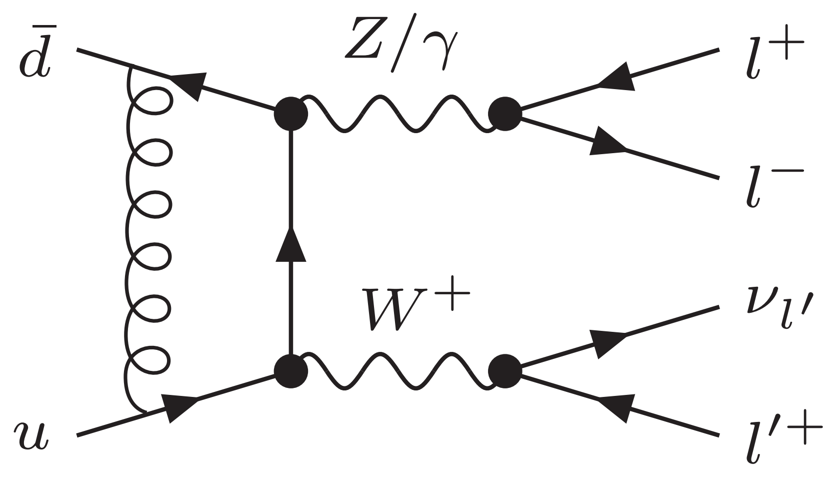


$\to {\rm d}\sigma^{\rm B}$

NLO (pp→WZ)



$\to {\rm d}\sigma^{\rm R}$



$\to {\rm d}\sigma^{\rm V}$

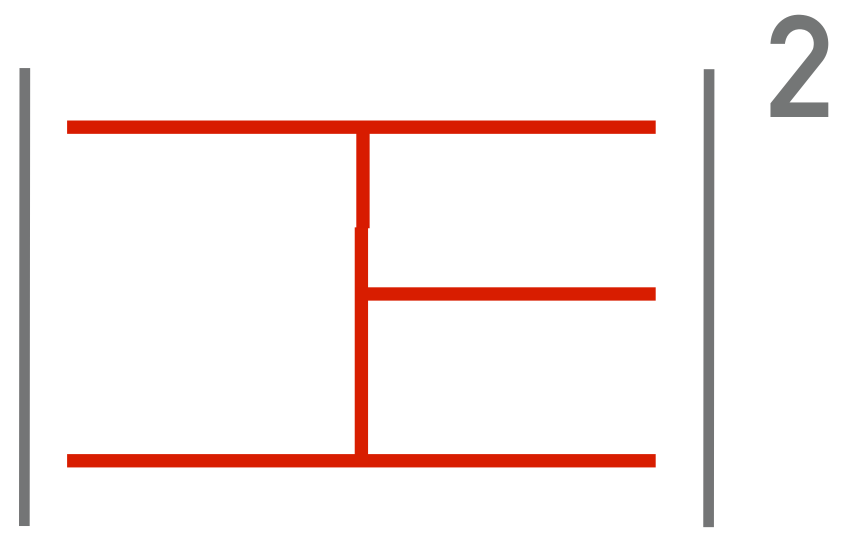
Subtraction terms?

$$\sigma_{\rm NLO} = \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} {\rm d}\sigma^{\rm R} + \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm V}$$

$$= \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} ({\rm d}\sigma^{\rm R} - {\rm d}\sigma^{\rm S}) + \int_{\Phi_{\rm B}} \left({\rm d}\sigma^{\rm V} + \int_1 {\rm d}\sigma^{\rm S}\right)_{\epsilon = 0}$$

- use factorization properties of squared amplitudes
- singularities appear when final-state parton soft or colliner
- singularity structure of amplitudes universal and known

schematically:



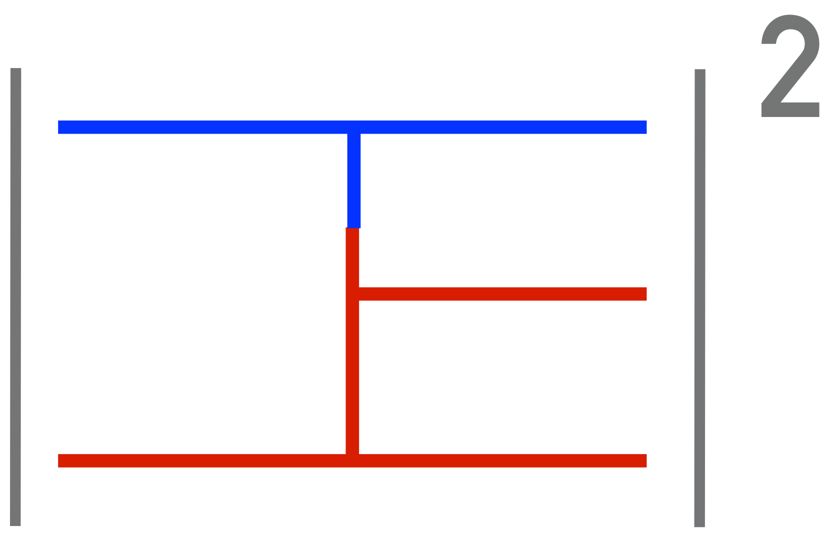
Subtraction terms?

$$\sigma_{\mathrm{NLO}} = \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B+1}}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{V}}$$
$$= \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B+1}}} \left(\mathrm{d}\sigma^{\mathrm{R}} - \mathrm{d}\sigma^{\mathrm{S}}\right) + \int_{\Phi_{\mathrm{B}}} \left(\mathrm{d}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)_{\epsilon = 0}$$

- use factorization properties of squared amplitudes
- singularities appear when final-state parton soft or colliner
- singularity structure of amplitudes universal and known

splitting parton

schematically:



Subtraction terms?

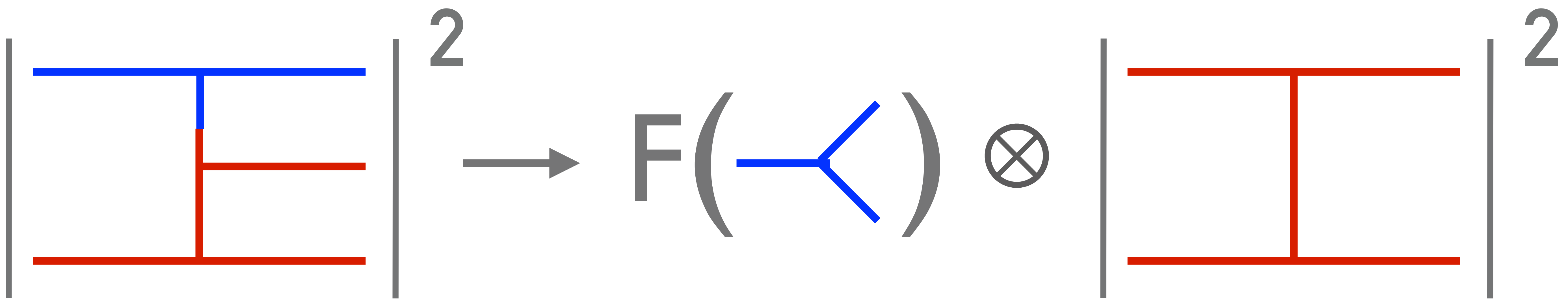
$$\sigma_{\rm NLO} = \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} {\rm d}\sigma^{\rm R} + \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm V}$$

$$= \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} \left({\rm d}\sigma^{\rm R} - {\rm d}\sigma^{\rm S}\right) + \int_{\Phi_{\rm B}} \left({\rm d}\sigma^{\rm V} + \int_1 {\rm d}\sigma^{\rm S}\right)_{\epsilon=0}$$

- use factorization properties of squared amplitudes
- singularities appear when final-state parton soft or colliner
- singularity structure of amplitudes universal and known

splitting parton getting soft or colliner

schematically:



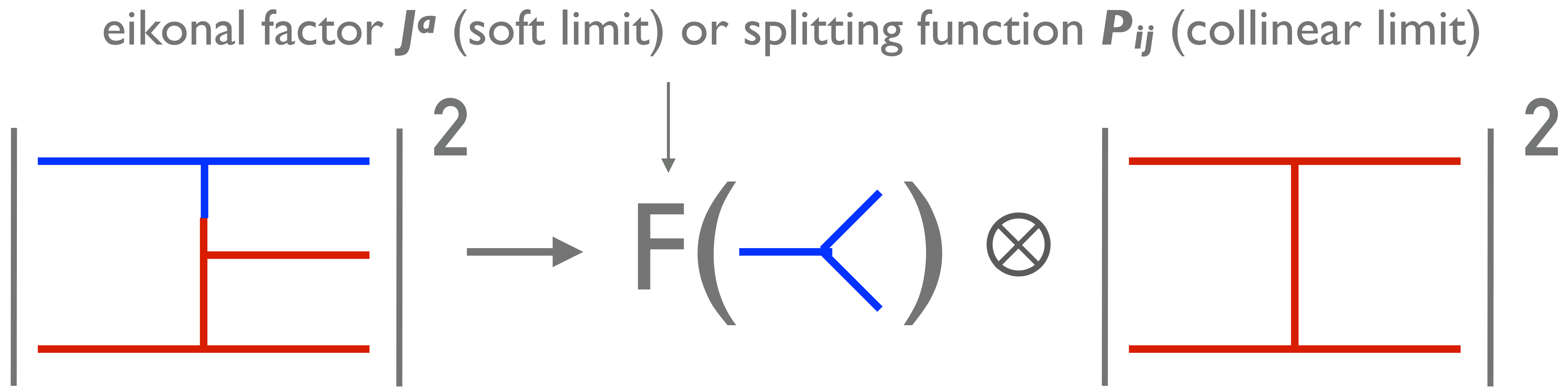
Subtraction terms?

$$\sigma_{\mathrm{NLO}} = \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B}+1}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{V}}$$

$$= \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B}+1}} \left(\mathrm{d}\sigma^{\mathrm{R}} - \mathrm{d}\sigma^{\mathrm{S}}\right) + \int_{\Phi_{\mathrm{B}}} \left(\mathrm{d}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)_{\epsilon = 0}$$

- use factorization properties of squared amplitudes
- singularities appear when final-state parton soft or colliner
- singularity structure of amplitudes universal and known

schematically:



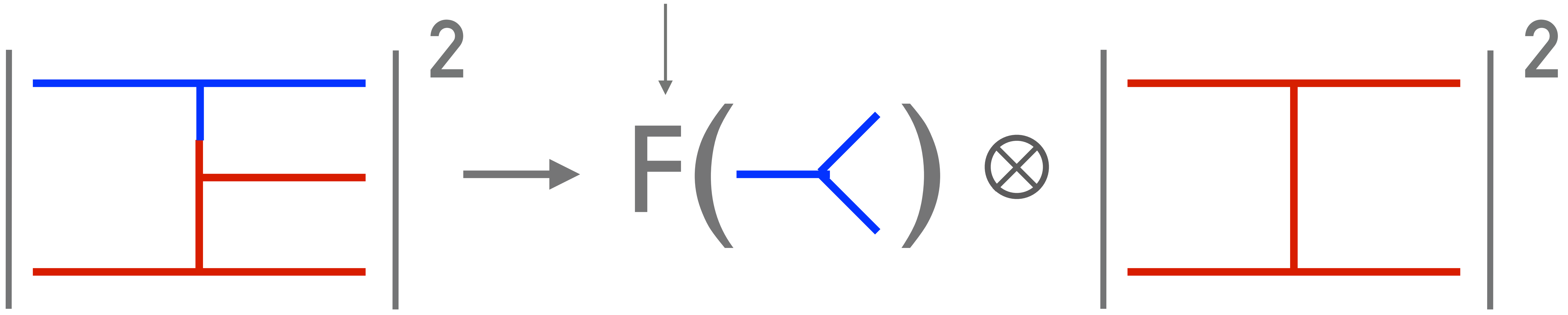
Subtraction terms?

$$\sigma_{\mathrm{NLO}} = \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B}+1}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{V}}$$

$$= \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B}+1}} \underbrace{\left(\mathrm{d}\sigma^{\mathrm{R}} - \mathrm{d}\sigma^{\mathrm{S}}\right)}_{\text{finite}} + \int_{\Phi_{\mathrm{B}}} \left(\mathrm{d}\sigma^{\mathrm{V}} + \int_{1} \mathrm{d}\sigma^{\mathrm{S}}\right)_{\epsilon = 0}$$

eikonal factor J^a (soft limit) or splitting function P_{ij} (collinear limit)

schematically:



Subtraction terms?

$$\sim \left(\frac{A_0}{\epsilon^2} + \frac{B_0}{\epsilon} + C_0\right)$$

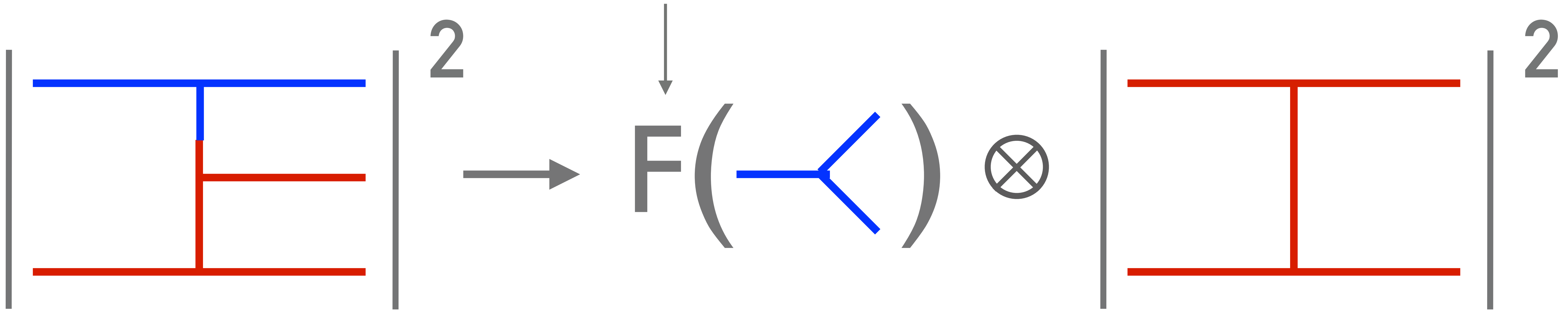
$$\sim \left(-\frac{A_0}{\epsilon^2} - \frac{B_0}{\epsilon} + D_0\right)$$

$$\sigma_{\rm NLO} = \int_{\Phi_{\rm B}} d\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} d\sigma^{\rm R} + \int_{\Phi_{\rm B}} d\sigma^{\rm V}$$

$$= \int_{\Phi_{\rm B}} d\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} \underbrace{\left(d\sigma^{\rm R} - d\sigma^{\rm S}\right)}_{\text{finite}} + \int_{\Phi_{\rm B}} \underbrace{\left(d\sigma^{\rm V} + \int_1 d\sigma^{\rm S}\right)_{\epsilon = 0}}_{= C_0 + D_0}$$

eikonal factor J^a (soft limit) or splitting function P_{ij} (collinear limit)

schematically:



Subtraction terms?

$$\sigma_{\rm NLO} = \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} {\rm d}\sigma^{\rm R} + \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm V}$$

$$= \int_{\Phi_{\rm B}} {\rm d}\sigma^{\rm B} + \int_{\Phi_{\rm B+1}} \left({\rm d}\sigma^{\rm R} - {\rm d}\sigma^{\rm S}\right) + \int_{\Phi_{\rm B}} \left({\rm d}\sigma^{\rm V} + \int_1 {\rm d}\sigma^{\rm S}\right)_{\epsilon = 0}$$

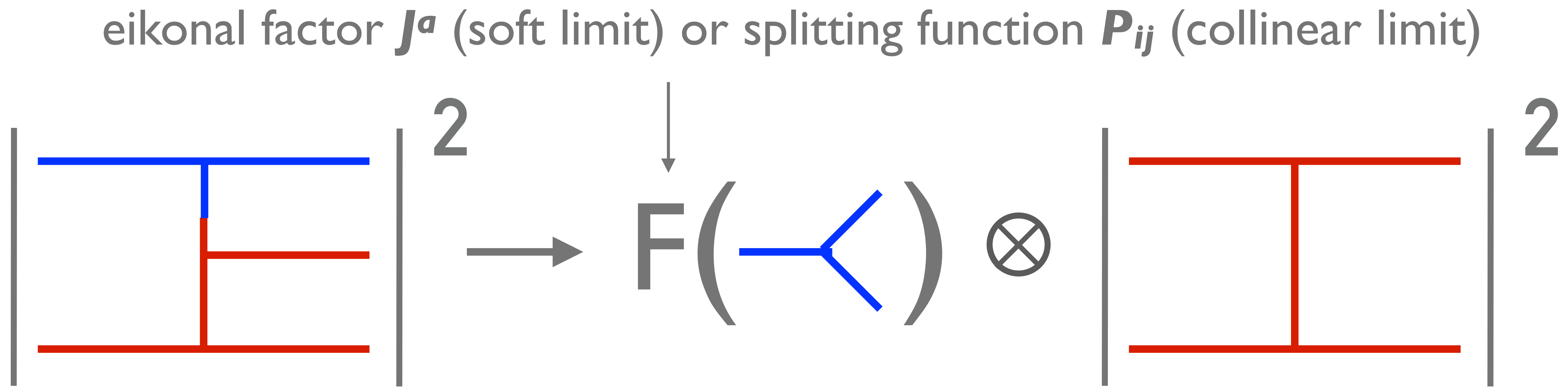
${\rm d}\sigma^{\rm S}$: subtraction term

→ Dipole [Catani, Seymour '96] ⇒ combines soft & collinear limit in dipole function

→ FKS [Frixione, Kunszt, Signer '96] ⇒ partitions phase space into soft, coll. & soft+coll.

→ Antenna [Gehrmann et al. '05] ⇒ like dipole, but 1 Antenna ≃ 1/2 Dipole

schematically:



Automation

★ Automation at LO (tree-level amplitudes & phase space) understood for very long time

Automation

★ Automation at LO (tree-level amplitudes & phase space) understood for very long time

★ Automation at NLO

- Automation of 1-loop amplitudes (OPP, OpenLoops,…)

MadLoop | OpenLoops | Gosam | Recola | Helac-NLO | BlackHat | NJet | NLOX | ...

- Automation of subtraction methods (Dipole, FKS, …) & NLO(+PS) calculations

MadGraph5_aMC@NLO | Munich/Matrix | Powheg | Sherpa | Herwig++ | WHIZARD | ...

Automation

★ Auto … ng time

★ Auto

• Au

Ma … NLOX …

• Au … ns

M … D …

Process		Syntax	Cross section (pb) LO 13 TeV	Cross section (pb) NLO 13 TeV
Single Higgs production				
g.1	$pp \to H$ (HEFT)	`p p > h`	$1.593 \pm 0.003 \cdot 10^{1}$ $^{+34.8\%}_{-26.0\%}$ $^{+1.2\%}_{-1.7\%}$	$3.261 \pm 0.010 \cdot 10^{1}$ $^{+20.2\%}_{-17.9\%}$ $^{+1.1\%}_{-1.6\%}$
g.2	$pp \to Hj$ (HEFT)	`p p > h j`	$8.367 \pm 0.003 \cdot 10^{0}$ $^{+39.4\%}_{-26.4\%}$ $^{+1.2\%}_{-1.4\%}$	$1.422 \pm 0.006 \cdot 10^{1}$ $^{+18.5\%}_{-16.6\%}$ $^{+1.1\%}_{-1.4\%}$
g.3	$pp \to Hjj$ (HEFT)	`p p > h j j`	$3.020 \pm 0.002 \cdot 10^{0}$ $^{+59.1\%}_{-34.7\%}$ $^{+1.4\%}_{-1.7\%}$	$5.124 \pm 0.020 \cdot 10^{0}$ $^{+20.7\%}_{-21.0\%}$ $^{+1.3\%}_{-1.5\%}$
g.4	$pp \to Hjj$ (VBF)	`p p > h j j $$ w+ w- z`	$1.987 \pm 0.002 \cdot 10^{0}$ $^{+1.7\%}_{-2.0\%}$ $^{+1.9\%}_{-1.4\%}$	$1.900 \pm 0.006 \cdot 10^{0}$ $^{+0.8\%}_{-0.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.5	$pp \to Hjjj$ (VBF)	`p p > h j j j $$ w+ w- z`	$2.824 \pm 0.005 \cdot 10^{-1}$ $^{+15.7\%}_{-12.7\%}$ $^{+1.5\%}_{-1.0\%}$	$3.085 \pm 0.010 \cdot 10^{-1}$ $^{+2.0\%}_{-3.0\%}$ $^{+1.5\%}_{-1.1\%}$
g.6	$pp \to HW^{\pm}$	`p p > h wpm`	$1.195 \pm 0.002 \cdot 10^{0}$ $^{+3.5\%}_{-4.5\%}$ $^{+1.9\%}_{-1.5\%}$	$1.419 \pm 0.005 \cdot 10^{0}$ $^{+2.1\%}_{-2.6\%}$ $^{+1.9\%}_{-1.4\%}$
g.7	$pp \to HW^{\pm} j$	`p p > h wpm j`	$4.018 \pm 0.003 \cdot 10^{-1}$ $^{+10.7\%}_{-9.3\%}$ $^{+1.2\%}_{-0.9\%}$	$4.842 \pm 0.017 \cdot 10^{-1}$ $^{+3.6\%}_{-3.7\%}$ $^{+1.2\%}_{-1.0\%}$
g.8*	$pp \to HW^{\pm} jj$	`p p > h wpm j j`	$1.198 \pm 0.016 \cdot 10^{-1}$ $^{+26.1\%}_{-19.4\%}$ $^{+0.8\%}_{-0.6\%}$	$1.574 \pm 0.014 \cdot 10^{-1}$ $^{+5.0\%}_{-6.5\%}$ $^{+0.9\%}_{-0.6\%}$
g.9	$pp \to HZ$	`p p > h z`	$6.468 \pm 0.008 \cdot 10^{-1}$ $^{+3.5\%}_{-4.5\%}$ $^{+1.9\%}_{-1.4\%}$	$7.674 \pm 0.027 \cdot 10^{-1}$ $^{+2.0\%}_{-2.5\%}$ $^{+1.9\%}_{-1.4\%}$
g.10	$pp \to HZj$	`p p > h z j`	$2.225 \pm 0.001 \cdot 10^{-1}$ $^{+10.6\%}_{-9.2\%}$ $^{+1.1\%}_{-0.8\%}$	$2.667 \pm 0.010 \cdot 10^{-1}$ $^{+3.5\%}_{-3.6\%}$ $^{+1.1\%}_{-0.9\%}$
g.11*	$pp \to HZjj$	`p p > h z j j`	$7.262 \pm 0.012 \cdot 10^{-2}$ $^{+26.2\%}_{-19.4\%}$ $^{+0.7\%}_{-0.6\%}$	$8.753 \pm 0.037 \cdot 10^{-2}$ $^{+4.8\%}_{-6.3\%}$ $^{+0.7\%}_{-0.6\%}$
g.12*	$pp \to HW^{+}W^{-}$ (4f)	`p p > h w+ w-`	$8.325 \pm 0.139 \cdot 10^{-3}$ $^{+0.0\%}_{-0.3\%}$ $^{+2.0\%}_{-1.6\%}$	$1.065 \pm 0.003 \cdot 10^{-2}$ $^{+2.5\%}_{-1.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.13*	$pp \to HW^{\pm}\gamma$	`p p > h wpm a`	$2.518 \pm 0.006 \cdot 10^{-3}$ $^{+0.7\%}_{-1.4\%}$ $^{+1.9\%}_{-1.5\%}$	$3.309 \pm 0.011 \cdot 10^{-3}$ $^{+2.7\%}_{-2.0\%}$ $^{+1.7\%}_{-1.4\%}$
g.14*	$pp \to HZW^{\pm}$	`p p > h z wpm`	$3.763 \pm 0.007 \cdot 10^{-3}$ $^{+1.1\%}_{-1.5\%}$ $^{+2.0\%}_{-1.6\%}$	$5.292 \pm 0.015 \cdot 10^{-3}$ $^{+3.9\%}_{-3.1\%}$ $^{+1.8\%}_{-1.4\%}$
g.15*	$pp \to HZZ$	`p p > h z z`	$2.093 \pm 0.003 \cdot 10^{-3}$ $^{+0.1\%}_{-0.6\%}$ $^{+1.9\%}_{-1.5\%}$	$2.538 \pm 0.007 \cdot 10^{-3}$ $^{+1.9\%}_{-1.4\%}$ $^{+2.0\%}_{-1.5\%}$
g.16	$pp \to Ht\bar{t}$	`p p > h t t~`	$3.579 \pm 0.003 \cdot 10^{-1}$ $^{+30.0\%}_{-21.5\%}$ $^{+1.7\%}_{-2.0\%}$	$4.608 \pm 0.016 \cdot 10^{-1}$ $^{+5.7\%}_{-9.0\%}$ $^{+2.0\%}_{-2.3\%}$
g.17	$pp \to Htj$	`p p > h tt j`	$4.994 \pm 0.005 \cdot 10^{-2}$ $^{+2.4\%}_{-4.2\%}$ $^{+1.2\%}_{-1.3\%}$	$6.328 \pm 0.022 \cdot 10^{-2}$ $^{+2.9\%}_{-1.8\%}$ $^{+1.5\%}_{-1.6\%}$
g.18	$pp \to Hb\bar{b}$ (4f)	`p p > h b b~`	$4.983 \pm 0.002 \cdot 10^{-1}$ $^{+28.1\%}_{-21.0\%}$ $^{+1.5\%}_{-1.8\%}$	$6.085 \pm 0.026 \cdot 10^{-1}$ $^{+7.3\%}_{-9.6\%}$ $^{+1.6\%}_{-2.0\%}$
g.19	$pp \to Ht\bar{t}j$	`p p > h t t~ j`	$2.674 \pm 0.041 \cdot 10^{-1}$ $^{+45.6\%}_{-29.2\%}$ $^{+2.6\%}_{-2.9\%}$	$3.244 \pm 0.025 \cdot 10^{-1}$ $^{+3.5\%}_{-8.7\%}$ $^{+2.5\%}_{-2.9\%}$
g.20*	$pp \to Hb\bar{b}j$ (4f)	`p p > h b b~ j`	$7.367 \pm 0.002 \cdot 10^{-2}$ $^{+45.6\%}_{-29.1\%}$ $^{+1.8\%}_{-2.1\%}$	$9.034 \pm 0.032 \cdot 10^{-2}$ $^{+7.9\%}_{-11.0\%}$ $^{+1.8\%}_{-2.2\%}$

MadGraph5_aMC@NLO: sample from 172 processes

...slide borrowed from Massimiliano Grazzini

Automation

★ Automation at LO (tree-level amplitudes & phase space) understood for very long time

★ Automation at NLO

- Automation of 1-loop amplitudes (OPP, OpenLoops,…)

MadLoop | OpenLoops | Gosam | Recola | Helac-NLO | BlackHat | NJet | NLOX | ...

- Automation of subtraction methods (Dipole, FKS, …) & NLO(+PS) calculations

MadGraph5_aMC@NLO | Munich/Matrix | Powheg | Sherpa | Herwig++ | WHIZARD | ...

→ NLO has become the minimal standard now in (most) LHC analyses

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➙ NLO has become the minimal standard now in (most) LHC analyses

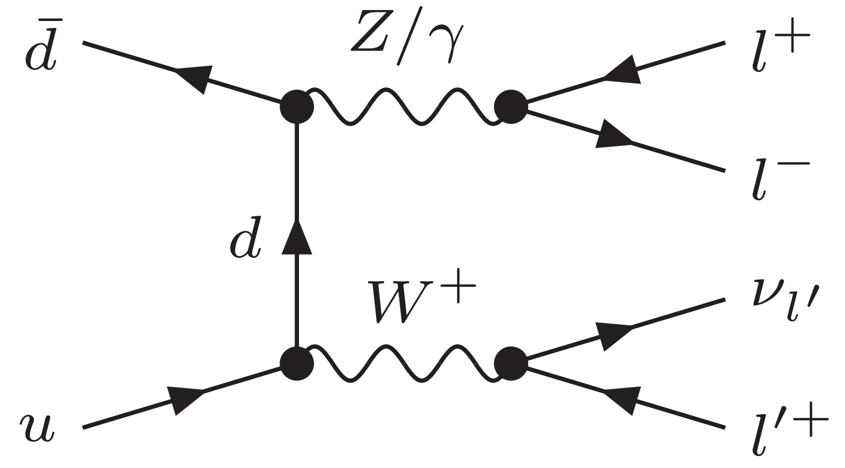
★ NNLO(+PS) only as process libraries in (public) codes

Matrix | MCFM | MiNNLO$_{PS}$ | Geneva | NNLOjet (not public) | Stripper (not public) | ...

How to do a NNLO calculation

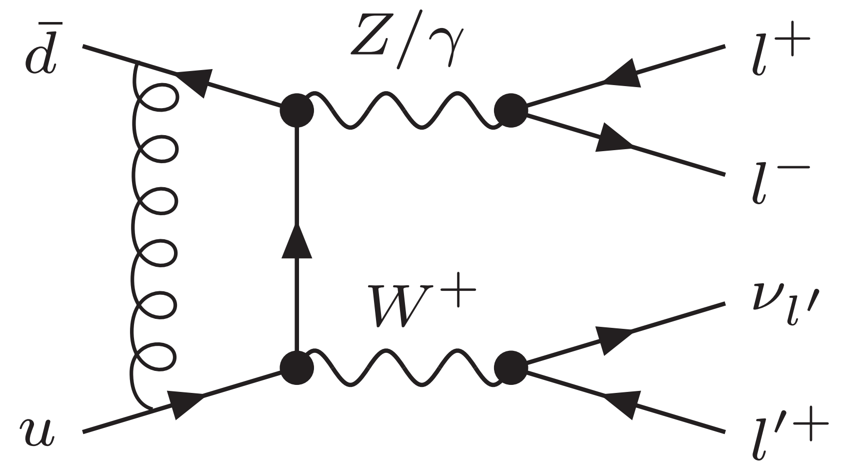
NNLO local subtraction

LO (pp→WZ)

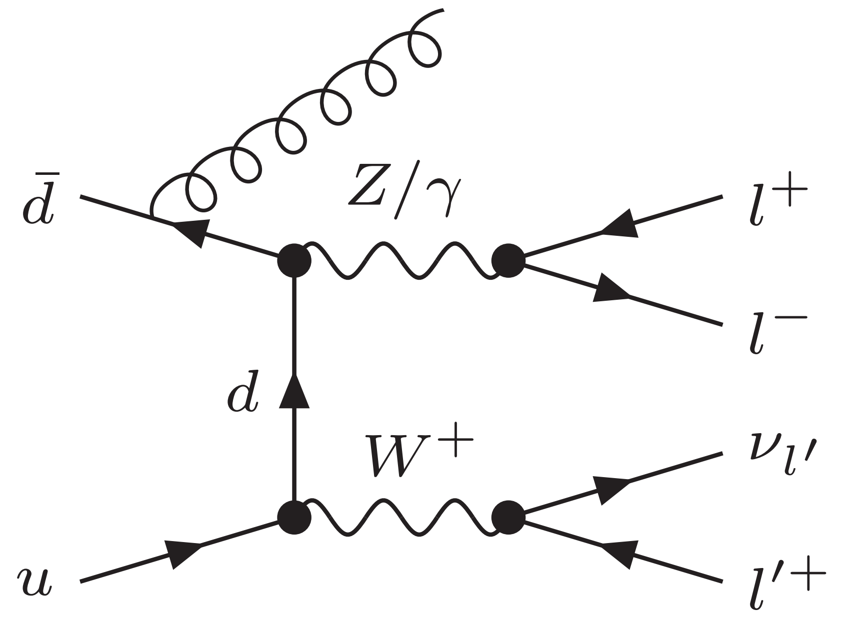


$\to \mathrm{d}\sigma^{\mathrm{B}}$

NLO (pp→WZ)

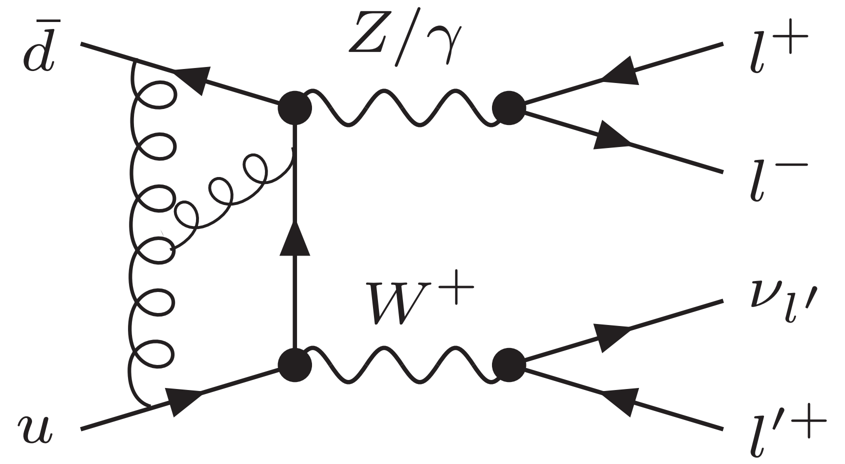


$\to \mathrm{d}\sigma^{\mathrm{V}}$

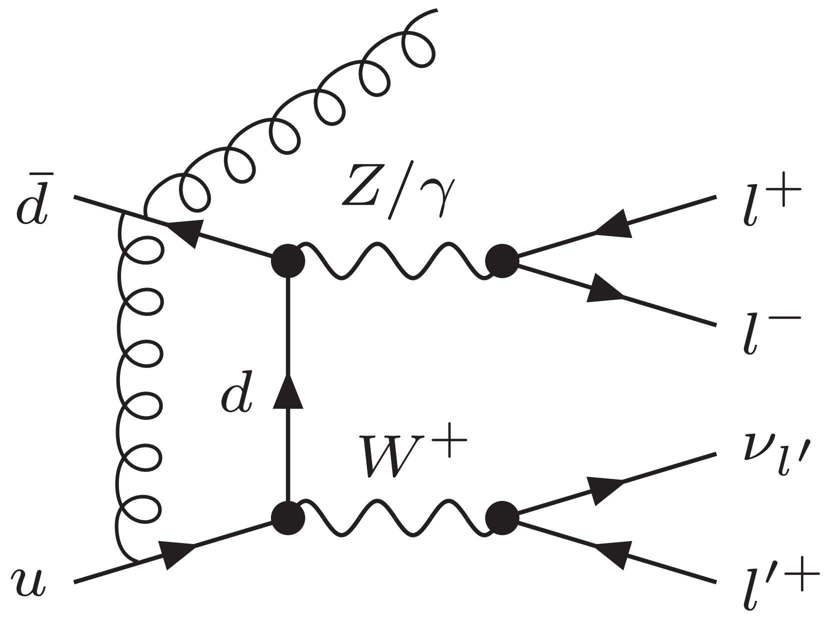


$\to \mathrm{d}\sigma^{\mathrm{R}}$

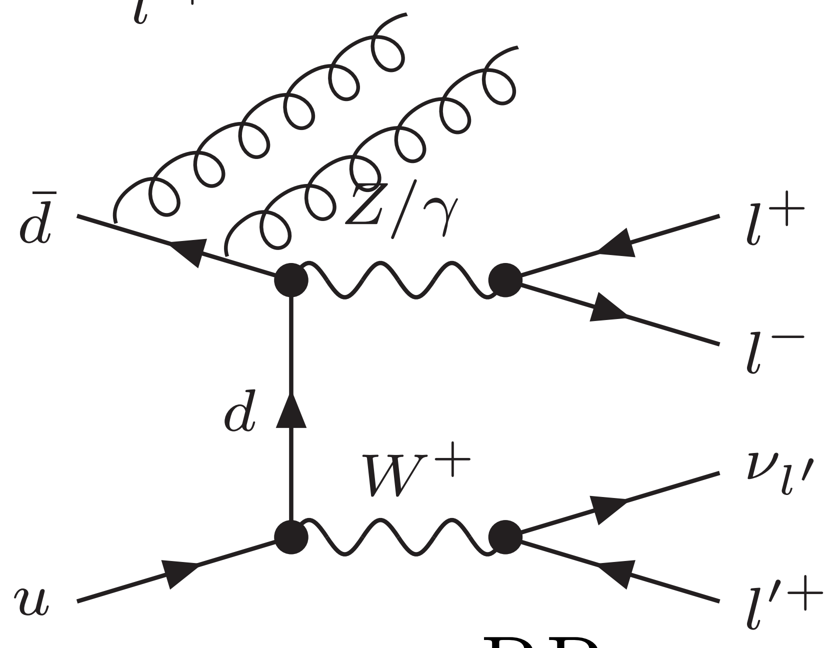
NNLO (pp→WZ)



$\to \mathrm{d}\sigma^{\mathrm{VV}}$



$\to \mathrm{d}\sigma^{\mathrm{RV}}$

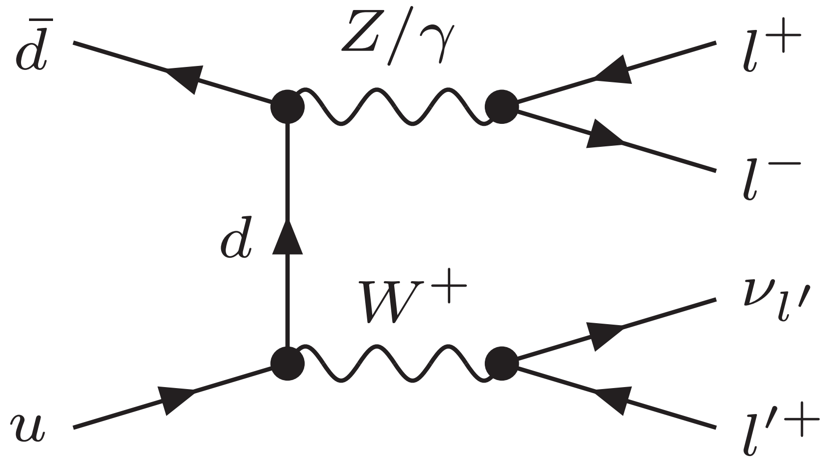


$\to \mathrm{d}\sigma^{\mathrm{RR}}$

NNLO local subtraction

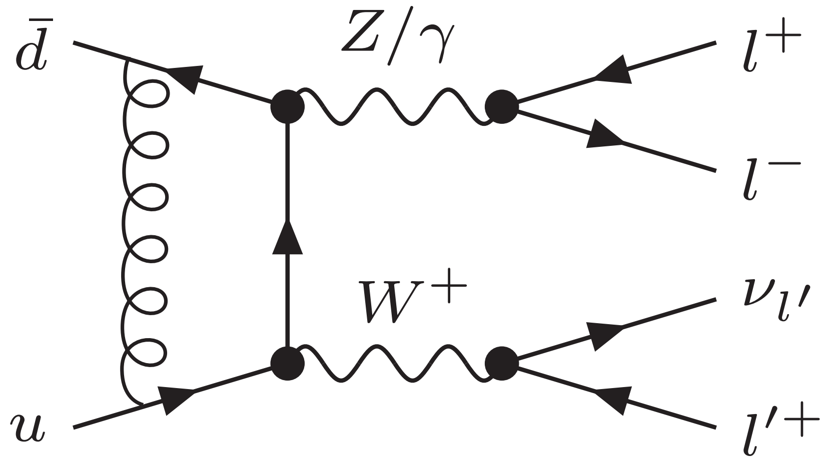
$$\sigma_{\text{NNLO}} = \int_{\Phi_B} d\sigma^{\text{B}} + \int_{\Phi_{B+1}} d\sigma^{\text{R}} + \int_{\Phi_B} d\sigma^{\text{V}} + \int_{\Phi_{B+2}} d\sigma^{\text{RR}} + \int_{\Phi_{B+1}} d\sigma^{\text{RV}} + \int_{\Phi_B} d\sigma^{\text{VV}}$$

LO (pp→WZ)

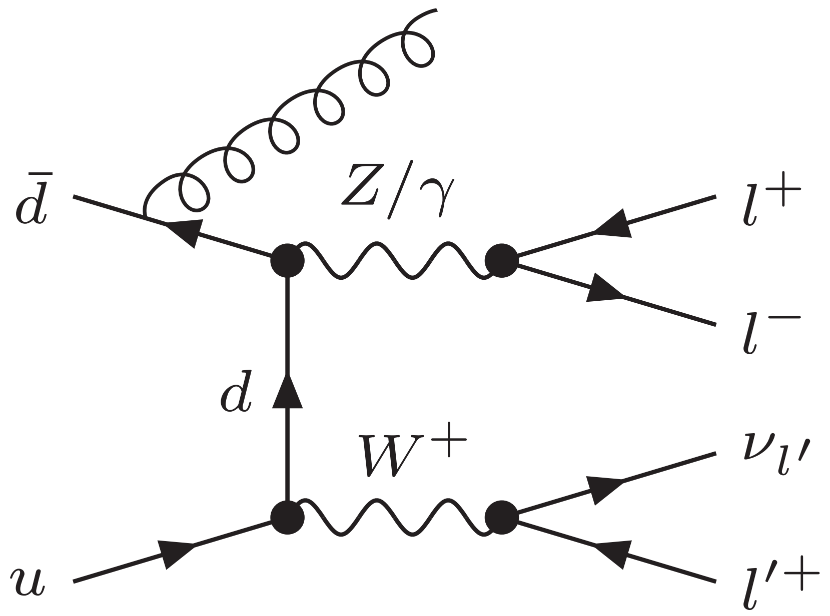


$\rightarrow d\sigma^{\text{B}}$

NLO (pp→WZ)

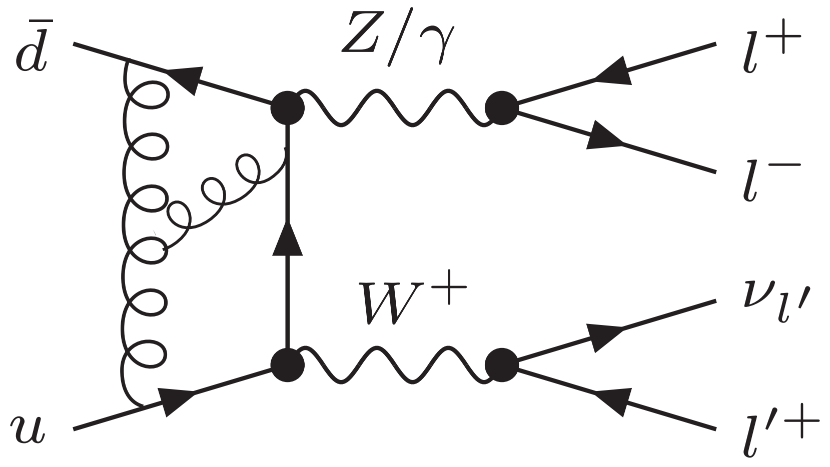


$\rightarrow d\sigma^{\text{V}}$

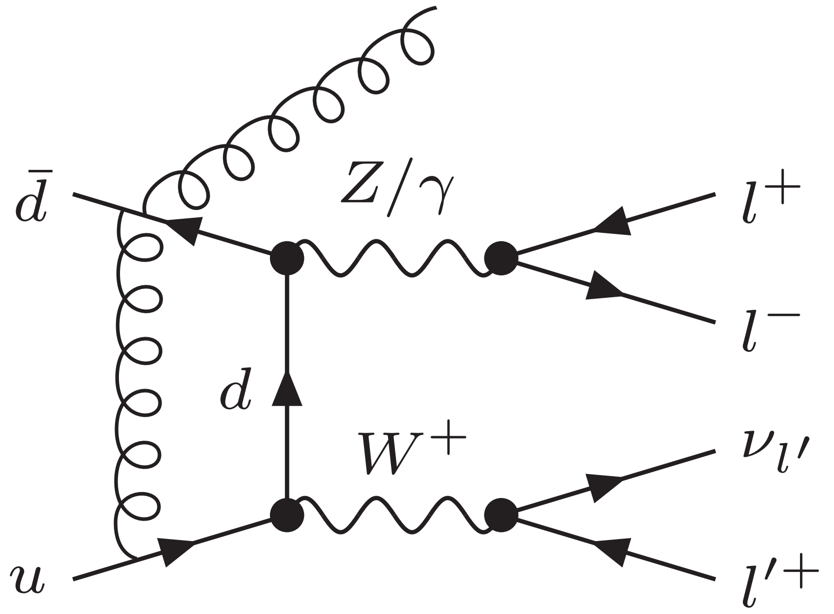


$\rightarrow d\sigma^{\text{R}}$

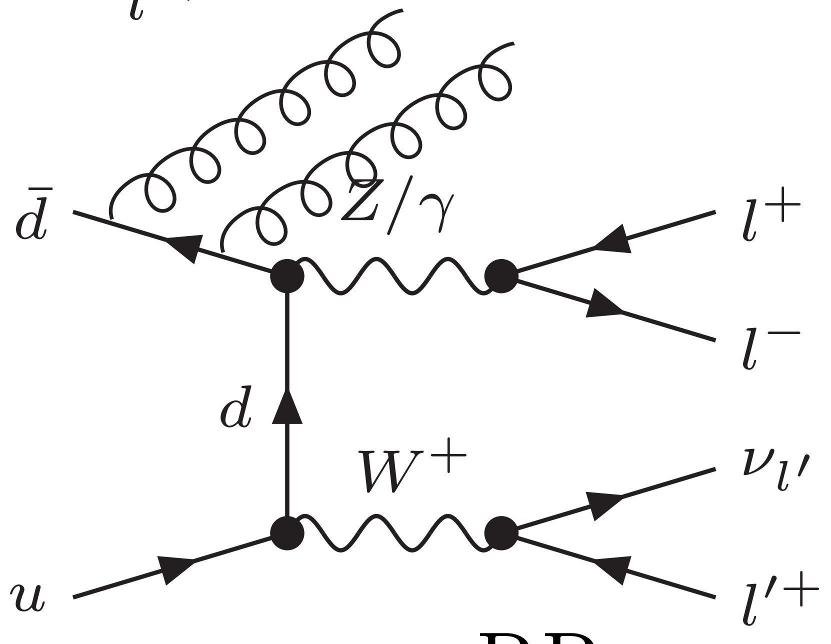
NNLO (pp→WZ)



$\rightarrow d\sigma^{\text{VV}}$



$\rightarrow d\sigma^{\text{RV}}$



$\rightarrow d\sigma^{\text{RR}}$

NNLO local subtraction

$$\sigma_{\rm NLO} = \int_{\Phi_B} \mathrm{d}\sigma^{\rm B} + \int_{\Phi_{B+1}} \left(\mathrm{d}\sigma^{\rm R} - \mathrm{d}\sigma^{\rm S}\right) + \int_{\Phi_B} \left(\mathrm{d}\sigma^{\rm V} + \int_1 \mathrm{d}\sigma^{\rm S}\right)$$

NNLO local subtraction

$$\sigma_{\text{NNLO}} = \int_{\Phi_B} \mathrm{d}\sigma^{\text{B}} + \int_{\Phi_{B+1}} \left(\mathrm{d}\sigma^{\text{R}} - \mathrm{d}\sigma^{\text{S}}\right) + \int_{\Phi_B} \left(\mathrm{d}\sigma^{\text{V}} + \int_1 \mathrm{d}\sigma^{\text{S}}\right)$$
$$+ \int_{\Phi_{B+2}} \left(\mathrm{d}\sigma^{\text{RR}} - \mathrm{d}\sigma^{\text{S}_{21}} - \mathrm{d}\sigma^{\text{S}_{22}}\right) + \int_{\Phi_{B+1}} \left(\mathrm{d}\sigma^{\text{RV}} - \mathrm{d}\sigma^{\text{S}_1} + \int_1 \mathrm{d}\sigma^{\text{S}_{21}}\right) + \int_{\Phi_B} \left(\mathrm{d}\sigma^{\text{VV}} + \int_1 \mathrm{d}\sigma^{\text{S}_1} + \int_2 \mathrm{d}\sigma^{\text{S}_{22}}\right)$$

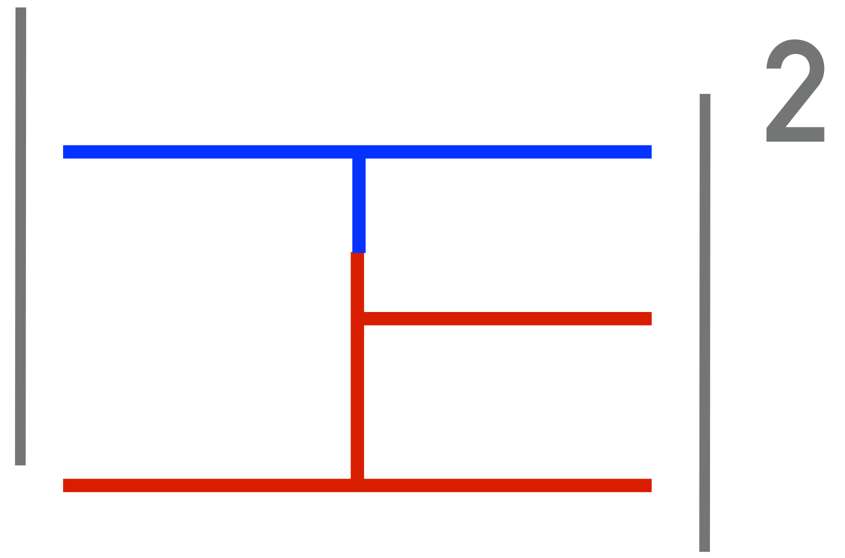
NNLO local subtraction

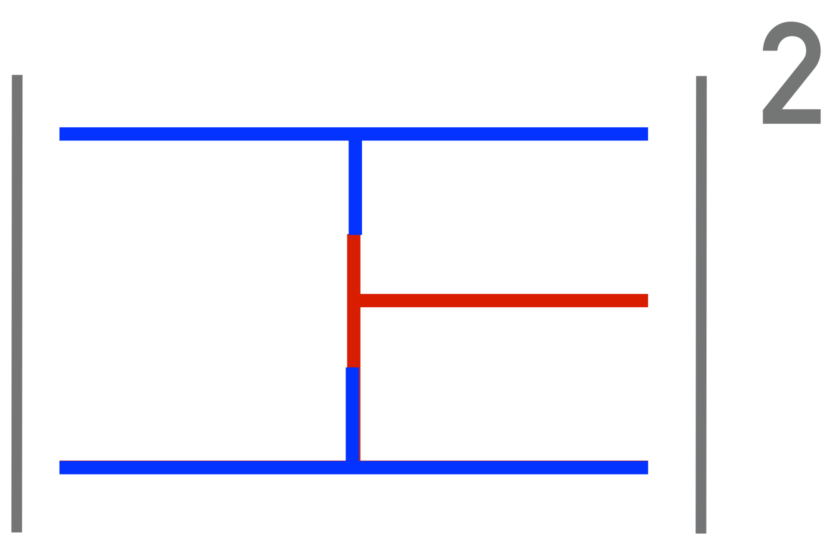
$$\sigma_{\text{NNLO}} = \int_{\Phi_B} d\sigma^{\text{B}} + \int_{\Phi_{B+1}} \left(d\sigma^{\text{R}} - d\sigma^{\text{S}}\right) + \int_{\Phi_B} \left(d\sigma^{\text{V}} + \int_1 d\sigma^{\text{S}}\right)$$

$$+ \int_{\Phi_{B+2}} \left(d\sigma^{\text{RR}} - d\sigma^{\text{S}_{21}} - d\sigma^{\text{S}_{22}}\right) + \int_{\Phi_{B+1}} \left(d\sigma^{\text{RV}} - d\sigma^{\text{S}_1} + \int_1 d\sigma^{\text{S}_{21}}\right) + \int_{\Phi_B} \left(d\sigma^{\text{VV}} + \int_1 d\sigma^{\text{S}_1} + \int_2 d\sigma^{\text{S}_{22}}\right)$$

1 unresolved (soft/colliner)
& 1 resolved emission (NLO-like)

2 unresolved
(soft/colliner) emissions





NNLO local subtraction

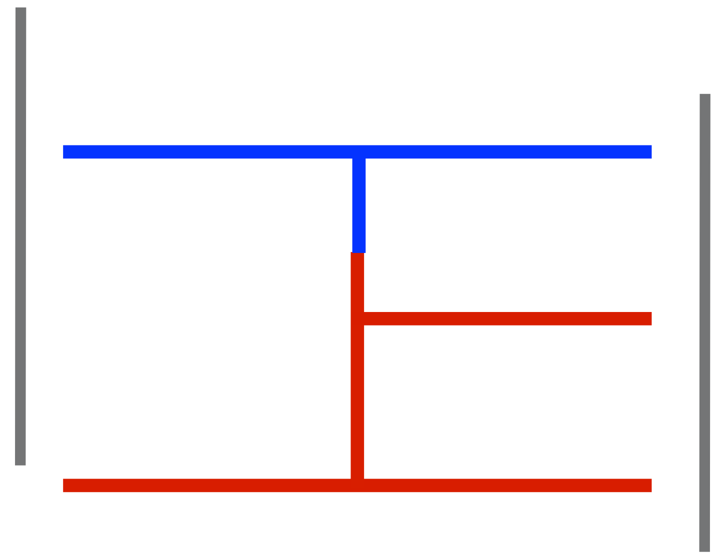
$$\sigma_{\text{NNLO}} = \int_{\Phi_B} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{B+1}} \left(\mathrm{d}\sigma^{\mathrm{R}} - \mathrm{d}\sigma^{\mathrm{S}}\right) + \int_{\Phi_B} \left(\mathrm{d}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)$$

$$+ \int_{\Phi_{B+2}} \left(\mathrm{d}\sigma^{\mathrm{RR}} - \mathrm{d}\sigma^{\mathrm{S}_{21}} - \mathrm{d}\sigma^{\mathrm{S}_{22}}\right) + \int_{\Phi_{B+1}} \left(\mathrm{d}\sigma^{\mathrm{RV}} - \mathrm{d}\sigma^{\mathrm{S}_1} + \int_1 \mathrm{d}\sigma^{\mathrm{S}_{21}}\right) + \int_{\Phi_B} \left(\mathrm{d}\sigma^{\mathrm{VV}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}_1} + \int_2 \mathrm{d}\sigma^{\mathrm{S}_{22}}\right)$$

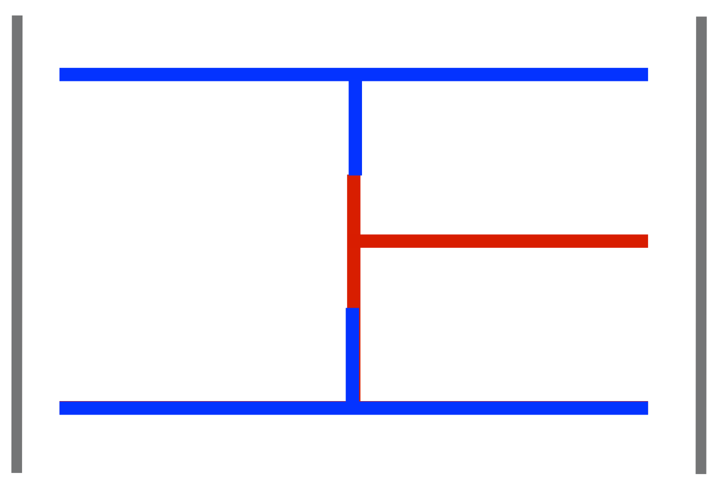
1 unresolved (soft/colliner)
& 1 resolved emission (NLO-like)

2 unresolved
(soft/colliner) emissions

cancels $1/\epsilon^n$ poles of RV



2



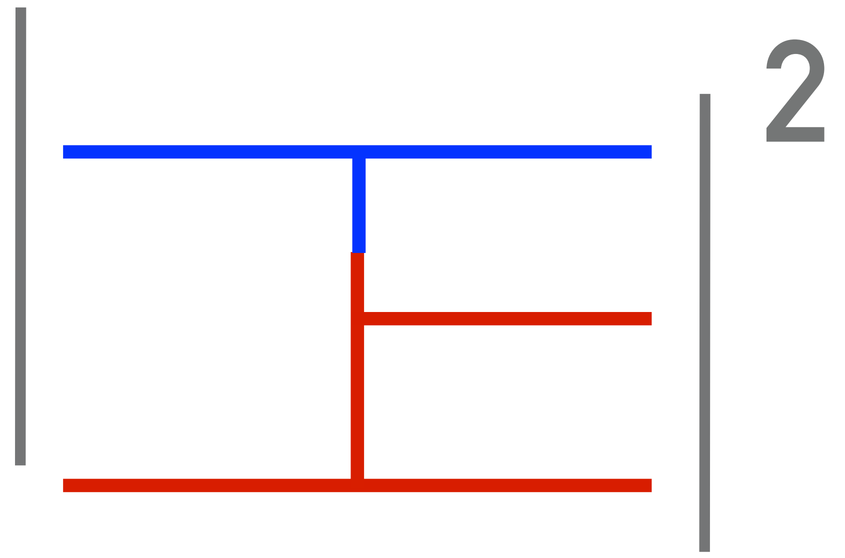
2

NNLO local subtraction

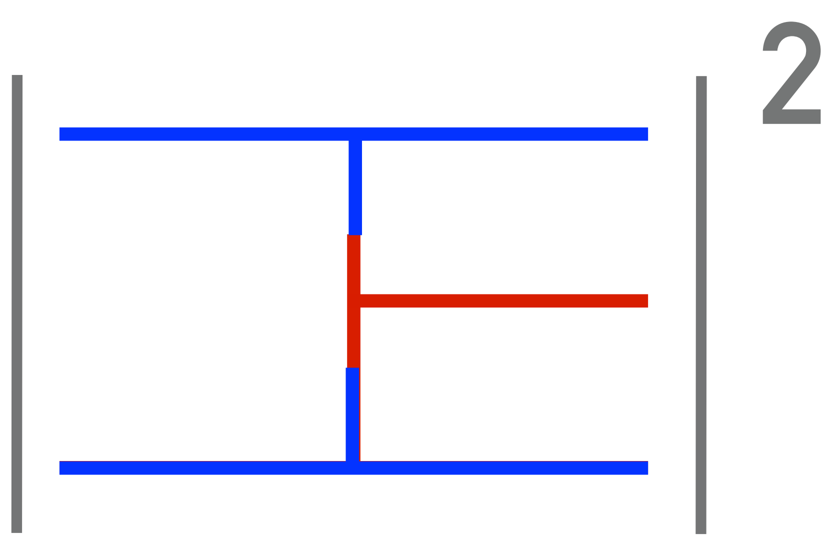
$$\sigma_{\text{NNLO}} = \int_{\Phi_B} d\sigma^{B} + \int_{\Phi_{B+1}} \left(d\sigma^{R} - d\sigma^{S}\right) + \int_{\Phi_B} \left(d\sigma^{V} + \int_1 d\sigma^{S}\right)$$

$$+ \int_{\Phi_{B+2}} \left(d\sigma^{RR} - d\sigma^{S_{21}} - d\sigma^{S_{22}}\right) + \int_{\Phi_{B+1}} \left(d\sigma^{RV} - d\sigma^{S_1} + \int_1 d\sigma^{S_{21}}\right) + \int_{\Phi_B} \left(d\sigma^{VV} + \int_1 d\sigma^{S_1} + \int_2 d\sigma^{S_{22}}\right)$$

I unresolved (soft/colliner)
& I resolved emission (NLO-like)



2 unresolved
(soft/colliner) emissions



subtracts one
unresolved
emission
(NLO-like)

cancels $1/\epsilon^n$ poles of RV

NNLO local subtraction

$$\sigma_{\mathrm{NNLO}} = \int_{\Phi_B} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{B+1}} \left(\mathrm{d}\sigma^{\mathrm{R}} - \mathrm{d}\sigma^{\mathrm{S}}\right) + \int_{\Phi_B} \left(\mathrm{d}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)$$

$$+ \int_{\Phi_{B+2}} \left(\mathrm{d}\sigma^{\mathrm{RR}} - \mathrm{d}\sigma^{\mathrm{S}_{21}} - \mathrm{d}\sigma^{\mathrm{S}_{22}}\right) + \int_{\Phi_{B+1}} \left(\mathrm{d}\sigma^{\mathrm{RV}} - \mathrm{d}\sigma^{\mathrm{S}_1} + \int_1 \mathrm{d}\sigma^{\mathrm{S}_{21}}\right) + \int_{\Phi_B} \left(\mathrm{d}\sigma^{\mathrm{VV}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}_1} + \int_2 \mathrm{d}\sigma^{\mathrm{S}_{22}}\right)$$

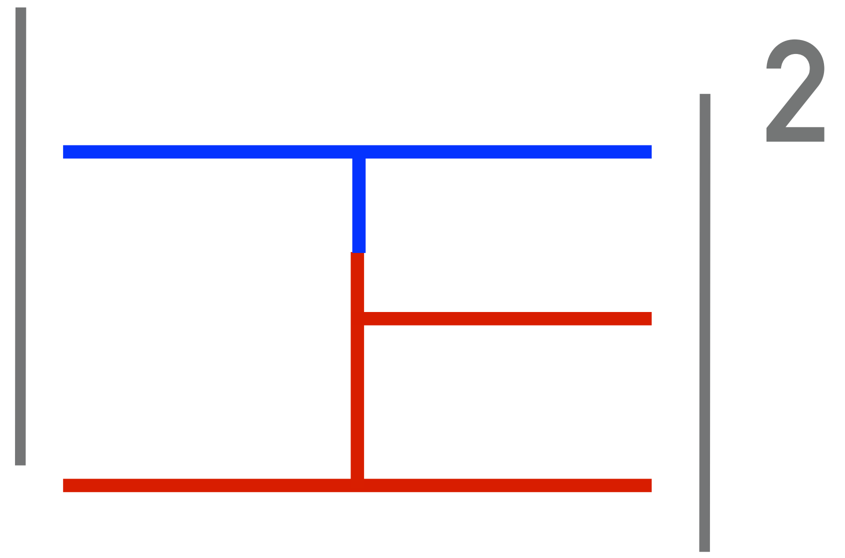
1 unresolved (soft/colliner) & 1 resolved emission (NLO-like)

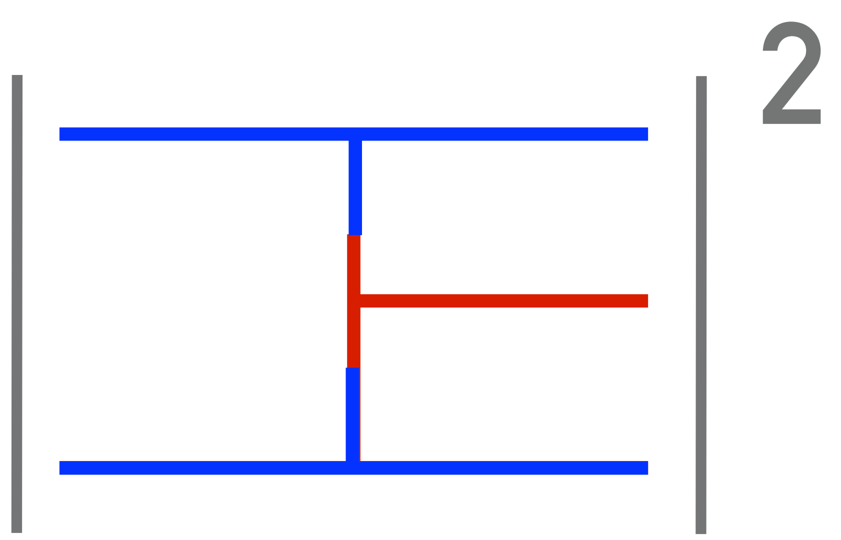
2 unresolved (soft/colliner) emissions

subtracts one unresolved emission (NLO-like)

cancels $1/\epsilon^n$ poles of RV

sum has to cancel all $1/\epsilon^n$ poles of VV





NNLO local subtraction

$$\sigma_{\mathrm{NNLO}} = \int_{\Phi_B} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{B+1}} \left(\mathrm{d}\sigma^{\mathrm{R}} - \mathrm{d}\sigma^{\mathrm{S}}\right) + \int_{\Phi_B} \left(\mathrm{d}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)$$

$$+ \int_{\Phi_{B+2}} \left(\mathrm{d}\sigma^{\mathrm{RR}} \boxed{- \mathrm{d}\sigma^{\mathrm{S}_{21}}} \boxed{- \mathrm{d}\sigma^{\mathrm{S}_{22}}}\right) + \int_{\Phi_{B+1}} \left(\mathrm{d}\sigma^{\mathrm{RV}} \boxed{- \mathrm{d}\sigma^{\mathrm{S}_1}} \boxed{+ \int_1 \mathrm{d}\sigma^{\mathrm{S}_{21}}}\right) + \int_{\Phi_B} \left(\mathrm{d}\sigma^{\mathrm{VV}} \boxed{+ \int_1 \mathrm{d}\sigma^{\mathrm{S}_1} + \int_2 \mathrm{d}\sigma^{\mathrm{S}_{22}}}\right)$$

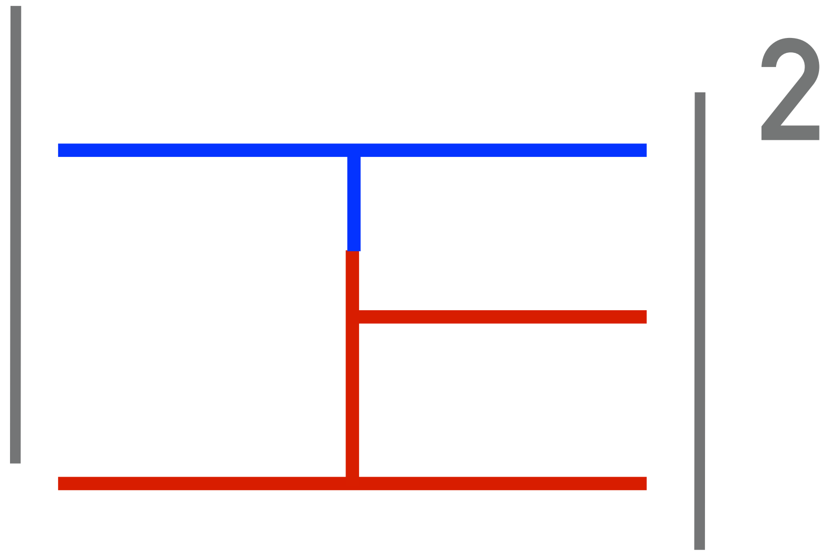
1 unresolved (soft/colliner) & 1 resolved emission (NLO-like)

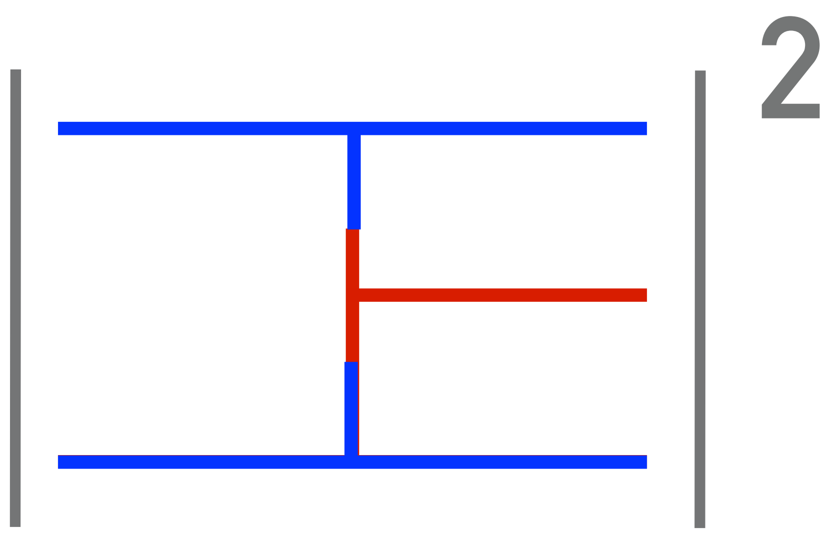
2 unresolved (soft/colliner) emissions

subtracts one unresolved emission (NLO-like)

cancels $1/\epsilon^n$ poles of RV

sum has to cancel all $1/\epsilon^n$ poles of VV





Antenna subtraction
Kosower; Gehrmann, Gehrmann-de Ridder, Glover

Stripper
Czakon

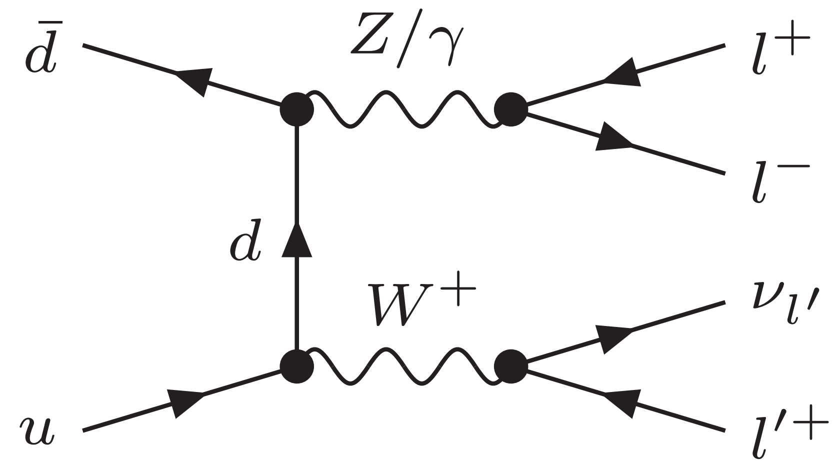
Sector-improved residue subtraction
Boughezal, Melnikov, Petriello

CoLorFul subtraction
Del Duca, Somogyi, Troscanyi

Local analytic sector subtraction
Bertolotti, Magnea, Maina, Pelliccioli, Ratti, Signorile-Signorile, Torrielli, Uccirati

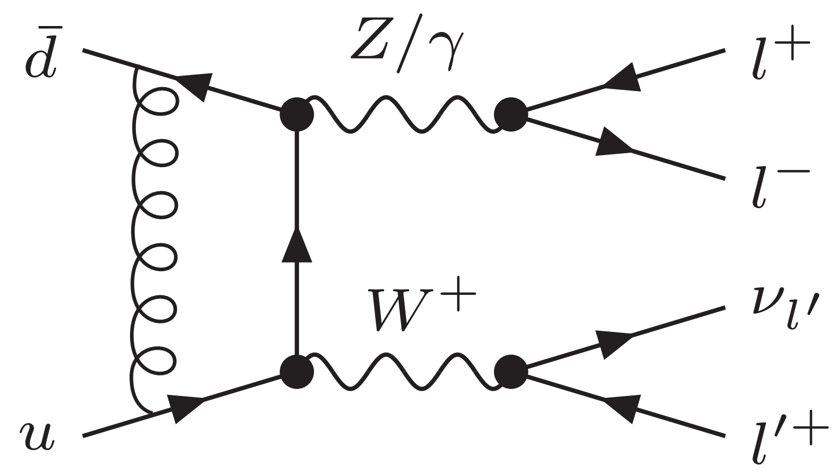
NNLO through X+jet at NLO + Slicing

LO (pp→WZ)

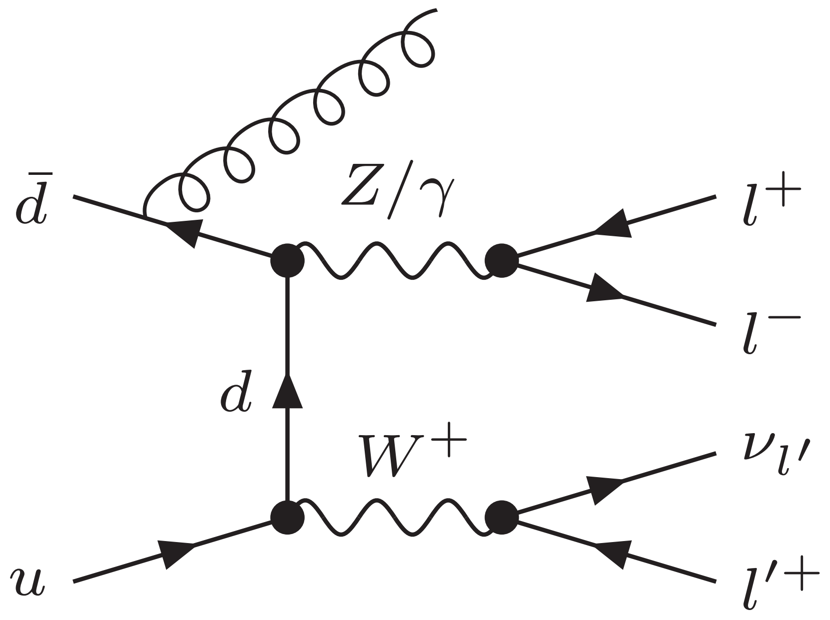


$\to \mathrm{d}\sigma^{\mathrm{B}}$

NLO (pp→WZ)

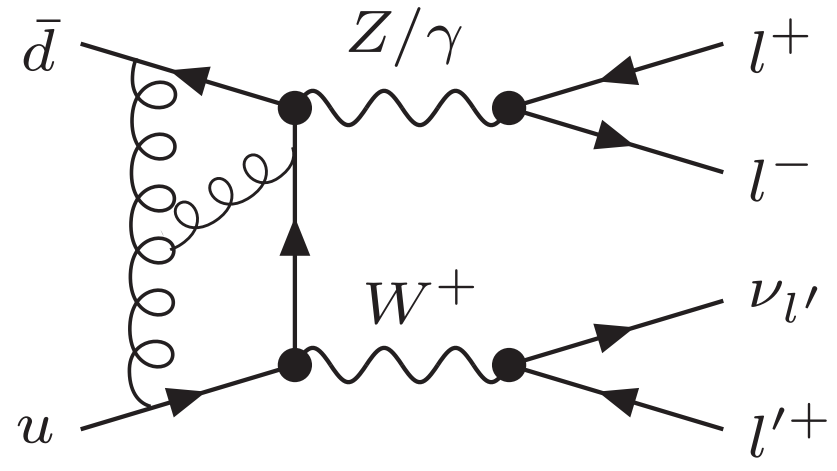


$\to \mathrm{d}\sigma^{\mathrm{V}}$

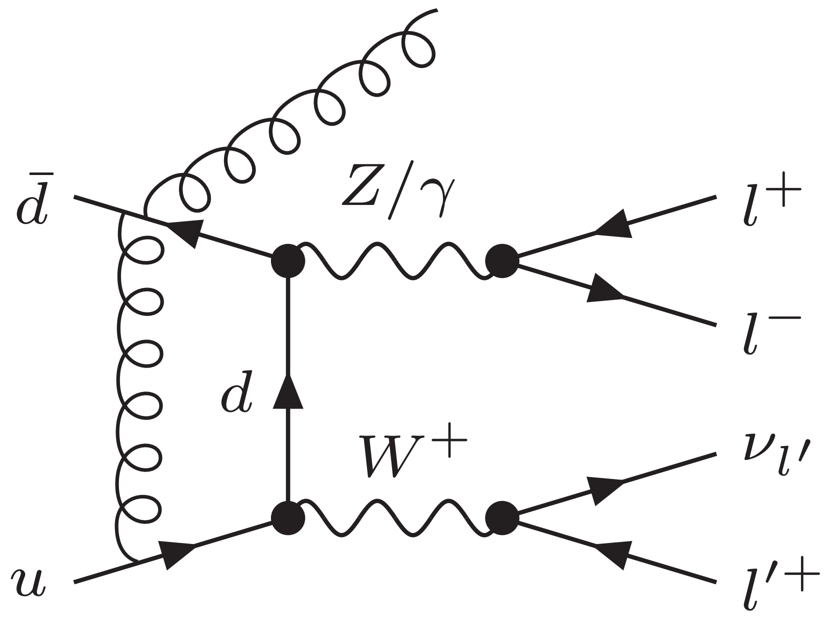


$\to \mathrm{d}\sigma^{\mathrm{R}}$

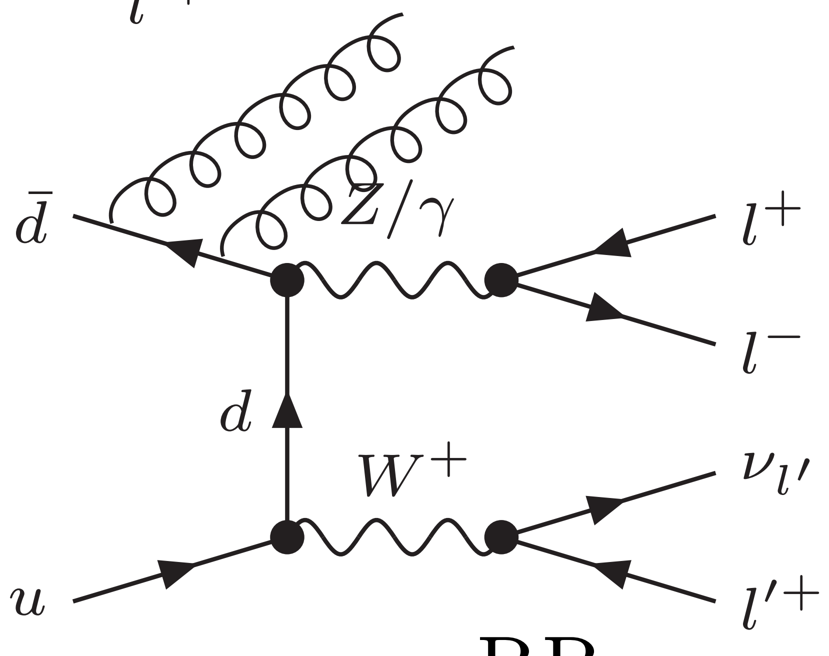
NNLO (pp→WZ)



$\to \mathrm{d}\sigma^{\mathrm{VV}}$



$\to \mathrm{d}\sigma^{\mathrm{RV}}$



$\to \mathrm{d}\sigma^{\mathrm{RR}}$

NNLO through X+jet at NLO + Slicing

$$\sigma_{\rm NLO}^{\rm X+jet} = \int_{\Phi_{\rm R}} {\rm d}\sigma^{\rm R} + \int_{\Phi_{\rm RV+1}} \left({\rm d}\sigma^{\rm RR} - {\rm d}\sigma^{\rm S}\right) + \int_{\Phi_{\rm RV}} \left({\rm d}\sigma^{\rm RV} + \int_1 {\rm d}\sigma^{\rm S}\right)$$

$d\sigma^{S}$: subtraction term

→ Dipole [Catani, Seymour '96]

→ FKS [Frixione, Kunszt, Signer '96]

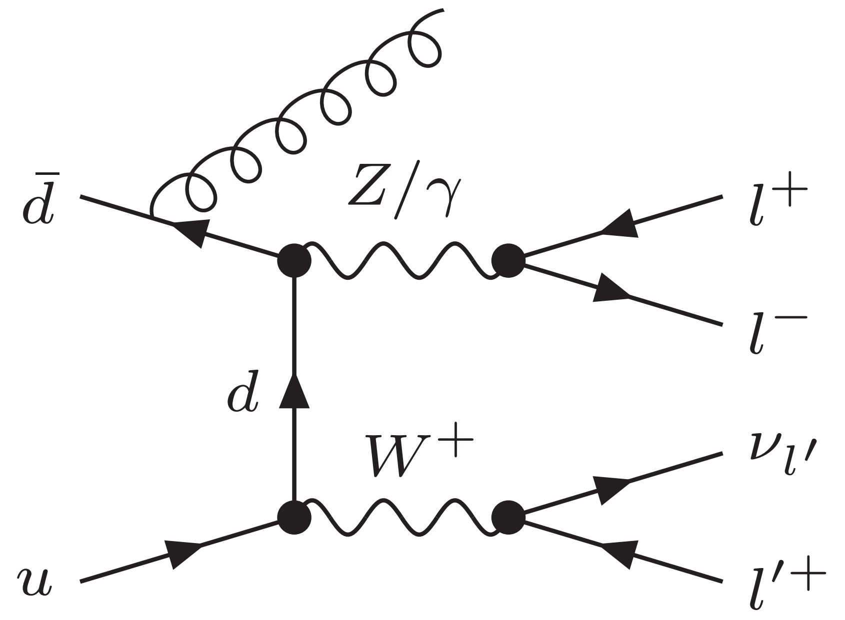
→ Antenna [Gehrmann et al. '05]

→ ...

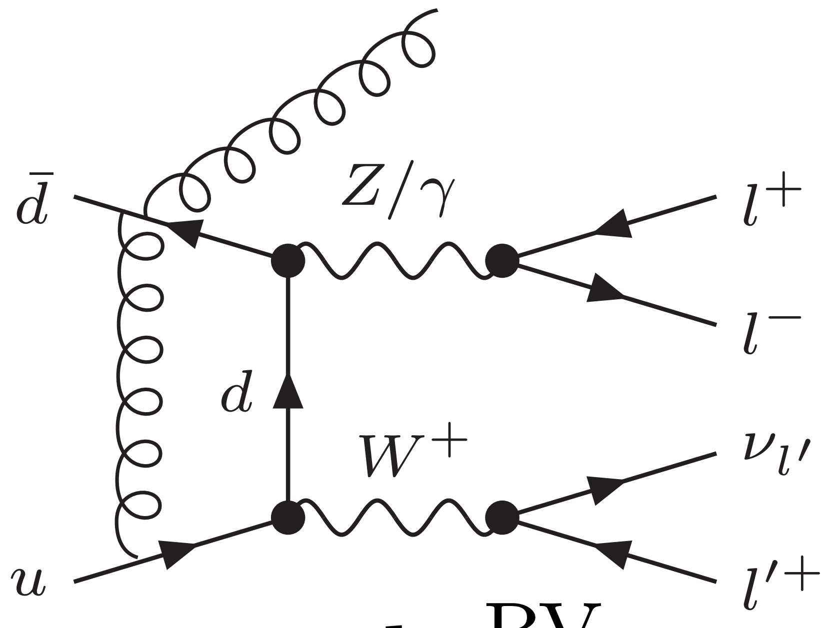
~~LO (pp→WZ)~~

~~N~~LO (pp→WZ+jet)

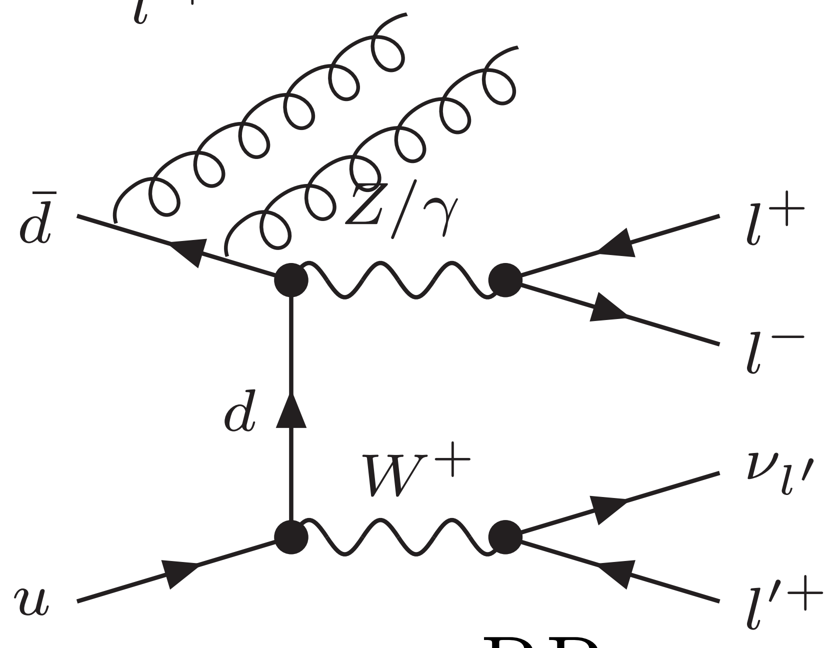
~~N~~NLO (pp→WZ+jet)



$\rightarrow d\sigma^{R}$



$\rightarrow d\sigma^{RV}$

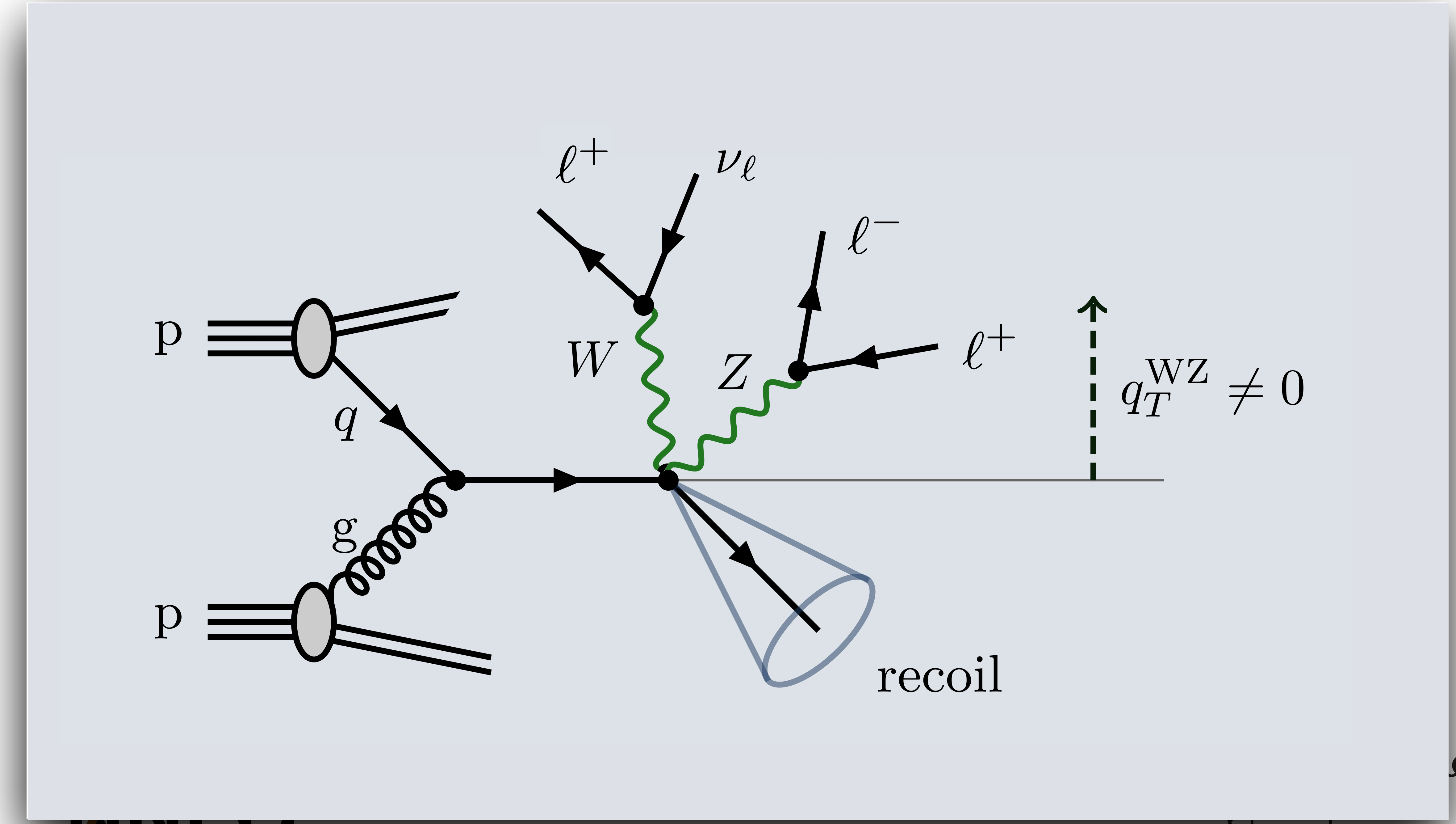


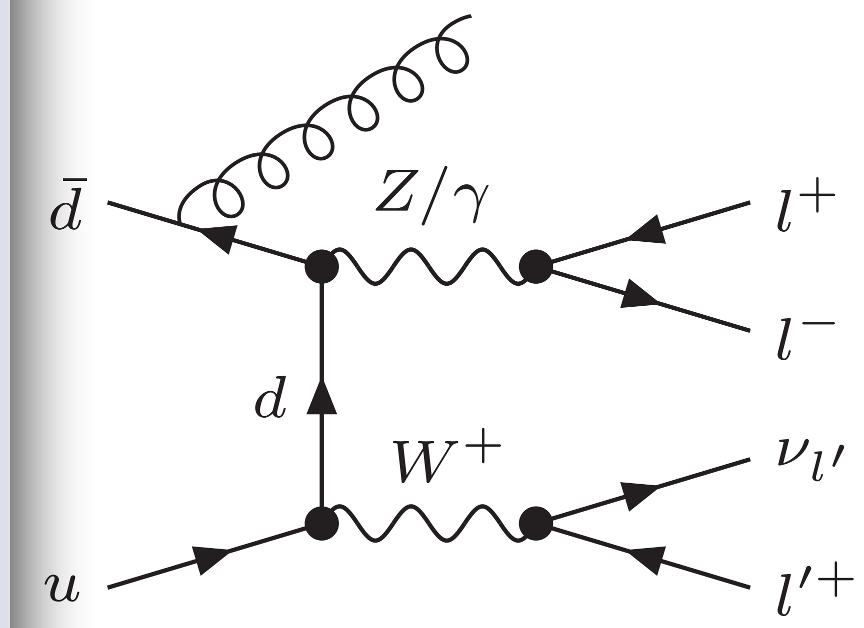
$\rightarrow d\sigma^{RR}$

NNLO through X+jet at NLO + Slicing

$$\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}} = \left[\int_{\Phi_{\mathrm{R}}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{RV+1}}} (\mathrm{d}\sigma^{\mathrm{RR}} - \mathrm{d}\sigma^{\mathrm{S}}) + \int_{\Phi_{\mathrm{RV}}} \left(\mathrm{d}\sigma^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)\right]_{\frac{q_T}{Q} \equiv r > r_{\mathrm{cut}}}$$

$$\xrightarrow{r_{\mathrm{cut}} \ll 1} \left[A \cdot \log^4(r_{\mathrm{cut}}) + B \cdot \log^3(r_{\mathrm{cut}}) + C \cdot \log^2(r_{\mathrm{cut}}) + D \cdot \log(r_{\mathrm{cut}})\right] \otimes \mathrm{d}\sigma^{\mathrm{B}}$$

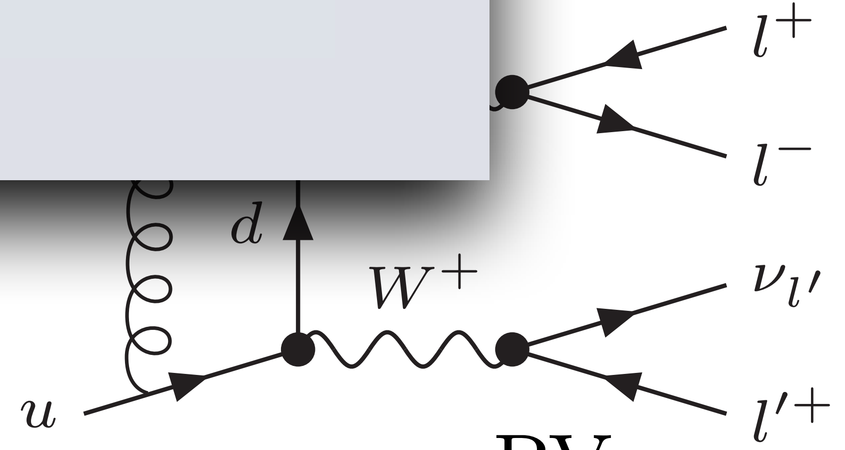




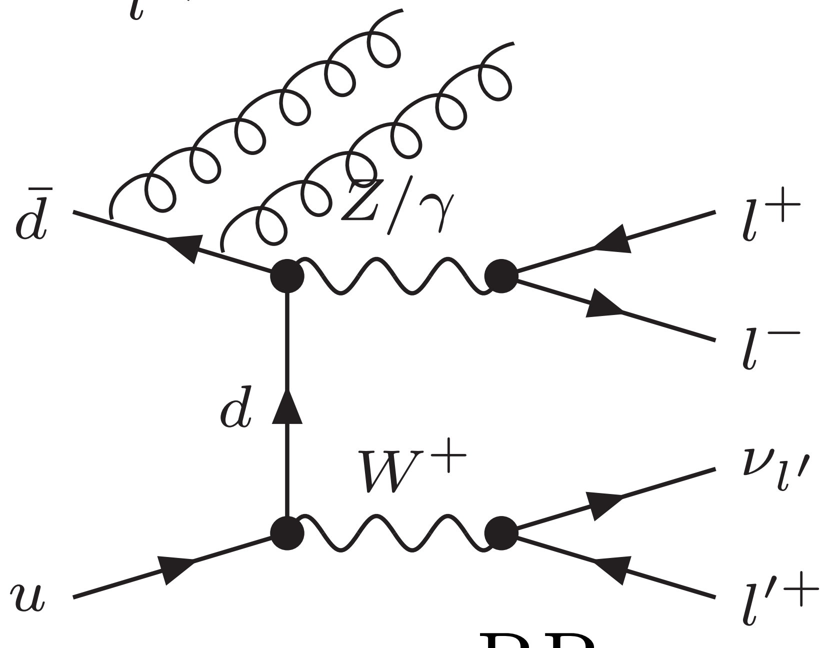
$\to \mathrm{d}\sigma^{\mathrm{R}}$

NNLO

(pp→WZ+jet)



$\to \mathrm{d}\sigma^{\mathrm{RV}}$



$\to \mathrm{d}\sigma^{\mathrm{RR}}$

NNLO through X+jet at NLO + Slicing

$$\sigma^{\rm X+jet}_{\rm NLO} = \left[\int_{\Phi_{\rm R}} {\rm d}\sigma^{\rm R} + \int_{\Phi_{\rm RV+1}} ({\rm d}\sigma^{\rm RR} - {\rm d}\sigma^{\rm S}) + \int_{\Phi_{\rm RV}} \left({\rm d}\sigma^{\rm RV} + \int_1 {\rm d}\sigma^{\rm S}\right)\right]_{\frac{q_T}{Q} \equiv r > r_{\rm cut}}$$

$$\xrightarrow{r_{\rm cut} \ll 1} \left[A \cdot \log^4(r_{\rm cut}) + B \cdot \log^3(r_{\rm cut}) + C \cdot \log^2(r_{\rm cut}) + D \cdot \log(r_{\rm cut})\right] \otimes {\rm d}\sigma^{\rm B}$$

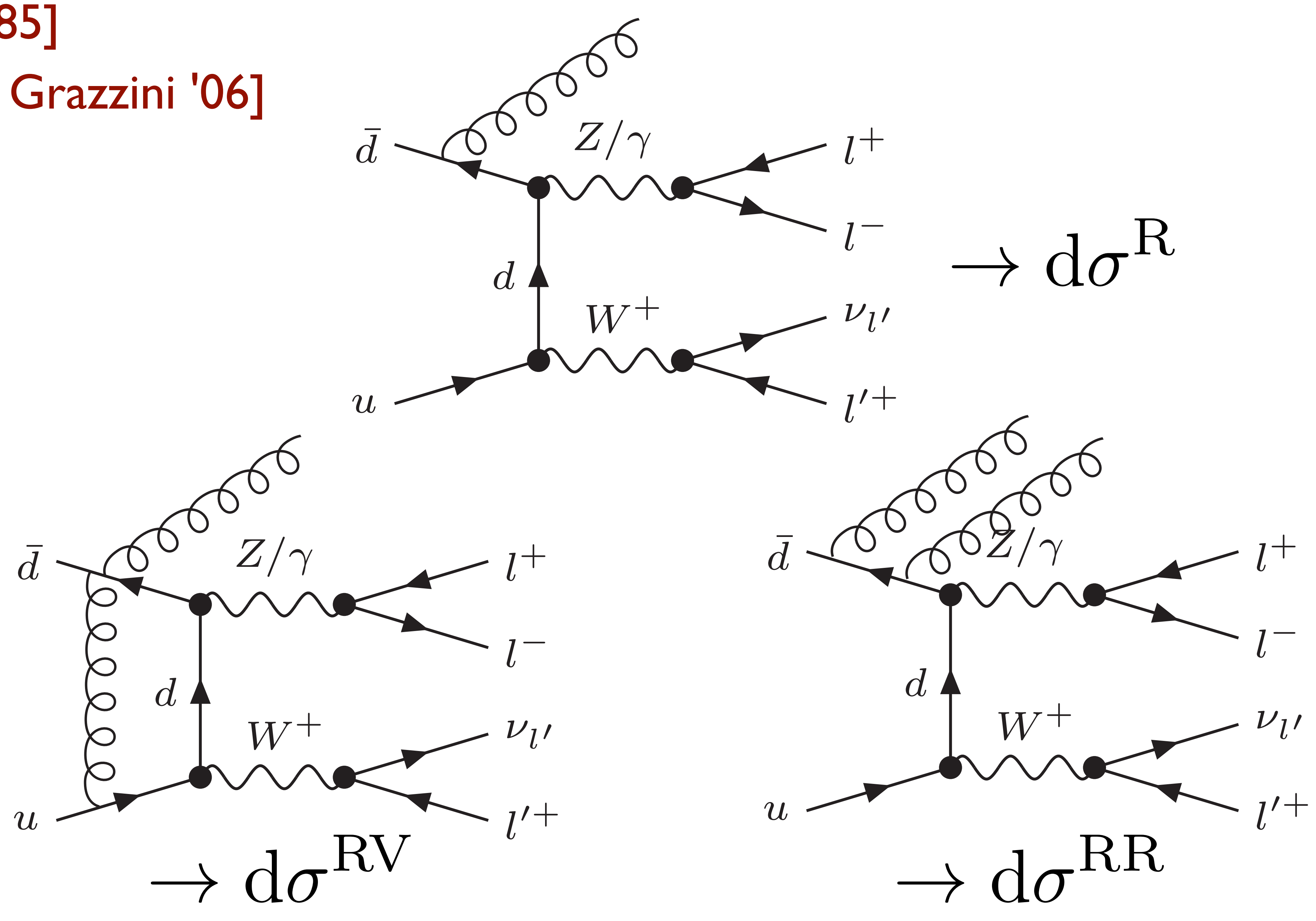
$$= \int_{r > r_{\rm cut}} \left[d\sigma^{\rm (res)}\right]_{\rm f.o.} \equiv \Sigma_{\rm NNLO}(r_{\rm cut}) \otimes {\rm d}\sigma^{\rm B}$$

[Collins, Soper, Sterman '85]
[Bozzi, Catani, de Florian, Grazzini '06]

~~LO~~ (pp→WZ)

~~N~~LO (pp→WZ+jet)

~~N~~NLO (pp→WZ+jet)



NNLO through X+jet at NLO + Slicing

$$\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}} = \left[\int_{\Phi_{\mathrm{R}}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{RV+1}}} (\mathrm{d}\sigma^{\mathrm{RR}} - \mathrm{d}\sigma^{\mathrm{S}}) + \int_{\Phi_{\mathrm{RV}}} \left(\mathrm{d}\sigma^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)\right]_{\frac{q_T}{Q} \equiv r > r_{\mathrm{cut}}}$$

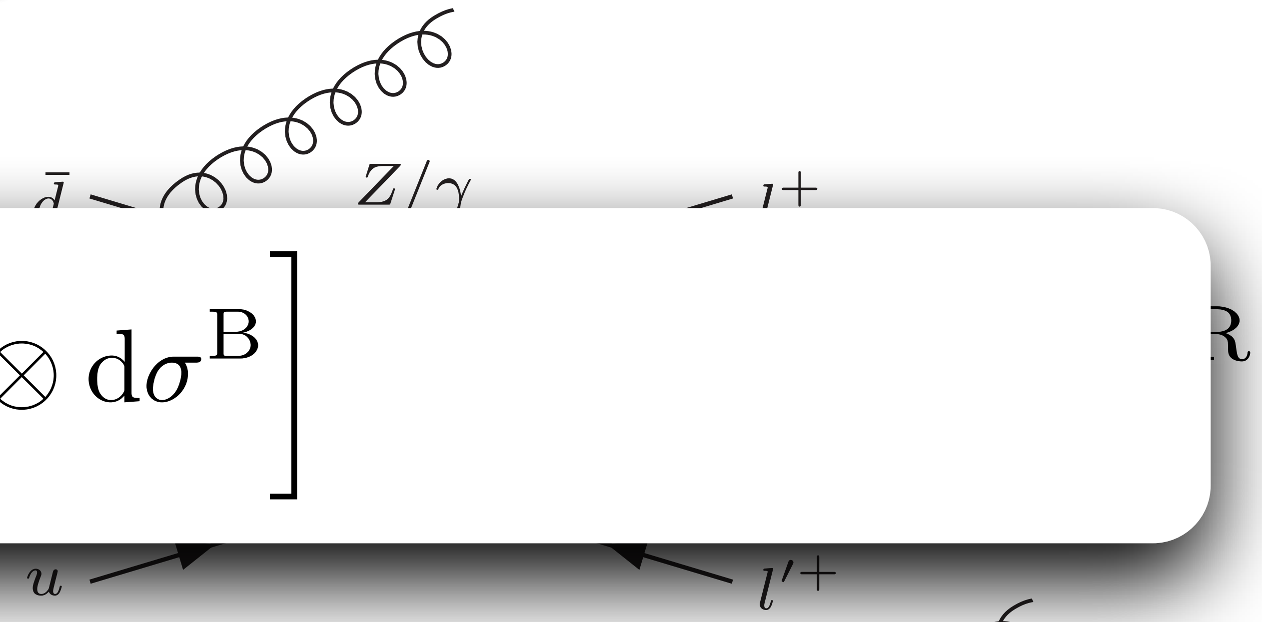
$$\xrightarrow{r_{\mathrm{cut}} \ll 1} \left[A \cdot \log^4(r_{\mathrm{cut}}) + B \cdot \log^3(r_{\mathrm{cut}}) + C \cdot \log^2(r_{\mathrm{cut}}) + D \cdot \log(r_{\mathrm{cut}})\right] \otimes \mathrm{d}\sigma^{\mathrm{B}}$$

$$= \int_{r > r_{\mathrm{cut}}} \left[d\sigma^{(\mathrm{res})}\right]_{\mathrm{f.o.}} \equiv \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes \mathrm{d}\sigma^{\mathrm{B}}$$

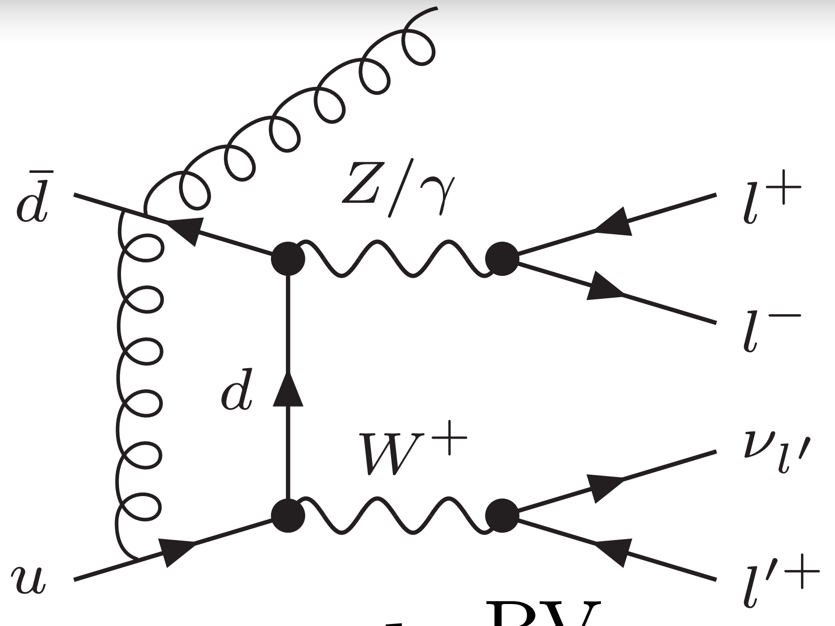
~~LO (pp→WZ)~~

$$\mathrm{d}\sigma^{\mathrm{X}}_{\mathrm{NNLO}} = \left[\mathrm{d}\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}}\Big|_{r > r_{\mathrm{cut}}} - \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes \mathrm{d}\sigma^{\mathrm{B}}\right]$$

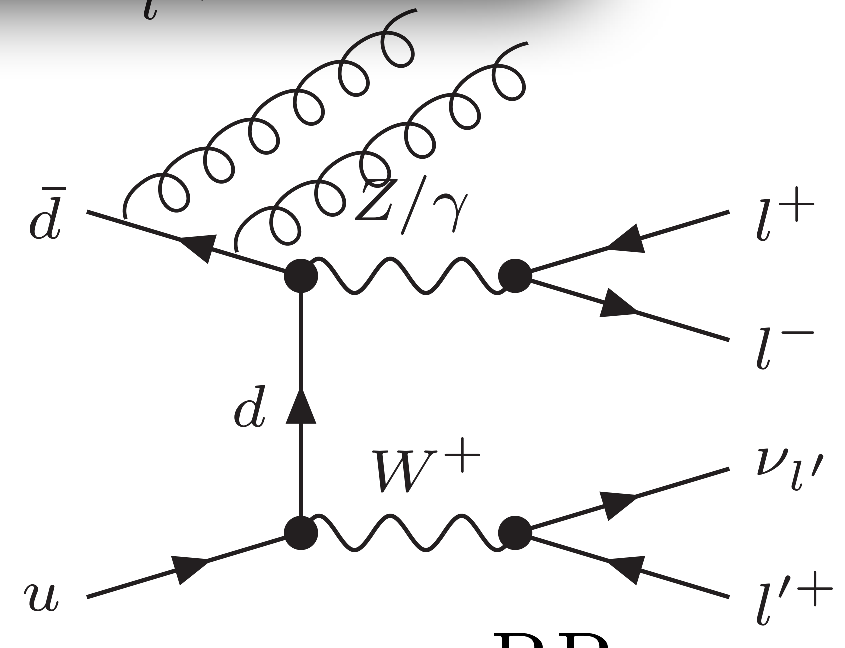
~~N~~L
(pp ... jet)



~~N~~NLO
(pp→WZ+jet)



$\to \mathrm{d}\sigma^{\mathrm{RV}}$



$\to \mathrm{d}\sigma^{\mathrm{RR}}$

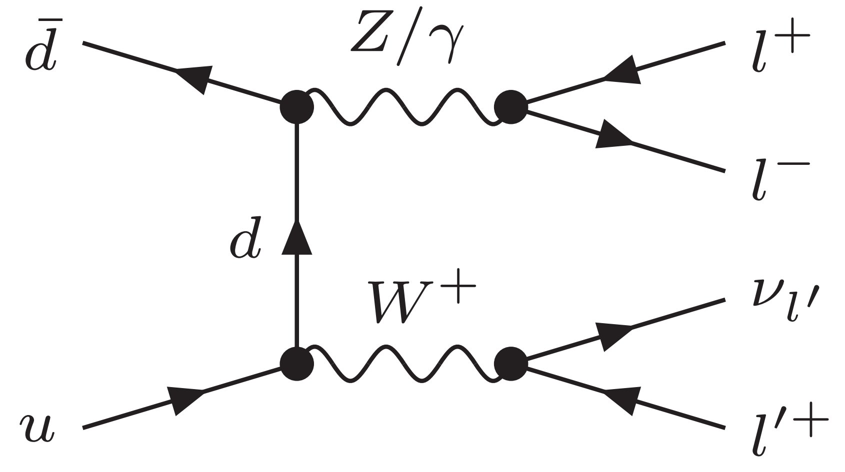
NNLO through X+jet at NLO + Slicing

$$\mathrm{d}\sigma^{\mathrm{X}}_{\mathrm{NNLO}} = \left[\mathrm{d}\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}} \Big|_{r > r_{\mathrm{cut}}} - \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes \mathrm{d}\sigma^{\mathrm{B}} \right] +$$

q_T subtraction

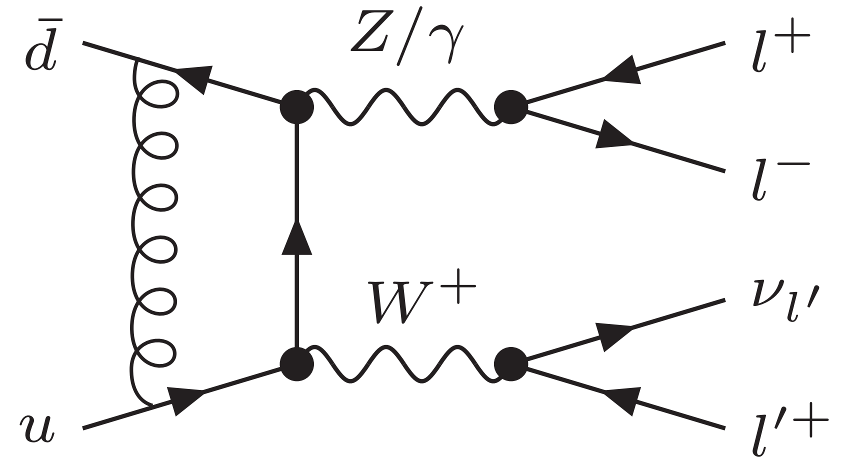
[Catani, Grazzini '07]

LO (pp→WZ)

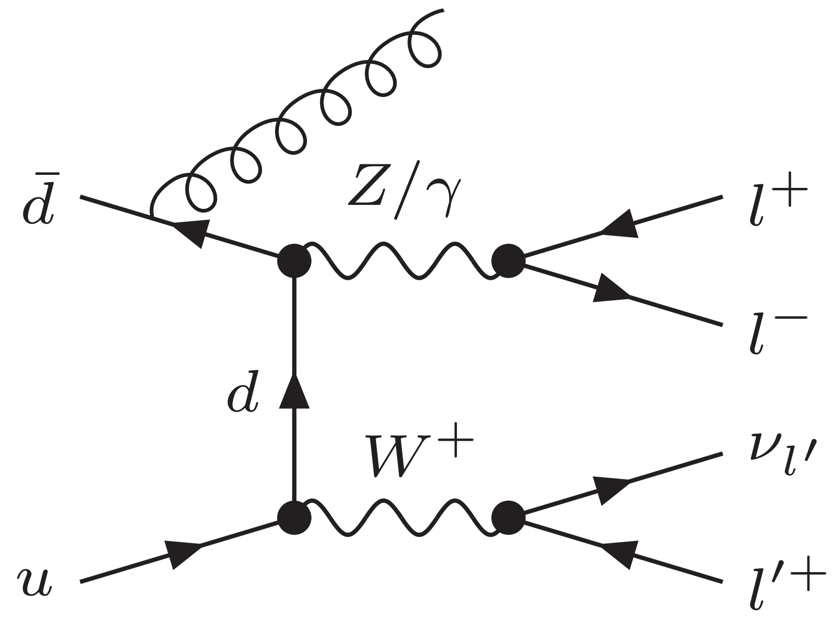


$\rightarrow \mathrm{d}\sigma^{\mathrm{B}}$

NLO (pp→WZ)

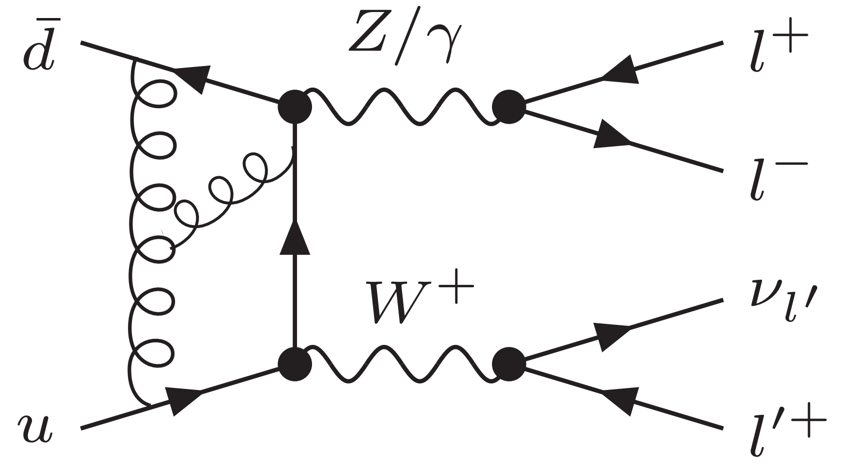


$\rightarrow \mathrm{d}\sigma^{\mathrm{V}}$

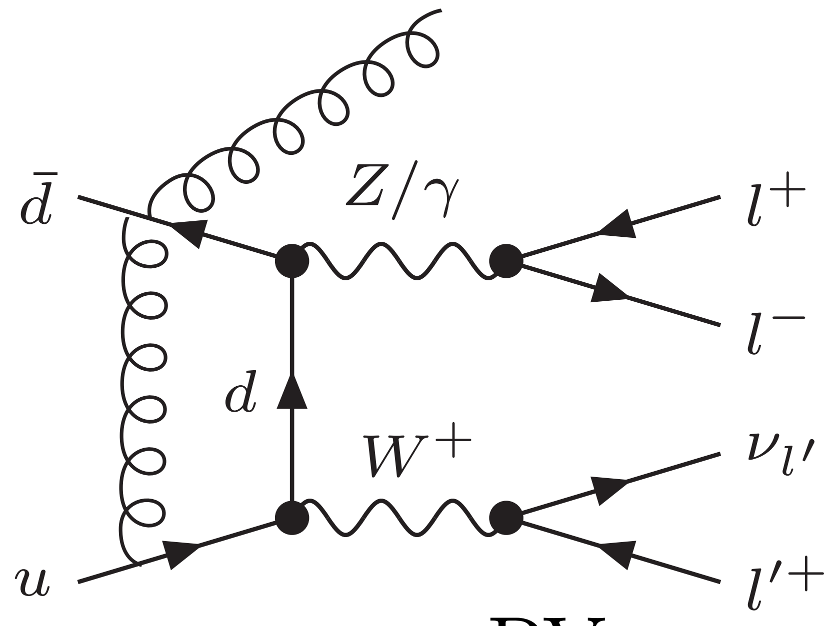


$\rightarrow \mathrm{d}\sigma^{\mathrm{R}}$

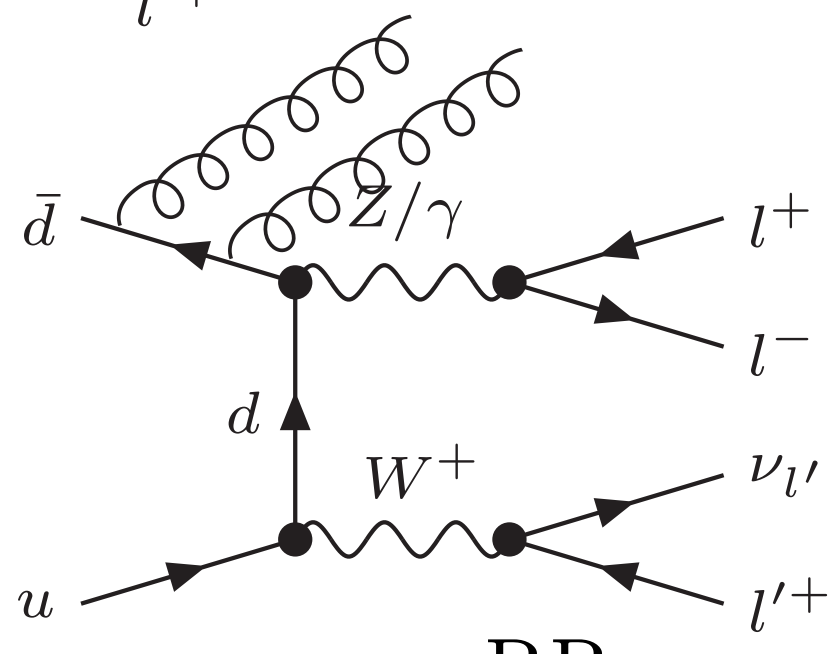
NNLO (pp→WZ)



$\rightarrow \mathrm{d}\sigma^{\mathrm{VV}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{RV}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{RR}}$

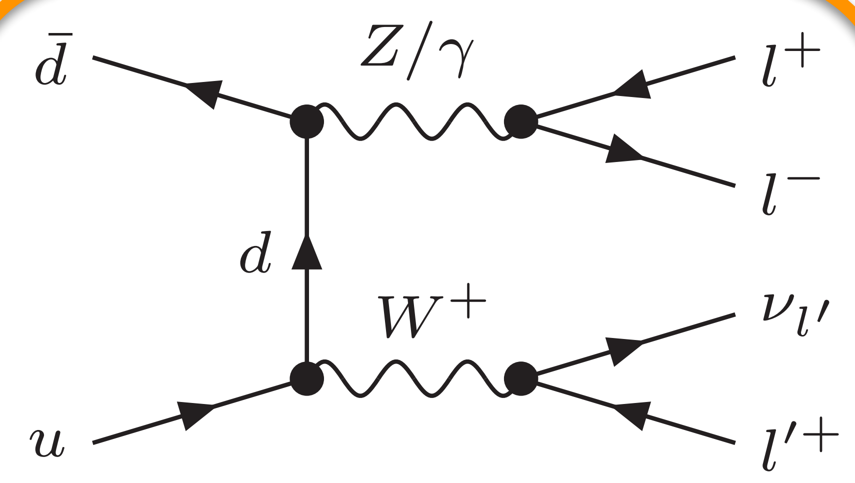
NNLO through X+jet at NLO + Slicing

$$d\sigma^{X}_{NNLO} = \left[d\sigma^{X+jet}_{NLO} \Big|_{r > r_{cut}} - \Sigma_{NNLO}(r_{cut}) \otimes d\sigma^{B} \right] + \mathcal{H}_{NNLO} \otimes d\sigma^{B}$$

q_T subtraction

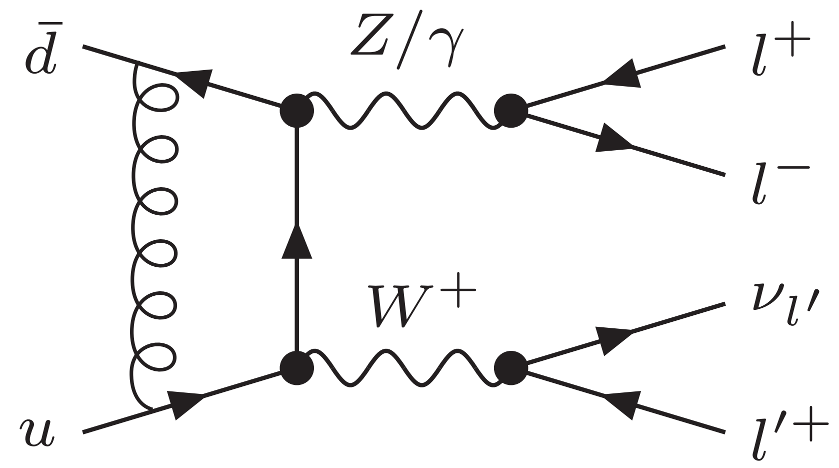
[Catani, Grazzini '07]

LO (pp→WZ)

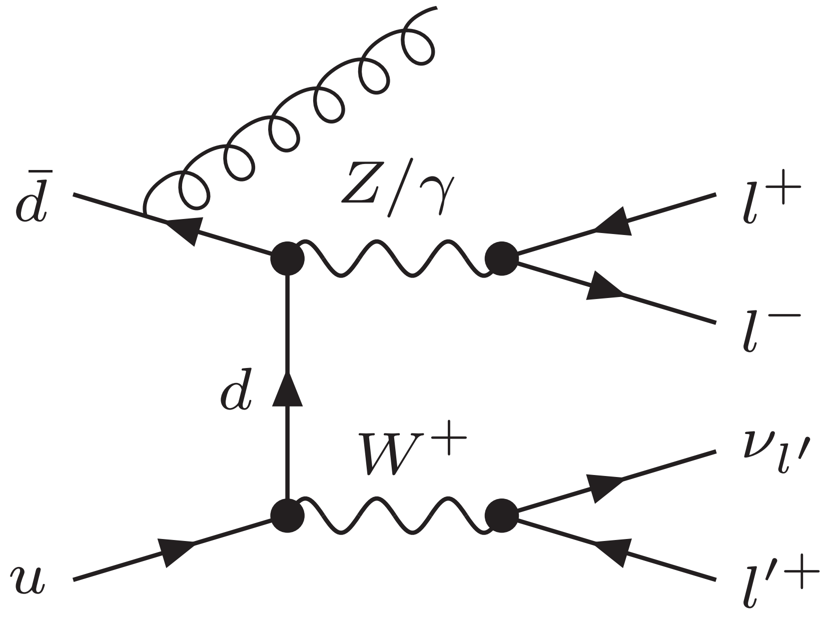


$\to d\sigma^{B}$

NLO (pp→WZ)

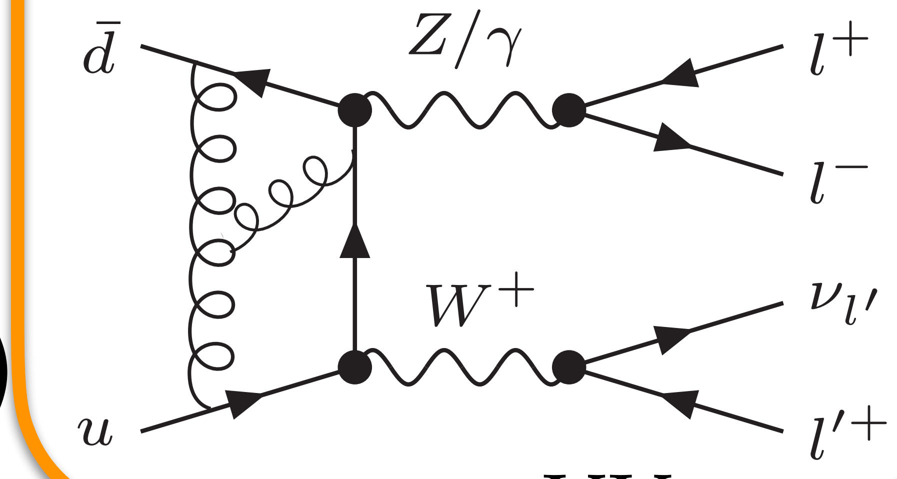


$\to d\sigma^{V}$

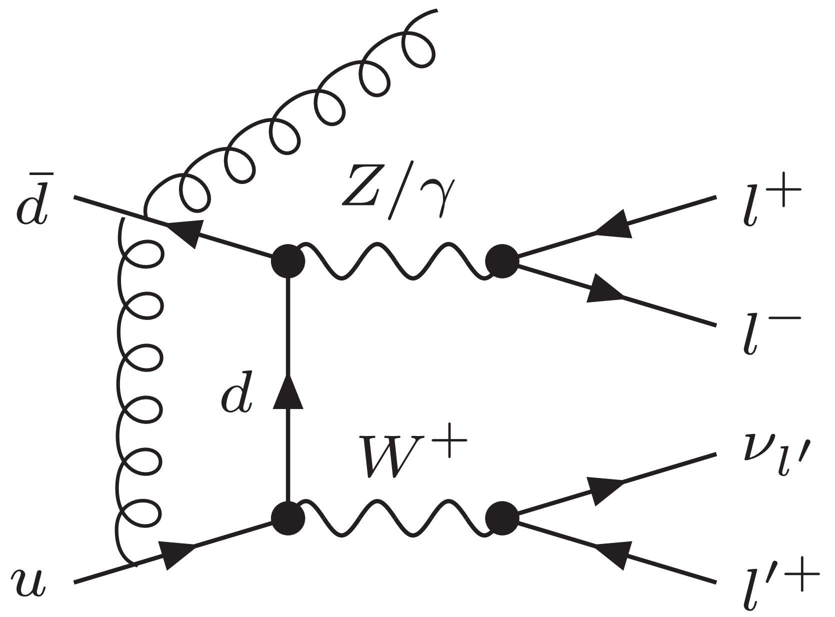


$\to d\sigma^{R}$

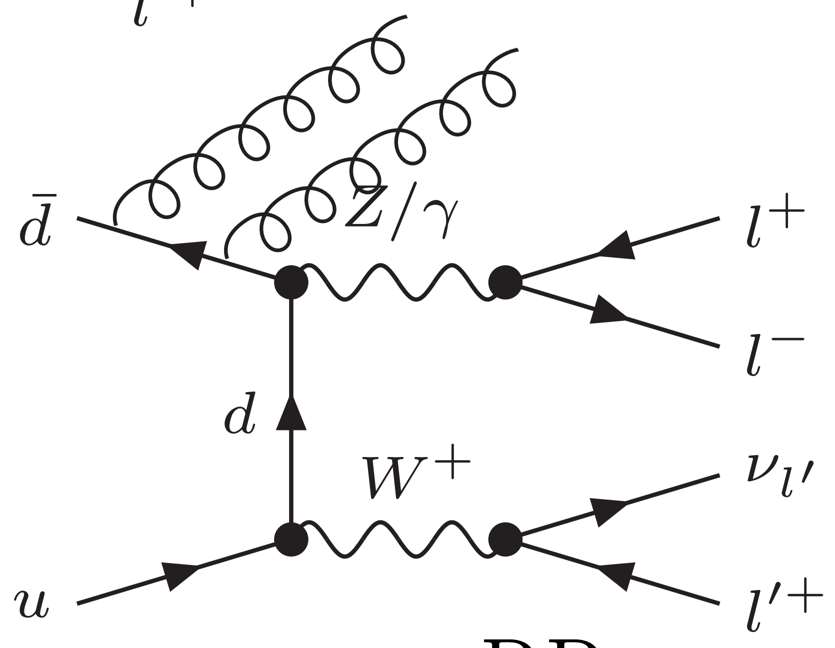
NNLO (pp→WZ)



$\to d\sigma^{VV}$



$\to d\sigma^{RV}$

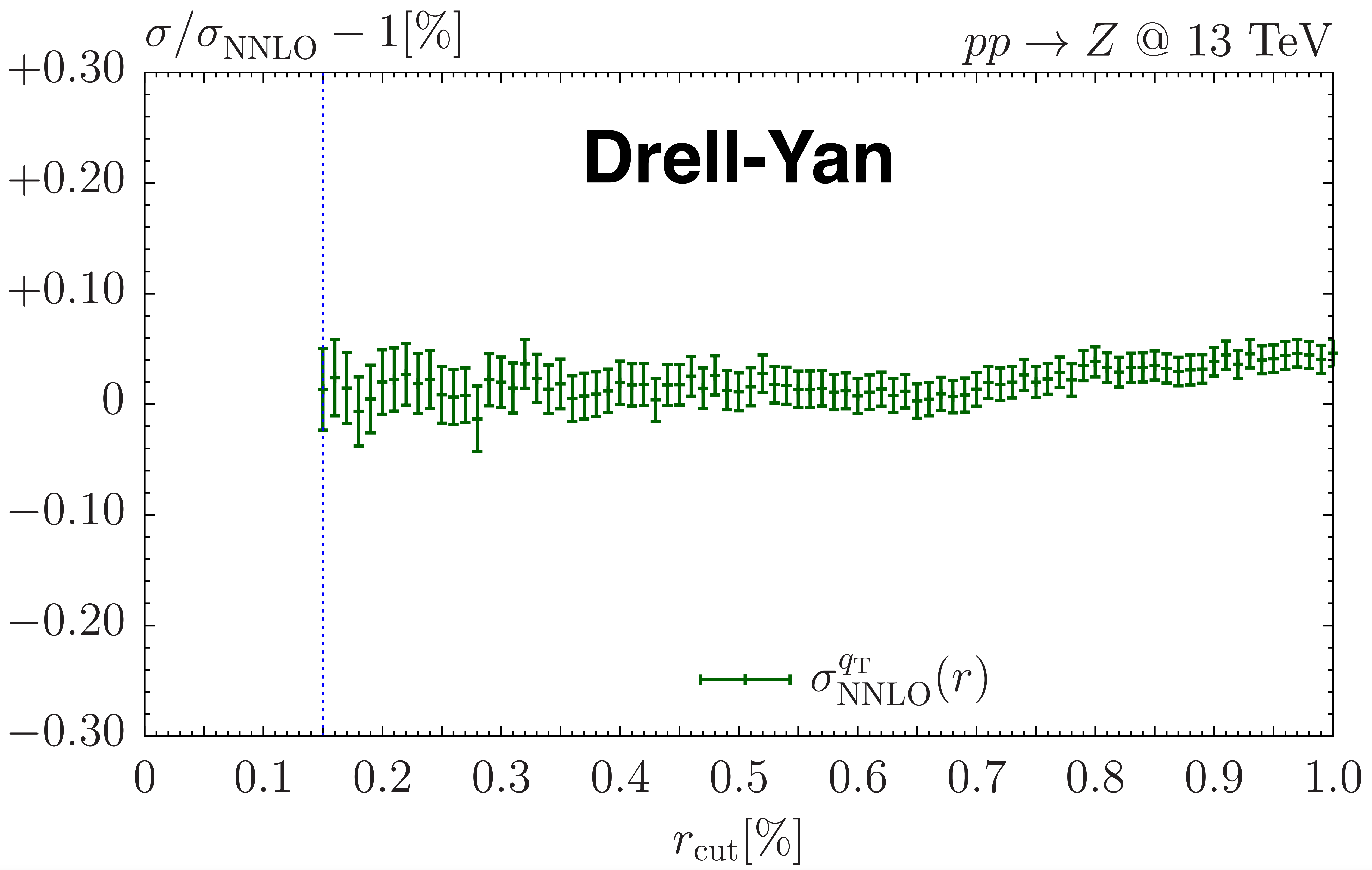


$\to d\sigma^{RR}$

$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

automatically computed in every single MATRIX NNLO run

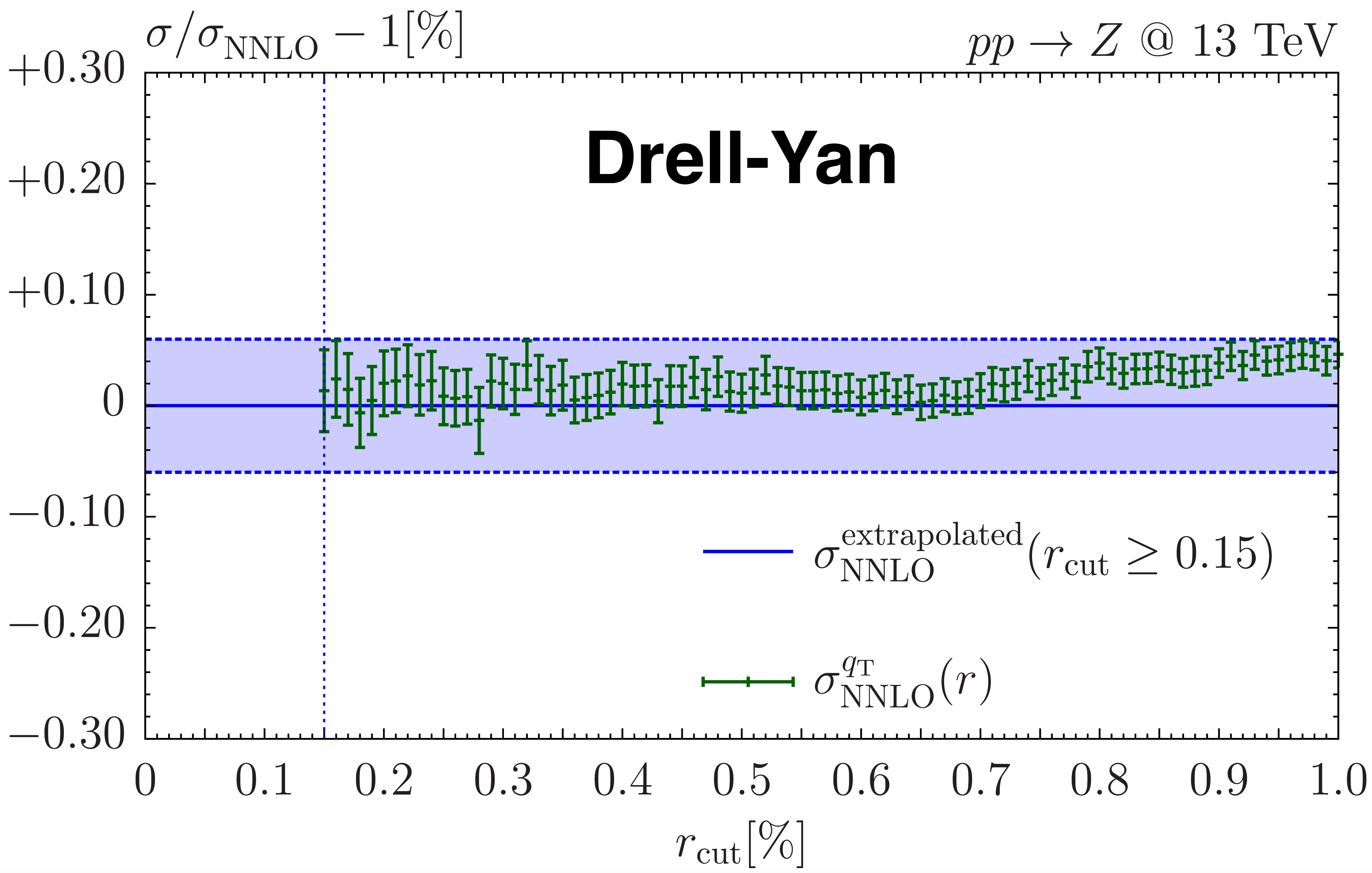


$$\mathrm{d}\sigma^{\mathrm{X}}_{\mathrm{NNLO}} = \left[\mathrm{d}\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}}\Big|_{r>r_{\mathrm{cut}}} - \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes \mathrm{d}\sigma^{\mathrm{B}}\right] + \mathcal{H}_{\mathrm{NNLO}} \otimes \mathrm{d}\sigma^{\mathrm{B}}$$

$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

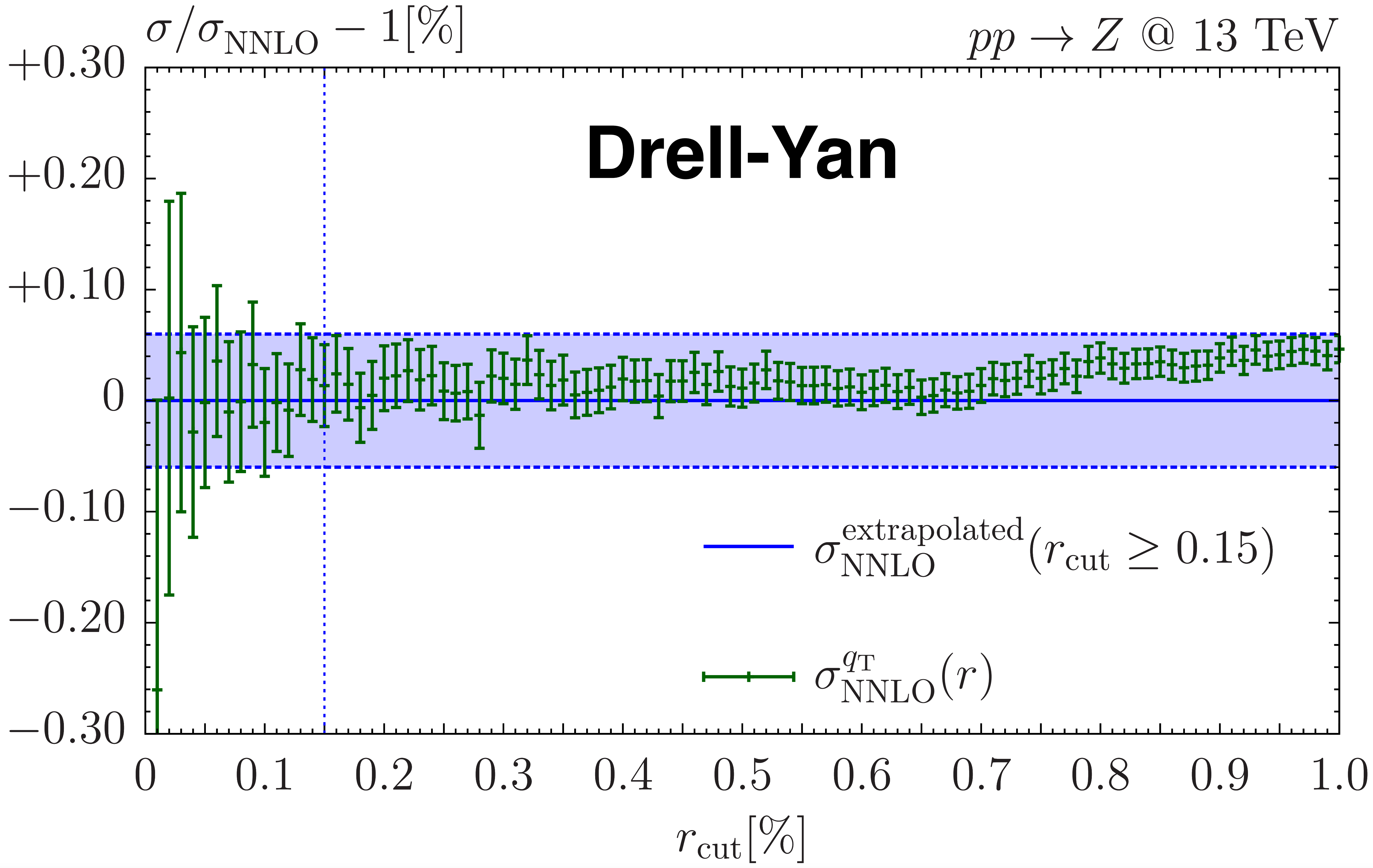
simple quadratic fit ($A * r_{cut}^2 + B * r_{cut} + C$) to extrapolate to r_{cut}=0



$$d\sigma^{X}_{NNLO} = \left[d\sigma^{X+jet}_{NLO}\Big|_{r>r_{cut}} - \Sigma_{NNLO}(r_{cut}) \otimes d\sigma^{B} \right] + \mathcal{H}_{NNLO} \otimes d\sigma^{B}$$

$r_{cut} \rightarrow 0$ extrapolation in MATRIX

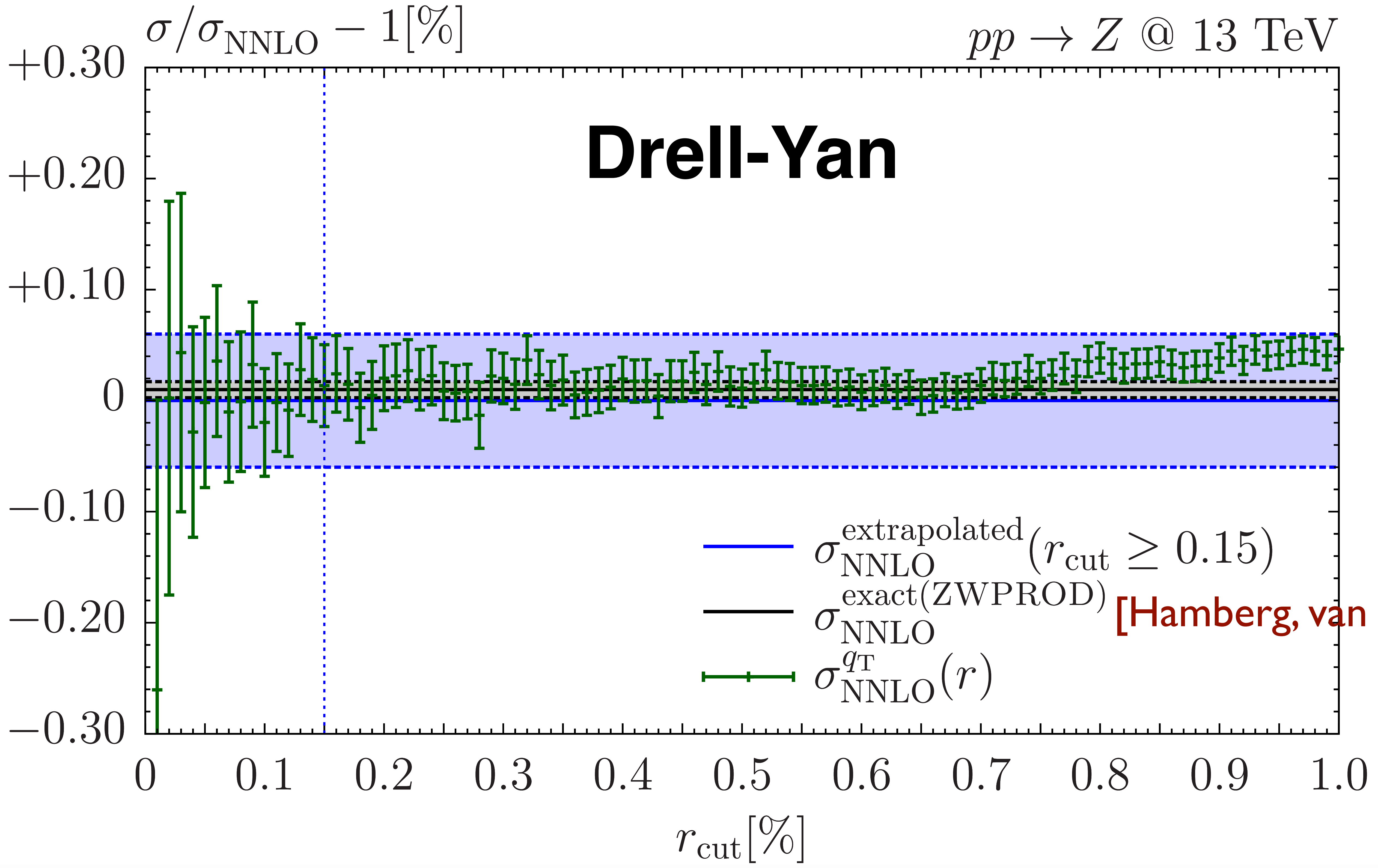
[Grazzini, Kallweit, MW '17]



$$\mathrm{d}\sigma^{\mathrm{X}}_{\mathrm{NNLO}} = \left[\mathrm{d}\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}}\Big|_{r > r_{\mathrm{cut}}} - \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes \mathrm{d}\sigma^{\mathrm{B}}\right] + \mathcal{H}_{\mathrm{NNLO}} \otimes \mathrm{d}\sigma^{\mathrm{B}}$$

$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

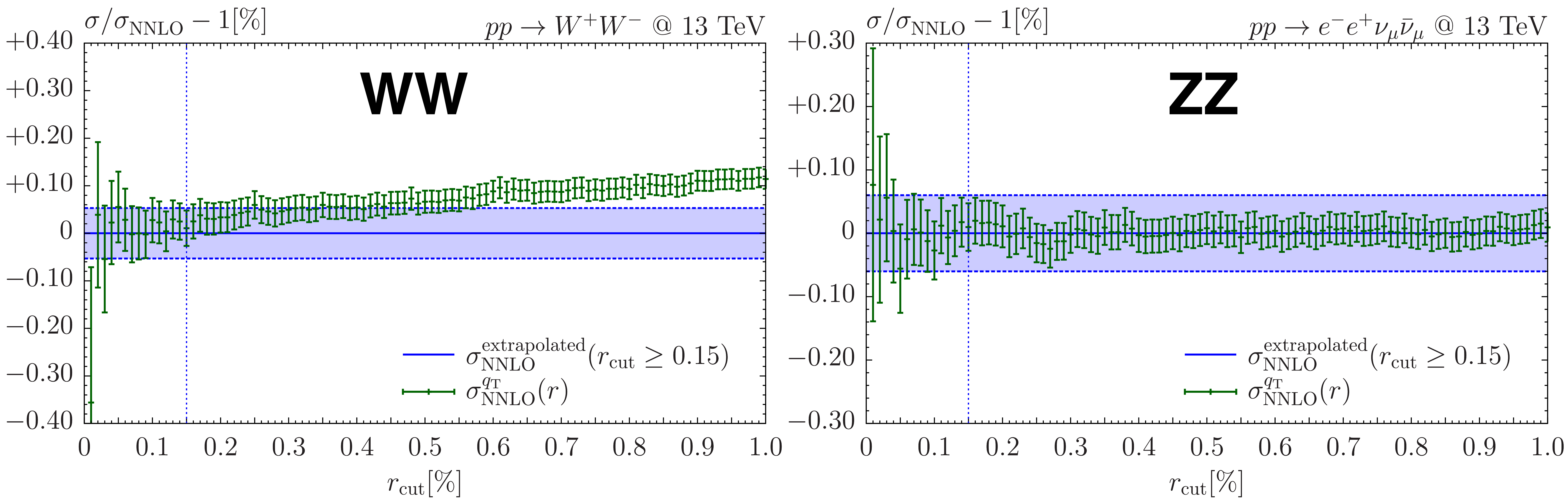


[Hamberg, van Neerven, Matsuura '91]

$$d\sigma^{\rm X}_{\rm NNLO} = \left[d\sigma^{\rm X+jet}_{\rm NLO} \Big|_{r > r_{\rm cut}} - \Sigma_{\rm NNLO}(r_{\rm cut}) \otimes d\sigma^{\rm B} \right] + \mathcal{H}_{\rm NNLO} \otimes d\sigma^{\rm B}$$

$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]



Questions?

Explosion of NNLO results

...slide borrowed from Gavin Salam

NNLO QCD timeline

[based on slide by M. Grazzini at QCD@LHC 2019 and an update by Alexander Huss LH@2021]

Example #1: R-ratio

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \qquad [\alpha_s \equiv \alpha_s(\sqrt{s_{e^+e^-}})]$$

$$= R_0 \left(1 + 0.32\alpha_s + 0.14\alpha_s^2 - 0.47\alpha_s^3 - 0.59316\alpha_s^4 + \cdots\right)$$

Baikov et al., 1206.1288
(numbers for γ-exchange only)

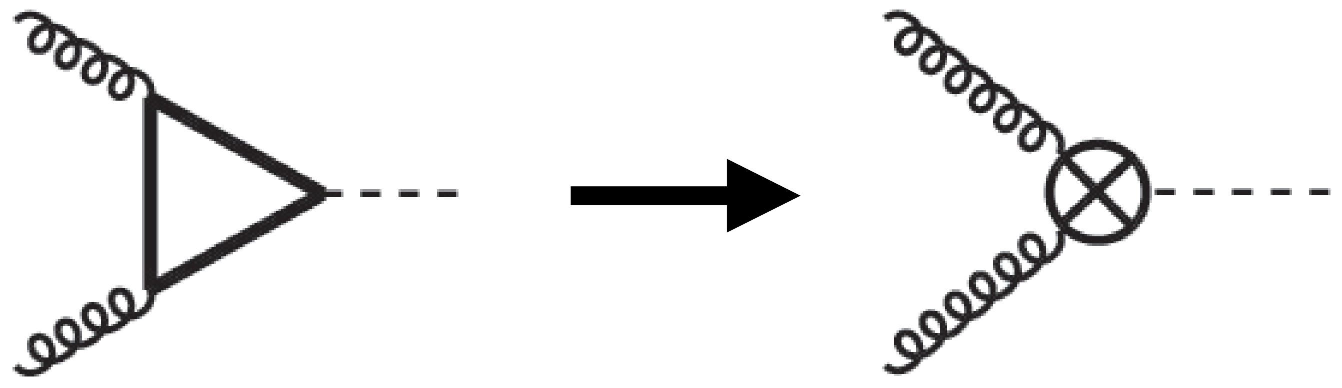
This is one of the few quantities calculated to N4LO

Good convergence of the series at every order
(at least for $\alpha_s(M_Z) = 0.118$)

...slide borrowed from Gavin Salam

Example #2: Higgs production

$$\sigma(pp \to H) = (961\,\mathrm{pb}) \times (\alpha_s^2 + 10.4\alpha_s^3 + 38\alpha_s^4 + 48\alpha_s^5 + \cdots)$$



$\alpha_s \equiv \alpha_s(M_H/2)$

$\sqrt{s_{pp}} = 13\,\mathrm{TeV}$

Anastasiou et al., 1602.00695 (ggF, hEFT)

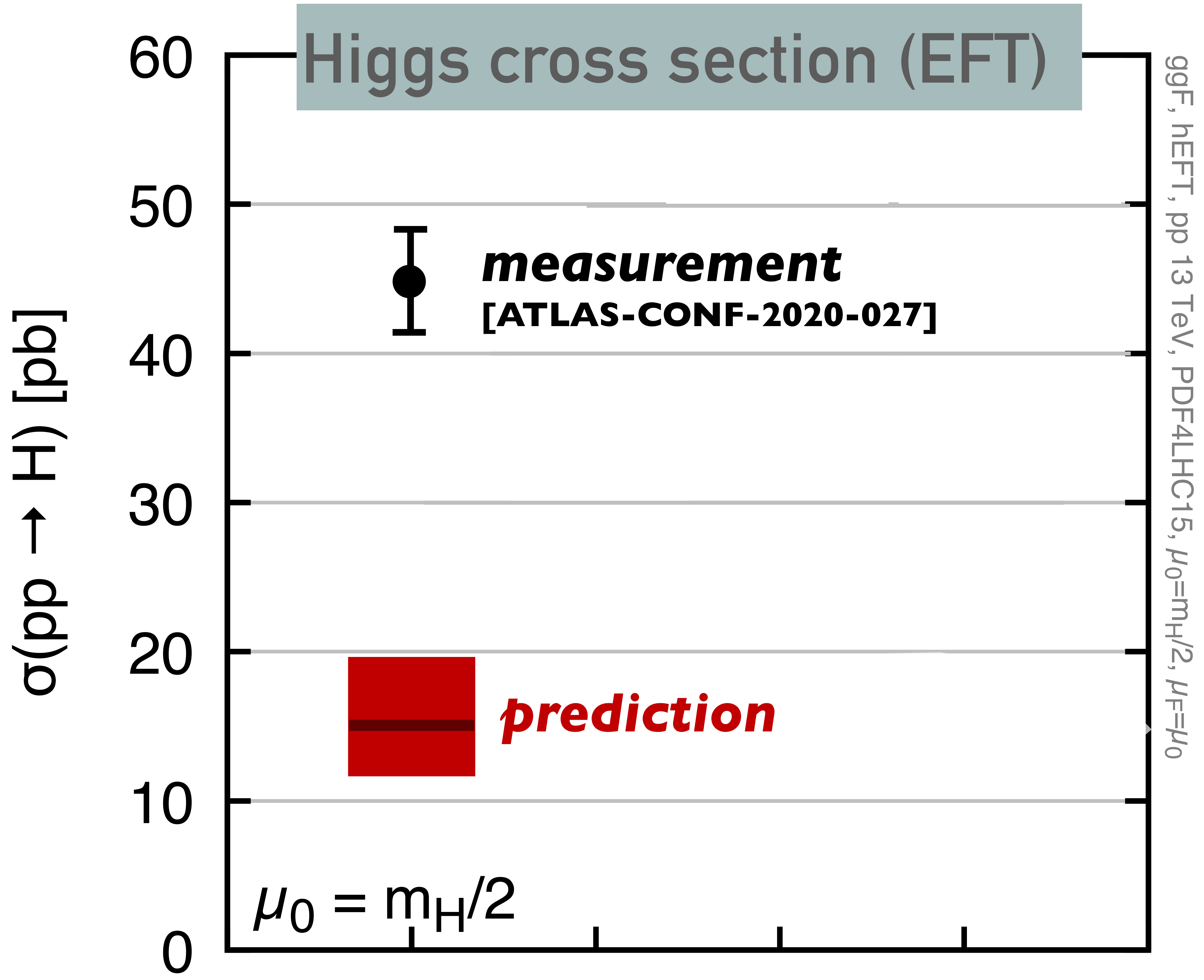
pp→H (via gluon fusion) is one of only few hadron-collider processes known at N3LO

The series does not converge well

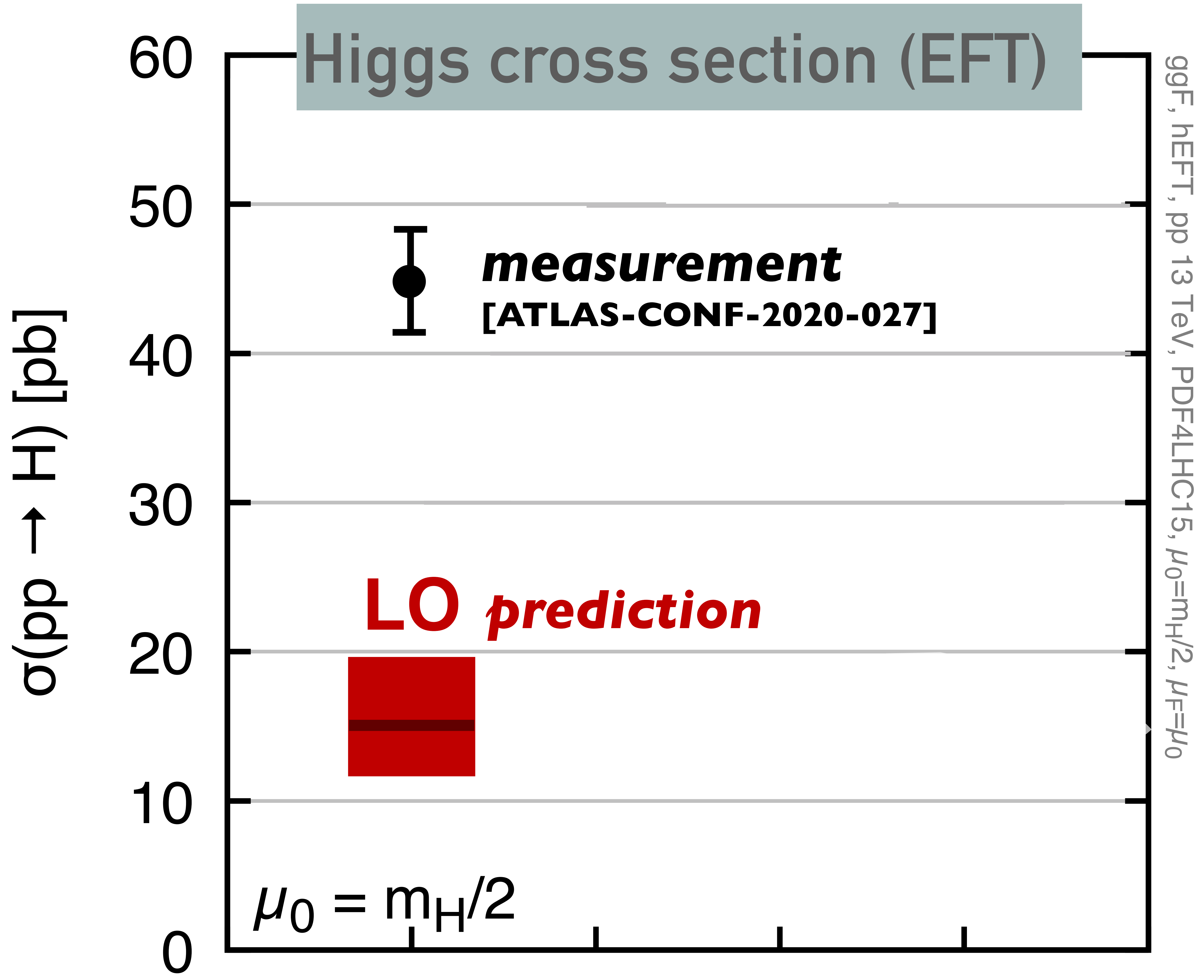
(explanations for why are only moderately convincing)

...slide borrowed from Gavin Salam

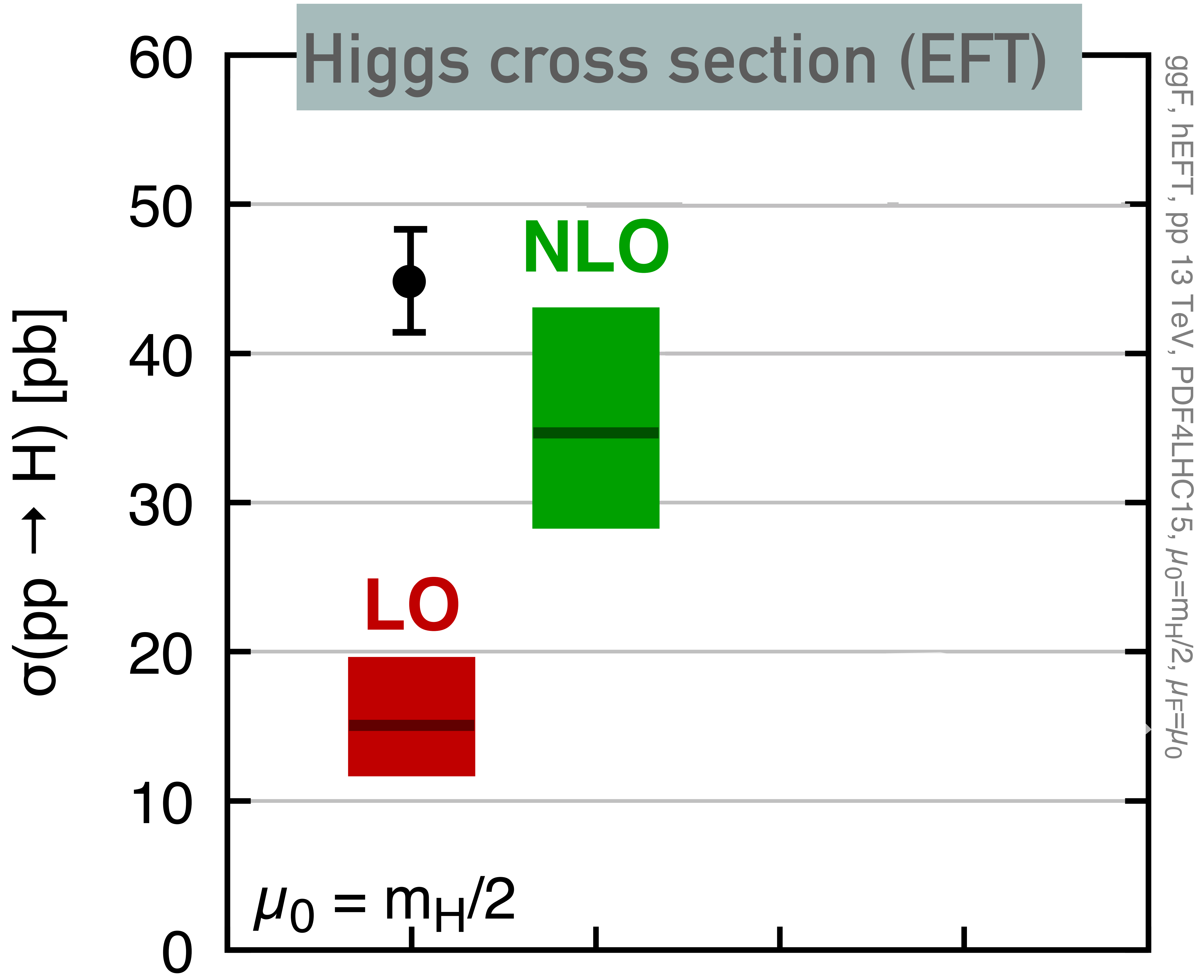
Example #2: Higgs production



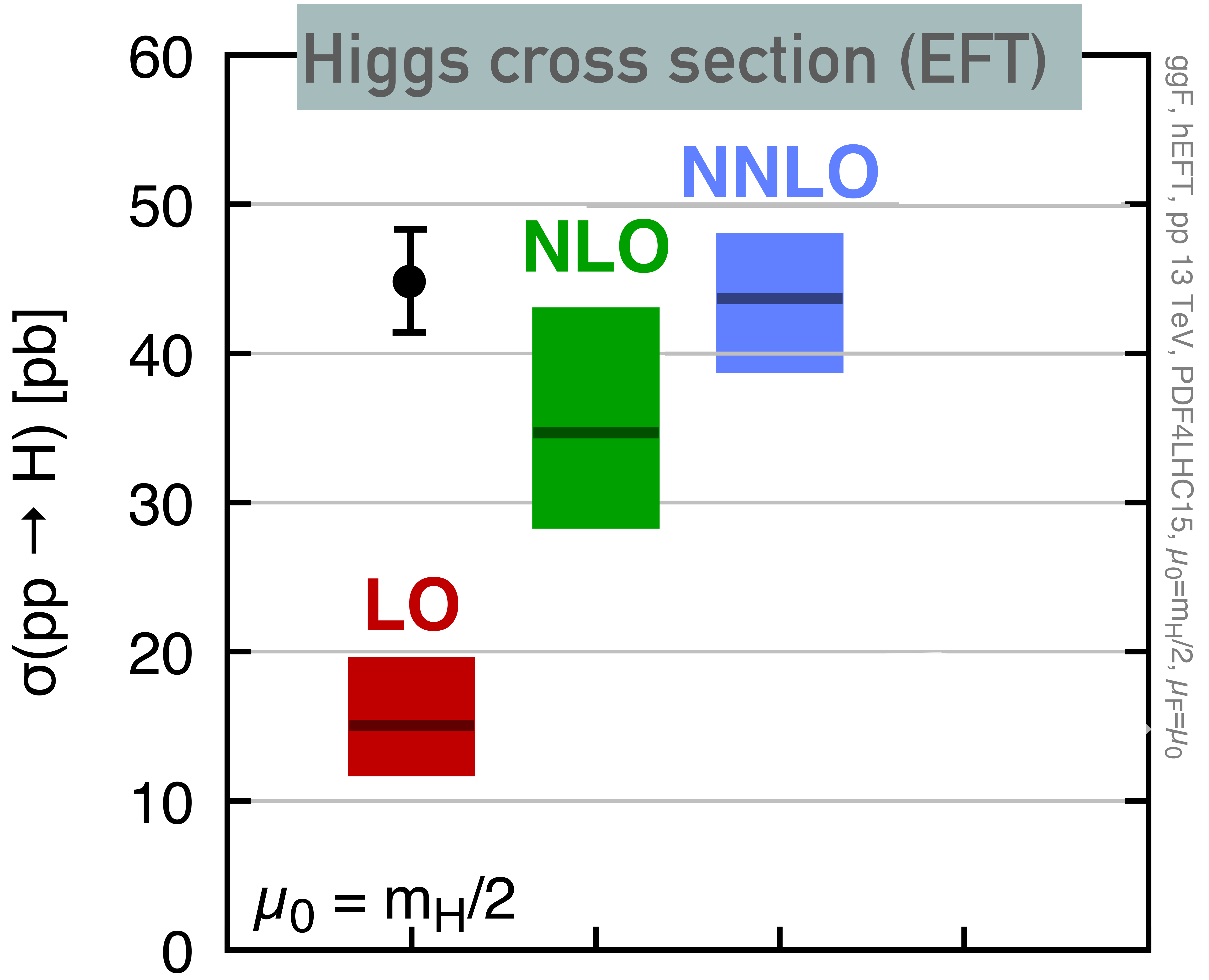
Example #2: Higgs production



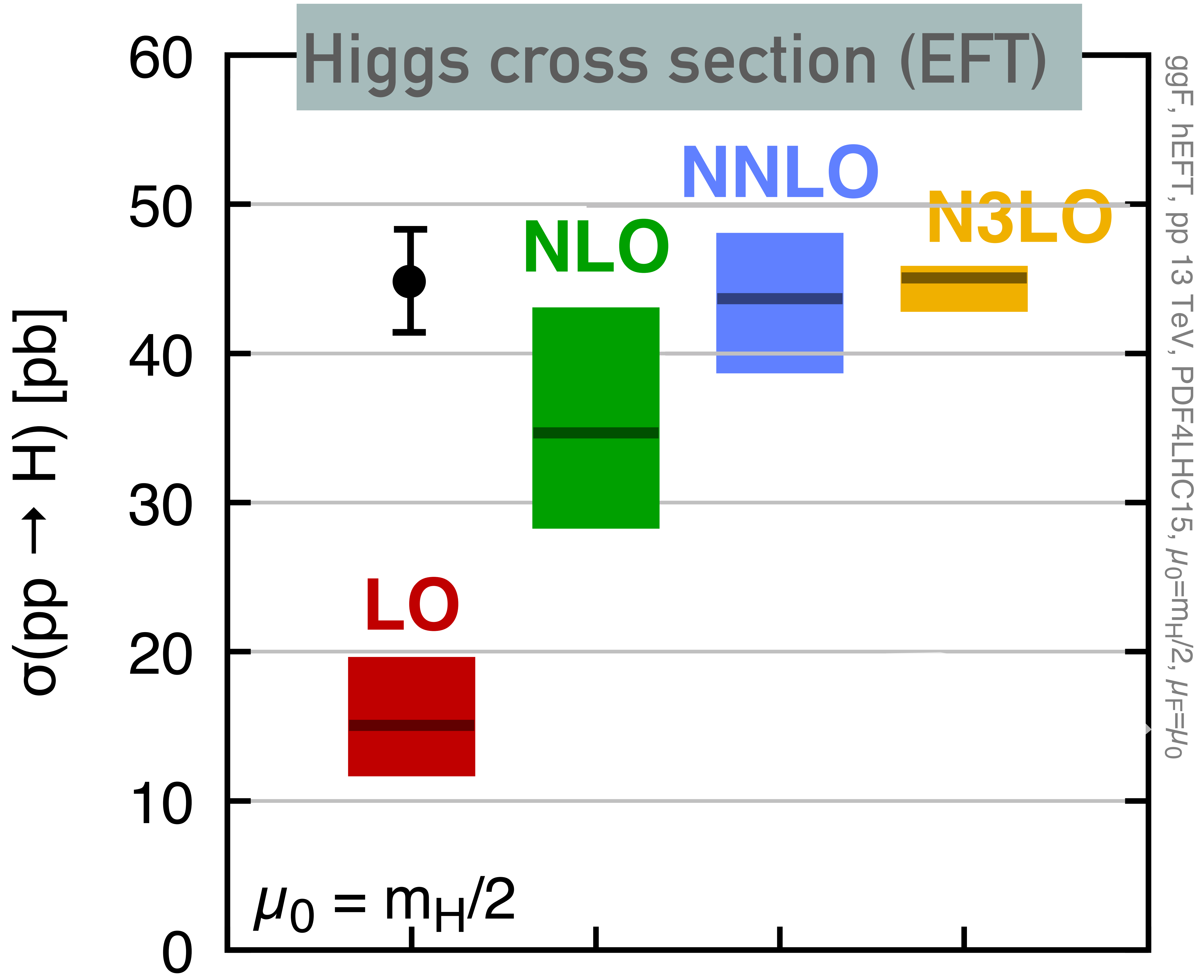
Example #2: Higgs production



Example #2: Higgs production



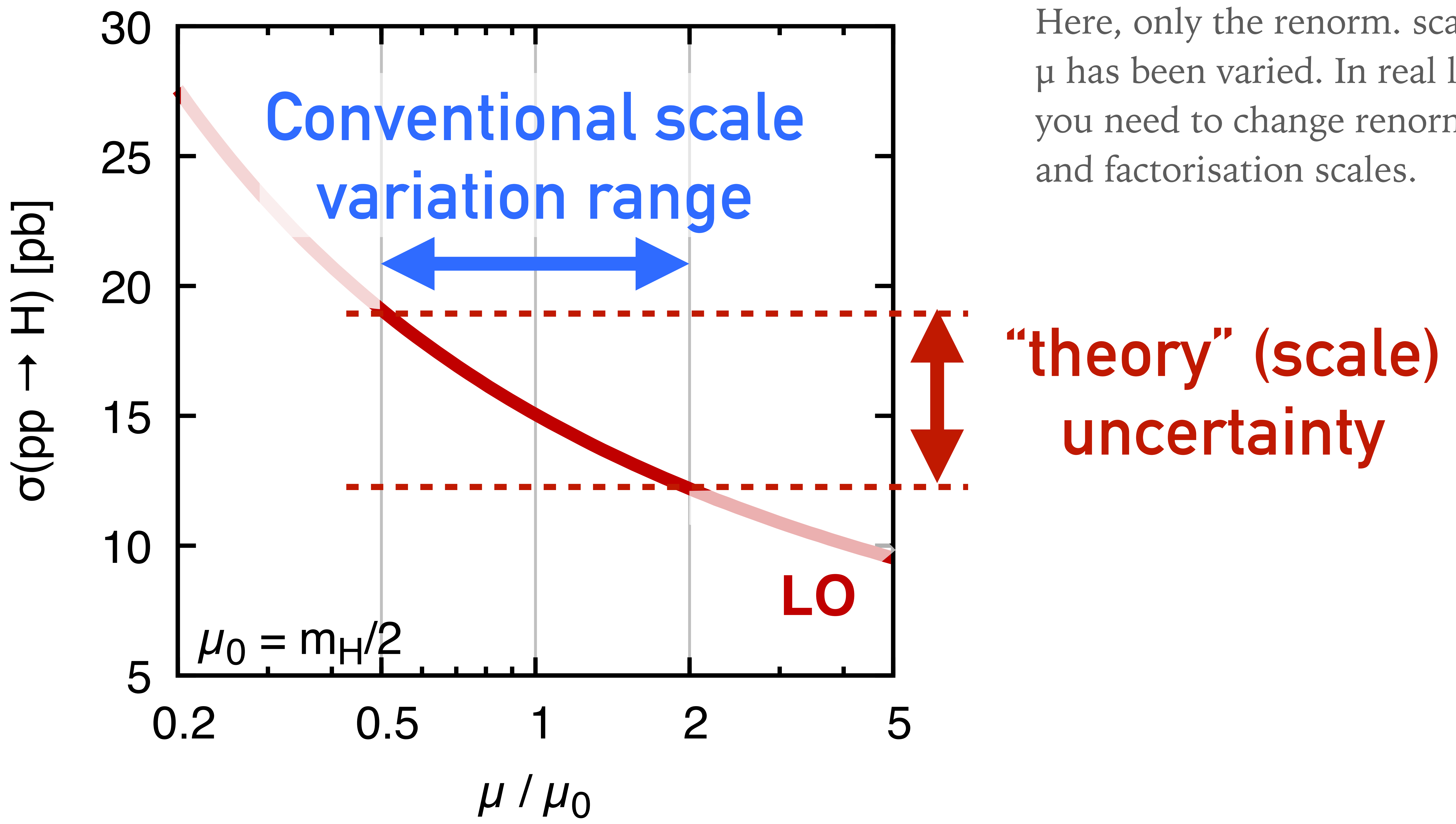
Example #2: Higgs production



[Anastasiou et al. '15], [Mistlberger '18]

→ see Bernhards's lecture

Scale dependence as the "THEORY UNCERTAINTY"



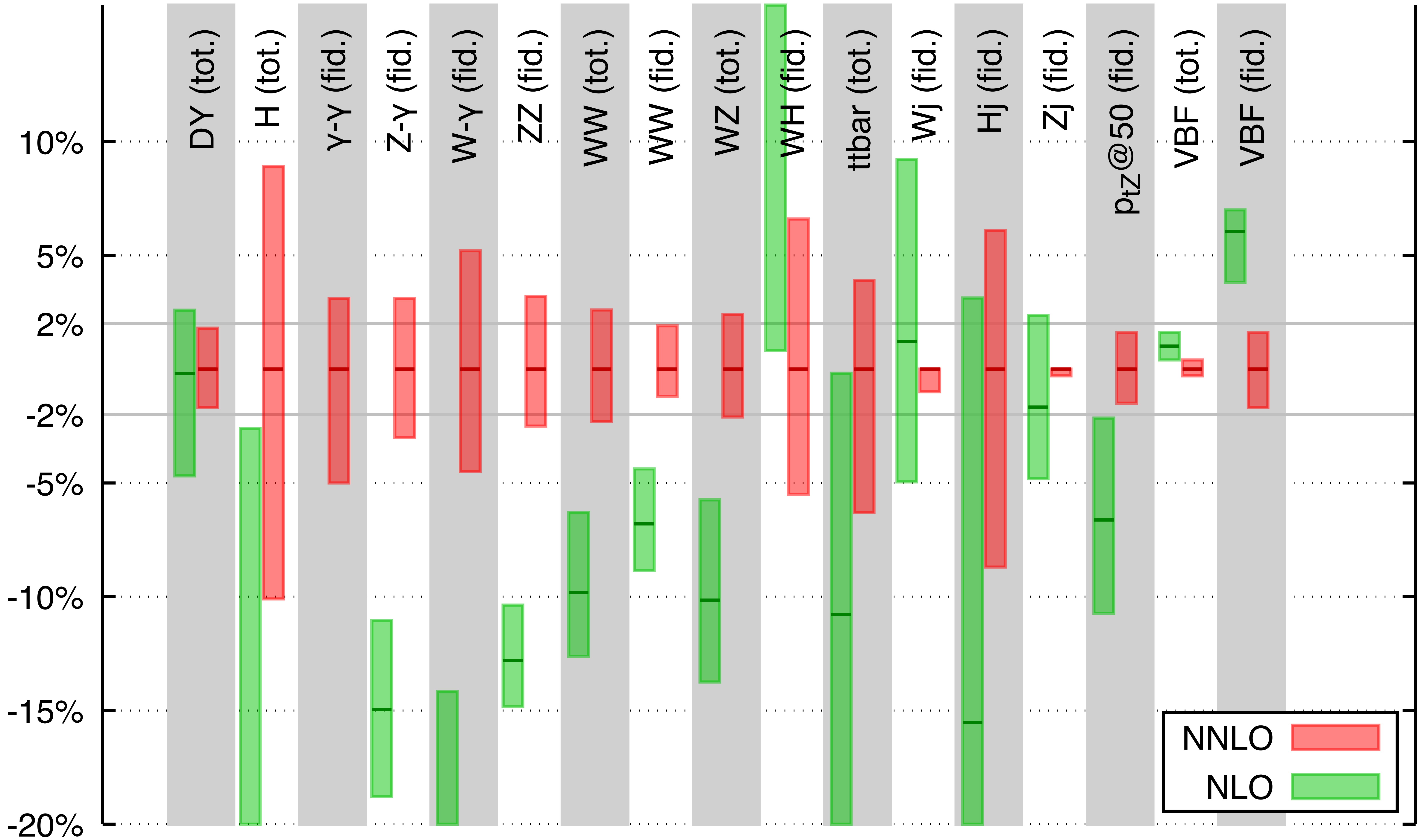
Here, only the renorm. scale μ has been varied. In real life you need to change renorm. and factorisation scales.

Convention: "theory uncertainty" (i.e. from missing higher orders) is estimated by change of cross section when varying μ in range 1/2 → 2 around central value

13

...slide borrowed from Gavin Salam

Precision at NNLO



For many processes NNLO scale band is ~±2%
But only in 3/17 cases is NNLO (central) within NLO scale band…

...slide borrowed from Gavin Salam

NNLO frontier 2 ➞ 3 processes

- massless/one mass (full 2-loop):

- **pp→γγγ** [Chawdhry, Czakon, Mitov, Poncelet '19], [Kallweit, Sotnikov, MW '20]
- **pp→γγ+jet** [Chawdhry, Czakon, Mitov, Poncelet '21]
- **pp→3-jet** [Czakon, Mitov, Poncelet '21]
- **pp→bbW** (m_b=0) [Hartano, Poncelet, Popescu, Zoia '22]
- **pp→γ+2-jet** [Badger, Czakon, Hartano et al. '23]

- massive (with approximated 2-loop):

- **pp→ttH** (soft approx.) [Catani, Devoto, Grazzini et al. '22]
- **pp→bbW** (small m_b) [Buonocore, Devoto, Grazzini et al. '23]
- **pp→ttW** (both) [Buonocore, Devoto, Kallweit et al. '22]

Example #3

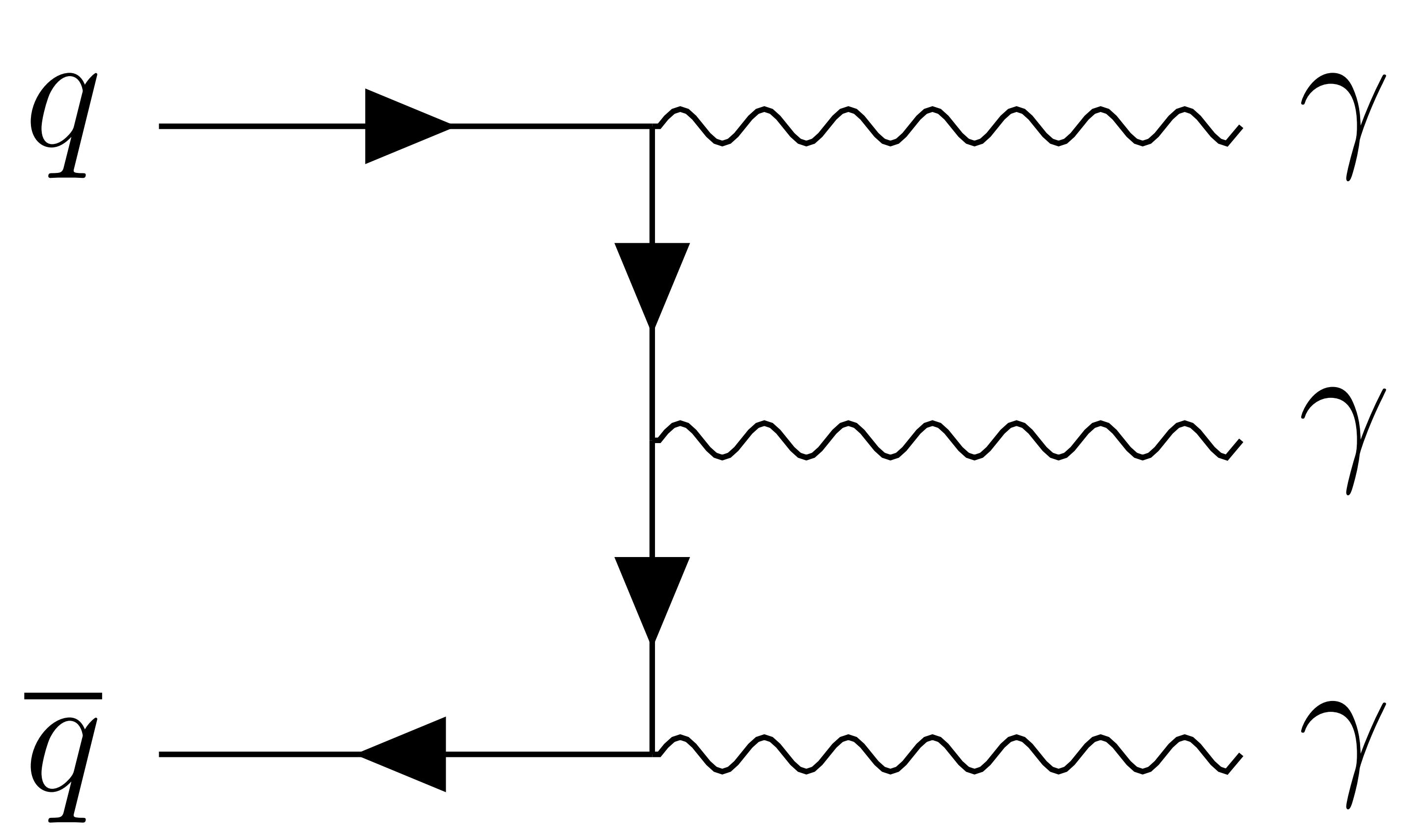
- massless/one mass (full 2-loop):

- **pp→γγγ** [Chawdhry, Czakon, Mitov, Poncelet '19], [Kallweit, Sotnikov, MW '20]
- **pp→γγ+jet** [Chawdhry, Czakon, Mitov, Poncelet '21]
- **pp→3-jet** [Czakon, Mitov, Poncelet '21]

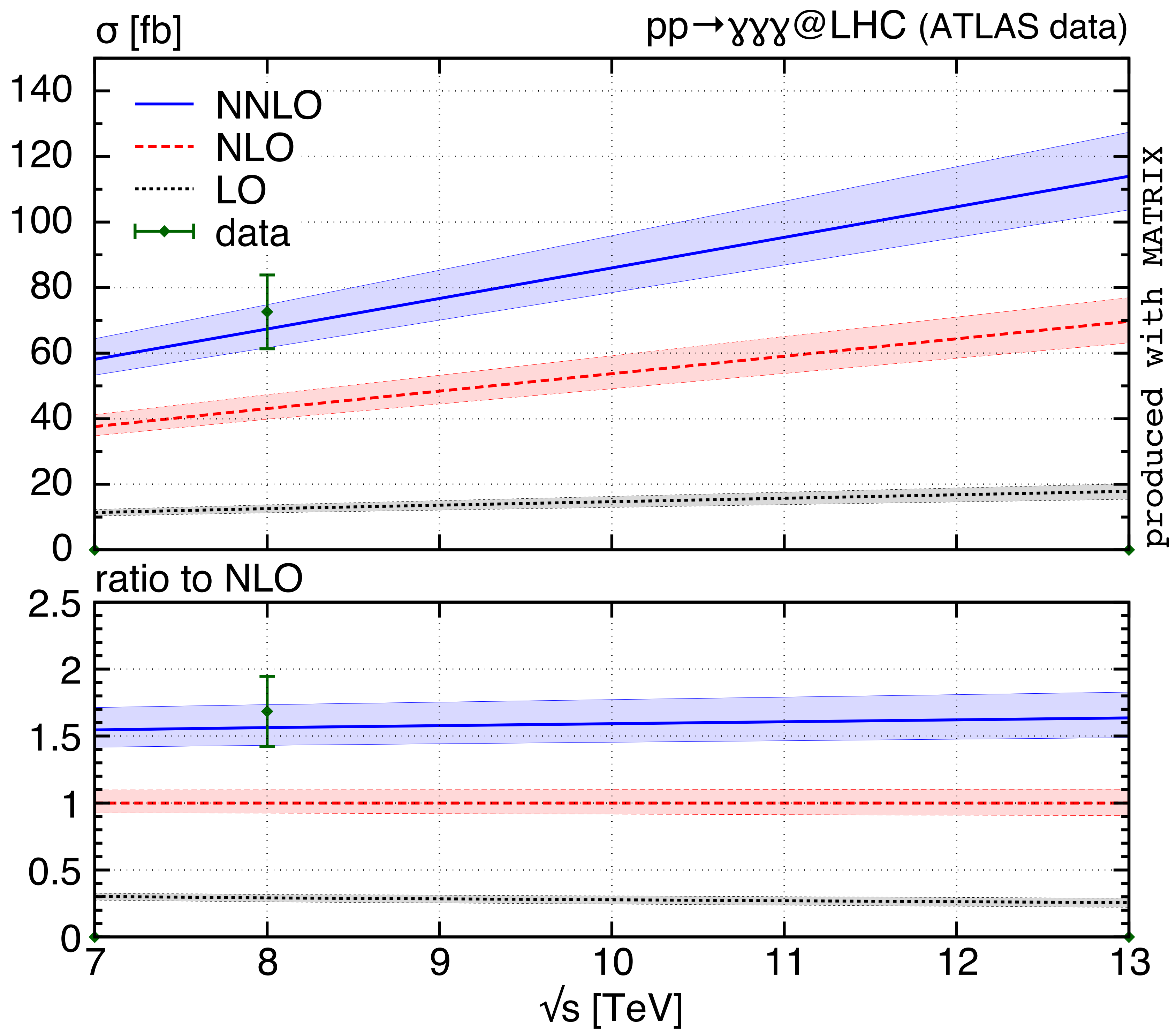
- massive (with approximated 2-loop):

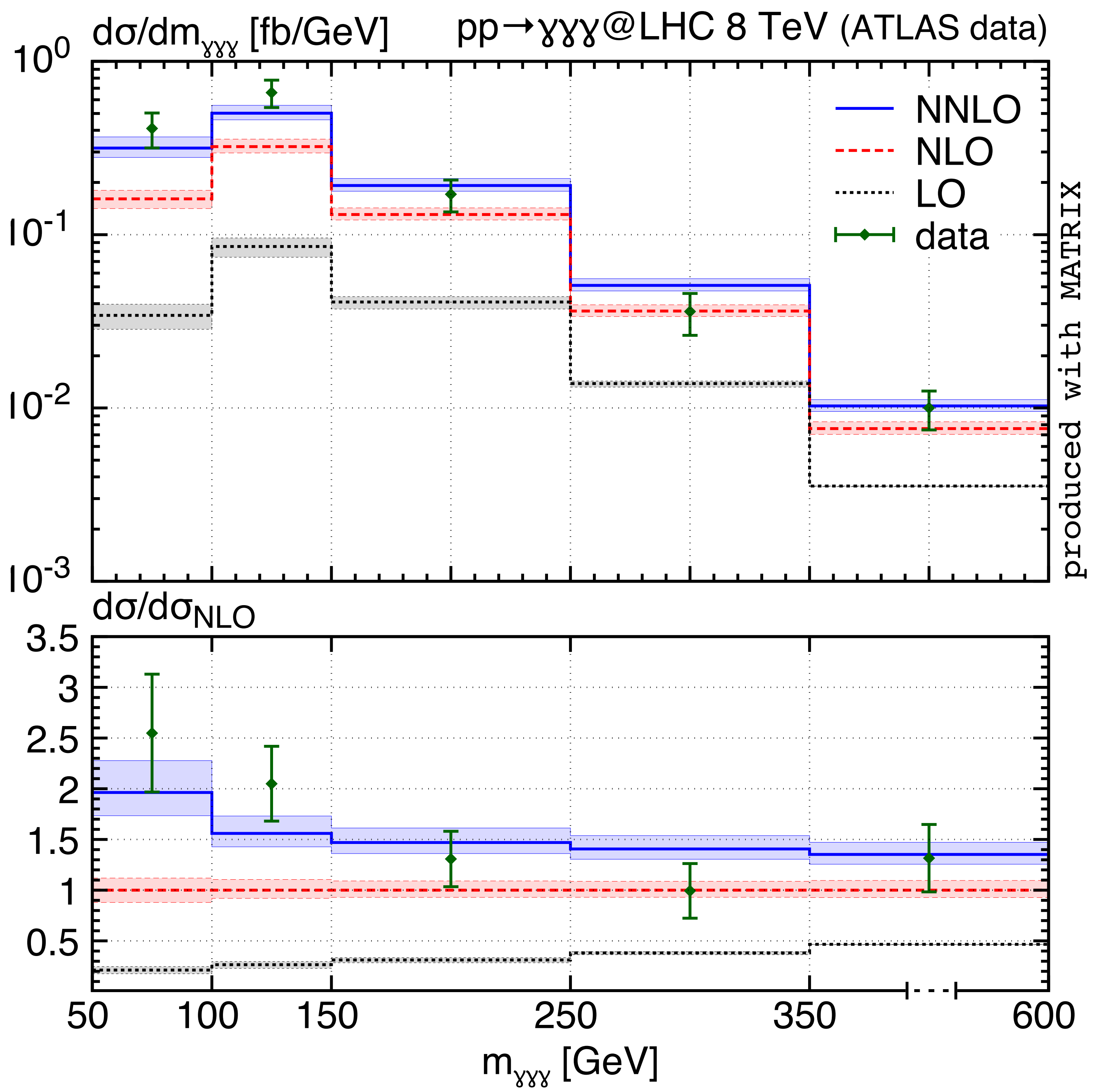
- **pp→ttH** (soft approx.) [Catani, Devoto, Grazzini et al. '22]
- **pp→bbW** (small m_b) [Buonocore, Devoto, Grazzini et al. '23]
- **pp→ttW** (both) [Buonocore, Devoto, Kallweit et al. '22]

First 2→3 process at NNLO QCD



✦ two-loop five-point function [Abreu, Page, Pascual, Sotnikov '20]





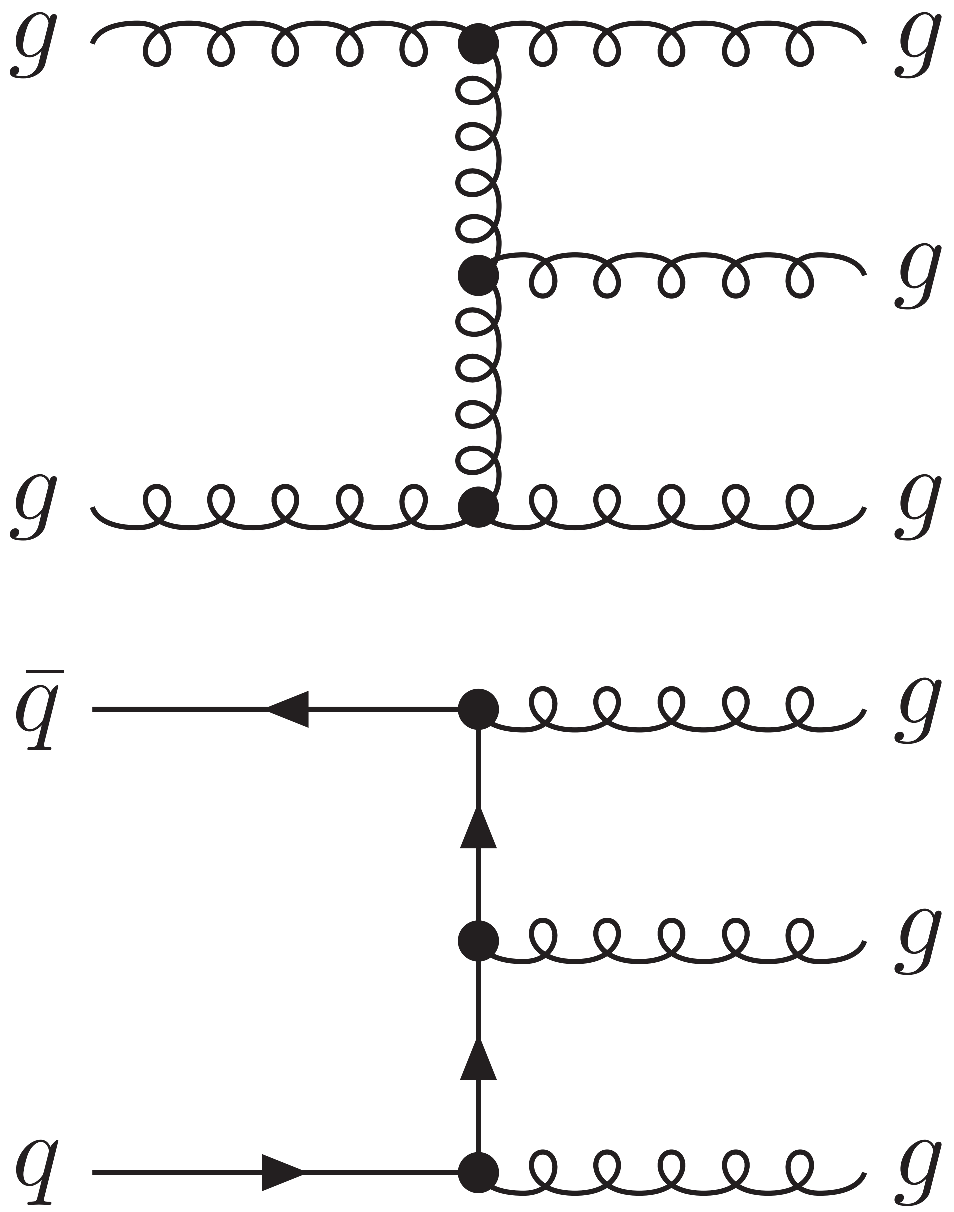
Example #4

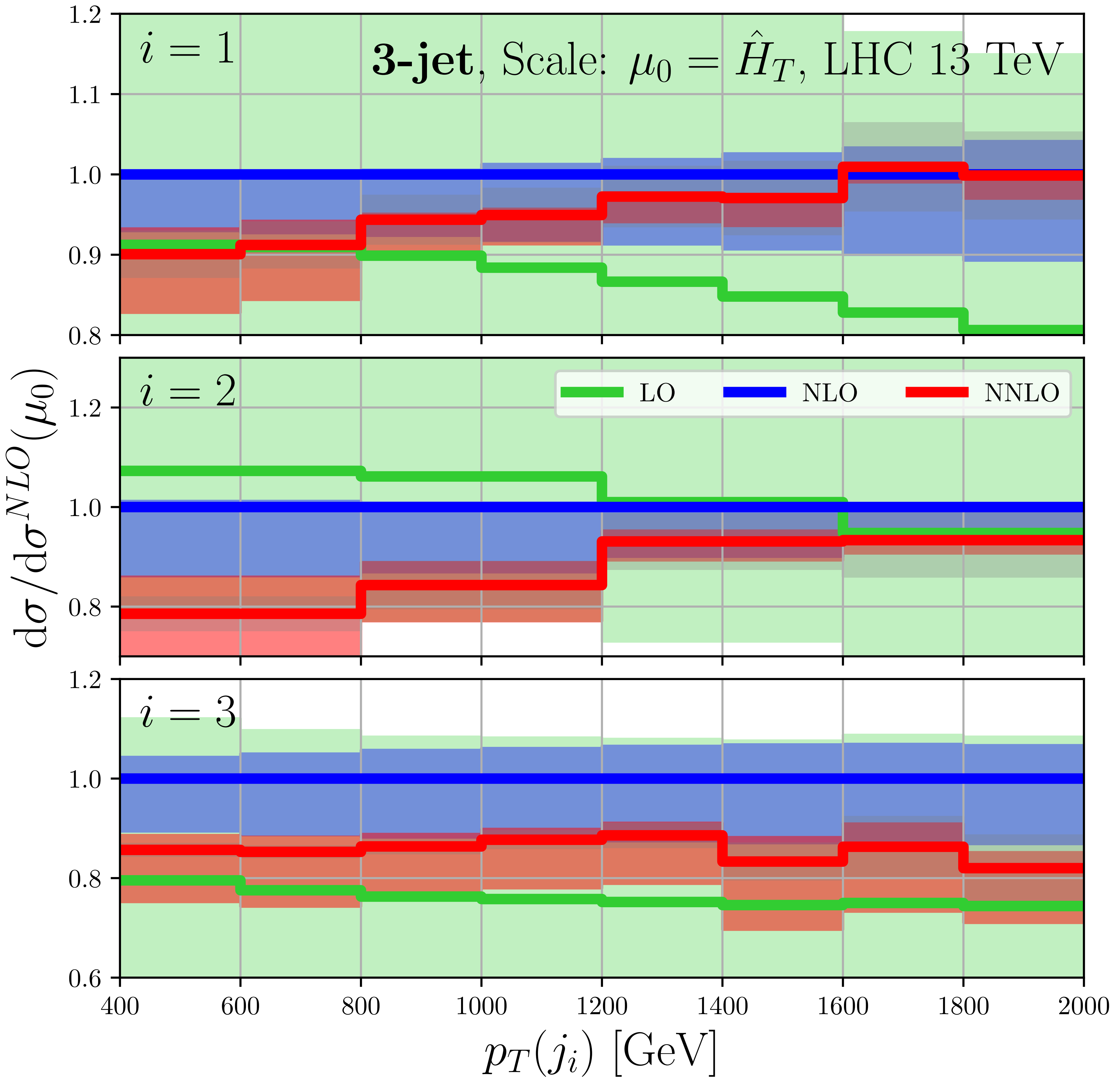
- massless/one mass (full 2-loop):

- pp→γγγ [Chawdhry, Czakon, Mitov, Poncelet '19], [Kallweit, Sotnikov, MW '20]
- pp→γγ+jet [Chawdhry, Czakon, Mitov, Poncelet '21]
- **pp→3-jet** [Czakon, Mitov, Poncelet '21]

enables α_S fits through 3-jet/2-jet!

"Tour de force in Quantum Chromodynamics"





LH '17 wishlist

$pp \to 3\,\text{jets}$	NLO_{QCD}	$\text{N}^2\text{LO}_{\text{QCD}}$

Example #5

- massless/one mass (full 2-loop):

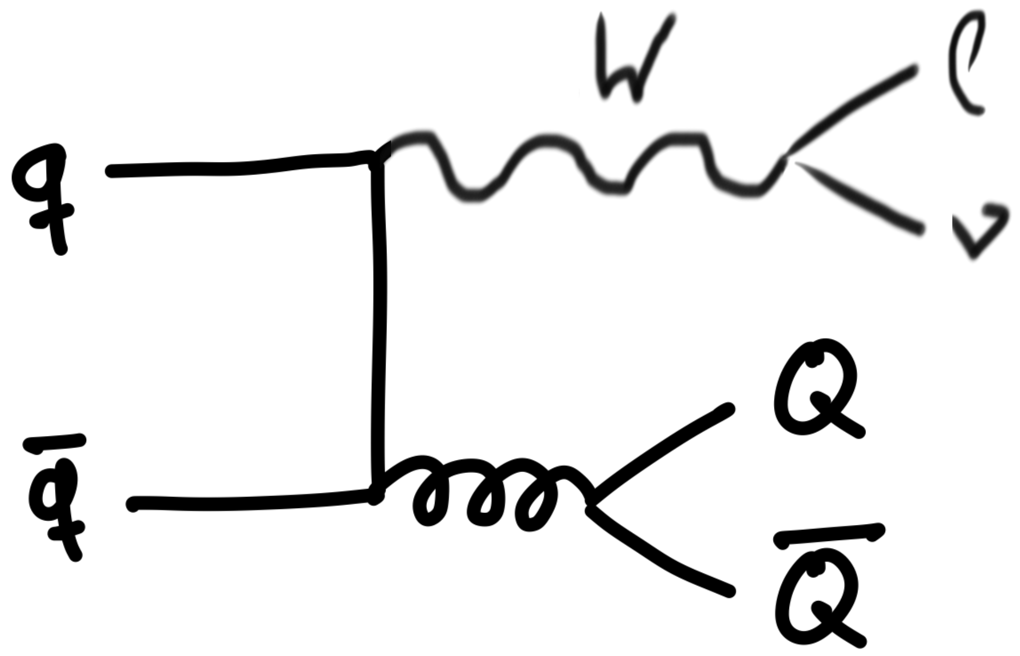
- **pp→γγγ** [Chawdhry, Czakon, Mitov, Poncelet '19], [Kallweit, Sotnikov, MW '20]
- **pp→γγ+jet** [Chawdhry, Czakon, Mitov, Poncelet '21]
- **pp→3-jet** [Czakon, Mitov, Poncelet '21]
- **pp→bbW** (m_b=0) [Hartano, Poncelet, Popescu, Zoia '22]
- **pp→γ+2-jet** [Badger, Czakon, Hartano et al. '23]

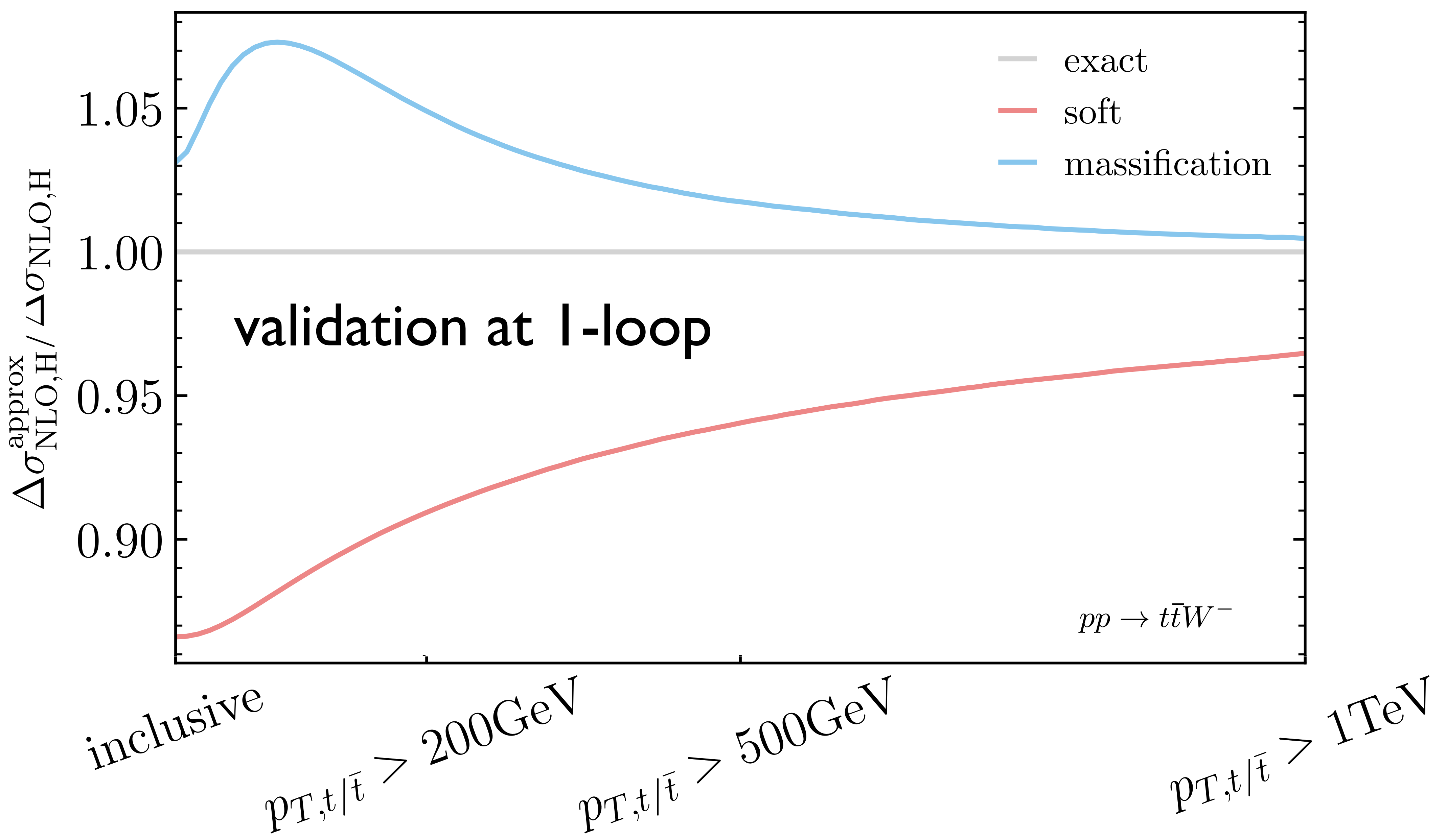
- massive (with approximated 2-loop):

- **pp→ttH** (soft approx.) [Catani, Devoto, Grazzini et al. '22]
- **pp→bbW** (small m_b) [Buonocore, Devoto, Grazzini et al. '23]
- **pp→ttW** (both) [Buonocore, Devoto, Kallweit et al. '22]

✦ two approximations for two-loop

1. W assumed to be soft and factorizing
2. tops assumed to have small mass
   small-mass expansion [Mitov, Moch '06]





$$2\mathrm{Re}\langle R^{(0)} | R^{(2)} \rangle = \sum_{i=1}^{4} \kappa_i \log^i(m_t/\mu_R) + 2\mathrm{Re}\langle R_0^{(0)} | R_0^{(2)} \rangle + \mathcal{O}(m_b/\mu)$$

massive amplitude — massless amplitude [Badger, Hartano, Krys, Zoia '21]

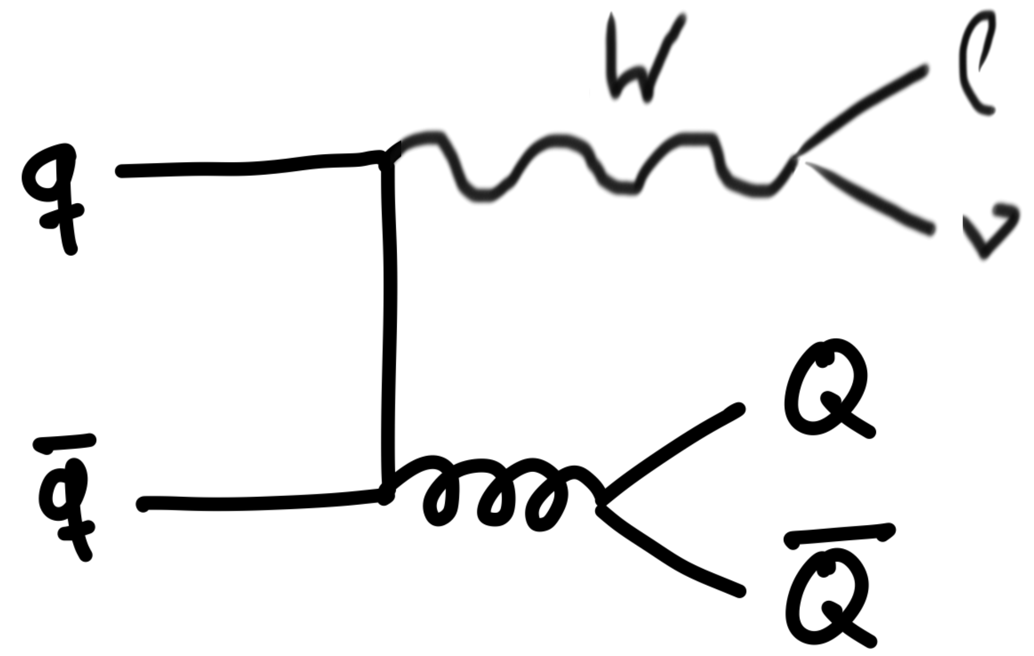
Example #5

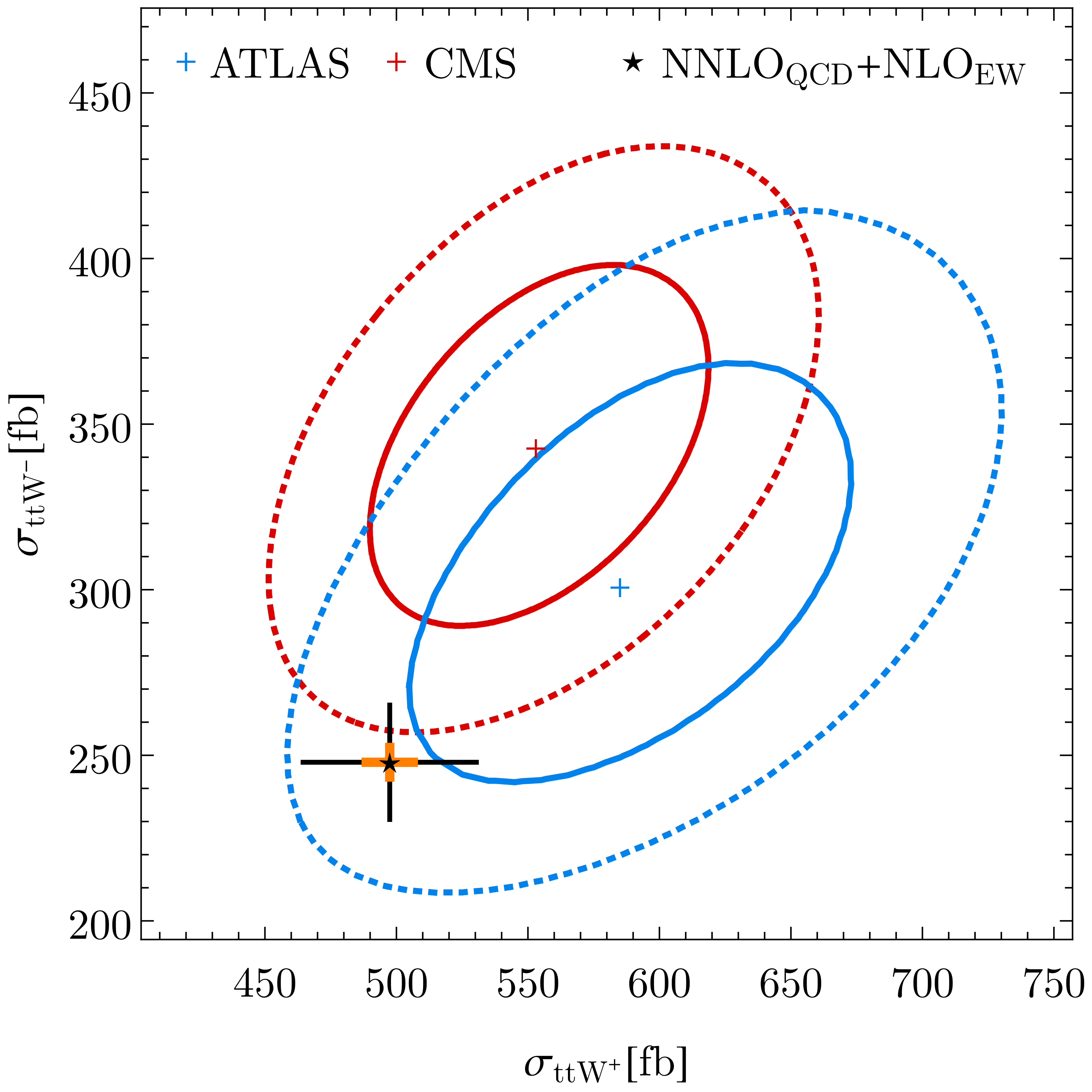
- massless/one mass (full 2-loop):

- **pp→γγγ** [Chawdhry, Czakon, Mitov, Poncelet '19], [Kallweit, Sotnikov, MW '20]
- **pp→γγ+jet** [Chawdhry, Czakon, Mitov, Poncelet '21]
- **pp→3-jet** [Czakon, Mitov, Poncelet '21]
- **pp→bbW** (m_b=0) [Hartano, Poncelet, Popescu, Zoia '22]
- **pp→γ+2-jet** [Badger, Czakon, Hartano et al. '23]

- massive (with approximated 2-loop):

- **pp→ttH** (soft approx.) [Catani, Devoto, Grazzini et al. '22]
- **pp→bbW** (small m_b) [Buonocore, Devoto, Grazzini et al. '23]
- **pp→ttW** (both) [Buonocore, Devoto, Kallweit et al. '22]





N^3LO QCD frontier 2 → 1 processes

- *inclusive N^3LO calculations:*

- **pp→H** [Anastasiou et al. '15], [Mistlberger '18]
- **pp→Z/W** [Duhr, Dulat, Mistlberger '20 '20]
- **pp→Hjj (VBF)** [Dreyer, Karlberg '16]
- **pp→HHjj (VBF)** [Dreyer, Karlberg '18]

- *differential N^3LO calculations:*

- **pp→H** [Cieri, Chen, Gehrmann, Glover, Huss '18], [Dulat, Mistlberger, Pelloni '18], [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni '21], [Billis, Dehnadi, Ebert, Michel, Tackmann '21]
- **pp→$\ell\ell$** [Chen, Gehrmann, Glover, Huss, Yang, Zhu '21], [Camarda, Cieri, Ferrera '21], [Neumann, Campbell '22]
- **pp→$\ell\nu$** [Chen, Gehrmann, Glover, Huss, Yang, Zhu '22], [Neumann, Campbell '23]
- **H→bb** [Mondini, Schiavi, Willams '19]

Example #6

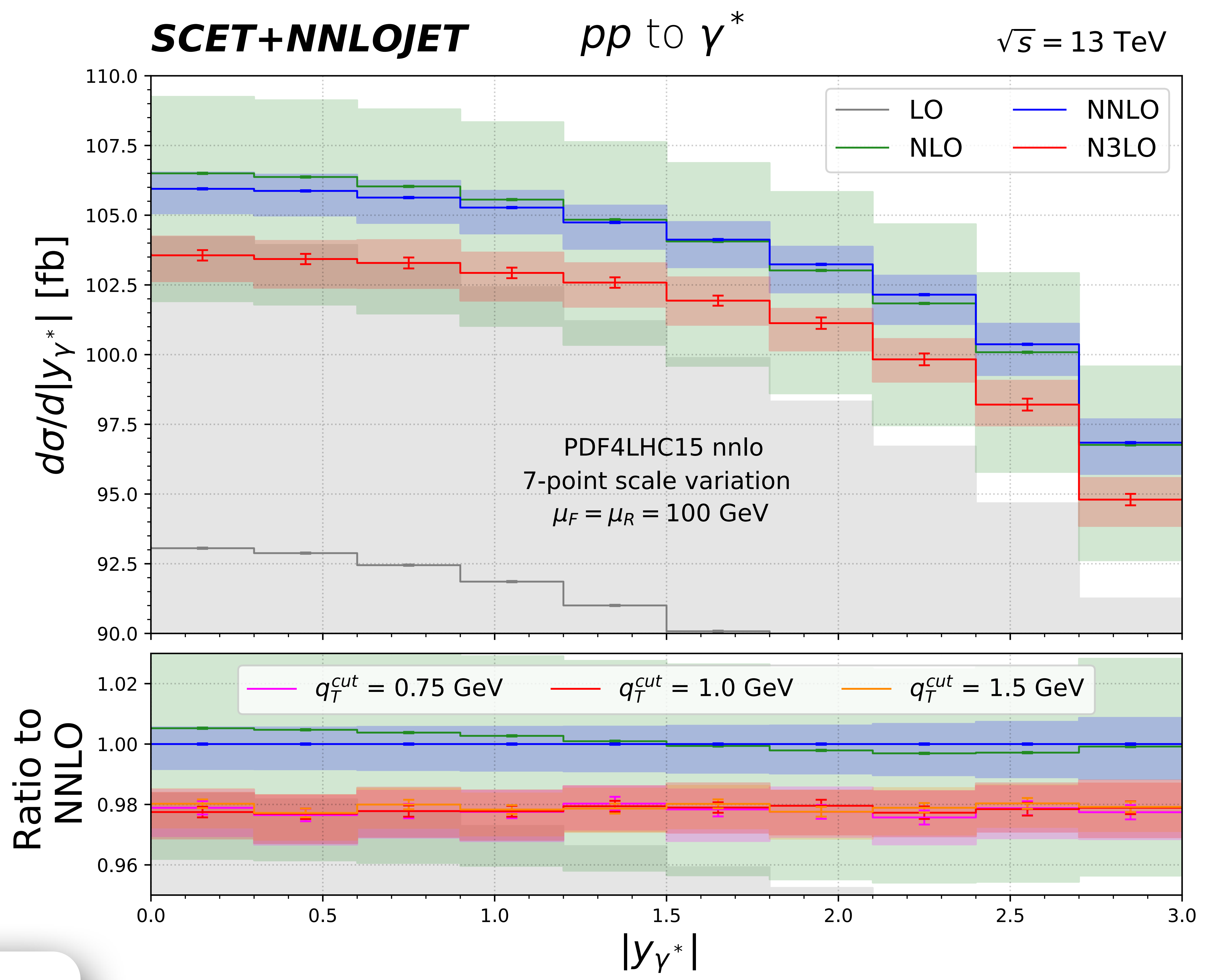
- inclusive N³LO calculations:

- pp→H
- pp→Z/W
- pp→Hjj (VBF)
- pp→HHjj (VBF)

- differential N³LO calculations:

- pp→H
- pp→$\ell\ell$ [Chen, Gehrmann, Glover, Huss, Yang, Zhu '21]

$$\sigma^{\mathrm{Z}}_{\mathrm{N^3LO}} = \left[\sigma^{\mathrm{Z+jet}}_{\mathrm{NNLO}} \Big|_{r > r_{\mathrm{cut}}} - \Sigma_{\mathrm{N^3LO}}(r_{\mathrm{cut}}) \otimes d\sigma^{\mathrm{B}} \right] + \mathscr{H}_{\mathrm{N^3LO}} \otimes d\sigma^{\mathrm{B}}$$



- ✦ NNLO for Z+jet via Antenna subtraction
- ✦ N³LO via q_T slicing

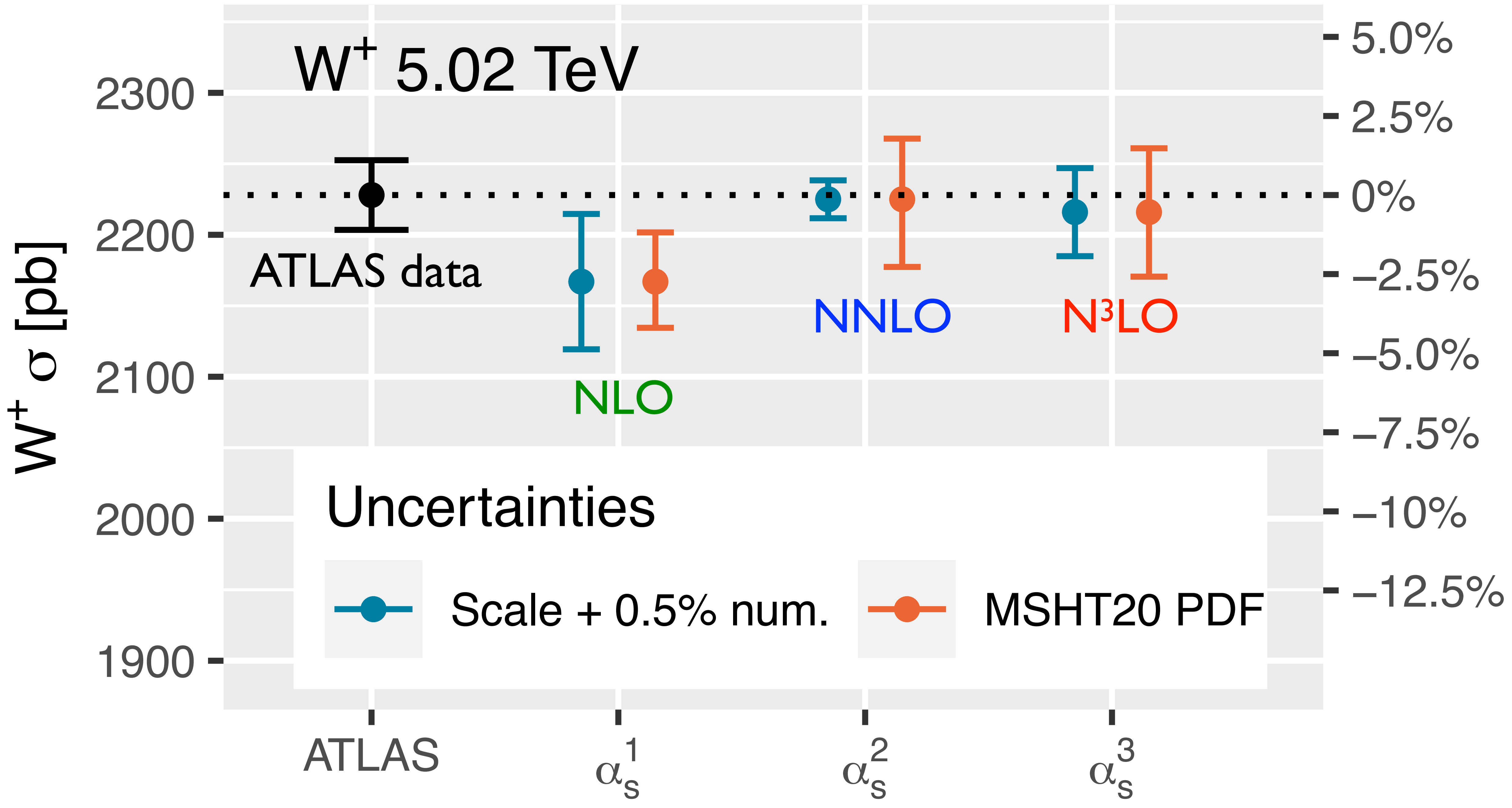
Example #7

- inclusive N^3LO calculations:

- **pp→H**
- **pp→Z/W**
- **pp→Hjj (VBF)**
- **pp→HHjj (VBF)**

- differential N^3LO calculations:

- **pp→H**
- **pp→ℓℓ**
- **pp→ℓν** [Neumann, Campbell '23]
- **H→bb**

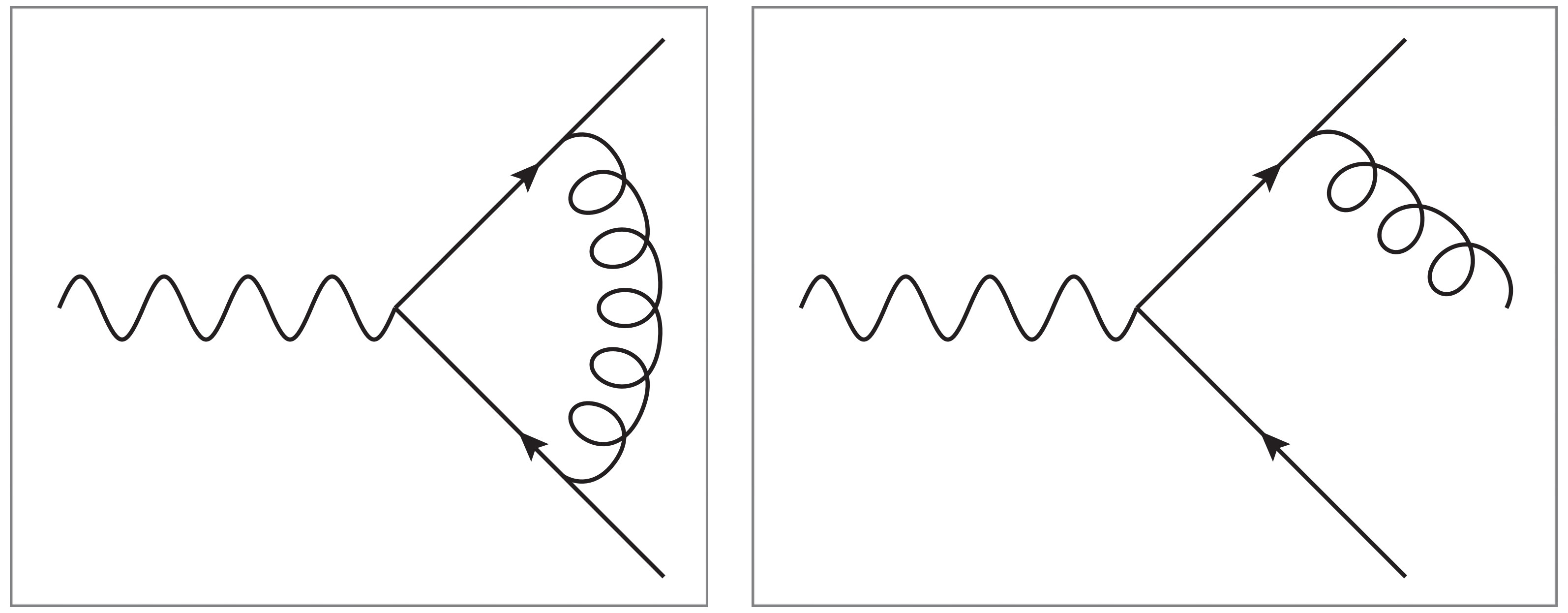


- ✦ NNLO for W+jet via 1-jettiness slicing
- ✦ N^3LO via q_T slicing

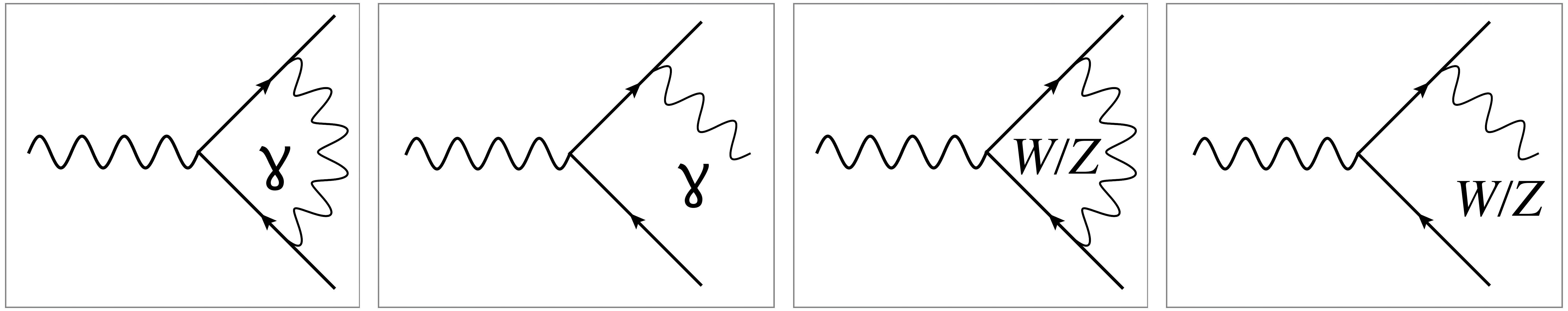
EW corrections

★ EW corrections just like (abelian version of) QCD corrections, and yet different…

NLO QCD



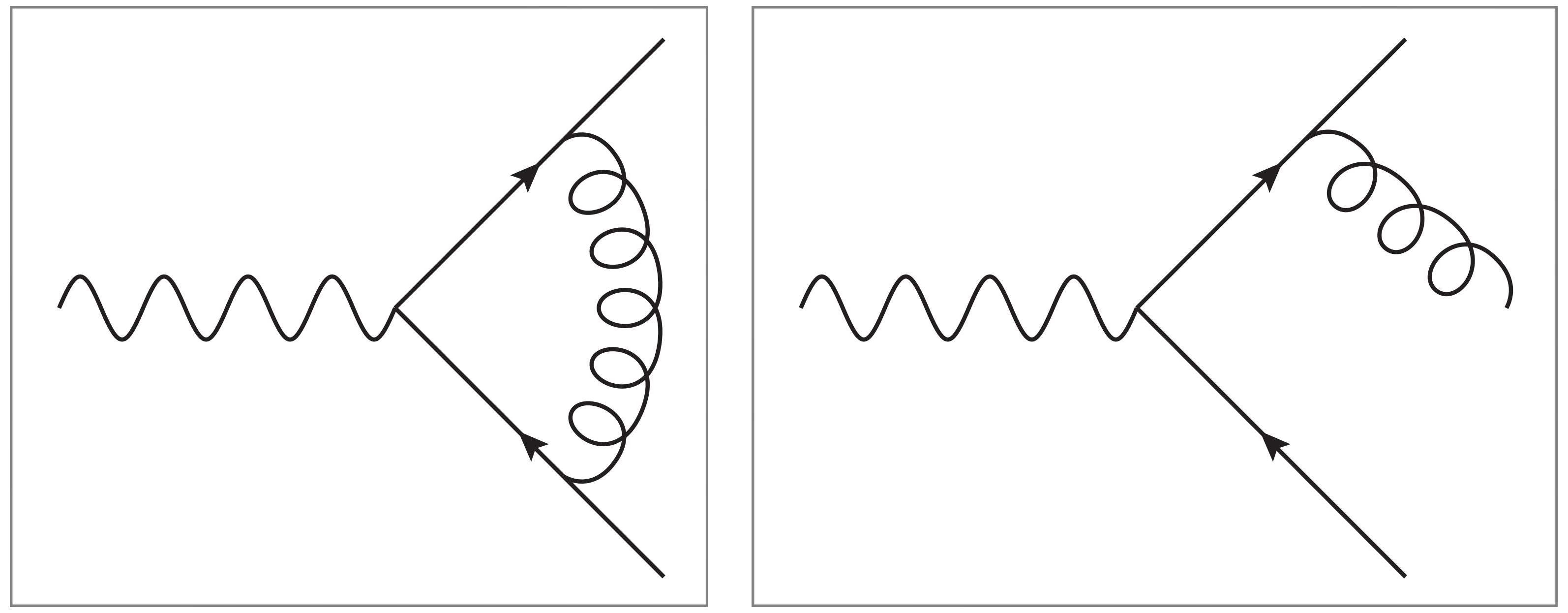
NLO EW



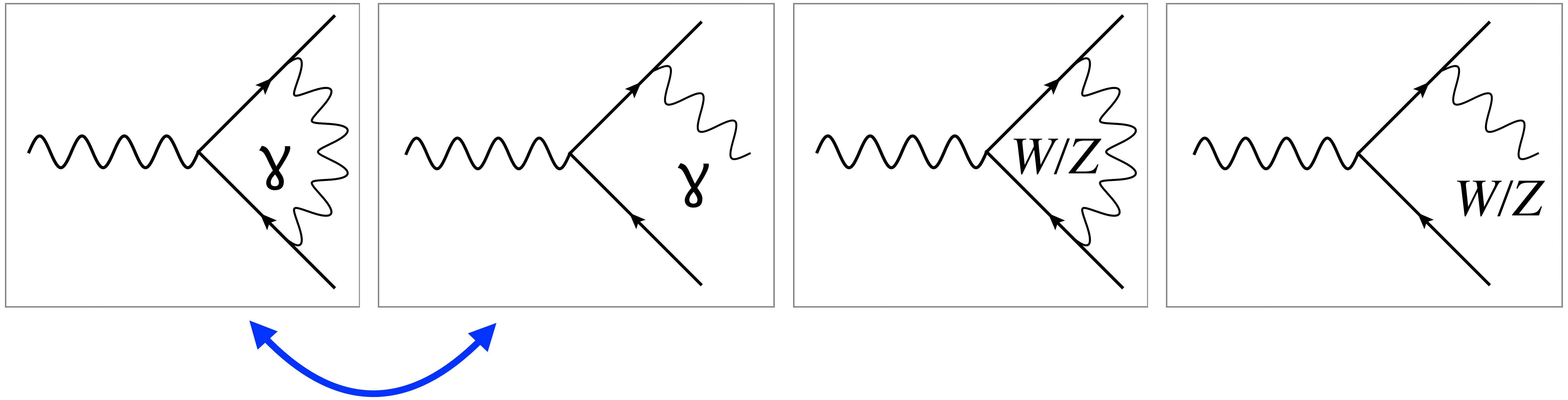
EW corrections

★ EW corrections just like (abelian version of) QCD corrections, and yet different…

NLO QCD



NLO EW



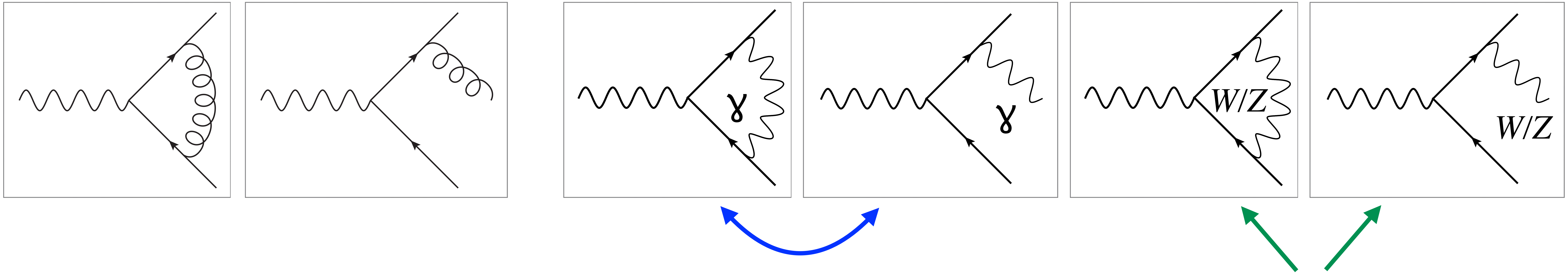
cancellation of IR singularities

EW corrections

★ EW corrections just like (abelian version of) QCD corrections, and yet different…

NLO QCD

NLO EW



cancellation of IR singularities

IR singularities regulated by $m_{Z/W}$

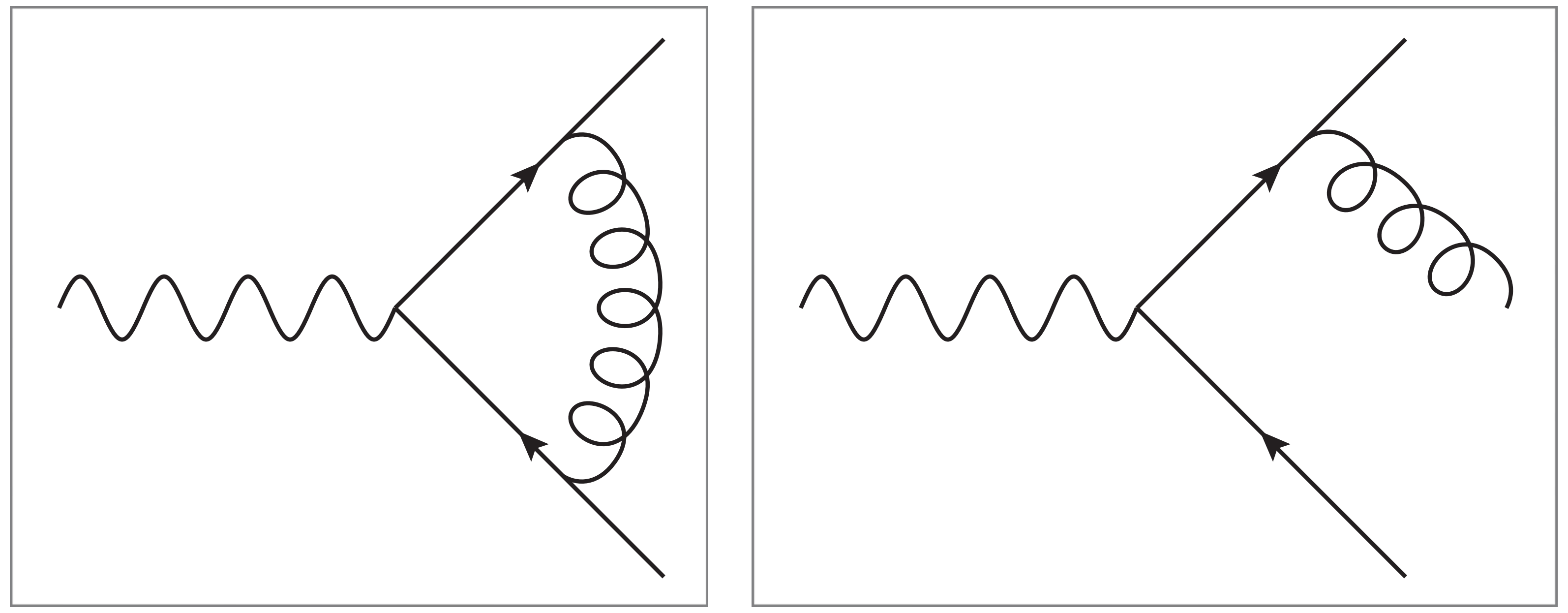
➞ separately finite

➞ real Z's/W's can be measured

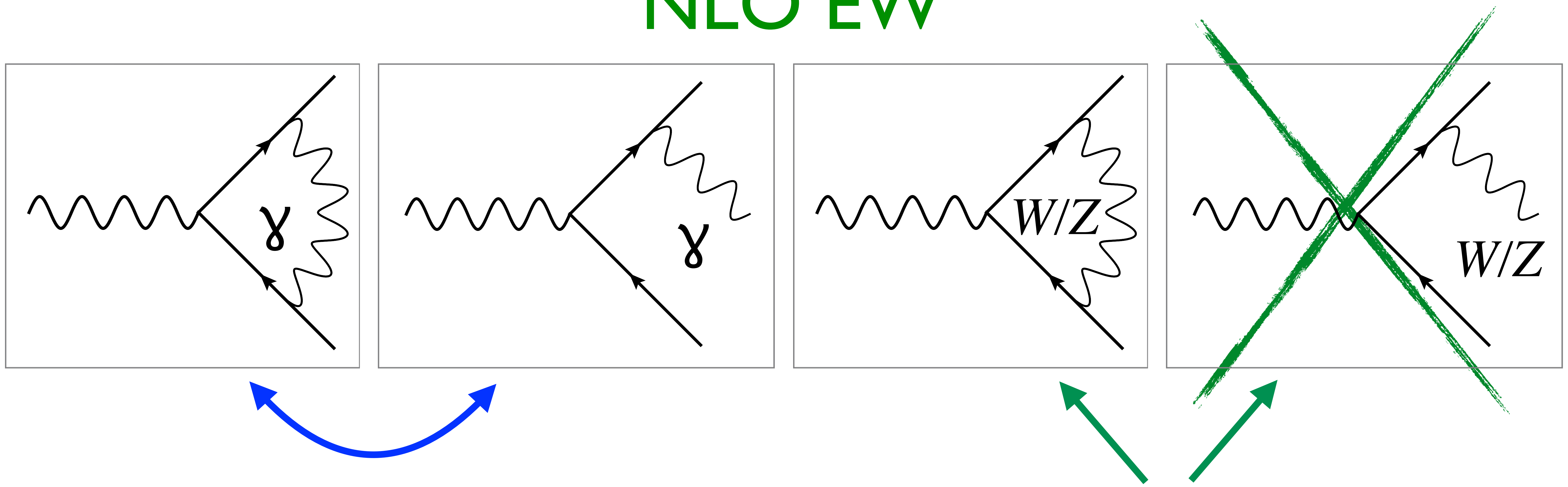
EW corrections

★ EW corrections just like (abelian version of) QCD corrections, and yet different…

NLO QCD



NLO EW



cancellation of IR singularities

IR singularities regulated by $m_{Z/W}$

→ separately finite

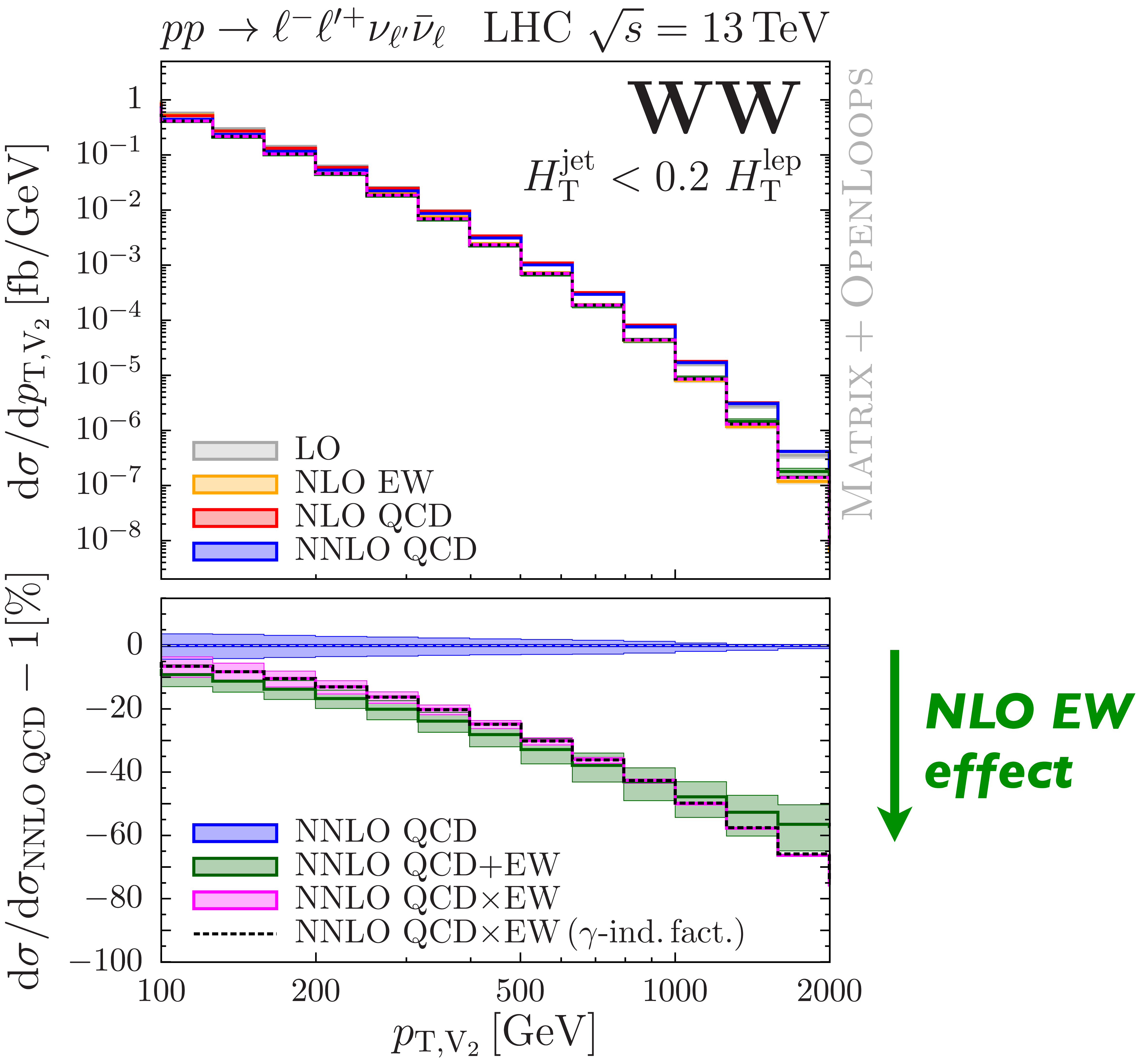
→ real Z's/W's can be measured

→ large EW Sudakov logs:

$$\alpha^n \log^k \left(s / m^2_{Z/W} \right), \quad k \leq 2n$$

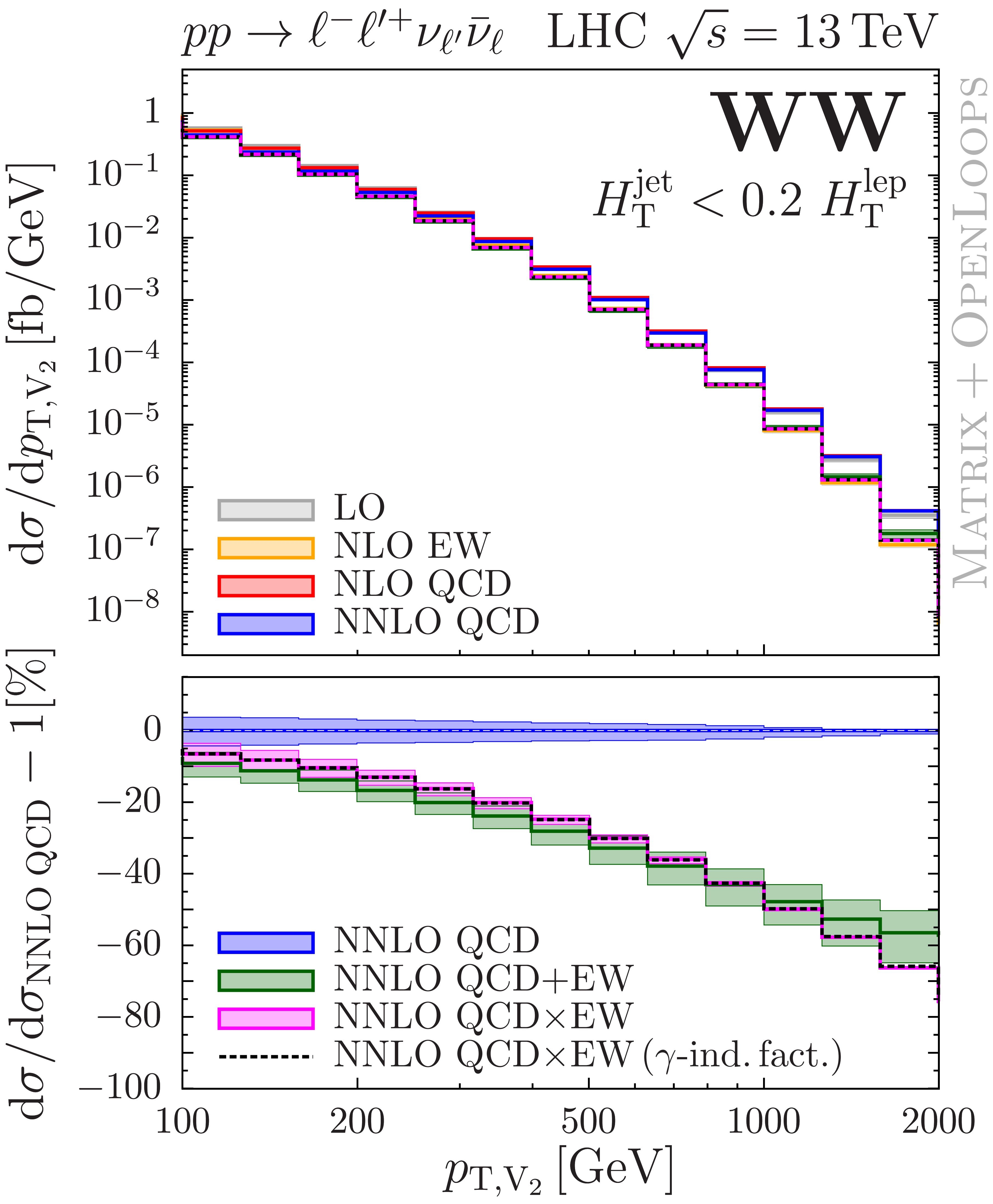
Example #8

[Grazzini, Kallweit, Lindert, Pozzorini, MW '19]

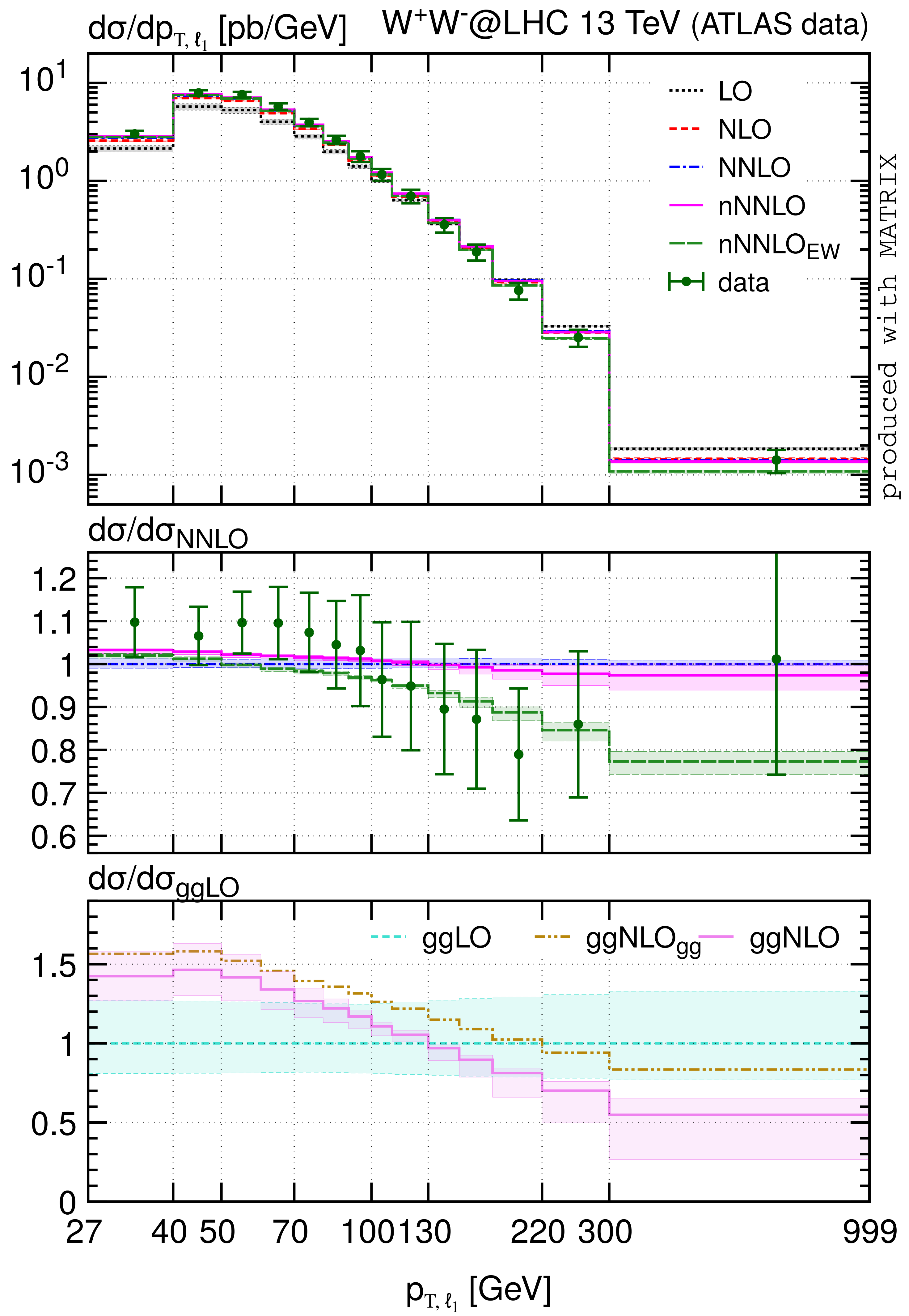


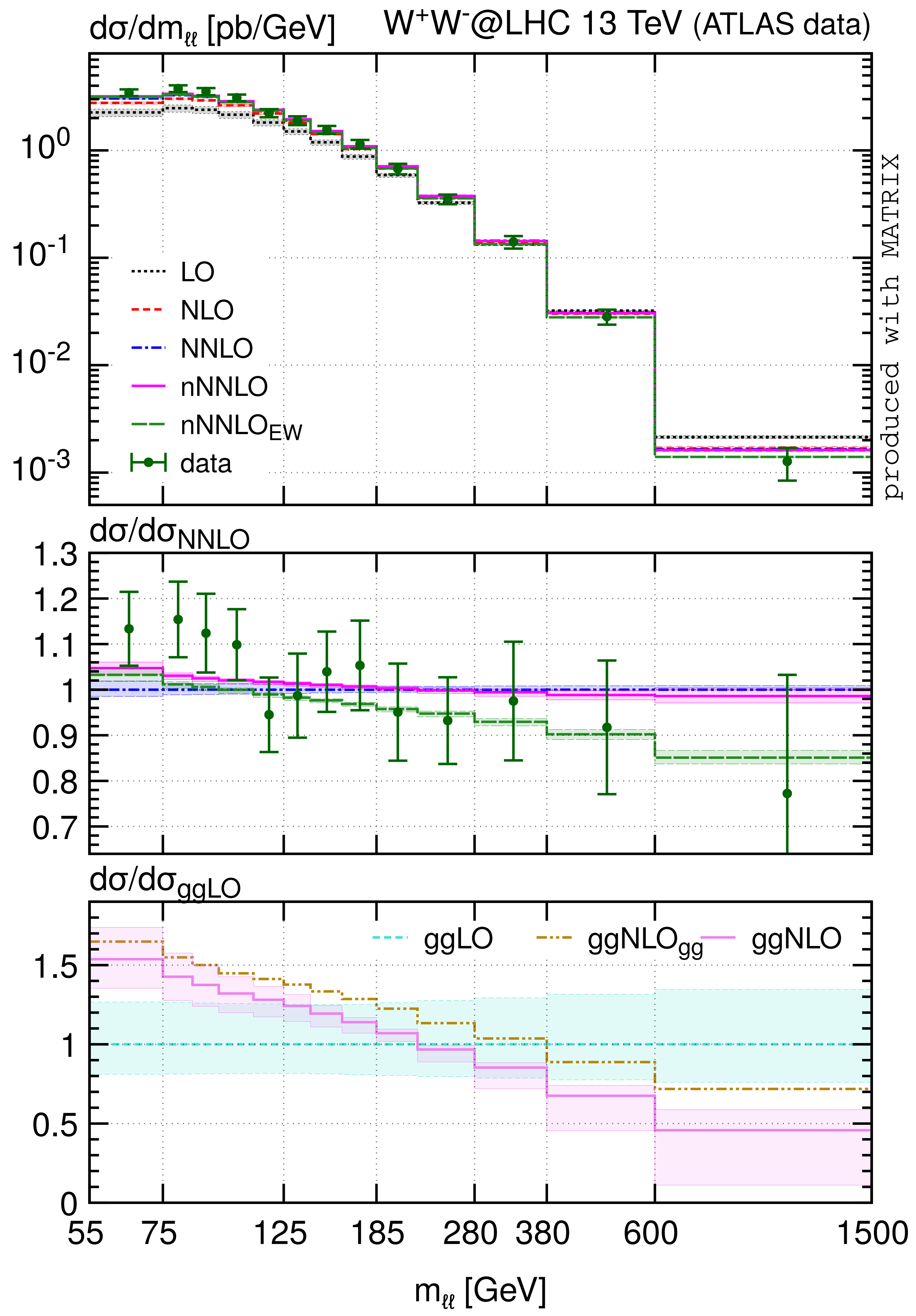
Example #8

[Grazzini, Kallweit, Lindert, Pozzorini, MW '19]



[Grazzini, Kallweit, MW, Yook '20], [Grazzini, Kallweit, Linder, Pozzorini '19]





Summary so far

- High energy colliders allow us to probe fundamental interactions among elementary particle in a controlled environment at very short distances, but it requires that SM Physics has to be described with:

  ★ physical observables that can be reliably calculated and measured at the same time

  ★ accurate+precise predictions (and measurements)
  -- very difficult & highly advanced technology

Summary so far

Theory predictions reached an accuracy considered impossible some years ago:

LO	fully automated Edge: 10-12 particles in the final state
NLO	fully automated Edge: 4-6 particles in the final state
NNLO	dedicated calculations, few public codes essentially all 2 → 2 reactions, several 2 → 3 recenly
N^{3}LO	first few calculations only 2 → 1 reactions so far, but differential recently

Many Theory Aspects NOT Talked About

★ Resummation and Event Generation

(highly sophisticated methods to describe soft physics in collider processes; high-accuracy analytic resummation; improving shower accuracy; matching to NNLO; ...)

★ How to do loop calculations in detail

(five-point functions for $2\to3$ processes currently being solved; four-point functions with internal masses for $2\to2$ processes; ...)

★ Extraction of SM parameters (couplings, masses, ...)

★ ...

Many Theory Aspects NOT Talked About

★ Resummation and Event Generation

(highly sophisticated methods to describe soft physics in collider processes; high-accuracy analytic resummation; improving shower accuracy; matching to NNLO; ...)

★ How to do loop calculations in detail → ***in Ben's lectures***

(five-point functions for $2 \to 3$ processes currently being solved; four-point functions with internal masses for $2 \to 2$ processes; ...)

★ Extraction of SM parameters (couplings, masses, ...)

★ ...

Many Theory Aspects NOT Talked About

★ Resummation and Event Generation → ***tomorrow in lecture 3***

(highly sophisticated methods to describe soft physics in collider processes; high-accuracy analytic resummation; improving shower accuracy; matching to NNLO; ...)

★ How to do loop calculations in detail

(five-point functions for 2→3 processes currently being solved; four-point functions with internal masses for 2→2 processes; ...)

★ Extraction of SM parameters (couplings, masses, ...)

Thank you very much for your attention!

Questions?

Hands on !

- download PDF of this talk!
- two options:
  1. use your own laptop locally

     ➙ need to install LHAPDF from https://lhapdf.hepforge.org/ (including the needed PDF set)

  2. use your remote ssh login (for Mac/Windoof users highly recommended)

     `$ ssh bndXXX@bnd01.iihe.ac.be` ➙ enter password

     (**`$ source /cvmfs/sft.cern.ch/lcg/views/setupViews.sh LCG_102 x86_64-centos7-gcc11-opt`** ➙ should not be needed, check: **`gcc --version`** ➙ 11.2.0)

```
mars — bnd005@bnd01:~ — ssh bnd005@bnd01.iihe.ac.be — 107×44
[mars:~] ssh bnd005@bnd01.iihe.ac.be
bnd005@bnd01.iihe.ac.be's password:
Last login: Fri Aug 23 08:01:24 2024 from ip-088-152-010-164.um26.pools.vodafone-ip.de
[bnd005@bnd01 ~]$ source /cvmfs/sft.cern.ch/lcg/views/setupViews.sh LCG_102 x86_64-centos7-gcc11-opt
[bnd005@bnd01 ~]$
```

Hands on !

- download & setup MATRIX from https://matrix.hepforge.org/

```
$ mkdir Matrix_tutorial
$ cd Matrix_tutorial
$ wget https://matrix.hepforge.org/download/MATRIX_v2.1.0.tar.gz
$ tar xf MATRIX_v2.1.0.tar.gz
$ cd MATRIX_v2.1.0/
```

```
mars — bnd005@bnd01:/var/bnd/users/bnd005/Matrix_Tutorial/MATRIX_v2.1.0 — ssh bnd005@bnd01.iihe.ac.be — 107×44
[mars:~] ssh bnd005@bnd01.iihe.ac.be
bnd005@bnd01.iihe.ac.be's password:
Last login: Fri Aug 23 09:27:38 2024 from ip-088-152-010-164.um26.pools.vodafone-ip.de
[bnd005@bnd01 ~]$ source /cvmfs/sft.cern.ch/lcg/views/setupViews.sh LCG_102 x86_64-centos7-gcc11-opt
[bnd005@bnd01 ~]$ mkdir Matrix_Tutorial
[bnd005@bnd01 ~]$ cd Matrix_Tutorial
[bnd005@bnd01 Matrix_Tutorial]$ wget https://matrix.hepforge.org/download/MATRIX_v2.1.0.tar.gz
--2024-08-23 09:28:51--  https://matrix.hepforge.org/download/MATRIX_v2.1.0.tar.gz
    [                <=>                                              ] 64,598,385  18.8MB/s   in 3.3s

2024-08-23 09:28:55 (18.8 MB/s) - »MATRIX_v2.1.0.tar.gz« gespeichert [64598385]
[bnd005@bnd01 Matrix_Tutorial]$ tar xf MATRIX_v2.1.0.tar.gz
[bnd005@bnd01 Matrix_Tutorial]$ cd MATRIX_v2.1.0/
[bnd005@bnd01 MATRIX_v2.1.0]$
```

Hands on !

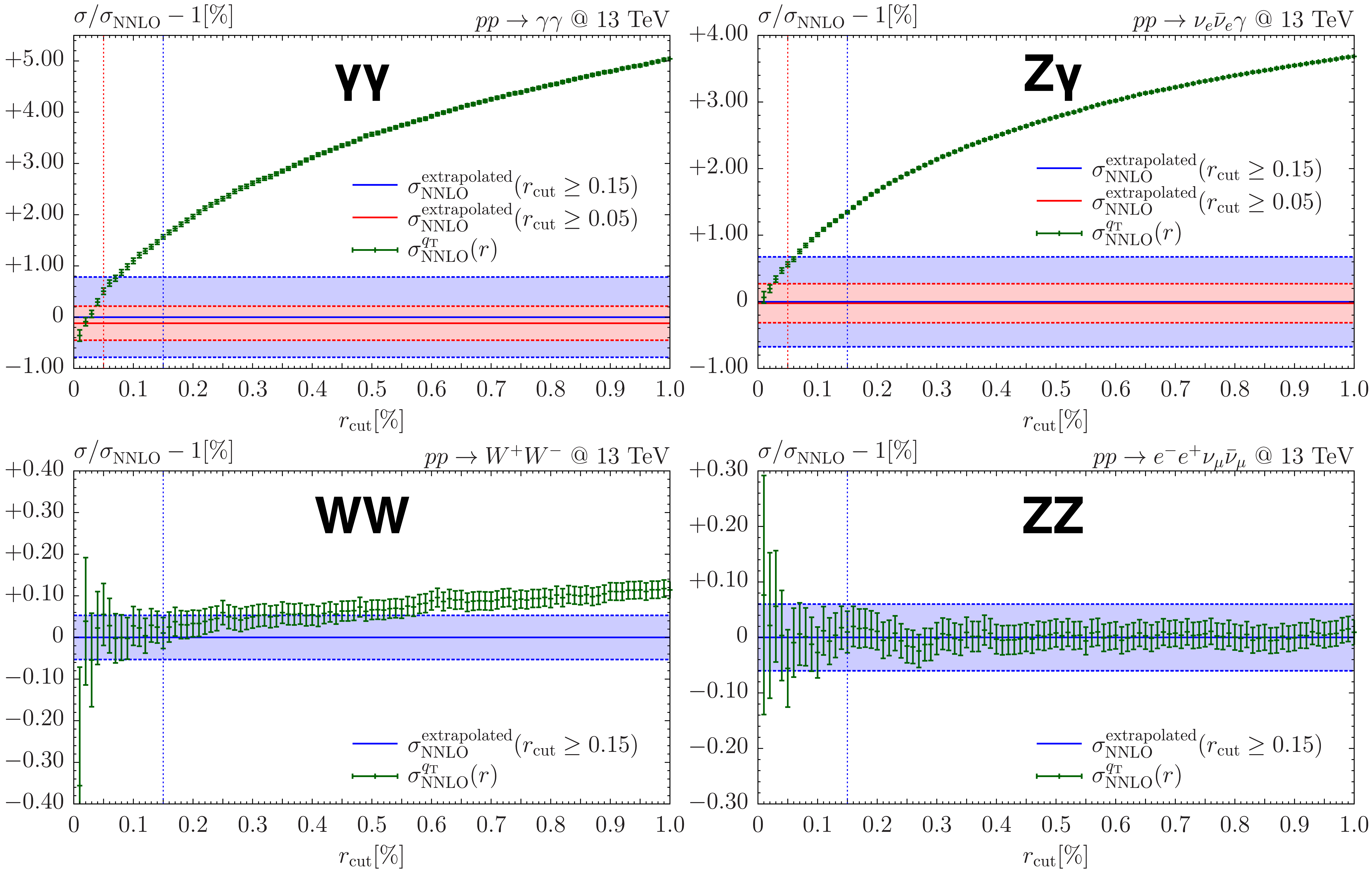
- start compilation script

```
$ ./matrix
```

Extra Slides

$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]



r_{cut}→0 extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]