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QCD and Monte Carlo event generators (Lecture 3 — Resummation & MCs)

BND summer school 2024 Blankenberge (Belgium), September 2-12th, 2024

Recap of Lecture 1

$$
\sigma_{\rm had}=\sum_{ij}\int {\rm d}x_1\,{\rm d}x_2\,f_i(x_1,\mu_{\rm F})\,f_j
$$

- \bigstar Perturbative (higher-order) QCD calculations vital for partonic (hard) cross section • LO just gives a rough order-of-magnitude estimate
	- NLO is largely automated by now and the minimum requirement for a reliable description of the physical cross sections at the LHC
	- NNLO has been substantially advanced in the past years and is required for precision data/theory comparisons & to reduce theory uncertainties at the LHC (current bottleneck: mostly 2-loop amplitudes)

 \rightarrow all relevant $2 \rightarrow 2$ and first $2 \rightarrow 3$ reactions known at NNLO

• N³LO frontier passed for $2 \rightarrow 1$ processes (Higgs & Drell-Yan)

- \bigstar LHC Master Formula is based on factorizing long-distance form short-distance physics
	- $(x_2,\mu_F) \times \sigma_{ij}(x_1P_1,x_2P_2,\mu_F) + \mathcal{O}(\Lambda/Q)$

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- ★ EW corrections?
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★ EW corrections just like (abelian version of) QCD corrections, and yet different… NLO QCD NLO EW ∞ O_O

EW corrections

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EW corrections

cancellation of IR singularities

EW corrections

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EW corrections

$\overline{\mathbf{r}}$ [Grazzini, Kallweit, Lindert, Pozzorini, MW '19]

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 \rightarrow all relevant $2 \rightarrow 2$ and first $2 \rightarrow 3$ reactions known at NNLO

- N³LO frontier passed for $2 \rightarrow 1$ processes (Higgs & Drell-Yan)
- \bigstar EW corrections important due to photon radiation & EW Sudakov logarithms (in tails)
- \bigstar LHC Master Formula is based on factorizing long-distance form short-distance physics
	- $(x_2,\mu_F) \times \sigma_{ij}(x_1P_1,x_2P_2,\mu_F) + \mathcal{O}(\Lambda/Q)$

-
- main problems to solve:

4 nested soft.-coll.

²→³ *[×] + complex conj. Local subtraction*

2→2 *[×] + complex conj.* [Gehrmann-De Ridder, Gehrmann, Glover '05]

2→2 [Caola, Melnikov, Röntsch '17]

5 [Catani, Grazzini '07, MATRIX]

qT-subtraction

N-jettiness

Projection-to-Born

non-local/slicing

CoLorFul [Del Duca, Somogyi, Troscanyi '05]

[Gaunt, Stahlhofen Tackmann, Walsh '15] [Boughezal, Focke, Lui, Petriello '15, MCFM]

[Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]

Outline

Lecture 2: Hands-on session on MATRIX

 \mathbf{m} Lecture 3Ure JOO

★ Fixed-order calculations

- **QCD basics (Lagrangian, Feynman rules, strong coupling)**
- LHC Factorization/Master Formula (PDFs, partonic cross section)
- NLO QCD (methods, slicing vs. subtraction vs. analytic)
- NNLO QCD (methods, timeline)
- EW corrections (NLO, Sudakov logarithms, mixed QCD-EW corrections?)

Lecture 1 Lecture

- ★ Monte Carlo Event Generation & Resummation
	- **Resummation**
	- Parton Shower Generators (formalism, hadronization, MPI)
	- NLO+PS Matching (MC@NLO, Powheg, merging)
	- NNLO+PS Matching (MiNNLO, Geneva)

Resummation safe observables but….

een real and virtual contributions is unbalan between real and virtual contributions is unbalanced between real and virtual contributions is unbalanced

→ large logarithmic terms invalidate the perturbative expansion *nof the cross section*

 \bigstar Gluon radiation produces a double-log behaviour (one collinear and one soft logarithm) θ ω θ ω

$$
C_d C_{\overline{a}} \in F_{\overline{b}} C_F \quad a \neq q
$$

 $C_a C_{\overline{a}} \in C_A$ *a* \oplus \oplus g

R esummation

riangle production of colorless particles (system F , invariant mass M)

···

 α_s : ln(p_T^2/M^2), ln²(p_T^2/M^2) α_s^2 : ln(p_T^2/M^2), ln²(p_T^2/M^2), ln³(p_T^2)

P production of colorless particles (system F , invariant mass M) ρ ^T *p*² /*M*² *F*₂ *p*² /*M*² *F*₂ *p*²

$$
\ln^2(p_T^2/M^2)
$$

$$
\ln^2(p_T^2/M^2), \ln^3(p_T^2/M^2), \ln^4(p_T^2/M^2)
$$

Resummation

-
- **P** problem: p_T distribution of F diverges at $p_T \rightarrow 0$
- **P** reason: large logs $\ln p_T^2/M^2$ for $p_T \ll M$

···

Resummation

-
- **P** problem: p_T distribution of F diverges at $p_T \rightarrow 0$
- **P** reason: large logs $\ln p_T^2/M^2$ for $p_T \ll M$
	- α_s : ln(p_T^2/M^2), ln²(p_T^2/M^2)

In solution: all order resummation

Transverse-momentum resummation **INGREDIENTS FOR A CALCULATION (generic 2→2 process)** Tranc[®] <u>17ansverse-momentum resummatic</u>

★ Factorization of soft and collinear radiation in matrix elements allows for resummation \sim illustrate the interval \sim concepts in \mathbb{R} care where we have a state particles are \mathbb{R}^n . The internal particles are \mathbb{R}^n **2** $\begin{array}{|c|c|c|c|}\hline \textbf{L} & \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I} \ \hline \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I} \ \hline \textbf{I} & \textbf{I} \ \hline \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I} & \textbf{I$ **Tree 2→2** concept in the concept of t care when the care when th **2** — | ² **2** → <u>**2 1 1 2 e 2** *s* **e 2** *split***</u>** *universal* ➙ eikonal factor *Ja* (soft) or splitting function *Pij* (collinear)

Transverse-momentum resummation **INGREDIENTS FOR A CALCULATION (generic 2→2 process)** Tranc[®] <u>17ansverse-momentum resummatic</u> **Tree 2 Property 2** |
|
| **2 Tree**

★ Factorization of soft and collinear radiation in matrix elements allows for resummation **¹** Multiple emissions of soft collinear GCD radiation fulfills factorization \sim illustrate the interval \sim \bigotimes care where we have a state particles are \mathbf{p} and \mathbf{p} are \mathbf{p} and \mathbf{p} **2** $\begin{picture}(180,10) \put(0,0){\line(1,0){10}} \put(10,0){\line(1,0){10}} \put(10,0){\line($ **Tree 2→2** concept in the concept of t care when the care when th particles are \mathcal{P} and \mathcal{P} are \mathcal{P} and \mathcal{P} **2** — | ² **2→2** *[×] + complex conj.* **upic carries 2→2** *universal* ➙ eikonal factor J^q (soft) or splitting function *Pij* (collinear) $I - \lambda$ Multiple ϵ μ is equipped foot μ and ϵ α β \sim illustrate the interval \sim concepts, we do not a set \mathbb{R}^n care where we have a state particles are \mathbb{R}^n . The internal particles are \mathbb{R}^n **2** F() **LO Tree 2→2** $\mathcal{L} \cap \mathbb{R}$ care when \mathcal{L} **2 2→4 2→4 2→4 2→4 NNL 1-loop** |
|
| 2008 complex of softward(in the CCD radiation furtile factorization 2 **2→2** *[×] + complex conj.* **1-loop 2→2 2→2** $F(\prec)$ ENNISS⁽⁾ **1-loop ²→³** *[×] + complex conj.* **2-loop 2→2** *[×] + complex conj.* **NNLO 1-loop** ² **2** *x* 2¹ **1** *splitting function P_{ij} (collinear)*

2² *CO* radiation furtus factorization $\frac{2}{2}$ **2→2** *[×] + complex conj.* **1-loop 2→2** … **NNLOC 1-loop** ²/₂ (collinear)

2² *x* **1** *x* **2-loop 2→2** *[×] + complex conj.* 1-loop 1-loo **2→2** $F(\prec)$ Parton showed a show turn its factorization Parton shower at work Parton shower at work Parton shower at working and the shower at working the set of the shower at working $\frac{1}{2}$ possible splittings: \sim \sim \sim \sim \sim \sim or \sim or or

5

Transverse-momentum resummation **INGREDIENTS FOR A CALCULATION (generic 2→2 process)** Tranc[®] <u>17ansverse-momentum resummatic</u> **Tree 2 Property 2** |
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★ Factorization of soft and collinear radiation in matrix elements allows for resummation **¹** Multiple emissions of soft collinear GCD radiation fulfills factorization $F(\prec)\otimes F(\prec)\otimes\cdots\otimes F(\prec)$ \sim illustrate the interval \sim \otimes care where we have a state particles are \mathbf{p} and \mathbf{p} are \mathbf{p} and \mathbf{p} **2** $\begin{picture}(180,10) \put(0,0){\line(1,0){10}} \put(10,0){\line(1,0){10}} \put(10,0){\line($ **Tree 2→2** concept in the concept of t care when the care when th particles are \mathcal{P} and \mathcal{P} are \mathcal{P} and \mathcal{P} **2** — | ² **2→2** *[×] + complex conj.* **upic carries 2→2** *universal* ➙ eikonal factor J^q (soft) or splitting function *Pij* (collinear) $I - \lambda$ Multiple ϵ μ is equipped foot μ and ϵ α β \sim illustrate the interval \sim concepts, we do not a set \mathbb{R}^n care where we have a state particles are ̶ just F() **LO 2 Tree 2→2** $\mathcal{L} \cap \mathbb{R}$ care when \mathcal{L} particles are \overline{p} and \overline{p} are \overline{p} and \overline{p} are \overline{p} and \overline{p} are \overline{p} $= exp(-S)$ **2 2→4 2→4 2→4 2→4 NNL 1-loop** |
|
| 2008 complex of softward(in the CCD radiation furtile factorization 2 **2→2** *[×] + complex conj.* $F(\prec)$ ENNISS⁽⁾ **1-loop ²→³** *[×] + complex conj.* **2-loop 2→2** *[×] + complex conj.* **NNLO 1-loop** ² **2** *x* 2¹ **1** *splitting function P_{ij} (collinear)*

2² *CO* radiation furtus factorization $\frac{2}{2}$ **2→2** *[×] + complex conj.* … **NNLOC 1-loop** ²/₂ (collinear)

2² *x* **1** *x* **2-loop 2→2** *[×] + complex conj.* $F(\prec)$ *Sudakov form factor*

 \rightarrow Multiple emissions of soft/collinear QCD radiation fulfills factorization \overline{C} multiple multiple emission of the multiple solution collinear \overline{OCD} radiation fulfills factorization \mathbf{p}_T *Z* $\mathbf{p}_T^{\mathsf{L}}$ τ *i* collinear QCD radiation fuitilis to **1-loop**

 \bigstar However, also the phase space need sto be factorized p*T n* **2→2**

 \rightarrow go to impact-parameter space (in case of p_T), where radiation factorizes, to implement momentum conservation \sim go to impact-parameter space (in case or p_{1}), where

Latter Universal
 \rightarrow eikonal factor J^a (soft) or ➙ eikonal factor *Ja* (soft) or splitting function *Pij* (collinear)

In the case of the case of the part of the case of the

Transverse-momentum resummation **INGREDIENTS FOR A CALCULATION (generic 2→2 process)** Tranc[®] <u>17ansverse-momentum resummatic</u>

$$
\delta^{(2)}(\mathbf{p}_T - \mathbf{p}_{T1} - \dots \mathbf{p}_{Tn}) \qquad e^{i\mathbf{b}\cdot\mathbf{p}_T}
$$

$$
e^{i\mathbf{b}\cdot\mathbf{p}_T}\prod_{i=1}^n e^{-i\mathbf{b}\cdot\mathbf{p}_{Ti}}
$$

d(res)

=

universal

*b*2

⁰ */b*²

+ *B*

 i,j

nsverse-momentum resummation (2.7) using the impact- \blacksquare = *p*^T 1at *dbJ*1(*b p*T) *eS*(*b*0*/b*) [Collins, Soper, Sterman '85] Transverse-momentum resummation

⇣

G[*b*]

$$
\frac{d\sigma(p_{\rm T})}{d\Phi_{\rm F}} = p_{\rm T} \int_0^{\infty} db J_1(b p_{\rm T}) e^{-S(b_0/b)} \mathcal{L}_b(Qb/b_0)
$$
\n
$$
S(p_{\rm T}) = 2 \int_{p_{\rm T}}^Q \frac{dq}{q} \left(A(\alpha_s(q)) \ln \frac{Q^2}{q^2} + B(\alpha_s(q)) \right),
$$
\n
$$
S(p_{\rm T}) = 2 \int_{k=1}^Q \frac{dq}{q} \left(A(\alpha_s(q)) \ln \frac{Q^2}{q^2} + B(\alpha_s(q)) \right),
$$
\n
$$
S(c)
$$

*b*2

the following, for ease of notation, we will drop the ^B dependence in *L*. $\overline{\text{event generators (Lect)}}$ **a**) QCD and Monte Carlo event generators (Lecture 3) September 1, 2024 September 1, 2024 QCD and Monte Carlo event generators (Lecture 3)

nsverse-momentum resummation (2.7) using the impact- \blacksquare = *p*^T 1at *dbJ*1(*b p*T) *eS*(*b*0*/b*) [Collins, Soper, Sterman '85] Transverse-momentum resummation

From the Weyl transform (MIP-Munich)	$\frac{d\sigma(p_T)}{dp_F} = p_T \int_0^\infty db J_1(b p_T) e^{-S(b_0/b)} \mathcal{L}_b(Q_{\text{max}}^{\text{max}})$	University of the Weyl transform																																			
Use	$S(p_T) = 2 \int_{p_T}^Q \frac{dq}{q} \left(A(\alpha_s(q)) \ln \frac{Q^2}{q^2} + B(\alpha_s(q)) \right),$	University of the Weyl transform																																			
Use	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	S_c	<math display="block</td>

, *B*(1)

Transverse-momentum resummation

[Parisi, Petronzio '79], [Dokshitzer, Diakonov, Troian '80], [Curci, Greco, Srivastava '79], [Bassetto,

- ★ developed already 40 years ago Ciafaloni, Marchesini '80], [Kodaira, Trentadue '82], [Collins, Soper, Sterman '85]
- ★ newer formulations and advancement up to NNLL [Catani, de Florian, Grazzini '01], [Bozzi, Catani, de Florian, Grazzini '06 '07]
- \star recent reformulation in direct space, conserving momentum & keeping relevant subleading terms in p_T [Monni, Re, Torrielli '16], [Ebert, Tackmann '17]
- ★ Current state-of-the-art: N3LL & partial N4LL Tackmann, et al.; reSolve: Coradeschi, Cridge; Resbos: Isaacson, Yuan, et al.;]

[Matrix+RadISH: Kallweit, Re, Rottoli, MW; CuTe+MCFM: Becher, Campbell, Neumann, et al.; RadISH: Monni, Re, Rottoli, Torrielli; NangaParbat: Bacchetta, Bertone, Bozzi, et al.; Artemide: Scimemi, Vladimirov; DYTurbo: Catani, Grazzini, Ferrera, Cieri, Camarda, et al.; SCETlib: Billis, Ebert, Michel,

(several seminal works in SCET not discussed here)

Matching of resummation & fixed-order

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M_{\odot} ^I unitarity (due to *L* æ *L*^Õ Matching of resummation & fixed-order $\sum_{n=1}^{\infty}$ T (pdf)

 $\mathrm{d}\sigma\!/\mathrm{d}p_{\mathrm{T}}^{\mathrm{H}}\,[\!\,\mathrm{pb}]$

$$
\equiv \left[\sigma^{(\text{tot})}\right]_{f.o.}
$$

f.o.+l.a.

.

the most accurate predictions for specific observables -- $\ddot{}$ Resummed computations, properly matched to fixed order, are able to provide

Resummation: Example #I $Resummation: Example H1$

[ATLAS-CONF-2023-013]

MATRIX+RADISH Resummation: Example #2

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Questions?

Parton Shower (PS) +

Hadronization

realistic LHC event

shower accuracy (low precision)

no NXLO precision

Parton Shower Event Generators

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-

Parton Shower Event Generators

- \bigstar Parton shower event generators build the foundation of theoretical tools in experimental analyses to connect measurements & predictions
- Used to unfold from detector-level events to fiducial cross sections.
- \bigstar Parton showers build the core of the event simulation, combined with hadronization and multi-parton-interaction (MPI) models.
- \bigstar Parton showers provide the most flexible predictions, applicable, in principle, simultaneously to all IR-safe observables. However, unlike observable-specific resummation approaches they are limited to a lower logarithmic accuacy (so far)
- new approaches evolving to improve logarithmic accuracy of parton showers: [Forshaw, Holguin, Plätzer '20] [Nagy, Soper '19] [Dasgupta, et al. '20; Hamilton, et al. '20; Karlberg, et al. '21, …], [Höche et al. '22 '24]

Silvia Ferrario Ravasio Loopfest XXII 7

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Start with $q\bar{q}$ state produced at a hard scale v_0 . τ \mathcal{L} \mathcal{L} and \mathcal{L} and \mathcal{L} and \mathcal{L} and \mathcal{L} \mathcal{L} $q\bar{q}$ used shower paradigm (typically the invariant mass $v_0 \sim Q_{q\bar{q}}$)

Parton Shower **Parton Showers in a nutshell**

Dipole showers [Gustafson, Pettersson, '88] are the most

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- Ω Start with $q\bar{q}$ state produced at a hard scale v_0 .
- $\overline{\mathcal{U}}$ Throw a random number to determine down to what **scale** state persists unchanged

$$
\Delta(v_0, v) = \exp\left(-\int_v^{v_0} dP_{q\bar{q}}(\Phi)\right)
$$

no-emission probability between the v_0 and v_1

$\overline{}$ Solve for scale v_1 : $\Delta(v_0, v_1) \equiv n_{\text{random}}$

g2 At some point, **state splits** (2→3, i.e. emits gluon) at a scale $v_1 < v_0$. The kinematic (rapidity and azimuth) of the gluon is chosen according to

Ω Start with $q\bar{q}$ state produced at a hard scale v_0 .

 $\overline{\mathcal{U}}$ Throw a random number to determine down to what **scale** state persists unchanged

$$
dP_{q\bar{q}}(\Phi(\nu_1)) \qquad \Phi = \{\nu, \eta, \varphi\}
$$

g2 At some point, **state splits** (2→3, i.e. emits gluon) at a scale $v_1 < v_0$.

 $\sqrt{2}$ The gluon is part of two dipoles (qg), (gq).

independently, using v_1 as starting scale. Iterate the above procedure for both dipoles

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Ω Start with $q\bar{q}$ state produced at a hard scale v_0 .

 $\overline{\mathcal{U}}$ Throw a random number to determine down to what **scale** state persists unchanged

self-similar evolution continues until it reaches a nonscale

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 $d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}$

$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \frac{\Delta(\nu_0, \Lambda)}{\Delta(\nu_0, \Lambda)} + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$

no-emission

draw lines in the second s

$$
\left|\frac{\text{Born - }B}{\text{Der}(\text{diag})}\right|^2 \otimes F(\text{diag}) \longrightarrow \left|\frac{\text{Der}(\text{diag})}{\text{diag}}\right|^2
$$

$$
\mathrm{d}\sigma_{\rm PS} = \mathrm{d}\Phi_B B \times \left\{ \Delta(\nu_0,\Lambda) + \mathrm{d}\Phi_1 \Delta(\nu_0,\nu_1) \, \mathcal{P}(d\Phi_1) \right\}
$$

integrates to unity \rightarrow "unitarity" of parton shower (parton shower affects kinematics, not inclusive cross section)

-
-
-
-

$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$

$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \times \left\{ \Delta(\nu_0, \Lambda) \right\} \right\}
$$

$$
\left|\frac{\text{Born}-B}{\text{Dir}}\right|^2 \otimes F(-\left\langle \frac{1}{2} \otimes F(-\left\langle \frac{1}{2} \right\rangle) \longrightarrow \left\vert \frac{1}{\text{Dir}}\right\rangle
$$

 $d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \times \left\{ \Delta(\nu_1, \Lambda) + d\Phi_2 \Delta(\nu_1, \nu_2) \mathcal{P}(d\Phi_2) \right\} \right\}$

$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$

\n
$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \times \left\{ \Delta(\nu_1, \Lambda) + d\Phi_2 \Delta(\nu_1, \nu_2) \mathcal{P}(d\Phi_2) \right\} \right\}
$$

\nno-emission
\none emission
\nsecond emission
\nsecond emission
\nBorn - B
\n
$$
\left| \frac{\text{Born - B}}{\text{max of the F}} \right| \mathcal{P} \left(\frac{\text{Born - B}}{\text{max of the F}} \right)
$$

$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \times \left\{ \Delta(\nu_1, \Lambda) + d\Phi_2 \Delta(\nu_1, \nu_2) \mathcal{P}(d\Phi_2) \right\} \right\}
$$

$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$

$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu) \mathcal{P}(d\Phi_1) \times \left\{ \Delta(\nu_0, \Lambda) + \frac{\Delta(\nu_0, \Lambda)}{\Lambda} \right\} \right\}
$$

$$
\left| \frac{\text{Born} - B}{\cdot} \right|^2 \otimes F(\prec) \otimes F(\prec) \otimes
$$

Hadronization & underlying event

hadronization

- parton shower stops at a cutoff $\Lambda_{\rm QCD} \sim 0.2 \, {\rm GeV}$ and hadronization starts
- preconfinement: colour naturally arrangers in \rightarrow colour colour naturally arranged ➙ colour singlets close
- When the order of the order of the order of α • phenomenological models: enological models:

★ besides perturbative showering procedure, event generators include non-perturbative models to simulate hadronization & underlying event/multi-parton interactions (MPI)

MPI UNDI
Underlying events of the contract of the contra
Under the contract of the

• apart from primary hard scattering (several) secondary secondary collisions from other partons inside the proton may occur 12 GeV apart from primary hard scattering (several)

Parton showers at work

Problems of parton showers Problems of parton showers but the solution section and collected and collinear section an

★ parton showers rely on soft/collinear approximation for radiation (like resummation)

→ valid only in when radiation is soft/collinear

- \star in regions where hard QCD radiation such as at large p_T or a \angle boson, a part
does not provide a physical descriptio **boson will be OK only at low pT** \star in regions where hard QCD radiation is probed, such as at large p_T of a Z boson, a parton shower does not provide a physical description
	- \star by contrast, the shower provides a physical picture at low p_T

Questions?

Parton Shower (PS) +

Hadronization

realistic LHC event

shower accuracy (low precision)

no NXLO precision

- \sqrt{N} N^XLO (high precision)
- no event
	- no shower accuracy

proton proton

 \mathbb{R}^{\bullet}

Hard Process

Parton Shower (PS) + Hadronization

realistic LHC event

shower accuracy (low precision)

no NXLO precision

Combination NXLO+PS

- NXLO (high precision)
- realistic LHC event
- shower accuracy

NLO+PS matching: MC@NLO atch
 $3 \times \left\{ 2 \right\}$ (2)(p*T*

reminder shower formula: $\sigma_{\rm PS}$

NLO cross section: $d\sigma_{\rm NLO} \equiv \int d\sigma^{(1)} + d\sigma^{(2)}$ ²

$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$

$$
\sigma_{NLO} \equiv \left\{ d\sigma^{(1)} + d\sigma^{(2)} \right\} = d\Phi_B \left(B + V + \int d\Phi_{rad} R \right)
$$

NLO+PS matching: MC@NLO

 NLO cross section: $d\sigma_{NLO} \equiv \left\{ d\sigma^{(1)} + d\sigma^{(2)} \right\}$

$$
[\mathbf{X} \, I_{\text{MC}}^{(n)} + [\mathrm{d} \Phi_B \, \mathrm{d} \Phi_{\text{rad}} \, R] \times I_{\text{MC}}^{(n+1)}
$$

 $d\sigma_{\text{MC@NLO}}^{\text{naive}} = [d\Phi_B (B + V)] \times I_{\text{MC}}^{(n)}$ naive try:

 $I^{(k)}_{\rm MC}$: corresponds to the shower emission propability from a k-body kinematics **MC**

MC@NLO: additive matching (similar to analytic resummation & local subtraction): [Frixione, Webber '02]

$$
\text{remainder shower formula:} \quad \mathbf{d}\sigma_{\text{PS}} = \mathbf{d}\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + \mathbf{d}\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$
\n
$$
\text{NLO cross section:} \quad \mathbf{d}\sigma_{\text{NLO}} \equiv \left\{ \mathbf{d}\sigma^{(1)} + \mathbf{d}\sigma^{(2)} \right\} = \mathbf{d}\Phi_B \left(B + V + \int \mathbf{d}\Phi_{\text{rad}} R \right)
$$

NLO+PS matching: MC@NLO

remind

naive try:
$$
d\sigma_{\text{MC@NLO}}^{\text{naive}} = [d\Phi_B (B + V)] \times I_{\text{MC}}^{(n)} + [d\Phi_B d\Phi_{\text{rad}} R] \times I_{\text{MC}}^{(n+1)}
$$

\n \rightarrow double counting! $[\Phi_B B \times I_{\text{MC}}^{(n)}]$ and $[d\Phi_B d\Phi_{\text{rad}} R]$ both include the first radiation

 $I^{(k)}_{\rm MC}$: corresponds to the shower emission propability from a k-body kinematics **MC**

[Frixione, Webber '02] *MC@NLO:* additive matching (similar to analytic resummation & local subtraction):

der shower formula:
$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$

NLO cross section: $d\sigma_{NLO} \equiv \left\{ d\sigma^{(1)} + d\sigma^{(2)} \right\} = d\Phi_B \left(B + V + \int d\Phi_{rad} R \right)$

reminder shower formula: $d\sigma_{PS} = d\Phi_B B \times$

 NLO cross section: $d\sigma_{NLO} \equiv \left\{ d\sigma^{(1)} + d\sigma^{(2)} \right\}$

NLO+PS matching: MC@NLO

[Frixione, Webber '02] *MC@NLO:* additive matching (similar to analytic resummation & local subtraction):

 $\sigma_{\rm MCO} = 0$ (d $\sigma_{\rm MCO} = 0$) and $\sigma_{\rm HCO} = 0$ ($B + V + 0$) and $\sigma_{\rm rad}$ (M) and $V_{\rm MCO}$

solution: local MC counter term: $MC \simeq B \times [d\Phi_1/d\Phi_{rad} \mathcal{P}(d\Phi_1)]$ (depends on shower that you interface to)

$$
\left\{\Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1)\right\}
$$

$$
d\sigma^{(2)}\right\} = d\Phi_B \left(B + V + \int d\Phi_{\text{rad}} R\right)
$$

$$
V + \int d\Phi_{\text{rad}}MC \left[\int d\Phi_{\text{rad}}W C \right] \times I_{\text{MC}}^{(n)} + \left[d\Phi_B d\Phi_{\text{rad}} (R - MC) \right] \times I_{\text{MC}}^{(n+1)}
$$

reminder shower formula: $d\sigma_{PS} = d\Phi_R B \times$

 NLO cross section: $d\sigma_{NLO} \equiv \left\{ d\sigma^{(1)} + d\sigma^{(2)} \right\}$

NLO+PS matching: MC@NLO

[Frixione, Webber '02] *MC@NLO:* additive matching (similar to analytic resummation & local subtraction):

 σ_{no} double counting: $\sigma_{\text{MCGNLO}} = \left[\, \mathrm{d}\Phi_B \left(B + V + \int \mathrm{d}\Phi_{\text{rad}} M C \, \right) \, \right] \times I_{\text{MCGNLO}}^{(n)}$

solution: local MC counter term: $MC \simeq B \times |d\Phi_1/d\Phi_2|$

$$
\left\{\Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1)\right\}
$$

$$
d\sigma^{(2)}\right\} = d\Phi_B \left(B + V + \int d\Phi_{\text{rad}} R\right)
$$

$$
V + \int d\Phi_{\text{rad}} MC \Bigg] \times I_{\text{MC}}^{(n)} + \left[d\Phi_B d\Phi_{\text{rad}} (R - MC) \right] \times I_{\text{MC}}^{(n+1)}
$$

\n
$$
= \left\{ \left(1 - \int d\Phi_1 \mathcal{P}(d\Phi_1) \right) + d\Phi_1 \mathcal{P}(d\Phi_1)
$$

\n
$$
= \left\{ 1 - \int d\Phi_{\text{rad}} \frac{MC}{B} + d\Phi_{\text{rad}} \frac{MC}{B} \right\}
$$

reminder shower formula: $d\sigma_{PS} = d\Phi_B B \times$

 NLO cross section: $d\sigma_{NLO} \equiv \left\{ d\sigma^{(1)} + d\sigma^{(2)} \right\}$

NLO+PS matching: MC@NLO

$$
\begin{aligned}\n\left[\mathrm{d}\sigma_{\text{MC@NLO}}\right]_{\text{NLO}} &= \left[\mathrm{d}\Phi_B \left(B + V + \frac{\mathrm{d}\Phi_{\text{rad}}\mathcal{M}C}{\mathrm{d}\Phi_{\text{rad}}}\right)\right] \times I_{\text{MC}}^{(n)} + \left[\mathrm{d}\Phi_B \,\mathrm{d}\Phi_{\text{rad}}\left(R - \frac{\mathrm{d}C}{\mathrm{d}\Phi}\right)\right] \times I_{\text{MC}}^{(n-1)} \\
&= \mathrm{d}\sigma_{\text{NLO}} \\
&= \left\{\left(1 - \int \mathrm{d}\Phi_1 \mathcal{P}(d\Phi_1)\right) + \mathrm{d}\Phi_1 \mathcal{P}(d\Phi_1)\right\} \\
\text{counter term: } \mathcal{M}C \simeq B \times \left[\mathrm{d}\Phi_1/\mathrm{d}\Phi_{\text{rad}}\mathcal{P}(d\Phi_1)\right] \\
&= \left\{1 - \int \mathrm{d}\Phi_{\text{rad}}\mathcal{R} + \frac{\mathrm{d}\Phi_{\text{rad}}\mathcal{M}C}{\mathrm{d}\Phi_{\text{rad}}}\right\}\n\end{aligned}
$$

solution: local MC counter

[Frixione, Webber '02] *MC@NLO:* additive matching (similar to analytic resummation & local subtraction):

$$
\left\{\Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1)\right\}
$$

$$
d\sigma^{(2)}\right\} = d\Phi_B \left(B + V + \int d\Phi_{\text{rad}} R\right)
$$

NLO+PS matching: MC@NLO

remind

[Frixione, Webber '02] *MC@NLO:* additive matching (similar to analytic resummation & local subtraction):

Her shower formula:
$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$

\nNLO cross section:
$$
d\sigma_{NLO} \equiv \left\{ d\sigma^{(1)} + d\sigma^{(2)} \right\} = d\Phi_B \left(B + V + \int d\Phi_{rad} R \right)
$$

$$
d\sigma_{\text{MC@NLO}} = \left[d\Phi_B \left(B + V + \int d\Phi_{\text{rad}} MC\right)\right] \times I_{\text{MC}}^{(n)} + \left[d\Phi_B d\Phi_{\text{rad}} (R - MC)\right] \times I_{\text{MC}}^{(n+1)}
$$

$I^{(k)}_{\rm MC}$: corresponds to the shower emission propability from a k-body kinematics **MC**

NLO+PS matching: MC@NLO

remind

[Frixione, Webber '02] *MC@NLO:* additive matching (similar to analytic resummation & local subtraction):

Her shower formula:
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d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$

\nNLO cross section:
$$
d\sigma_{NLO} \equiv \left\{ d\sigma^{(1)} + d\sigma^{(2)} \right\} = d\Phi_B \left(B + V + \int d\Phi_{rad} R \right)
$$

$$
d\sigma_{\text{MC@NLO}} = \left[d\Phi_B \left(B + V + \int d\Phi_{\text{rad}} M C \right) \right] \times I_{\text{MC}}^{(n)} + \left[d\Phi_B d\Phi_{\text{rad}} (R - MC) \right] \times I_{\text{MC}}^{(n+1)}
$$

only sum is positive definite for physical observables, S- and H-events can be seperately negative

S-events H-events

NLO+PS matching: Powheg

 $\mathsf{remainder}\ \mathsf{shown}\ \mathsf{formula:}\qquad \mathsf{d}\sigma_\mathrm{PS} = \mathsf{d}\Phi_B B \ \times\ \Big\{\ \Delta(\nu_0,\Lambda) + \mathsf{d}\Phi_1\Delta(\nu_0,\nu_1)\,\mathscr{P}(d\Phi_1) \ \Big\}.$

reminder shower formula:

Powheg: generate first emission through matrix elements: [Nason '04], [Frixione, Nason, Oleari '07]

NLO+PS matching: Powheg

$$
d\sigma_{\text{PWG}} = d\Phi_B \tilde{B} \times \left\{ \Delta_{\text{pwg}}(\Lambda_{\text{pwg}}) + d\Phi_{\text{rad}} \Delta_{\text{pwg}}(p_{T,\text{rad}}) \frac{R}{B} \times I_{\text{MC}}^{(n+1)} \right\}
$$

 $d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}$

reminder shower formula: $d\sigma_{PS} = d\Phi_B B \times$

Powheg: generate first emission through matrix elements: [Nason '04], [Frixione, Nason, Oleari '07]

NLO+PS matching: Powheg

$$
\tilde{B} = B + V + \int d\Phi_{rad} R
$$

$$
\equiv \left\{ \frac{d\sigma^{(1)}}{d\Phi_{\rm m}} + \frac{d\sigma^{(2)}}{d\Phi_{\rm m}} \right\}
$$

 $d\Phi_{\rm B}$

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$$
\left\{\Delta(\nu_0,\Lambda)+\mathrm{d}\Phi_1\Delta(\nu_0,\nu_1)\,\mathcal{P}(d\Phi_1)\right\}
$$

$$
d\sigma_{\text{PWG}} = d\Phi_B \tilde{B} \times \left\{ \Delta_{\text{pwg}}(\Lambda_{\text{pwg}}) + d\Phi_{\text{rad}} \Delta_{\text{pwg}}(p_{T,\text{rad}}) \frac{R}{B} \times I_{\text{MC}}^{(n+1)} \right\}
$$

NLO+PS results

$35.8 \text{ fb}^1 (13 \text{ TeV})$ \rightarrow $35.8 \text{ fb}^1 (13 \text{ TeV})$ \rightarrow 10^{-1} $- 35.8 \text{ fb}^{-1} (13 \text{ TeV})$ 1

Multi-jet merging

...slide borrowed from Massimilano Grazzini

➙ *need to take care double counting!*

 \rightarrow merging is ad-hoc combination of n-jet different samples

◆ main idea:

-
- $\,$ resolution variable r_i typically related to transverse momentum of the emission
-

LO+PS merging

✦ LO+PS merging methods:

- hard emissions $(r_i > Q_{\text{cut}})$ described by matrix elements, soft emissions $(r_i < Q_{\text{cut}})$ by shower

- $\,$ merging scale \mathcal{Q}_{cut} cannot be pushed too low as large $\log(\mathcal{Q}_{\text{cut}}/\mathcal{Q})$ in matrix elements

CKKW | MLM | UMEPS | MEPS@LO (Sherpa) | …

NLO+PS merging

✦ idea very similar at NLO, but need to account for overlap in matrix elements

✦ X@NLO+PS, X+2,..,nj@LO+PS merging method:

✦ X+0,…,nj@NLO+PS:

 $C@NLO$ for Hi \triangleleft MC@NLO for Higgs+1-jet for $r_0 > Q_{\text{cut}}$

 $C@NLO$ for Hi restrict extra emis \triangleleft MC@NLO for Higgs+1-jet for $r_0 > Q_{\text{cut}}$ \rightarrow restrict extra emission $r_1 < Q_{\text{cut}}$

- $MCGNH$ *Qn*+1 *< Q*cut ✦ start from MC@NLO for Higgs+0-jet \rightarrow restrict first emission $r_0 < Q_{\text{cut}}$
- MC@NLO *pp* ! *h* + jet $C@NLO$ for Hi restrict extra emis \triangleleft MC@NLO for Higgs+1-jet for $r_0 > Q_{\text{cut}}$ \rightarrow restrict extra emission $r_1 < Q_{\text{cut}}$
- ← MC@NLO for Higgs+2-jet for $r_1 > Q_{\text{cut}}$ *Qn*+2 *< Q*cut

- $MCGNH$ *Qn*+1 *< Q*cut ✦ start from MC@NLO for Higgs+0-jet \rightarrow restrict first emission $r_0 < Q_{\text{cut}}$
- MC@NLO *pp* ! *h* + jet $C@NLO$ for Hi restrict extra emis \triangleleft MC@NLO for Higgs+1-jet for $r_0 > Q_{\text{cut}}$ \rightarrow restrict extra emission $r_1 < Q_{\text{cut}}$
- ← MC@NLO for Higgs+2-jet for $r_1 > Q_{\text{cut}}$ \overline{C} restrict exti \rightarrow restrict extra emission $r_2 < Q_{\text{cut}}$

- $m \cdot f_{\text{max}} \sim M C \otimes N \cup \bigcap$ *Q* **C C** *Qc Qc***ut C** *Qc* ✦ start from MC@NLO for Higgs+0-jet \rightarrow restrict first emission $r_0 < Q_{\text{cut}}$
- MC@NLO *pp* ! *h* + jet $C@NLO$ for Hi restrict extra emis \triangleleft MC@NLO for Higgs+1-jet for $r_0 > Q_{\text{cut}}$ \rightarrow restrict extra emission $r_1 < Q_{\text{cut}}$
- ← MC@NLO for Higgs+2-jet for $r_1 > Q_{\text{cut}}$ \overline{C} restrict exti \rightarrow restrict extra emission $r_2 < Q_{\text{cut}}$
- $\bigcup_{n=0}^{\infty}$ *pp* ! *h* + 2jets for *Q*uived, iterate ◆ LO+PS for Higgs+3-jet for $r_2 > Q_{\text{cut}}$ (MEPS@NLO/Sherpa specific) → no restriction on further radiation (keep PS)

- $m \cdot f_{\text{max}} \sim M C \otimes N \cup \bigcap$ *Q* **C C** *Qc Qc***ut C** *Qc* ✦ start from MC@NLO for Higgs+0-jet \rightarrow restrict first emission $r_0 < Q_{\text{cut}}$
- MC@NLO *pp* ! *h* + jet $C@NLO$ for Hi restrict extra emis \triangleleft MC@NLO for Higgs+1-jet for $r_0 > Q_{\text{cut}}$ \rightarrow restrict extra emission $r_1 < Q_{\text{cut}}$
- ← MC@NLO for Higgs+2-jet for $r_1 > Q_{\text{cut}}$ \overline{C} restrict exti \rightarrow restrict extra emission $r_2 < Q_{\text{cut}}$
- $\bigcup_{n=0}^{\infty}$ *pp* ! *h* + 2jets for *Q*uived, iterate ◆ LO+PS for Higgs+3-jet for $r_2 > Q_{\text{cut}}$ (MEPS@NLO/Sherpa specific) → no restriction on further radiation (keep PS)
-
- sum all contributions in ✦ sum all together ➙ Higgs+0,1,2j@NLO+PS

- $m \cdot f_{\text{max}} \sim M C \otimes N \cup \bigcap$ *Q* **C C** *Qc Qc***ut C** *Qc* ✦ start from MC@NLO for Higgs+0-jet \rightarrow restrict first emission $r_0 < Q_{\text{cut}}$
- MC@NLO *pp* ! *h* + jet $C@NLO$ for Hi restrict extra emis \triangleleft MC@NLO for Higgs+1-jet for $r_0 > Q_{\text{cut}}$ \rightarrow restrict extra emission $r_1 < Q_{\text{cut}}$
- ← MC@NLO for Higgs+2-jet for $r_1 > Q_{\text{cut}}$ \overline{C} restrict exti \rightarrow restrict extra emission $r_2 < Q_{\text{cut}}$
- $\bigcup_{n=0}^{\infty}$ *pp* ! *h* + 2jets for *Q*uived, iterate ◆ LO+PS for Higgs+3-jet for $r_2 > Q_{\text{cut}}$ (MEPS@NLO/Sherpa specific) → no restriction on further radiation (keep PS)
- - sum all contributions in ✦ sum all together ➙ Higgs+0,1,2j@NLO+PS
	- eg. *p*?(*h*)*>*200 GeV ✦ high pT receives multiple contributions

- $m \cdot f_{\text{max}} \sim M C \otimes N \cup \bigcap$ *Q* **C C** *Qc Qc***ut C** *Qc* ✦ start from MC@NLO for Higgs+0-jet \rightarrow restrict first emission $r_0 < Q_{\text{cut}}$
- MC@NLO *pp* ! *h* + jet $C@NLO$ for Hi restrict extra emis \triangleleft MC@NLO for Higgs+1-jet for $r_0 > Q_{\text{cut}}$ \rightarrow restrict extra emission $r_1 < Q_{\text{cut}}$
- ← MC@NLO for Higgs+2-jet for $r_1 > Q_{\text{cut}}$ \overline{C} restrict exti \rightarrow restrict extra emission $r_2 < Q_{\text{cut}}$
- $\bigcup_{n=0}^{\infty}$ *pp* ! *h* + 2jets for *Q*uived, iterate ◆ LO+PS for Higgs+3-jet for $r_2 > Q_{\text{cut}}$ (MEPS@NLO/Sherpa specific) → no restriction on further radiation (keep PS)
- sum all contributions in ✦ sum all together ➙ Higgs+0,1,2j@NLO+PS
- eg. *p*?(*h*)*>*200 GeV ✦ high pT receives multiple contributions
- ◆ smoother than MC@NLO for Higgs+0-jet

NLO+PS merging: Example #2 25 merging[.] F The final ingredient to reach the NNLO accuracy is the inclusion of the inclusion of the inclusion of the reweighting \mathbf{r}_i

$P_{\perp}(h) \simeq H^{\perp 1}$ -jet NLO+PS merging: Example #2 $p_{\perp}(h) \simeq H+1$ -jet 25 merging[.] F The final ingredient to reach the NNLO accuracy is the inclusion of the inclusion of the inclusion of the reweighting \mathbf{r}_i

pH

$p_1(h) \simeq H+1$ -jet

pH

$p_1(h) \simeq H+1$ -jet

Minlo output (rescaled by a global factor such that the total inclusive cross section is the same as

NLO+PS merging: Example #2 $N_{\rm H}$ other results with plots of the same kind. As expected, for the same kind. As expect \mathbf{u} in full agreement, both for the intervalues and scale uncertainty and scale uncertainty \mathbf{v} $\frac{1}{2}$ (NLOTE) merging. Example HZ

Inclusive Jet Multiplicity) [pb] ATLAS data *N*jet jets MePs@Nlo MePs@Nlo *µ*/2 . . . 2*µ* $10⁴$ MEnloPS $\bigwedge\big[$ $MENLOPS$ $\mu/2 \ldots 2\mu$ $\, + \,$ \geq Mc@NLo $\smash{\smash{\smash{\,\lrcorner\!\!\lrcorner\!\!\lrcorner\!\!\lrcorner\!\!\lrcorner\!\!\lrcorner\!\!\lrcorner\!\!\lrcorner\!\!\lrcorner\!\!\lrcorner}}\mspace{2.5mu}$ *s*10 ³ $p_\perp^{\rm jet}$ $\frac{C}{\mu}$ > 20 GeV $(\times 10)$ $p_\perp^{\rm jet}$ $J_{\perp}^{\text{et}} > 30 \,\text{GeV}$ 10 ² . 10 ¹ MEPS@NLO (Sherpa). 0 1 2 3 4 5 *N*jet

Questions?

NNLO+PS: What do we want to achieve? NNLO+PS: What do we wall

 \blacktriangleright NNLO accuracy for observables inclusive on radiation. $[d\sigma/dy_F]$

- appropriate scale choice for each kinematics regime

In preserve the PS accuracy (leading log - LL)

- possibly, no merging scale required.

-
- \blacktriangleright NLO(LO) accuracy for $F + 1(2)$ jet observables (in the hard region). $[d\sigma/dp_{T,j_1}]$
- **If all resummation from the Parton Shower (PS)** $[\sigma(p_{T,j} < p_{T,\text{veto}})]$
	-

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NNLOPS: MiNLO+reweighting Geneva

MiNNLOPS UNNLOPS

- ✦ LL accuracy (+ simple NLL terms) from PS
- ✦ no new unphysical scale (i.e. physically sound)
- ✦ numerically very intensive
- ✦ applied beyond 2→1 processes
- ✦ LL accuracy from PS (at most! no NNLL nonesense!)
- ✦ slicing cutoff (missing power corrections)
- ✦ numerical cancellations in slicing parameter
- ✦ applied beyond 2→1 processes

[Alioli, Bauer, Berggren, Tackmann, Walsh '15 + Zuberi '13]

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

- ✦ LL accuracy (+ simple NLL terms) from PS
- ✦ no new unphysical scale (i.e. physically sound)
- ✦ numerically efficient
- ✦ applied beyond 2→1 and even beyond colour singlet

[Höche, Prestel '14 '15]

✦ extension of UNLOPS merging of event samples

- ✦ two-loop corrections entirely in 0-jet bin
- ✦ only applied to 2→1 processes

[Hamilton, Nason, Oleari, Zanderighi '12, + Re '13], [Karlberg, Re, Zanderighi '14]

NNLO+PS methods

there was also some recent progress on NNLO+PS for sector showers [Campbell, Höche, Li, Preuss, Slands '21]

Geneva MiNLO+rew / MiNNLOPS

Ided in POWHEG, startin from F+jet

on of NNLO corrections: *i-dim. event reweighting in Born phase space* relevant terms derived from resummation formula NPS ∼ $\left(\frac{d\sigma_F^{\text{NNLO}}}{d\Phi_B}\right)$ $\left(\frac{d\sigma_F^{\text{MiNLO}}}{d\Phi_B}\right)$ = $c_0 + c_1 \alpha_s + c_2 \alpha_s^2$ $c_0 + c_1 \alpha_s + d_2 \alpha_s^2$ $= 1 +$ $c_2 - d_2$ *c*0

$$
LO = \tilde{B}^{\text{Mi(N)NLO}} \times \left\{ \Delta_{\text{pwg}} + d\Phi_{\text{rad}} \Delta_{\text{pwg}} \frac{R_{FJ}}{B_{FJ}} \right\}
$$

 ζ h modification of the \bar{B} function

$$
\tilde{B}^{\text{MiNLO}} \sim e^{-S} \left\{ d\sigma_{FJ}^{(1)} \left(1 + S^{(1)} \right) + d\sigma_{FJ}^{(2)} \right\}
$$

$$
^{5} \text{ M} \Phi \varphi_{\text{FS}} \sim \bar{B}^{\text{MiNLO}} + e^{-S} \left\{ \left(D - D^{(1)} - D^{(2)} \right) \times \right\}
$$

ࠂ

GENES IN MINE CY \cdot ucting IR-finite events

 $z = E_m/E_s$
 $z = E_m/E_s$ \mathcal{L}^{Ψ} essentially q_{T} slicing, but spread by splitting punction \mathscr{P} **Plunction** $\mathcal{P}(\Phi_{E})$ is a probability that in the result of the result of \mathcal{P}_{FJ} \blacklozenge essentially q_T slicing, but spread by splitting function

Marius Wiesemann (MPP Munich) $\frac{11}{2}$ OCD and Monte Carlo event generators (Lecture 3) September 7, 2024 **108** *^N* ! 0 $z = E_m/E_s$ $z = E_m/E_s$ ϕ $\frac{m}{2}$ $\frac{S}{Q}$
Comparison to high-precision Drell-Yan data [CMS '22 - arXiv:2205.04897]

[Monni, Re, MW '20]

MiNNLOPS: different matching observables

[from L. Rottoli's talk at Ringberg 2024]

[Ebert, Rottoli, MW, Zanderighi, Zanoli '23]

ATLAS data $pp \rightarrow ZW^{\pm} \rightarrow \ell \ell \ell'$ $\rightarrow \ell\ell\ell'\nu_{\ell'}$ @LHC 13 TeV

Figure 7 compares our MiNNLOPS predictions to recent ATLAS data [130]. The re-

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*s*fid. [fb]

 $(n \cap \mathbb{R})$

|
|
|

colour singlet: heavy quark pair: $\mathrm{d}\sigma_\mathrm{res}^F\thicksim$ d d*pT* ${e^{-S} \tH}$ (*C* ⊗ *f*) (*C* ⊗ *f*)} ${\rm d}\sigma^{F}_{\rm res} \sim$ d d*pT* $\{e^{-S} \text{Tr}(\mathbf{H}\Delta)(C \otimes f)(C \otimes f)\}$ [Catani, Grazzini, Torre '14] $\mathsf{u} \rho_T$ derived to NNLO in [Catani, Devoto, Grazzini, Mazzitelli, '23]

- [Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20]
- ✦ substantial complication due to final-state radiation and interferences

: operator/matrix in **Δ** colour space that encodes soft emissions of $t\bar{t}$ and interferences

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✦ compare resummation formulas (very schematic):

MiNNLOps: heavy quark production

[Czakon, Mitov '12], [Czakon, Fiedler, Mitov '13] [Czakon, Fiedler, Heymes, Mitov '15 '16]

tt ¯ production **Top pair production** • Top pair production is the **main source** of top quarks at LHC

 $t\bar{t}$ and tW *tt*

 H^{\pm} \bar{t} *tW* H^{\pm}

$$
t\bar{t} \rightarrow b\bar{b} W^{-}W^{+}
$$
\nFully leptonic

\n
$$
W^{+}W^{-} \rightarrow l\bar{\nu}_{l}\bar{l}\nu_{l}
$$
\nSemi-leptonic

\n
$$
W^{+}W^{-} \rightarrow l\bar{\nu}_{l}q\bar{q}'
$$
\nHadronic

\n
$$
W^{+}W^{-} \rightarrow q\bar{q}'q'\bar{q}
$$
\n(where $q = \{u, c\}$ and $q' = \{d, s\}$)

\n
$$
W^{+}W^{-} \rightarrow q\bar{q}'q'\bar{q}
$$
\n
$$
q = \{u, c\}
$$
\n
$$
q' = \{d, s\}
$$
\nHint:

tt ¯ production

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[Mazzitelli, Sotnikov, Wiesemann '24]

Marius Wiesemann (MPP Munich) QCD and Monte Carlo event generators (Lecture 3) September 7, 2024 **16** \blacklozenge same structure of singular/resummed cross section as $Q\bar Q,$ but need to account for recoil: colour singlet: heavy quark pair: heavy quark pair + colour singlet: ${\rm d}\sigma^{F}_{\rm res} \sim$ d d*pT* ${e^{-S} \tH}$ (*C* ⊗ *f*) (*C* ⊗ *f*)} ${\rm d}\sigma^{F}_{\rm res} \sim$ d d*pT* $d\sigma_{\text{res}}^F \sim \frac{u}{\sigma_{\text{res}}^F} \left\{ e^{-S} \operatorname{Tr}(\mathbf{H}\Delta) (C \otimes f) (C \otimes f) \right\}$ ${\rm d}\sigma^{F}_{\rm res} \sim$ d d*pT* [Devoto, Mazzitelli 'in preparation] Soft function for Heavy quark production in ARbitrary Kinematics

MiNNLOPS: heavy quark + colour singlet production

 $\{e^{-S} \text{Tr}(\mathbf{H}\Delta)(C \otimes f)(C \otimes f)\}$ account for recoil: The attempt and a $(C\otimes f)$ 201 $(\mathcal{Q}(f))$ 201

bb \overline{b}

[Mazzitelli, Sotnikov, MW '24]

Z production

★ bottom mass neither a large nor small scale: 4FS (massive bottom) and 5FS (massless bottom) viable

- ★ complication:
-

 \rightarrow approximated by small- m_h , expansion [Mitov, Moch '06], [Wang, Xia, Yang, Ye '23]

Z couples to initial-state light quarks and final-state heavy quarks & coupling depends on quark falvour

 \star 2-loop amplitude: most complicated ingredient & among most complicated 2-loop computed to date

$$
2\text{Re}\langle R^{(0)} | R^{(2)} \rangle = \sum_{m=1}^{4}
$$

$$
\kappa_i \log^i(m_b/\mu_R) + 2\text{Re}\langle R_0^{(0)} | R_0^{(2)} \rangle + \mathcal{O}(m_b/\mu)
$$

massive amplitude $i=1$ | massless amplitude power corrections

$$
i=1
$$
 |
coefficients of massification

[Abreu, Cordero, Ita, Klinkert, Page, Sotnikov '21]

[Mazzitelli, Sotnikov, MW '24]

Z+1b-jet distributions compared to CMS data [CMS 2112.09659]

Z production

-
- ★ parton showers include only QED (and QCD) radiation
-

 \bigstar multiple-radiation of heavy weak bosons not relevant at LHC energies (possibly at future colliders), since emissions regulated by boson mass \rightarrow not included in parton showers

thus, NLO+PS matching done at level of QED corrections (same methods as for QCD)

Public (N)NLO+PS Codes

SHERPA Sherpa

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Theses

Manual **Issue Tracker Git Repo**

Sherpa Homepage

simulate processes in virtually all configurations of interest, in particular for hadronic and e+e- colliders;

The standard reference for the use of the code is: J. Alwall et al, "The automated computation of tree-

simulations", arXiv:1405.0301 [hep-ph]. In addition to that, computations in mixed-coupling expansions

and/or of NLO corrections in theories other than QCD (eg NLO EW) require the citation of: R. Frederix et

more complete list of references can be found here: http://amcatnlo.web.cern.ch/amcatnlo/list_refs.htm

al, "The automation of next-to-leading order electroweak calculations", arXiv:1804.10017 [hep-ph]. A

starting from version 3.2.0, the latter include Initial State Radiation and beamstrahlung effects.

level and next-to-leading order differential cross sections, and their matching to parton shower

Sherpa is a Monte Carlo event generator for the Simulation of High-Energy Reactions of PArticles in lepton-lepton, lepton-photon, photon-photon, lepton-hadron and hadron-hadron collisions. Simulation programs - also dubbed event generators - like Sherpa are indispensable work horses for current particle physics phenomenology and are (at) the interface between theory and experiment.

iples of event

Latest version is 3.5.x

MG5_aMC_v3.5.5.tar.gz

released on 2023-05-12

All downloads

 $MG5$ _aMC_v2.9.20.tar.gz \downarrow

- - To browse the Sherpa manual online, see the manual
	- To find out more about the physics in Sherpa, see Publications and Theses.
- To get information about or contact the authors of Sherpa, see Sherpa Team
- To ask questions and browse answers about Sherpa, see the Sherpa Issue Tracker
- To be informed about patches and newer releases, subscribe to our announcement mailing list

The POWHEG BOX

Project

in shower Monte Carlo programs according to the POWHEG method. It is also a library, where previously included processes are made available to the users. It can be interfaced with all modern shower Monte Carlo programs that support the Les Houches Interface for User Generated Processes.

The WHIZARD Event Generator

The Generator of Monte Carlo Event **Generators for Tevatron, LHC, ILC, CLIC,** CEPC, FCC-ee, FCC-hh, SppC, the muon collider and other High Energy Physics **Experiments**

click here: <https://sherpa-team.gitlab.io/> **click here:** <https://whizard.hepforge.org/> **ARD?**

• HOME

• WHIZARD

- Main Page
- MANUAL, WIKI, NEWS
	- Manual (HTML)
	- Manual (PDF)
	- Miki Dogo

WHIZARD can evaluate NLO QCD corrections in the SM for arbitary lepton and hadron colliders. Tree-level matrix elements are generated automatically for arbitrary partonic processes by using the Optimized Matrix Element Generator O'Mega. Matrix elements obtained by alternative methods (e.g., including loop corrections) may be interfaced as well. The program is able to calculate numerically stable signal and background cross sections and generate unweighted event samples with reasonable efficiency for processes with up to eight final-state particles; more particles are possible. For more particles, there is the option to generate processes as decay cascades including complete spin

correlations. Different options for QCD parton showers are available.

 \rightarrow release candidate (since 2016) on git repo, only process: Drell-Yan production using τ_0

GENEVA: release candidate since 2016

Anmelden

ummed NNLO+NNLL' calculations. It produces LHEF events, hower generator (currently Pythia8) to produce fully

in bugs and missing features. We kindly ask that you report minary version prior to their usage in any publication.

click here: <https://gitlab.desy.de/geneva/geneva-public>

MiNNLOPS generators public in POWHEG BOX

The POWHEG BOX

Project

The POWHEG BOX is a general computer framework for implementing NLO calculations in shower Monte Carlo programs according to the POWHEG method. It is also a library, where previously included processes are made available to the users. It can be interfaced with all modern shower Monte Carlo programs that support the Les Houches Interface for User Generated Processes.

Index:

- · **Available NLO+PS** processes
- NNLOps using MiNNLOps
- Proper references
- Downloads
- Version 2
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- Bugs
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- **Contributing Authors**

 $Z\gamma$ generator ($Z \rightarrow \ell^+ \ell^-$, $Z \rightarrow \nu \bar{\nu} + a \, \text{TGC}$) [Lombardi, MW, Zanderighi '20, '21] *WW generator [Lombardi, MW, Zanderighi '21]*

 $M\dot{\textit{i}} N\textit{N} L O_{PS}$ has been extended to $2\to 2$ colour-singlet processes *in POWHEG-BOX-RES*

ZZ generator (qq¯*+gg) [Buonocre, Koole, Lombardi, Rottoli, MW, Zanderighi '21] WZ generator NNLOQCD+PS and NLOEW+PS [Lindert, Lombardi, MW, Zanderighi, Zanoli '23]*

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

H generator with full top-mass effects ω *NNLO [Niggetiedt, MW '24]*

Top-quark pair generator [Mazzitelli, Monni, Nason, Re, MW, Zanderighi '20 '21]

 $M\text{i}\text{\textit{NNLO}}$ processes (H, $\text{\textit{Z}}$, W) in POWHEG-BOX-V2

VH generator interfaced with H→*bb decay and SMEFT effects (t.b.a.) t.b.a: [Zanoli, Chiesa, Re, MW, Zanderighi '21], [Haisch, Scott, MW, Zanderighi, Zanoli '22] generator (t.b.a.) [Gavardi, Oleari, Re '22] bb generator (t.b.a.) [Mazzitelli, Ratti, MW, Zanderighi '24] bbZ generator (t.b.a.) [Mazzitelli, Sotnikov, MW '24]*

Summary

★ 50 years after the discovery of asymptotic freedom QCD has turned out to be a beautiful theory that allows us to provide predictions required at hadron colliders

 \star Parton Shower Event Generator bridge the gab between theory predictions and experimental measurements

★ Resummation for specific observables: NLL, NNLL, N3LL, …

★ Inclusion of higher-order corrections in parton showers: NLO+PS, NNLO+PS, …

➙ plenty of room for improvements (shower accuracy, shower uncertainties, non-perturbative effects, new NNLO+PS processes, …)

 \star Precision through perturbation theory: NLO, NNLO, N³LO, ...

What I didn't have time to cover

- \bigstar Different techniques for resummation (automation, SCET, other observables, soft resummation, …)
- ★ Jets in LHC collisions (jet algorithms, infrared-safe jet flavour, jet substructure, …)
- \bigstar Details on Higgs production and decay channels (heavy-top effective field theory, quark-mass effects, boosted Higgs analyses for VH, Higgs couplings, …)
- \star Improving the accuracy of parton showers

★ …

What I didn't have time to cover

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- \bigstar Improving the accuracy of parton showers

Thank you very much for your attention!

Oktober 2022 November 2022 Dezember 2022 January 2023 Mai 2023 June 2023 April 2024

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Questions?

Extra Slides

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

MiNNLOPS: main idea

NXLO+Parton Shower (PS) for pp → **F**

(symbolically)

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

◆ starting equation: $\frac{dC_F}{dC} = \frac{d}{dC} \{e^{-S}\mathscr{L}\} = e^{-S} \{S'\mathscr{L} + \mathscr{L}'\}$ $\mathscr{L} \sim H(C \otimes f)(C \otimes f)$ $\equiv D$ ${e^{-S}\mathscr{L}} = e^{-S} {S'\mathscr{L} + \mathscr{L'}}$

MiNNLOPS: derivation

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 \rightarrow *<u>CD</u> and Monte*

^c ^f[*b*]

 $\equiv D$

 \mathbf{D}

Lb(*Qb*0*/b*)*,* (E.1)

⇡

g¯3(*b*)*,* (E.2)

MiNNLOPS: derivation *f*_{*J*} *ps*: derivati ⌘ + 2 ⇣ *f*[*a*] *^c* (*P*ˆ(1) ⌦ *^f*) [*b*] *^c*⁰ + (*P*ˆ(1) ⌦ *^f*) [*a*] *^c ^f*[*b*] *c*0 + (*C*(1)(*K*F) ⌦ *^f*) [*a*] *^c* (*P*ˆ(0) ⌦ *^f*) [*b*] *^c*⁰ + (*P*ˆ(0) ⌦ *^f*) C *PS*: der ⌘ space formulation of transverse-momentum resummation. We start from the formula d(*p*T)

 $\overline{ }$ *^c* (*P*ˆ(0) ⌦ *^C*(1)(*K*F) ⌦ *^f*) [*b*] *^c*⁰ + (*P*ˆ(0) ⌦ *^C*(1)(*K*F) ⌦ *^f*) *^c*⁰ + (*P* ^ˆ(0) ⌦ *^f*) [*a*] *^c* (*C*(1)(*K*F) ⌦ *^f*) $V'20$

$$
e^{-S}\mathcal{L} = e^{-S} \{ S'\mathcal{L} + \mathcal{L}' \}
$$

\n
$$
\mathcal{L} \sim H(C \otimes f) (C)
$$

\n
$$
\equiv D
$$

\n
$$
V
$$
 (symbolically)
\n
$$
V
$$

$$
\leq \sum_{i,j}\bigg\{ \left(C^{[a]}_{ci}\otimes f^{[a]}_i\right)\bar{H}(Qb/b_0)\left(C^{[b]}_{c'j}\otimes f^{[b]}_j\right)\bigg\}
$$

= *p*^T 1at *dbJ*1(*b p*T) *eS*(*b*0*/b*)

Lb(*Qb*0*/b*)*,* (E.1)

 $\frac{1}{2}$

g¯3(*b*)*,* (E.2)

 $\frac{1}{2}$

$$
\left(A(\alpha_s(q)) \ln \frac{Q^2}{q^2} + B(\alpha_s(q))\right),
$$
\n
$$
\int^k A^{(k)}, \quad B(\alpha_s) = \sum_{k=1}^2 \left(\frac{\alpha_s}{2\pi}\right)^k B^{(k)}
$$

. (E.3)

H¯ (*Qb/b*0)

, (E.4)

$$
E = pT \int_0^\infty db J_1(b \, p_{\rm T}) \, e^{-S(b_0/b)} \mathcal{L}_b(Qb/b_0)
$$

g¯³ ⌘ *g*³ +

2⇣3(*A*(1))²

 $\frac{1}{2}$

d*|M*^F*|*

d^F

i,j

 $\frac{1}{2}$

ci ⌦ *^f*[*a*]

^s) accuracy and therefore this is not an issue

◆

Garlo event generators (Lecture λ is the thome sand λ is such that its integral in λ $\mathscr{L} \sim H($

, (2.3)

 \mathbf{b} and **b** a set of \overline{D} is the *g*^{*i*} \overline{D} (Symbolically) $\frac{m}{2}$, and $\frac{m}{2}$

$$
\mathscr{L} \sim H(C \otimes f)(C
$$
 (symbolically)

MiNNLOPS: derivation

$$
\begin{aligned}\n\blacklozenge \text{ starting equation:} \qquad & \frac{\text{d}o_F}{\text{d}p_T \text{d}\Phi_B} = \frac{\text{d}}{\text{d}p_T} \left\{ e^{-S} \mathcal{L} \right\} = e^{-S} \left\{ S' \mathcal{L} + \mathcal{L}' \right\} \qquad & \mathcal{L} \sim H(C \otimes f)(C \otimes f) \\
& \frac{\text{d}o_F}{\text{d}p_T \text{d}\Phi_B} = \frac{\text{d}}{\text{d}p_T} \left\{ \frac{\text{d}^2 \mathcal{L} - \text{d}^2}{\text{d}p_T} \right\} = \frac{\text{d}^2 \left\{ S' \mathcal{L} + \mathcal{L}' \right\}}{\text{d}p_T \text{d}\Phi_B} \qquad & \text{ (symbolically)}\n\end{aligned}
$$

$$
\begin{array}{ll}\n\text{\LARGE{}}\n\text{\LARGE{}}\n\end{array}\n\text{ starting equation:} \n\begin{array}{ll}\n\text{d}\sigma_F^{\text{res}} \\
\text{d}\rho_T \, d\Phi_B\n\end{array}\n=\n\frac{d}{dp_T}\left\{e^{-S}\mathscr{L}\right\} = e^{-S}\left\{\frac{S'\mathscr{L} + \mathscr{L}'}{g}\right\} \\
\text{ F\n\n
$$
\text{ F}\n\end{array}\n\begin{array}{ll}\n\text{ F}\n\end{array}\n\text{ (symbolically)} \\
\text{ F}\n\end{array}\n\begin{array}{ll}\n\text{d}\sigma_F^{\text{res}} + [\text{d}\sigma_{FJ}]_{\text{L},0} - [\text{d}\sigma_F^{\text{res}}]_{\text{L},0} \\
\text{F}\n\end{array}
$$
\n
$$
\begin{array}{ll}\n\text{d}\sigma_F = \text{d}\sigma_F^{\text{res}} + [\text{d}\sigma_{FJ}]_{\text{L},0} - [\text{d}\sigma_F^{\text{res}}]_{\text{L},0} \\
\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin{array}{ll}\n\text{F}\n\end{array}\n\begin
$$
$$

Z

 \blacklozenge combine with $F+{\rm jet}$ fixed order ${\rm d} \sigma_{FJ}$: $ch F + jet$ fixed

$$
\mathscr{L} \sim H(C \otimes f)(C
$$
 (symbolically)

$$
\text{ starting equation:} \qquad \frac{\text{d}o_F}{\text{d}p_T \text{d}\Phi_B} = \frac{\text{d}}{\text{d}p_T} \left\{ e^{-S} \mathcal{L} \right\} = e^{-S} \left\{ S' \mathcal{L} + \mathcal{L}' \right\} \qquad \mathcal{L} \sim H(C \otimes f) \left(C \otimes f \right)
$$
\n
$$
\equiv D \qquad \text{(symbolically)}
$$

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

MiNNLOPS: derivation

 \triangle combine with $F + \text{jet fixed order } d\sigma_{FI}$:

 $d\sigma_F = d\sigma_F^{\text{res}} + [d\sigma_{FJ}]_{f.o.} - [d\sigma_F^{\text{res}}]_{f.o.} = e^{-S} \left\{ D +$

$$
D + \frac{[d\sigma_{FJ}]_{f.o.}}{[e^{-S}]_{f.o.}} - \frac{[d\sigma_{F}^{res}]_{f.o.}}{[e^{-S}]_{f.o.}} \}
$$

$$
\frac{1 - S^{(1)...}}{1 - D^{(1)} - D^{(2)...}}
$$

$$
\mathscr{L} \sim H(C \otimes f)(C
$$
 (symbolically)

$$
\text{ starting equation:} \qquad \frac{\text{d}o_F}{\text{d}p_T \text{d}\Phi_B} = \frac{\text{d}}{\text{d}p_T} \left\{ e^{-S} \mathcal{L} \right\} = e^{-S} \left\{ S' \mathcal{L} + \mathcal{L}' \right\} \qquad \mathcal{L} \sim H(C \otimes f)(C \otimes f)
$$
\n
$$
\equiv D \qquad \text{(symbolically)}
$$

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

MiNNLOPS: derivation

 \triangle combine with $F + jet$ fixed order $d\sigma_{FI}$:

$$
d\sigma_F = d\sigma_F^{res} + [d\sigma_{FJ}]_{f.o.} - [d\sigma_F^{res}]_{f.o.} = e^{-S} \left\{ D + \frac{[d\sigma_{FJ}]_{f.o.}}{[e^{-S}]_{f.o.}} - \frac{[d\sigma_F^{res}]_{f.o.}}{[e^{-S}]_{f.o.}} \right\}
$$

= $e^{-S} \left\{ D + [d\sigma_{FJ}]_{f.o.} (1 - S^{(1)} - \cdots) - D^{(1)} - D^{(2)} - \cdots \right\}$

$$
\mathcal{L} \sim H(C \otimes f)(C
$$
 (symbolically)

$$
\begin{array}{ll}\n\text{ starting equation:} & \frac{\mathrm{d}\sigma_F^{\text{res}}}{\mathrm{d}p_T \,\mathrm{d}\Phi_B} = \frac{\mathrm{d}}{\mathrm{d}p_T} \left\{ e^{-S} \mathcal{L} \right\} = e^{-S} \left\{ S' \mathcal{L} + \mathcal{L}' \right\} & \mathcal{L} \sim H(C \otimes f)(C \otimes f) \\
& \leq D\n\end{array}
$$

 \blacklozenge combine with $F+{\rm jet}$ fixed order ${\rm d} \sigma_{FJ}$:

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

MiNNLOPS: derivation

$$
d\sigma_F^{\text{MiNNLO}} = e^{-S} \left\{ d\sigma_{FJ}^{(1)} (1 + S^{(1)}) + d\sigma_{FJ}^{(2)} + (D - D^{(1)} - D^{(2)}) + \text{regular} \right\}
$$

$$
d\sigma_{FJ}^{\text{MiNNLO}} = e^{-S} \left\{ d\sigma_{FJ}^{(1)} (1 + S^{(1)}) + d\sigma_{FJ}^{(2)} + (D - D^{(1)} - D^{(2)}) + \text{regular} \right\}
$$

$$
d\sigma_{FJ}^{\text{AiNNLO}} = e^{-S} \left\{ d\sigma_{FJ}^{(1)} (1 + S^{(1)}) + d\sigma_{FJ}^{(2)} + (D - D^{(1)} - D^{(2)}) + \text{regular} \right\}
$$

$$
d\sigma_F = d\sigma_F^{res} + [d\sigma_{FJ}]_{f.o.} - [d\sigma_F^{res}]_{f.o.} = e^{-S} \left\{ D + \frac{[d\sigma_{FJ}]_{f.o.}}{[e^{-S}]_{f.o.}} - \frac{[d\sigma_F^{res}]_{f.o.}}{[e^{-S}]_{f.o.}} \right\}
$$

= $e^{-S} \left\{ D + [d\sigma_{FJ}]_{f.o.} (1 - S^{(1)} - \cdots) - D^{(1)} - D^{(2)} - \cdots \right\}$

 \blacklozenge expand in $\alpha_s(p_T)$ & rearrange:

$$
\mathscr{L} \sim H(C \otimes f)(C
$$
 (symbolically)

$$
\begin{aligned}\n\text{ starting equation:} \qquad & \frac{\text{d}\sigma_F^{\text{res}}}{\text{d}p_T \,\text{d}\Phi_B} = \frac{\text{d}}{\text{d}p_T} \left\{ e^{-S} \mathcal{L} \right\} = e^{-S} \left\{ S' \mathcal{L} + \mathcal{L}' \right\} \\
& \qquad \mathcal{L} \sim H(C \otimes f)(C \otimes f) \\
& \qquad \text{(symbolically)} \\
& \equiv D\n\end{aligned}
$$

 \blacklozenge combine with $F+{\rm jet}$ fixed order ${\rm d} \sigma_{FJ}$:

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

MiNNLOPS: derivation

$$
d\sigma_F^{\text{MiNNLO}} = \frac{e^{-S} \left\{ d\sigma_{FJ}^{(1)}(1 + S^{(1)}) + d\sigma_{FJ}^{(2)} \right\} + (D - D^{(1)} - D^{(2)}) + \text{regular} \left\{ d\sigma_{FJ}^{(1)}(1 + S^{(1)}) + d\sigma_{FJ}^{(2)} \right\}}{\sim \alpha_s^2(p_T)}.
$$

$$
d\sigma_F = d\sigma_F^{res} + [d\sigma_{FJ}]_{f.o.} - [d\sigma_F^{res}]_{f.o.} = e^{-S} \left\{ D + \frac{[d\sigma_{FJ}]_{f.o.}}{[e^{-S}]_{f.o.}} - \frac{[d\sigma_F^{res}]_{f.o.}}{[e^{-S}]_{f.o.}} \right\}
$$

= $e^{-S} \left\{ D + [d\sigma_{FJ}]_{f.o.} (1 - S^{(1)} - \cdots) - D^{(1)} - D^{(2)} - \cdots \right\}$

 \blacklozenge expand in $\alpha_s(p_T)$ & rearrange:

MiNLO

$$
\mathscr{L} \sim H(C \otimes f)(C
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 (symbolically)

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$$
d\sigma_F^{\text{MiNNLO}} = e^{-S} \left\{ \underbrace{d\sigma_{FJ}^{(1)}(1+S^{(1)}) + d\sigma_{FJ}^{(2)}}_{\sim \alpha_s(p_T)} + \underbrace{(D-D^{(1)}-D^{(2)})}_{\geq \alpha_s^3(p_T)} + \underbrace{regular \atop \text{beyond accuracy}}_{\text{beyond accuracy}}
$$

$$
d\sigma_F = d\sigma_F^{res} + [d\sigma_{FJ}]_{f.o.} - [d\sigma_F^{res}]_{f.o.} = e^{-S} \left\{ D + \frac{[d\sigma_{FJ}]_{f.o.}}{[e^{-S}]_{f.o.}} - \frac{[d\sigma_F^{res}]_{f.o.}}{[e^{-S}]_{f.o.}} \right\}
$$

= $e^{-S} \left\{ D + [d\sigma_{FJ}]_{f.o.} (1 - S^{(1)} - \cdots) - D^{(1)} - D^{(2)} - \cdots \right\}$

 \blacklozenge expand in $\alpha_s(p_T)$ & rearrange:

✦ apply idea to POWHEG FJ calculation $d\sigma_{FJ} = d\Phi_{FJ} \tilde{B}^{FJ} \times \left\{ \Delta_{pwg}(\Lambda_{pw}) \right\}$ $\tilde{B}^{FJ} = B_{FJ} + V_{FJ} + \int d\Phi_{rad} R_{FJ}$ ≡ { $\mathrm{d}\sigma_{\!}$ (1) $d\Phi_{\rm B}$ + $\mathrm{d}\sigma_{\!}$ (2) $\frac{1}{d\Phi_B}$ $\mathbf c$ The multiple emission of soft and collinear gluons fulfils factorisation but in order \mathbb{R}^n to obtain an all order result also the phase space space factorised by phase space has to be properly factoris

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

MiNNLOPS: master formula

$$
V_{\rm{wg}} + d\Phi_{\rm{rad}} \Delta_{\rm{pwg}} (p_{T,\rm{rad}}) \frac{R_{FJ}}{B_{FJ}} \}
$$

G FJ calculation
\n
$$
\times \left\{\Delta_{\text{pwg}}(\Lambda_{\text{pwg}}) + d\Phi_{\text{rad}}\Delta_{\text{pwg}}(p_{T,\text{rad}}) \frac{R_{FJ}}{B_{FJ}}\right\}
$$
\n
$$
J + \left\{\frac{d\Phi_{\text{rad}}R_{FJ}}{d\Phi_{\text{B}}}\right\} \frac{1}{\left\{\frac{\sum_{\text{pgn} \text{png}} \frac{d\Phi_{\text{f}}}{dt} \frac{1}{B_{FJ}}}{P_{T}} \sum_{\text{png} \text{png}} \frac{1}{P_{T}} \sum_{\text{
$$

$$
\frac{n}{\text{1.2.5.5.5.5.5.7.5.7.02}} \cdot \frac{n}{\text{1.2.5.5.7.02}} \cdot \frac{n}{\text{1.2.5.7.02}} \cdot \frac{n}{\text{1.2.5.02}} \cdot \frac
$$

The multiple emission of soft and collinear gluons fulfils factorisation but in order \mathcal{C}

Resummation

\blacklozenge NNLO+PS by turning POWHEG weight (\tilde{B} function) NNLO accurate: d*σ* $\rm MiNNLO_{PS}$ *F* $\tilde{B}^{\text{MINNLO}_{\text{PS}}} = e^{-S} \left\{ B_{FJ} \left(1 + S^{(1)} \right) + V_{FJ} + \int d\Phi_{\text{rad}} R_{FJ} + \left(D - D^{(1)} - D^{(2)} \right) \right\}$

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[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

➙ *spreads NNLO corrections in the* $F + jet$ *phase space*

NNLO acuracy 1.1 no merging/slicing cut 1.1 shower accuracy (at least LL)

$$
= \mathrm{d}\Phi_{FJ} \tilde{B}^{\text{MINNLO}_{\text{PS}}} \times \left\{ \Delta_{\text{pwg}}(\Lambda_{\text{pwg}}) + \mathrm{d}\Phi_{\text{rad}} \Delta_{\text{pwg}}(p_{T,\text{rad}}) \frac{R_{FJ}}{B_{FJ}} \right\}
$$

MiNNLOPS: master formula

$$
{J} + |\mathrm{d}\Phi{\text{rad}} R_{FJ} + (D - D^{(1)} - D^{(2)}) \times F^{\text{corr.}}|
$$

\blacklozenge NNLO+PS by turning POWHEG weight (\tilde{B} function) NNLO accurate: d*σ* $\rm MiNNLO_{PS}$ *F* $\tilde{B}^{\text{MINNLO}_{\text{PS}}} = e^{-S} \left\{ B_{FJ} \left(1 + S^{(1)} \right) + V_{FJ} + \int d\Phi_{\text{rad}} R_{FJ} + \left(D - D^{(1)} - D^{(2)} \right) \right\}$

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[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

MiNNLOPS: master formula

$$
= d\Phi_{FJ} \tilde{B}^{\text{MiNNLO}_{PS}} \times \left\{ \Delta_{\text{pwg}} (\Lambda_{\text{pwg}}) + d\Phi_{\text{rad}} \Delta_{\text{pwg}} (p_{T,\text{rad}}) \frac{R_{FJ}}{B_{FJ}} \right\}
$$

$$
F_J (1 + S^{(1)}) + V_{FJ} + \int d\Phi_{\text{rad}} R_{FJ} + (D - D^{(1)} - D^{(2)}) \times F^{\text{corr.}} \right\}
$$

NNLO acuracy 1.1 Comerging/slicing cut 1.1 Shower accuracy (at least LL)

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

\blacklozenge NNLO+PS by turning POWHEG weight (\tilde{B} function) NNLO accurate: $\tilde{B}^{\text{MINNLO}_{\text{PS}}} = e^{-S} \left\{ B_{FJ} \left(1 + S^{(1)} \right) + V_{FJ} + \int d\Phi_{\text{rad}} R_{FJ} + \left(D - D^{(1)} - D^{(2)} \right) \right\}$ d*σ* $\rm MiNNLO_{PS}$ *F*

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MiNNLOPS: master formula

$$
= d\Phi_{FJ} \tilde{B}^{\text{MiNNLO}_{PS}} \times \left\{ \Delta_{\text{pwg}} (\Lambda_{\text{pwg}}) + d\Phi_{\text{rad}} \Delta_{\text{pwg}} (p_{T,\text{rad}}) \frac{R_{FJ}}{B_{FJ}} \right\}
$$

$$
F_J (1 + S^{(1)}) + V_{FJ} + \int d\Phi_{\text{rad}} R_{FJ} + (D - D^{(1)} - D^{(2)}) \times F^{\text{corr.}} \right\}
$$

NNLO acuracy $\sqrt{ }$ no merging/slicing cut \Box shower accuracy (at least LL)

 $\frac{1}{2}$ errossions. $\frac{1}{2} \frac{d\psi_0}{d\psi_0}$ $\left(\frac{d\psi_0}{d\psi_1} \right)$ $\left(\frac{d\psi_0}{d\psi_1} \right)$ $\left(\frac{d\psi_1}{d\psi_1} \right)$

[Monni, Nason, Re, MW, Zanderighi '19], [Monni, Re, MW '20]

MiNNLOPS: master formula

d*σ* $\rm MiNNLO_{PS}$

reminder 2 shower emissions:
$$
d\sigma_{PS} = d\Phi_B B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \times \left\{ \Delta(\nu_1, \Lambda) + d\Phi_2 \Delta(\nu_1, \nu_2) \mathcal{P}(d\Phi_2) \right\} \right\}
$$

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$$
d\sigma_F^{\text{MiNNLO}_{PS}} = d\Phi_{FJ} \tilde{B}^{\text{MiNNLO}_{PS}} \times \left\{ \Delta_{\text{pwg}}(\Lambda_{\text{pwg}}) + d\Phi_{\text{rad}}\Delta_{\text{pwg}}(p_{T,\text{rad}}) \frac{R_{FJ}}{B_{FJ}} \right\}
$$

$$
\tilde{B}^{\text{MiNNLO}_{PS}} = e^{-S} \left\{ B_{FJ} \left(1 + S^{(1)} \right) + V_{FJ} + \int d\Phi_{\text{rad}} R_{FJ} + \left(D - D^{(1)} - D^{(2)} \right) \times F^{\text{corr.}} \right\}
$$

$$
\simeq B \times \left\{ \Delta(\nu_0, \Lambda) + d\Phi_1 \Delta(\nu_0, \nu_1) \mathcal{P}(d\Phi_1) \right\}
$$
(2) *FJ* $+ (D(\tau_N) - D^{(1)}(\tau_N) - D^{(2)(\tau_N)}) \times F^{\text{corr}}$

 $p_T \rightarrow \tau_N$

$$
\tilde{B}^{\text{MiNNLO}_{\text{PS}}} \sim e^{-S(\tau_N)} \left\{ d\sigma_{FJ}^{(1)} \left(1 + S^{(1)}(\tau_N) \right) + d\sigma \right\}
$$

✦ MiNNLOPS viable for any N-jet resolution variable (in principle), e.g. N-jettiness:

$$
p_T \to \tau_N
$$

$$
\tilde{B}^{\text{MINNLO}_{PS}} \sim e^{-S(\tau_N)} \left\{ d\sigma_{FJ}^{(1)}(1 + S^{(1)}(\tau_N)) + d\sigma_{FJ}^{(2)} + \left(D(\tau_N) - D^{(1)}(\tau_N) - D^{(2)(\tau_N)} \right) \times F^{\text{corr}} \right\}
$$

 \triangle Differences in singular cross section (SCETI vs SCETII) leads to a richer logarithmic structure for τ_N :

✦ MiNNLOPS viable for any N-jet resolution variable (in principle), e.g. N-jettiness:

$$
d\sigma_F^{\text{res}}(\tau_N) = e^{-S(\tau_N)} \Big[\mathcal{L}(\tau_N) \Big(1 - \frac{\zeta_2}{2} [(S')^2 - S''] - \zeta_3 S' S'' + \frac{3\zeta_4}{16} (S'')^2 + \frac{\zeta_3}{3} S''' \Big) + \mathcal{L}'(\tau_N) \big(\zeta_2 S' + \zeta_3 S'' \big) + \mathcal{L}'(\tau_N) \big(\zeta_2 S' + \zeta_3 S'' \big) - \frac{\zeta_2}{2} \mathcal{L}''(\tau_N) + \mathcal{O}(\alpha_s^3) \Big] \qquad \text{[Ebert, Rottoli, MW, Zanderighi, Za}
$$

$$
d\sigma_F^{\text{res}}(p_T) = e^{-S(p_T)} \left[\mathcal{L}(p_T) \left(1 - \frac{\zeta_3}{4} S' S'' + \frac{\zeta_3}{12} S''' \right) - \frac{\zeta_3}{4} \frac{\alpha_s(p_T)}{\pi} S'' \hat{P} \otimes \mathcal{L}(p_T) + \mathcal{O}(\alpha_s^3) \right]
$$

[Monni, Nason, Re, MW, Zanderighi '19]

to be compared with:

$$
p_T \to \tau_N
$$

see also Matthew's talk for recent developments in Geneva [Alioli et al. '23]

$$
\tilde{B}^{\text{MINNLO}_{\text{PS}}} \sim e^{-S(\tau_N)} \left\{ d\sigma_{FJ}^{(1)}(1+S^{(1)}(\tau_N)) + d\sigma_{FJ}^{(2)} + \left(D(\tau_N) - D^{(1)}(\tau_N) - D^{(2)(\tau_N)} \right) \times F^{\text{corr}} \right\}
$$

✦ MiNNLOPS viable for any N-jet resolution variable (in principle), e.g. N-jettiness:

$$
p_T \to \tau_N
$$

$$
\tilde{B}^{\text{MINNLO}_{PS}} \sim e^{-S(\tau_N)} \left\{ d\sigma_{FJ}^{(1)}(1 + S^{(1)}(\tau_N)) + d\sigma_{FJ}^{(2)} + \left(D(\tau_N) - D^{(1)}(\tau_N) - D^{(2)(\tau_N)} \right) \times F^{\text{corr}} \right\}
$$

 \triangle Differences in singular cross section (SCETI vs SCETII) leads to a richer logarithmic structure for τ_N :

✦ MiNNLOPS viable for any N-jet resolution variable (in principle), e.g. N-jettiness:

$$
d\sigma_F^{\text{res}}(\tau_N) = e^{-S(\tau_N)} \Big[\mathcal{L}(\tau_N) \Big(1 - \frac{\zeta_2}{2} [(S')^2 - S''] - \zeta_3 S' S'' + \frac{3\zeta_4}{16} (S'')^2 + \frac{\zeta_3}{3} S''' \Big) + \mathcal{L}'(\tau_N) \big(\zeta_2 S' + \zeta_3 S'' \big) + \mathcal{L}'(\tau_N) \big(\zeta_2 S' + \zeta_3 S'' \big) - \frac{\zeta_2}{2} \mathcal{L}''(\tau_N) + \mathcal{O}(\alpha_s^3) \Big] \qquad \text{[Ebert, Rottoli, MW, Zanderighi, Za}
$$

$$
d\sigma_F^{\text{res}}(p_T) = e^{-S(p_T)} \left[\mathcal{L}(p_T) \left(1 - \frac{\zeta_3}{4} S' S'' + \frac{\zeta_3}{12} S''' \right) - \frac{\zeta_3}{4} \frac{\alpha_s(p_T)}{\pi} S'' \hat{P} \otimes \mathcal{L}(p_T) + \mathcal{O}(\alpha_s^3) \right]
$$

[Monni, Nason, Re, MW, Zanderighi '19]

to be compared with:

[from L. Rottoli's talk at Ringberg 2024]

MiNNLOPS: towards jet production [Ebert, Rottoli, MW, Zanderighi, Zanoli '23]

$$
d\sigma_{\text{res}}^{F} \sim \frac{d}{dp_{T}} \left\{ e^{-S} \operatorname{Tr}(\mathbf{H}\mathbf{\Delta}) \left(C \otimes f \right) \left(C \otimes f \right) \right\}
$$

$$
S = -\int \frac{dq^{2}}{q^{2}} \left[\frac{\alpha_{s}(q)}{2\pi} \left(A^{(1)} \log(M/q) + B^{(1)} \right) + \right.
$$

 $\text{Tr}(\textbf{H}\Delta) = \langle M | \Delta | M \rangle,$

 \otimes *f*) }

 $\alpha_s^2(q)$ $\frac{\alpha_s (q)}{(2\pi)^2} (A^{(2)} \log(M/q) + B^{(2)}) + ...$

$$
\Delta = \mathbf{V}^{\dagger} \mathbf{D} \mathbf{V}, \quad \mathbf{V} = \exp \left\{ -\int \frac{dq^2}{q^2} \left[\frac{\alpha_s(q)}{2\pi} \Gamma_t^{(1)} + \frac{\alpha_s^2(q)}{(2\pi)^2} \Gamma_t^{(2)} \right] \right\}
$$

matrix in colour space

MiNNLOps: heavy quark production

MiNNLOPS: heavy quark production

$$
d\sigma_{\text{res}}^{F} \sim \frac{d}{dp_{T}} \left\{ e^{-S} \operatorname{Tr}(\mathbf{H}\mathbf{\Delta}) \left(C \otimes f \right) \left(C \otimes f \right) \right\}
$$

$$
S = -\int \frac{dq^{2}}{q^{2}} \left[\frac{\alpha_{s}(q)}{2\pi} \left(A^{(1)} \log(M/q) + B^{(1)} \right) + \right.
$$

 $\text{Tr}(\textbf{H}\Delta) = \langle M | \Delta | M \rangle,$

$$
\Im f)\big\}
$$

MiNNLOPS: heavy quark production

✦ approximations keeping NNLO and (N)LL \bullet azimuthal average with $\left[\mathbf{D}\right]_{\phi}=1 \to \mathsf{modifies}\ H \to \overline{H}$ and $\left(C \otimes f\right) \to \overline{\left(C \otimes f\right)}$ at α_s^2 see [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan '19]

$$
d\sigma_{\text{res}}^F \sim \frac{d}{dp_T} \left\{ e^{-S} \operatorname{Tr}(\mathbf{H}\mathbf{\Delta}) \left(C \otimes f \right) \left(C \otimes f \right) \right\}
$$

$$
S = -\int \frac{dq^2}{q^2} \left[\frac{\alpha_s(q)}{2\pi} \left(A^{(1)} \log(M/q) + B^{(1)} \right) \right]
$$

 \otimes *f*) }

 $\text{Tr}(\textbf{H}\Delta) = \langle M | \Delta | M \rangle,$

$$
(A^{(1)} \log(M/q) + B^{(1)}) + \frac{\alpha_s^2(q)}{(2\pi)^2} (A^{(2)} \log(M/q) + B^{(2)}) + ...
$$

$$
\Delta = \mathbf{V}^\dagger \mathbf{D} \mathbf{V}, \quad \mathbf{V} = \exp\left\{-\int \frac{dq^2}{q^2} \left[\frac{\alpha_s(q)}{2\pi} \Gamma_t^{(1)} + \frac{\alpha_s^2(q)}{(2\pi)^2} \Gamma_t^{(2)}\right]\right\}
$$

✦ approximations keeping NNLO and (N)LL $\mathcal{L} \left(M | \Delta | M \right) \approx \langle M | M \rangle$ $=$ *H* $\langle M^{(0)}|\blacktriangle|M^{(0)}\rangle$ $\langle M^{(0)} | M^{(0)} \rangle$

 \bigotimes *f*) }

MiNNLOPS: heavy quark production

 \bullet azimuthal average with $\left[\mathbf{D}\right]_{\phi}=1 \to \mathsf{modifies}\ H \to \overline{H}$ and $\left(C \otimes f\right) \to \overline{\left(C \otimes f\right)}$ at α_s^2 see [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan '19]

$$
d\sigma_{res}^{F} \sim \frac{d}{dp_{T}} \left\{ e^{-S} \operatorname{Tr}(\mathbf{H}\mathbf{\Delta}) \left(C \otimes f \right) \left(C \otimes f \right) \right\}
$$

$$
S = -\int \frac{dq^{2}}{q^{2}} \left[\frac{\alpha_{s}(q)}{2\pi} \left(A^{(1)} \log(M/q) + B^{(1)} \right) \right]
$$

 $\text{Tr}(\textbf{H}\Delta) = \langle M | \Delta | M \rangle,$

$$
(A^{(1)} \log(M/q) + B^{(1)}) + \frac{\alpha_s^2(q)}{(2\pi)^2} (A^{(2)} \log(M/q) + B^{(2)}) + ...
$$

$$
\Delta = \mathbf{V}^\dagger \mathbf{D} \mathbf{V}, \quad \mathbf{V} = \exp\left\{-\int \frac{dq^2}{q^2} \left[\frac{\alpha_s(q)}{2\pi} \Gamma_t^{(1)} + \frac{\alpha_s^2(q)}{(2\pi)^2} \Gamma_t^{(2)}\right]\right\}
$$

absorb mistake at NNLO in *B*(2)

✦ approximations keeping NNLO and (N)LL $\mathcal{L} \left(M | \Delta | M \right) \approx \langle M | M \rangle$ \triangleleft expand $V = \exp \{-\}$ $=$ *H* $\langle M^{(0)}|\blacktriangle|M^{(0)}\rangle$ $\langle M^{(0)} | M^{(0)} \rangle$ dq^2 q^2 *αs*(*q*) 2*π* $\Gamma^{(1)}_{\textbf{t}}$ \equiv **V**_{NLL}

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 \otimes *f*) }

MiNNLOPS: heavy quark production

 \bullet azimuthal average with $\left[\mathbf{D}\right]_{\phi}=1 \to \mathsf{modifies}\ H \to \overline{H}$ and $\left(C \otimes f\right) \to \overline{\left(C \otimes f\right)}$ at α_s^2 see [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan '19]

absorb in $B^{(2)}$ coefficient

$$
\left(\frac{1}{t}\right) \times \left(1 - \int \frac{dq^2}{q^2} \frac{\alpha_s^2(q)}{(2\pi)^2} \Gamma_t^{(2)}\right) + \mathcal{O}(N^3LL)
$$

$$
d\sigma_{res}^{F} \sim \frac{d}{dp_{T}} \left\{ e^{-S} \operatorname{Tr}(\mathbf{H}\mathbf{\Delta}) \left(C \otimes f \right) \left(C \otimes f \right) \right\}
$$

$$
S = -\int \frac{dq^{2}}{q^{2}} \left[\frac{\alpha_{s}(q)}{2\pi} \left(A^{(1)} \log(M/q) + B^{(1)} \right) \right]
$$

 $\text{Tr}(\textbf{H}\Delta) = \langle M | \Delta | M \rangle,$

$$
(A^{(1)} \log(M/q) + B^{(1)}) + \frac{\alpha_s^2(q)}{(2\pi)^2} (A^{(2)} \log(M/q) + B^{(2)}) + ...
$$

$$
\Delta = \mathbf{V}^\dagger \mathbf{D} \mathbf{V}, \quad \mathbf{V} = \exp\left\{-\int \frac{dq^2}{q^2} \left[\frac{\alpha_s(q)}{2\pi} \Gamma_t^{(1)} + \frac{\alpha_s^2(q)}{(2\pi)^2} \Gamma_t^{(2)}\right]\right\}
$$

MiNNLOPS: heavy quark production

$$
d\sigma_{\text{res}}^F \sim \frac{d}{dp_T} \left\{ e^{-S} \operatorname{Tr}(\mathbf{H}\mathbf{\Delta}) \left(C \otimes f \right) \left(C \otimes f \right) \right\}
$$

$$
S = -\int \frac{dq^2}{q^2} \left[\frac{\alpha_s(q)}{2\pi} \left(A^{(1)} \log(M/q) + B^{(1)} \right) + \frac{\alpha_s^2(q)}{(2\pi)^2} \left(A^{(2)} \log(M/q) + B^{(2)} \right) + \dots \right]
$$

reminder:
$$
V_{\text{NLL}} \equiv \exp \left\{-\int \frac{dq^2}{q^2} \frac{\alpha_s(q)}{2\pi} \Gamma_t^{(1)}\right\}
$$

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(N)LL) we **ΓΙΔΥΕ**

*α*2 ✦ using those approximations (exact up to NNLO & (N)LL) we have:

$$
\tilde{B}^{(2)} = B^{(2)} + \frac{\langle M^{(0)} | \Gamma^{(2)\dagger} + \Gamma^{(2)} | M^{(0)} \rangle}{\langle M^{(0)} | M^{(0)} \rangle} + \frac{2 \operatorname{Re} \left\{ \langle M^{(1)} | \Pi^{(0)} \rangle \right\}}{\langle M^{(0)} | M^{(0)} \rangle}
$$

 \overline{A} and $e^{-S} \langle M | \Delta | M \rangle = e^{-\tilde{S}} \frac{\langle M^{(0)} | \mathbf{V}_{\text{NLL}}^{\dagger} \mathbf{V}_{\text{NLL}} | M^{(0)} \rangle}{\langle M^{(0)} | M^{(0)} \rangle} H + \mathcal{O}(\alpha_s^5)$ $\mathbf{C} = \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{C} \mathbf{C} \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{$ $M^{(0)} | \Gamma^{(1)} + \Gamma^{(1)} | M^{(0)} \rangle \text{Re} \left\{ \langle M^{(1)} | M^{(0)} \rangle \right\}$ $\frac{|\mathbf{\Gamma}^{(1)\dagger} + \mathbf{\Gamma}^{(1)}\,|\,M^{(0)}\rangle}{\langle M^{(0)}\,|\,M^{(0)}\rangle} - \frac{2\,\langle M^{(0)}\,|\, \mathbf{\Gamma}^{(1)\dagger} + \mathbf{\Gamma}^{(1)}\,|\,M^{(0)}\rangle\,\text{Re}\left\{\langle M^{(1)}\,|\,M^{(0)}\rangle\right\}}{\langle M^{(0)}\,|\,M^{(0)}\rangle^2}$ (0) | $M^{(0)}$ $)$ $\langle M^{(0)}|M^{(0)}\rangle^2$ $\langle M^{(0)} | M^{(0)} \rangle$ $H + \mathcal{O}(\alpha_s^5)$

MiNNLOPS: heavy quark production

⟨*M*|**Δ**|*M*⟩ ≈ ⟨*M*|*M*⟩ **use basis** $|M^{(0)}\rangle$ where $\Gamma^{(1)}$ diagonal $\qquad \qquad$

(N)LL) we **Γ**(**1**)

 \overline{A} and $e^{-S} \langle M | \Delta | M \rangle = e^{-\tilde{S}} \frac{\langle M^{(0)} | \mathbf{V}_{\text{NLL}}^{\dagger} \mathbf{V}_{\text{NLL}} | M^{(0)} \rangle}{\langle M^{(0)} | M^{(0)} \rangle} H + \mathcal{O}(\alpha_s^5)$ $\overline{}$ **^t** } **the set of the set of t** $\mathbf{C} = \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{C} \mathbf{C} \mathbf{C} \mathbf{C} + \mathbf{C} \mathbf{$ $M^{(0)}$ $\Gamma^{(1)\dagger}$ + $\mathbf{u} \cdot \mathbf{c}$ $\{ R \in \{ \langle M^{(1)} | M^{(0)} \rangle \} \}$ $r = \sum_{i} c_i e^{-S+S_i}$ $\bar{B}^{(1)} = B^{(1)} + \nu$. re-absorb mistake at NNLO in *B*(2) $\frac{|\mathbf{\Gamma}^{(1)\dagger} + \mathbf{\Gamma}^{(1)}\,|\,M^{(0)}\rangle}{\langle M^{(0)}\,|\,M^{(0)}\rangle} - \frac{2\,\langle M^{(0)}\,|\, \mathbf{\Gamma}^{(1)\dagger} + \mathbf{\Gamma}^{(1)}\,|\,M^{(0)}\rangle\,\text{Re}\left\{\langle M^{(1)}\,|\,M^{(0)}\rangle\right\}}{\langle M^{(0)}\,|\,M^{(0)}\rangle^2}$ (0) | $M^{(0)}$ $\langle M^{(0)}|M^{(0)}\rangle^2$ $\langle M^{(0)} | M^{(0)} \rangle$ $H + \mathcal{O}(\alpha_s^5)$ *i* c_i *e*− \overline{S} + S $-\overline{\overline{S}}$ *i* $\equiv e^{S_i}$ eigenvalues of $\mathbf{V}_{\rm NLL}^{\dagger}\mathbf{V}_{\rm NLL}$ exponent $\bar{B}^{(1)} = B^{(1)} + \gamma_i$

$$
d\sigma_{\text{res}}^{F} \sim \frac{d}{dp_{T}} \left\{ e^{-S} \operatorname{Tr}(\mathbf{H}\mathbf{\Delta}) (C \otimes f) (C \otimes f) \right\}
$$

$$
S = -\int \frac{dq^{2}}{q^{2}} \left[\frac{\alpha_{s}(q)}{2\pi} (A^{(1)} \log(M/q) + B^{(1)}) + \frac{\alpha_{s}^{2}(q)}{(2\pi)^{2}} (A^{(2)} \log(M/q) + B^{(2)}) + \dots \right]
$$

reminder:
$$
V_{\text{NLL}} = \exp \left\{-\int \frac{dq^2}{q^2} \frac{\alpha_s(q)}{2\pi} \Gamma_t^{(1)}\right\}
$$

*α*2 ✦ using those approximations (exact up to NNLO & (N)LL) we have:

$$
\tilde{B}^{(2)} = B^{(2)} + \frac{\langle M^{(0)} | \Gamma^{(2)} | + \Gamma^{(2)} | M^{(0)} \rangle}{\langle M^{(0)} | M^{(0)} \rangle} + \frac{2 \operatorname{Re} \left\{ \langle M^{(1)} | \Pi^{(0)} | M^{(0)} \rangle \right\}}{\langle M^{(0)} | M^{(0)} \rangle}
$$

\overline{b} *Z* production

- ★ complete calculation (five-point functions with massive b's) out of reach
- \star we exploit small-mass expansion in m_h (massification procedure)

[Mazzitelli, Sotnikov, MW '24]

Two-loop amplitude

 $(m_b/\mu_R) + 2Re\langle R_0^{(0)} | R_0^{(2)} \rangle + \mathcal{O}(m_b/\mu)$

massless amplitude power corrections

ients of massification

$$
1/\varepsilon \text{ poles in 5FS}
$$
\n
$$
2\text{Re}\langle R^{(0)} | R^{(2)} \rangle = \sum_{i=1}^{4} \kappa_i \log^i(n)
$$
\nmassive amplitude
\ncoefficients of m

[Mazzitelli, Sotnikov, MW '24] \sim Full corrections (five-point two-loop amplitudes with massive bis) out of reach \sim

- ★ complete calculation (five-point functions with massive b's) out of reach
- \bigstar we exploit small-mass expansion in m_b (massification procedure)

★ logarithmic terms exact (massless loops: [Mitov, Moch '06], massive loops: [Wang, Xia, Yang, Ye '23]) \blacksquare \blacks

Two-loop amplitude

-
- \bigstar infra-red safe mapping required from massive to massless momenta
- ★ massless two-loop in LC approx. & dropping Z coupling to closed quark loops (small at NLO) (based on [Chicherin, Sotnikov, Zoia '2110.07541], [Abreu, Cordero, Ita, Klinkert, Page, Sotnikov '2110.07541])

\overline{b} *Z* production Two-loop Corrections **Two-loop Corrections**

Poles in 5FS Logic of many structure in 5FS 2-loop finite reminder

 $log(m_h)$ in 4FS

 $(m_b/\mu_R) + 2Re\langle R_0^{(0)} | R_0^{(2)} \rangle + \mathcal{O}(m_b/\mu)$ $\Delta \text{N} \text{N}_0$ N_0 / $\text{T} \text{O}$ (n_b μ)

Example 33 and superior and development with the contracts

assification ● Massless two-loop reminder **(red)** computed from analytic results

$$
1/\varepsilon \text{ poles in 5FS}
$$
\n
$$
2\text{Re}\langle R^{(0)} | R^{(2)} \rangle = \sum_{i=1}^{4} \kappa_i \log^i(m_b/\mu_R) + 2\text{Re}\langle R_0^{(0)} | R_0^{(2)} \rangle + \mathcal{O}(m_b/\mu)
$$
\nmassive amplitude
cosficients of massification

MiNNLOPS: bb \overline{b} *Z* production $MINNIQ_{BC} b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

116 Gev. The scale in brackets in brackets in brackets in brackets in brackets in brackets indicates the di⊥er
116 Gev. The scale in brackets in brac

total cross section: $66 \text{ GeV} \le m_{e^+e^-} \le 116 \text{ GeV}$

[Mazzitelli, Sotnikov, MW '24] *MiNNLOPS: bb* \overline{b} *Z* production +60% NNLO correction ! total cross section: $66 \text{ GeV} \leq m_{e^+e^-} \leq 116 \text{ GeV}$ $MINNIQ_{BC} b\bar{b}Z$ production σ_{total} [pb] ratio to NLO $\text{NLO+PS } (m_{b\bar{b}\ell\ell}) \qquad \quad \textcolor{red}{\big|} \ \ 31.86(1)^{+16.3\%}_{-13.3\%}$ $+10.3\%$ 1.000
 -13.3% 1.000 $\text{MINLO}'\ (m_{b\bar{b}\ell\ell}) \qquad \qquad \qquad \quad \ 22.33(1)^{+28.2\%}_{-17.9\%}$ $+28.2\%$
 -17.9%
 -16.8% $\text{MINNLO}_{\text{PS}}~(m_{b\bar{b}\ell\ell}) \quad | \quad 50.58(4)^{+16.8\%}_{-12.2\%}$ $+10.8\%$ 1.587
-12.2% 1.587 $\rm NLO\!+\!PS~(H_T/2) ~~~~~~~~\,1.42(1)^{+19.2\%}_{-15.4\%}$ $+19.2\%$ 1.000
 -15.4% 1.000 $\text{MINNLO}_{\text{PS}}\ (H_T/2) \ \mid \ 58.60(5)^{+19.0\%}_{-13.2\%}$ 1.414
 -13.2% 1.414

116 Gev. The scale in brackets in brackets in brackets in brackets in brackets in brackets indicates the di⊥er
116 Gev. The scale in brackets in brac

[Mazzitelli, Sotnikov, MW '24] *MiNNLOPS: bb* \overline{b} *Z* production +60% NNLO correction ! +41% NNLO correction ! total cross section: $66 \text{ GeV} \le m_{e^+e^-} \le 116 \text{ GeV}$ $MINNIQ_{BC} b\bar{b}Z$ production σ_{total} [pb] ratio to NLO $\text{NLO+PS } (m_{b\bar{b}\ell\ell}) \qquad \quad \textcolor{red}{\big|} \ \ 31.86(1)^{+16.3\%}_{-13.3\%}$ $+10.3\%$ 1.000
 -13.3% 1.000 $\text{MINLO}'\ (m_{b\bar{b}\ell\ell}) \qquad \qquad \qquad \quad \ 22.33(1)^{+28.2\%}_{-17.9\%}$ $+28.2\%$
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 -16.8% $\text{MINNLO}_{\text{PS}}~(m_{b\bar{b}\ell\ell}) \quad | \quad 50.58(4)^{+16.8\%}_{-12.2\%}$ $+10.8\%$ 1.587
-12.2% 1.587 $\rm NLO\!+\!PS~(H_T/2) ~~~~~~~~\,1.42(1)^{+19.2\%}_{-15.4\%}$ $+19.2\%$ 1.000
 -15.4% 1.000 $\text{MINNLO}_{\text{PS}}\ (H_T/2) \ \mid \ 58.60(5)^{+19.0\%}_{-13.2\%}$ 1.414
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116 Gev. The scale in brackets in brac

MiNNLOPS: bb \overline{b} *Z* production $MINNIQ_{BC} b\bar{b}Z$ production

[Mazzitelli, Sotnikov, MW '24]

 \rightarrow MiNLO/multi-jet merging not suitable due to incomplete α_s^2 correction and large $\log(m_b)$ contribution in 2-loop (leading to miscancellation with $\log(m_b)$ from reals) (only a problem for bottom quarks and processes with $Q \gg m_b$) \rightarrow Philaco/multi-jet merging not suitable due to incomplete a_s correction and large
 $\log(m_b)$ contribution in 2-loop (leading to miscancellation with $\log(m_b)$ from reals)

total cross section: $66 \text{ GeV} \leq m_{e^+e^-} \leq 116 \text{ GeV}$

Comparison to CMS 7+h-jet analysis [CMS, 2112.09659] Comparison to CMS Z+b-jet analysis [CMS 2112.09659]

> Selection
 $\overline{35 \text{GeV}}$, $p_T(\text{subleading}) > 25 \text{GeV}$, $|\eta| < 2.4$ $71 < m_{\ell\ell} < 111\,\text{GeV}$ nadron jet*, p*_T $> 30\,\text{GeV}$, $|\eta| < 2.4$

 σ_{fiducial} [pb] $Z + \geq 1$ *b*-jet $Z + \geq 2$ *b*-jets $NLO+PS$ (5FS) $\begin{array}{|l} 7.03 \pm 0.47 & 0.77 \pm 0.07 \ \text{NLO+PS} \ (4\text{FS}) & 4.08 \pm 0.66 & 0.44 \pm 0.08 \end{array}$ $NLO+PS$ (4FS) $| 4.08 \pm 0.66$ 0.44 \pm 0.08
MINNLO_{PS} (4FS) 6.59 ± 0.86 0.77 \pm 0.10 $\begin{array}{l|c} \text{MINNLO}_{\text{PS}} \ (\text{4FS}) & 6.59 \pm 0.86 & 0.77 \pm 0.10 \ \text{CMS} & 6.52 \pm 0.43 & 0.65 \pm 0.08 \end{array}$ 6.52 ± 0.43 0.77 ± 0.07

 $f(x) = f(x)$ Z boson

Generator-level b jet
 b-hadron jet, $p_T > 30$ GeV, $|\eta| < 2.4$ Generator-level b jet

Comparison to CMS 7+h-jet analysis [CMS, 2112.09659] Comparison to CMS Z+b-jet analysis [CMS 2112.09659]

Object

Dressed leptons $p_T(\text{leading}) > 35 \,\text{GeV}, p_T(\text{subleading}) > 25 \,\text{GeV}, |\eta| < 2.4$

$$
\begin{array}{c|c|c|c|c|c} \hline \sigma_{\text{fiducial}}\text{[pb]} & & Z+\geq 1\text{ }b\text{-jet} & Z+\geq 2\text{ }b\text{-jets} \\\hline \text{NLO+PS (5FS)} & & 7.03\pm0.47 & 0.77\pm0.07 & \text{huge difference} \\\text{NLO+PS (4FS)} & & 4.08\pm0.66 & 0.44\pm0.08 & \text{and 5FS calculation} \\\hline \end{array}
$$

between 4FS and 5FS calculation

Comparison to CMS 7+h-jet analysis [CMS, 2112.09659] Comparison to CMS Z+b-jet analysis [CMS 2112.09659]

> Selection
 $\overline{35 \text{GeV}}$, $p_T(\text{subleading}) > 25 \text{GeV}$, $|\eta| < 2.4$ $71 < m_{\ell\ell} < 111\,\text{GeV}$ nadron jet*, p*_T $> 30\,\text{GeV}$, $|\eta| < 2.4$

 ζ of the momentum ζ of the fiducial value of ζ of the fiducial predictions ζ

 $f(x) = f(x)$ *C* boson

Cenerator-level b jet
 b -hadron jet, $p_T > 30$ GeV, $|\eta| < 2.4$ Generator-level b jet

Comparison to CMS 7+h-jet analysis [CMS, 2112.09659] Comparison to CMS Z+b-jet analysis [CMS 2112.09659]

Object

Dressed leptons $p_T(\text{leading}) > 35 \text{ GeV}, p_T(\text{subleading}) > 25 \text{ GeV}, |\eta| < 2.4$

$$
+ \geq 1 \text{ b-jet } Z + \geq 2 \text{ b-jets}
$$

 $NLO+PS$ (5FS) $\begin{array}{|l|l|l|} \hline 7.03 \pm 0.47 & 0.77 \pm 0.07 \ \hline 4.08 \pm 0.66 & 0.44 \pm 0.08 \ \hline \end{array}$ $NLO+PS$ (4FS) $\begin{array}{|l} \hline 4.08 \pm 0.66 & 0.44 \pm 0.08 \\ \hline 6.59 \pm 0.86 & 0.77 \pm 0.10 \end{array}$ $\begin{array}{|l|l|} \hline \text{MINNLO}_{\text{PS}} \text{ (4FS)} & \text{6.59}\pm \text{0.86} & \text{0.77}\pm \text{0.10}\ \hline \text{6.52}\pm \text{0.43} & \text{0.65}\pm \text{0.08} \end{array}$ 6.52 ± 0.43 0.77 ± 0.07

NNLO corrections make 4FS and 5FS compatible

Comparison to CMS 7+h-jet analysis [CMS, 2112.09659] Comparison to CMS Z+b-jet analysis [CMS 2112.09659]

> Selection
 $\overline{35 \text{GeV}}$, $p_T(\text{subleading}) > 25 \text{GeV}$, $|\eta| < 2.4$ $71 < m_{\ell\ell} < 111\,\text{GeV}$ nadron jet*, p*_T $> 30\,\text{GeV}$, $|\eta| < 2.4$

