Gravitational Wave Project

“Matched filtering”

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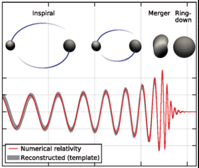
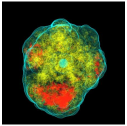
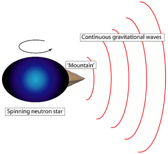
GHENT
UNIVERSITY

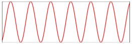
Gravitational-wave sources

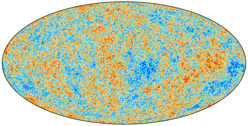


	Modelled	Unmodelled
Transient	Compact binary coalescences NS-NS, NS-BH, BBH	Bursts Supernova explosions
Persistent	Continuous waves Spinning deformed NS	Stochastic background Astrophysical + Cosmological

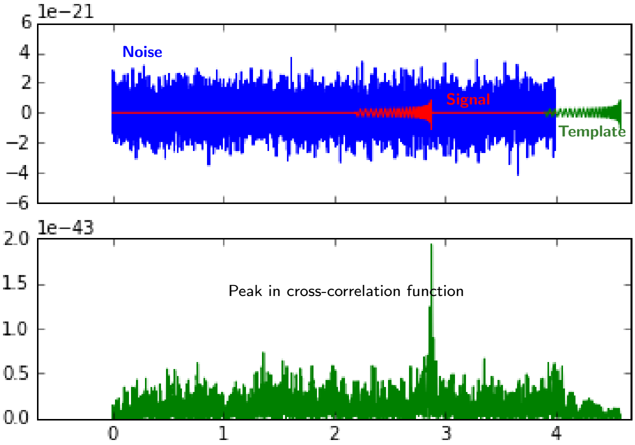








Matched filtering



In this project ...

- Code up a simple matched filtering algorithm

  Newtonian chirp waveform

  Gaussian **white** noise

- Search for a “fake” signal (with above assumptions)

  When?? [time]   How heavy?? [mass]

- Look at stretches of LIGO-Virgo-KAGRA data (`gwpy`)

  Determine its noise properties

- Try to generalize above algorithm for real data analysis

  Discuss complications

Pros and cons ...

Project likely on the easier side :)

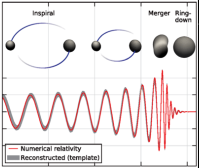
In line with the GW lectures by Elena Cuoco :)

Limited help available until until the very end :(

Python, `SciPy`, conda (`gwpy`) ... | cluster / laptop

Reference slides

Gravitational-wave sources



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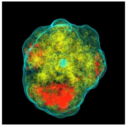
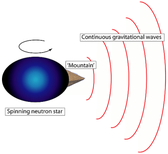
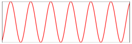
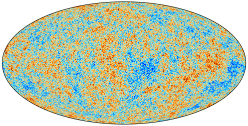








Gravitational-wave data analysis

Typically signal buried in noise significantly louder than itself.

In order to search for / study any signal,

one needs a thorough understanding of the detector noise.

Stationary Gaussian noise

Detector noise is **random**: detector output in absence of signal is a random number at every instant of time.

Stationary?

Its **statistical properties** do not change over time!

Caveat: they do!

Stationary Gaussian noise

Correlation of noise with itself at an earlier (or later) time

$$\langle n(t)n(t+\tau)\rangle \equiv \frac{1}{T}\int_{-T/2}^{T/2} dt\, n(t)\, n(t-\tau) = \kappa(\tau)$$

is a such a statistical property | "ergodicity"

In Fourier space **power spectral density**

$$\langle \tilde{n}(f)\tilde{n}^*(f')\rangle = S_n(f)\delta(f-f')\,, \quad \text{where} \quad S_n(f) \equiv \int_{-\infty}^{\infty} dt\, \kappa(t) e^{i2\pi f t}$$

$\kappa(\tau) = \kappa(-\tau) \Rightarrow S_n(f)$ is real dimensions of **time** unit: Hz^{-1}

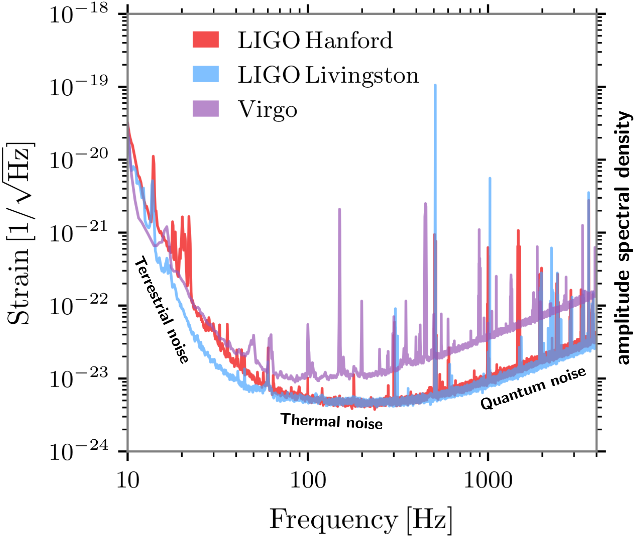
Stationary Gaussian noise

$$S_n(f) = \text{constant} \quad : \quad \text{white noise}$$

$$S_n(f) \propto \frac{1}{f} \quad : \quad \text{flicker noise}$$

$$S_n(f) \propto \frac{1}{f^2} \quad : \quad \text{random walk}$$

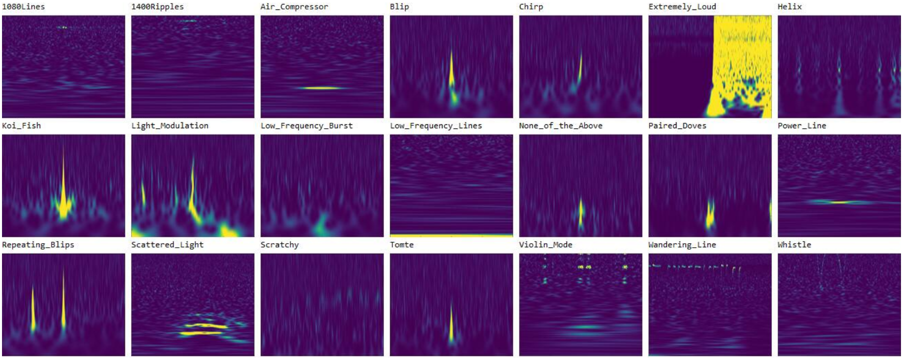
Gravitational-wave detector noise



Non-stationary effects

PSD slowly changing with time

transient "glitches"



Searching for modelled signal in detector data

detector data = possible signal + noise: $x(t) = h(t - t_{\rm arr}) + n(t)$

Correlation of template $q(t)$ with detector output.

$$c(\tau) \equiv \int_{-\infty}^{\infty} dt\, x(t)\, q(t+\tau)$$

lag τ: effectively concentrates all information in signal in one place

optimal filter $q(t)$ maximizes $c(\tau)$ when $h(t)$ in detector output

In frequency domain

$$c(\tau) = \int_{-\infty}^{\infty} df\, \tilde{x}(f)\, \tilde{q}^*(f) e^{-i2\pi f\tau}$$

Searching for modelled signal in detector data

$$S \equiv \langle c(\tau) \rangle = \int_{-\infty}^{\infty} df\, \tilde{h}(f)\, \tilde{q}^*(f) e^{-i2\pi f(\tau - t_{\mathrm{arr}})}$$

$$N^2 \equiv \langle |c - \langle c \rangle|^2 \rangle = \int_{-\infty}^{\infty} df\, S_n(f)\, |\tilde{q}(f)|^2$$

Noise-weighted inner product given $a(t)$, $b(t)$

$$\langle a|b \rangle \equiv 2 \int_0^{\infty} \frac{df}{S_n(f)} \left[\tilde{a}(f) \tilde{b}^*(f) + \tilde{a}^*(f) \tilde{b}(f) \right]$$

Signal-to-noise ratio (SNR):

$$\rho \equiv \frac{S}{N} = \frac{\langle h e^{i2\pi f t} | S_n q \rangle}{\langle S_n q | S_n q \rangle^{\frac{1}{2}}}$$

Searching for modelled signal in detector data

Signal-to-noise ratio (SNR):

$$\rho \equiv \frac{S}{N} = \frac{\langle h e^{i2\pi f t} | S_n q \rangle}{\langle S_n q | S_n q \rangle^{\frac{1}{2}}}$$

Optimal template that maximizes ρ is

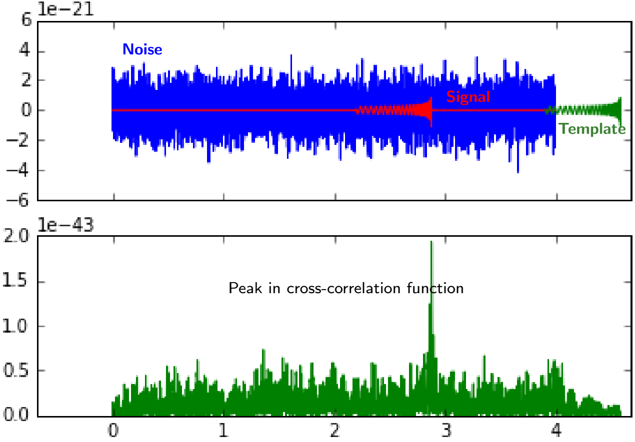
$$\tilde{q}(f) \sim \frac{\tilde{h}(f) e^{i2\pi f (t - t_{\mathrm{arr}})}}{S_n(f)}$$

- SNR maximized when $\tau = t_{\mathrm{arr}}$
- Optimal filter = signal weighted down by PSD (not just copy of signal)

Optimal SNR

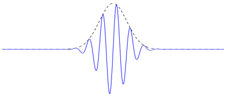
$$\rho_{\mathrm{opt}} = \frac{\langle h | h \rangle}{\langle h | h \rangle^{\frac{1}{2}}} = \langle h | h \rangle^{\frac{1}{2}} = 2 \left[\int_0^{\infty} df \, \frac{|h(f)|^2}{S_n(f)} \right]^{\frac{1}{2}}$$

Matched filtering

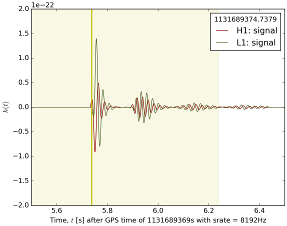


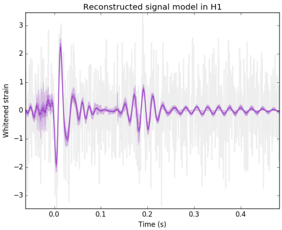
Searching for unmodelled signal in detector data

- Wavelet reconstruction: look for coherent excess power



Morlet-Gabor





Detector strain from an astrophysical signal

- Location of source w.r.t (arms of) the detector: (θ, ϕ)
- Polarization angle: ψ

$$F_+(\theta, \phi, \psi) = -\frac{1}{2}(1 + \cos^2\theta)\cos 2\phi \cos 2\psi - \cos\theta \sin 2\phi \sin 2\psi \quad \text{(A.10)}$$

$$F_\times(\theta, \phi, \psi) = +\frac{1}{2}(1 + \cos^2\theta)\cos 2\phi \sin 2\psi - \cos\theta \sin 2\phi \cos 2\psi \ . \quad \text{(A.11)}$$

These beam pattern functions are shown in Figure A.1.

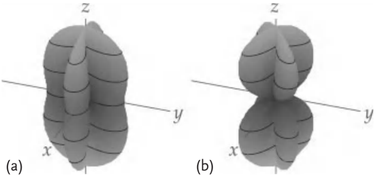


Figure A.1 The beam pattern functions $F_+^2(\theta, \phi, \psi = 0)$ (a) and $F_\times^2(\theta, \phi, \psi = 0)$ (b) for an interferometric gravitational-wave detector with orthogonal arms along the x- and y-axes.

detector strain $\quad h = F_+(\theta, \phi, \psi) h_+(t - t_{arr}) + F_\times(\theta, \phi, \psi) h_\times(t - t_{arr})$

Searching for unmodelled signal in detector data

Multiple reconstructions: which reconstruction to choose?

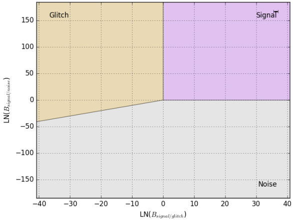
Bayesian Occam's razor: simpler reconstruction; fewer params

detector strain = signal + glitch + gaussian noise

signal model: $F^{+}_{\text{detector}}(\theta, \phi, \psi) h_{+}(t - t_{\text{arr}}) + F^{\times}_{\text{detector}}(\theta, \phi, \psi) h_{\times}(t - t_{\text{arr}})$

glitch model: independent sum of wavelets in each detector

noise model: gaussian noise of given (or measured) PSD

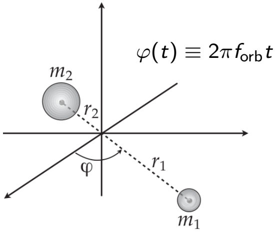


Bonus: model selection & glitch removal!

Compact binary coalescences

Two-body problem in GR

$$I_{ij} = \frac{1}{2}\,\mu\, r_{12}^2 \begin{bmatrix} 1+\cos 2\varphi & \sin 2\varphi & \\ \sin 2\varphi & 1-\cos 2\varphi & \\ & & 0 \end{bmatrix}$$



reduced mass $\mu \equiv \dfrac{m_1 m_2}{m_1 + m_2}$

$$h_{ij}^{\mathrm{TT}} \equiv \frac{2G}{c^4 d_L}\ddot{I}_{ij} \quad \Longrightarrow \quad \begin{cases} h_+(t) &= \dfrac{2GM\eta}{c^4 d_L}(\pi M f_{\mathrm{GW}})^2(1+\cos^2\iota)\cos 2\varphi(t) \\ h_\times(t) &= \dfrac{4GM\eta}{c^4 d_L}(\pi M f_{\mathrm{GW}})^2 \cos\iota \sin 2\varphi(t) \end{cases}$$

$$M \equiv m_1 + m_2$$

total mass

$$\eta \equiv \frac{m_1 m_2}{M^2}$$

symmetric mass ratio

Note: $f_{\mathrm{GW}} = 2 f_{\mathrm{orb}}$

Two-body problem in GR

$$L_{\rm GW} \quad = \quad \frac{1}{5}\frac{G}{c^5}\langle \dddot{I}_{ij}\dddot{I}^{ij}\rangle \quad = \quad \frac{32}{5}\frac{c^5}{G}\eta^2\left(\frac{\pi G M f_{\rm GW}}{c^3}\right)^{10/3}$$

Larmor formula

$$E = \frac{1}{2}\mu(\pi G M f_{\rm GW})^{2/3}$$

Newtonian energy

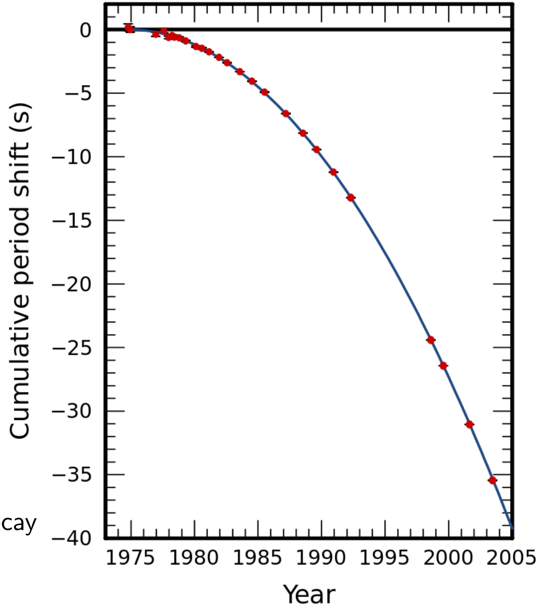
$$L_{\rm GW} = -\frac{dE}{dt}$$

energy conservation

$$\Rightarrow \qquad \frac{df_{\rm GW}}{dt} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} f_{\rm GW}^{11/3}$$

with **chirp mass** $\mathcal{M}_c \equiv \dfrac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

Leading order of the **post-Newtonian expansion**.



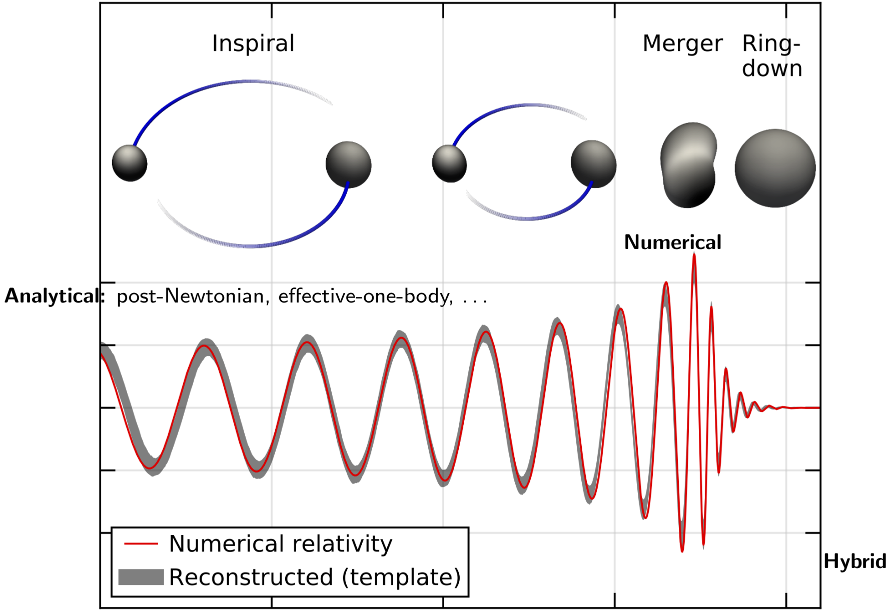
PSR B1913+16 orbital decay

Hulse-Taylor (1975)

Post-Newtonian expansion

Expansion in powers of $\left(\frac{v}{c}\right)^2$

$$P_{\mathrm{gw}} = \frac{32c^5}{5G}\nu^2 x^5 \left\{ 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} \right.$$
$$+ \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2$$
$$+ \left(-\frac{8191}{672} - \frac{583}{24}\nu\right) \pi x^{5/2} \qquad (5.$$
$$+ \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\log(16\,x)\right.$$
$$\left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right] x^3$$
$$\left. + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right) \pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right) \right\}$$


Inspiral
Merger
Ring-
down
Numerical
Analytical: post-Newtonian, effective-one-body, . . .
Numerical relativity
Reconstructed (template)
Hybrid

Waveform approximants

Post-Newtonian	`TaylorT`, `TaylorF`
Hybrid, phenomenological	`IMRPhenom`, `IMRPhenomP`, `IMRPhenomX`
Effective one-body	`SEOBNR`
reduced-order-model	`ROM`

Intrinsic parameters

- Component masses, $m_{1,2}$
- Component dimensionless spin angular momenta, $\vec{s}_{1,2}$ $\qquad \vec{s} \equiv \frac{\vec{J}}{m^2}$
- Tidal deformability parameters for neutron stars, $\lambda_{1,2}$

NS in binaries deform in gravitational field of companion
deformation depends on NS composition (equation-of-state)
leaves imprint on GW signal (waveform)

- Any residual eccentricity? radiated away Peters & Matthews (1963)

Two polarizations

Dominant $(2,2)$-mode to leading order:

$$h_+(t) = \frac{2M\eta}{d_L}(\pi M f_{\rm GW})^2(1+\cos^2\iota)\cos 2\varphi(t)$$
$$h_\times(t) = \frac{4M\eta}{d_L}(\pi M f_{\rm GW})^2\cos\iota\sin 2\varphi(t)$$

$$M \equiv m_1 + m_2 \qquad \eta \equiv \frac{m_1 m_2}{M^2}$$

- Distance, d_L
- Inclination angle, ι
- Phase at coalescence, ϕ_c
- Time of coalescence, t_c

Antenna beam pattern functions

- Two angles in the sky $(\theta, \phi) \longrightarrow (\alpha, \delta)$, right ascension and declination
- Polarization angle, ψ

$$F_+(\theta, \phi, \psi) = -\frac{1}{2}(1 + \cos^2\theta)\cos 2\phi \cos 2\psi - \cos\theta \sin 2\phi \sin 2\psi \quad \text{(A.10)}$$

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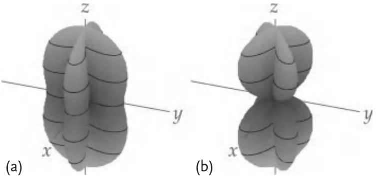

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CBC parameters

Intrinsic parameters: $\{m_1, m_2, \vec{s}_1, \vec{s}_2, \lambda_1, \lambda_2, \ldots\}$

Extrinsic parameters: $\{\alpha, \delta, d_L, \iota, \psi, \varphi_c, t_c\}$

$$F^{+}_{\text{detector}}(\theta, \phi, \psi) h_{+}(t - t_{\text{arr}}) + F^{\times}_{\text{detector}}(\theta, \phi, \psi) h_{\times}(t - t_{\text{arr}})$$

masses of the components
spin angular momenta
tidal deformability for neutron stars
~~eccentricity~~

distance d_L
sky position α, δ
euler angles $\{\theta, \phi, \psi\} = \text{fn.}(\{\iota, \psi, \varphi_c\})$
time of arrival

Bayesian parameter estimation

Obtain the posterior probability distribution on the parameter space given the data and a prior probability distribution.

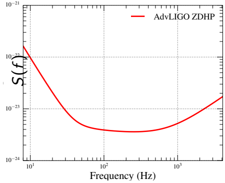
$$Posterior(\vec{\Omega}|\text{data}, I) = \frac{Prior(\vec{\Omega}|I)\,\mathcal{L}(\text{data}|\vec{\Omega}, I)}{Evidence(\text{data}, I)}$$

$$\vec{\Omega} = \{\mathcal{M}, q, \vec{s}_1, \vec{s}_2, \lambda_1, \lambda_2, \alpha, \sin\delta, d_L, \cos\iota, \psi, \varphi_c, t_c\}$$

$$\mathcal{L}(\text{data}|\vec{\Omega}, I) = P(\text{data}|\text{signal}(\vec{\Omega}), I)$$

$\text{data} = \text{signal}(\vec{\Omega}) + \text{noise}, n$

$$= \exp\left(-\frac{1}{2}\left\langle \text{data} - \text{signal}(\vec{\Omega})|\text{data} - \text{signal}(\vec{\Omega})\right\rangle\right)$$



$$\langle n|n\rangle \equiv \int df\, \frac{||n(f)||^2}{S^2(f)}$$

Gaussian noise

Bayesian parameter inference algorithms

Markov Chain Monte Carlo (MCMC)

Metropolis-Hastings

Parallel tempering (MCMC)

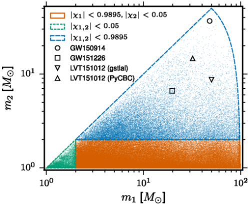
Nested sampling 1024 live points

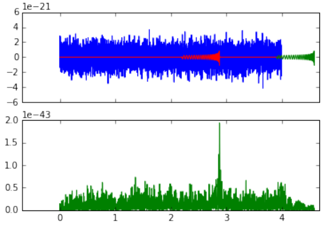
Data analysis workflow of CBCs

Searches

generate (real-time) triggers

LVC: Abbott+, PRX **6**, 041015 (2016)





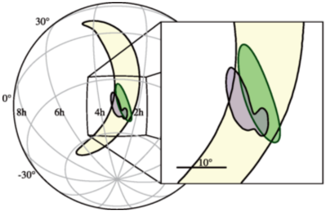
Implications

fundamental physics, astrophysics, cosmology

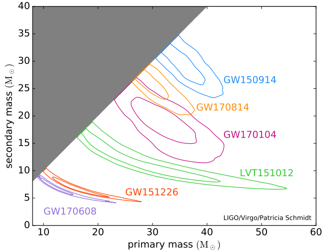
Parameter estimation

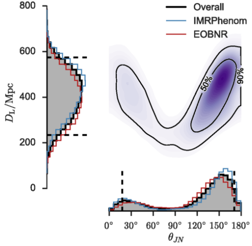
rigorous analysis of data around trigger

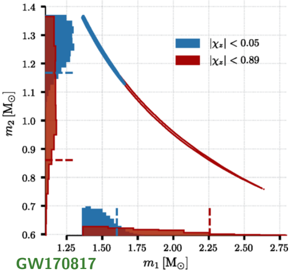
Low latency	High latency
quick	accurate
BayesSTAR	LALINFERENCE
RapidPE	BILBY



Parameter estimation results







GW170817

LVC: Abbott+, PRL **119**, 161101 (2017)