Scale Variations for Cross-Section Uncertainties

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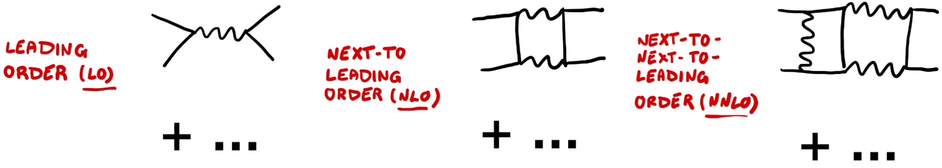
Cross-section are predicted from a Dyson series.

Feynman diagrams: terms in perturbative expansion of scattering matrix:

$$\langle \Psi_f | U | \Psi_i \rangle \sim$$



(Image adapted from Simone Devoto)

Renormalization introduces an arbitrary scale.

Classical Lagrangian $\mathcal{L}$ yields **unphysical** divergent loop diagrams.
Solve by regularizing the integral (dimreg, Λ cut-off)

$\rightarrow$ dictates counter terms to **renormalize** the classical theory:

$$\mathcal{L}_{\text{fund}} = \mathcal{L} + \mathcal{L}_{\text{c.t.}}$$

Renormalizing effectively varies the couplings dependent on **arbitrary scale** μ_R^2:

$$\frac{\mathrm{d}\alpha(\mu^2)}{\mathrm{d}\ln(\mu^2)} = \beta\left(\alpha(\mu^2)\right) \implies \alpha(\mu^2) = \frac{\alpha(\mu_R)^2}{1 - \alpha(\mu_R^2)\beta_0 \ln(\mu^2/\mu_0^2)} + \mathcal{O}(\alpha^2)$$

Factorization introduces an arbitrary scale.

QCD is:

- non-perturbative at low E (hadron formation),
- perturbative at high E (hard parton scattering)

LHC collides hadrons...

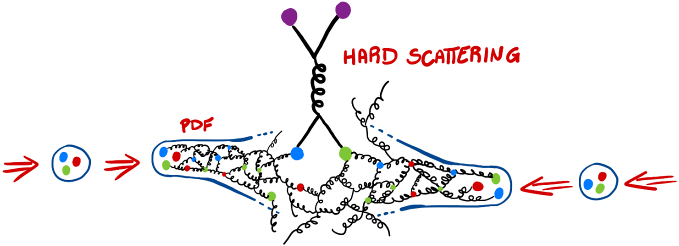
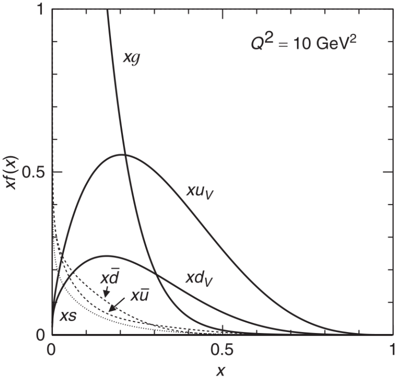


Image from Simone Devoto

Factorization introduces an arbitrary scale.

For hadron collisions: factorize the mixed behaviour:

$$\sigma_{\text{had}} = \sum_{ij\ (\text{partons})} \int \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{PDF}_i(x_1)\, \mathrm{PDF}_j(x_2) \times \sigma_{ij}(x_1 p_1, x_2 p_2)$$



Parton Distribution Functions (Thomson)

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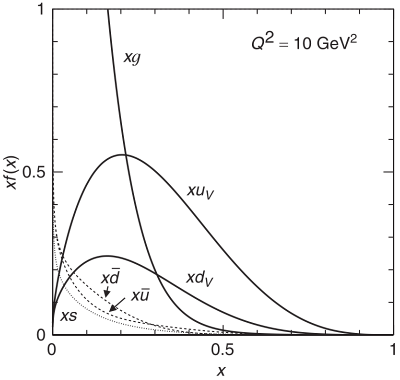
Q^2-dependent PDF **diverges at UV**.
$\rightarrow$ renormalize

(Collinear divergences cancel against PDF renormalization.)



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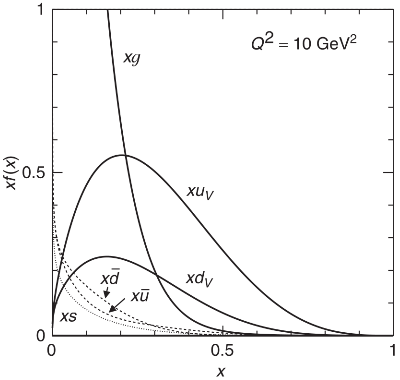
Analogous to RGE: **arbitrary scale** μ_F in perturbative solution of RGE.



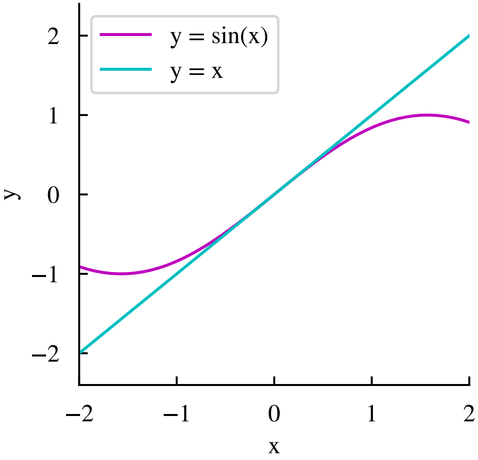
Parton Distribution Functions (Thomson)

The choice of μ_R and μ_F matters in computations.

The arbitrary scales

$$\mu_R \text{ and } \mu_F$$

have to be chosen
close to probed scale.



Scale Variations estimate the expansion error.

To estimate error, vary the scale μ around a central value μ^{central} and check how cross-section predictions vary: '**theory error**'.

7-point scale variation

$$\left\{\mu_R^{\text{central}}/2,\ \mu_R^{\text{central}},\ 2\,\mu_R^{\text{central}}\right\}\times\left\{\mu_F^{\text{central}}/2,\ \mu_F^{\text{central}},\ 2\,\mu_F^{\text{central}}\right\}$$

without $(\mu_i^{\text{central}}/2,\ 2\mu_j^{\text{central}})$.

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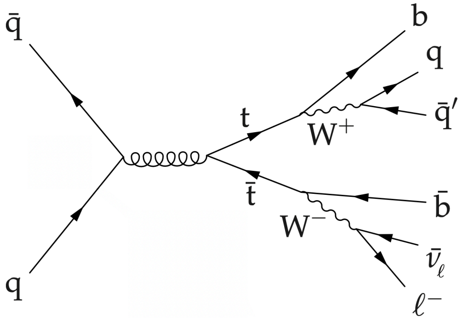
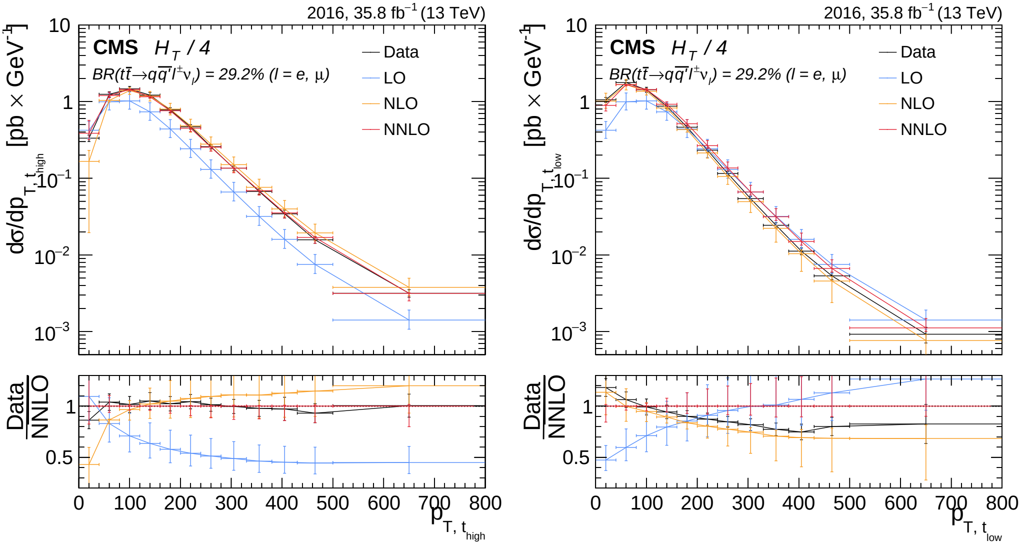
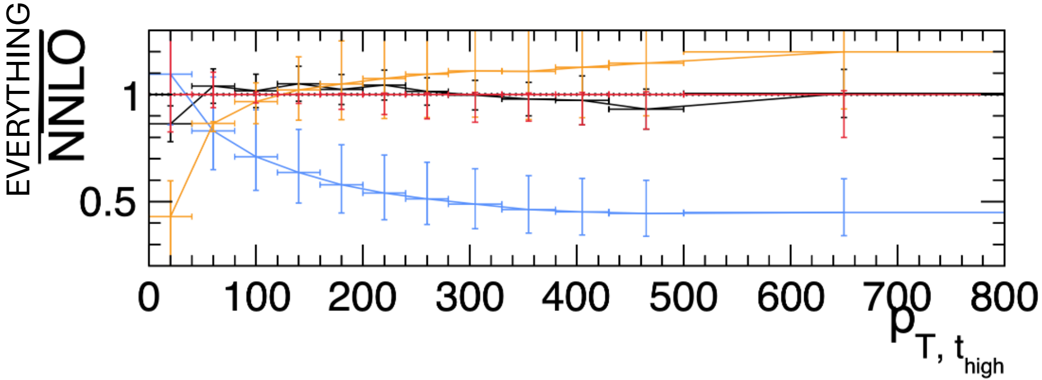
What scale μ^{central} to use??

Event Overview

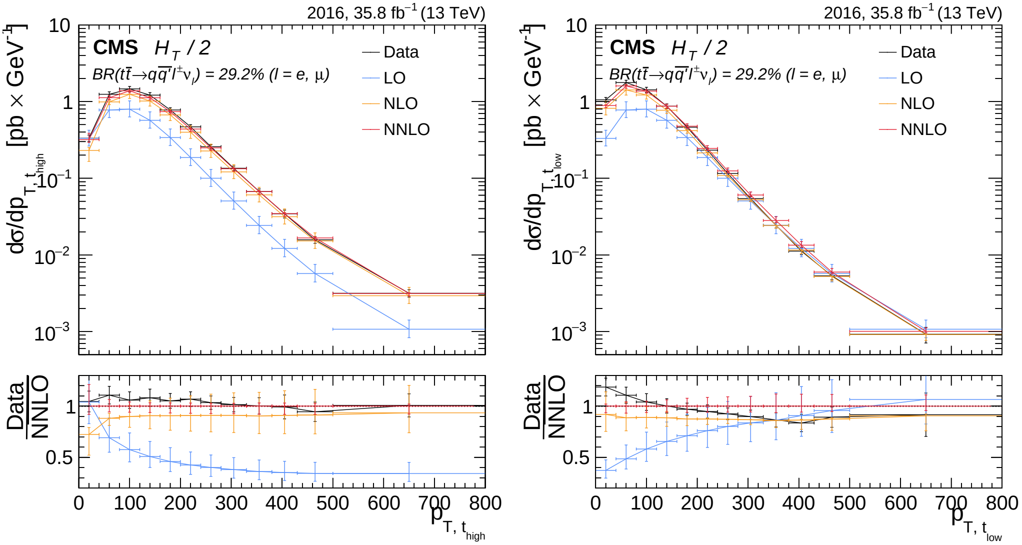




2016, 35.8 fb^{-1} (13 TeV)
CMS $H_T / 4$
$BR(t\bar{t} \to q\bar{q}' l^{\pm}\nu_l)$ = 29.2% (l = e, μ)
Data
LO
NLO
NNLO
dσ/dp$_{T, t_{high}}$ [pb × GeV^{-1}]
Data/NNLO
p$_{T, t_{high}}$
dσ/dp$_{T, t_{low}}$ [pb × GeV^{-1}]
p$_{T, t_{low}}$



- Using scale variations to represent our theoretical uncertainty.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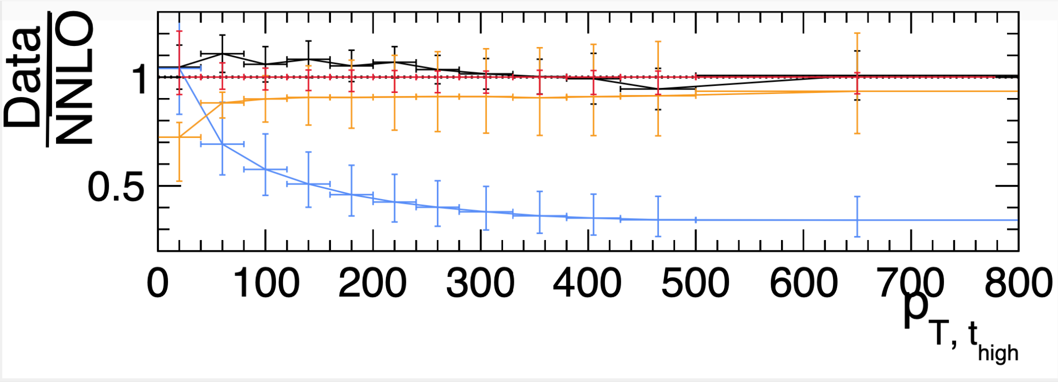
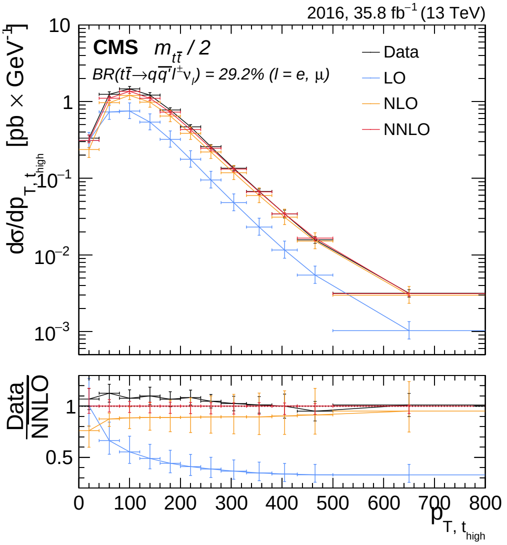


- We find that HT/2 proves a more realistic representation of the theoretical uncertainty.



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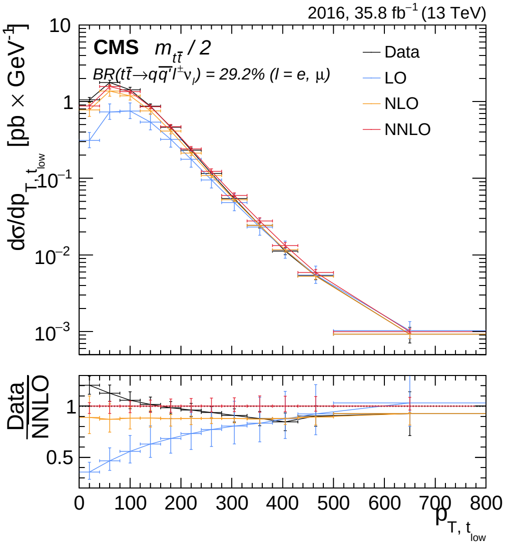

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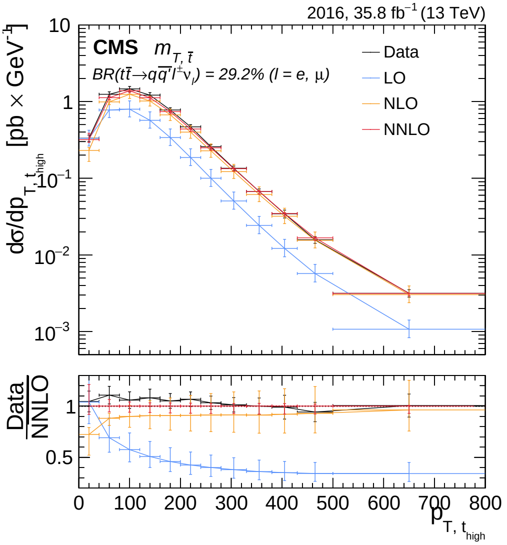

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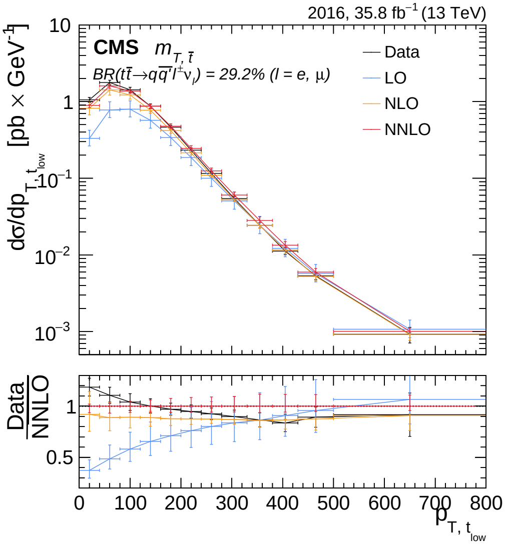

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What to take home

you don't compute theoretical uncertainties you estimate them!

Simone Devoto