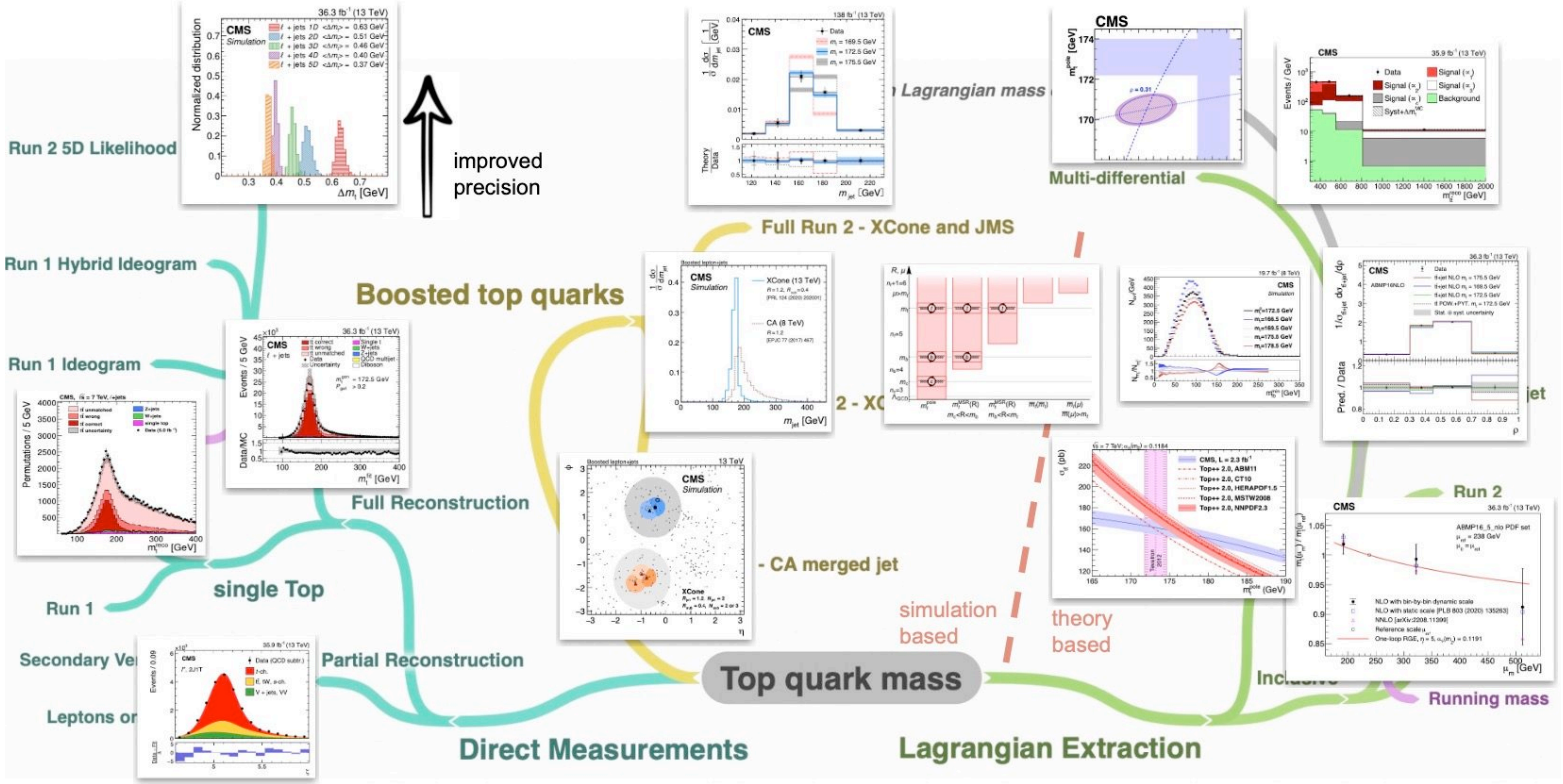
Understanding the top quark m~~e~~ass

Illustration from
CMS Physics Briefing

Experimental Particle Physics Seminar

University of Ghent
May 7th, 2024

Matteo M. Defranchis (CERN)



Next update: May 2029

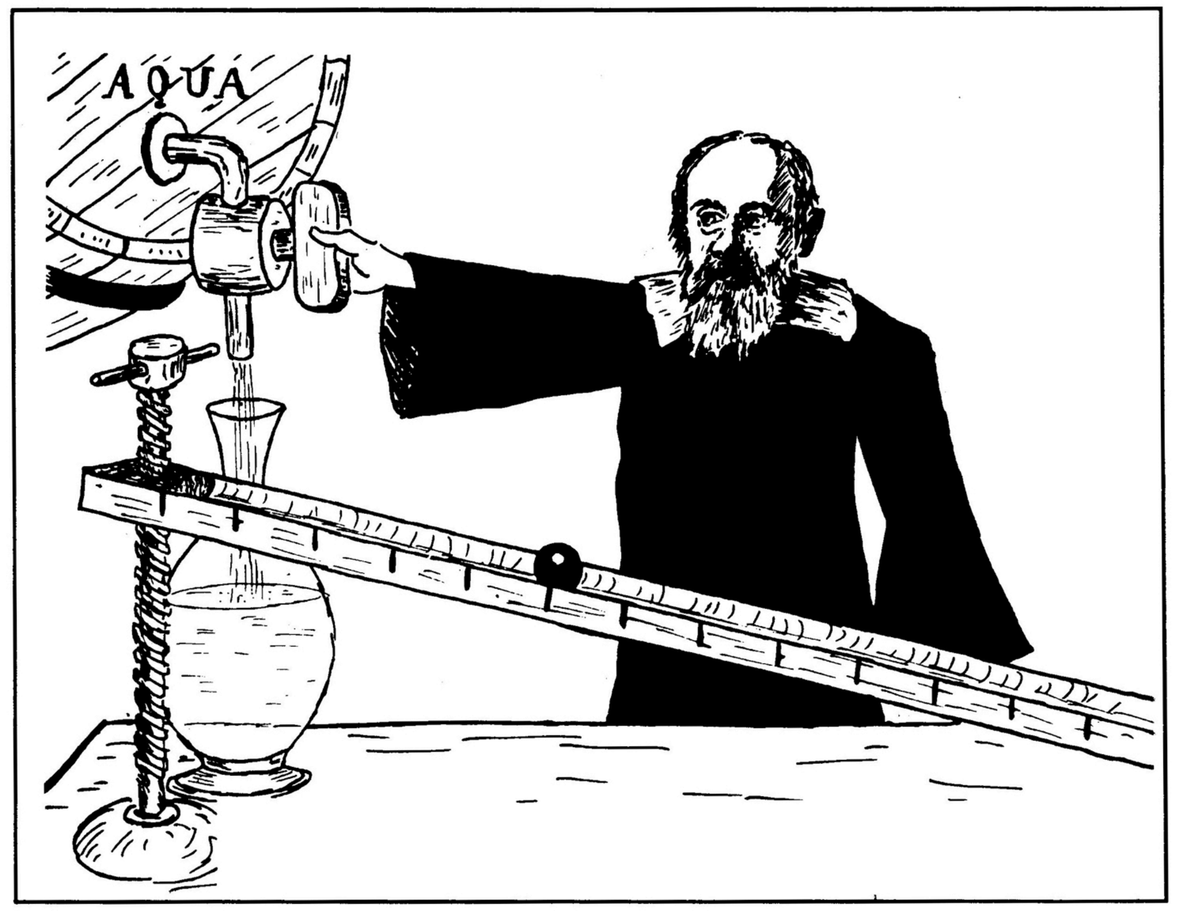
EPP Seminar - University of Ghent
May 14th, 2019

The mass of the top quark: which one?

a pseudo-historical approach

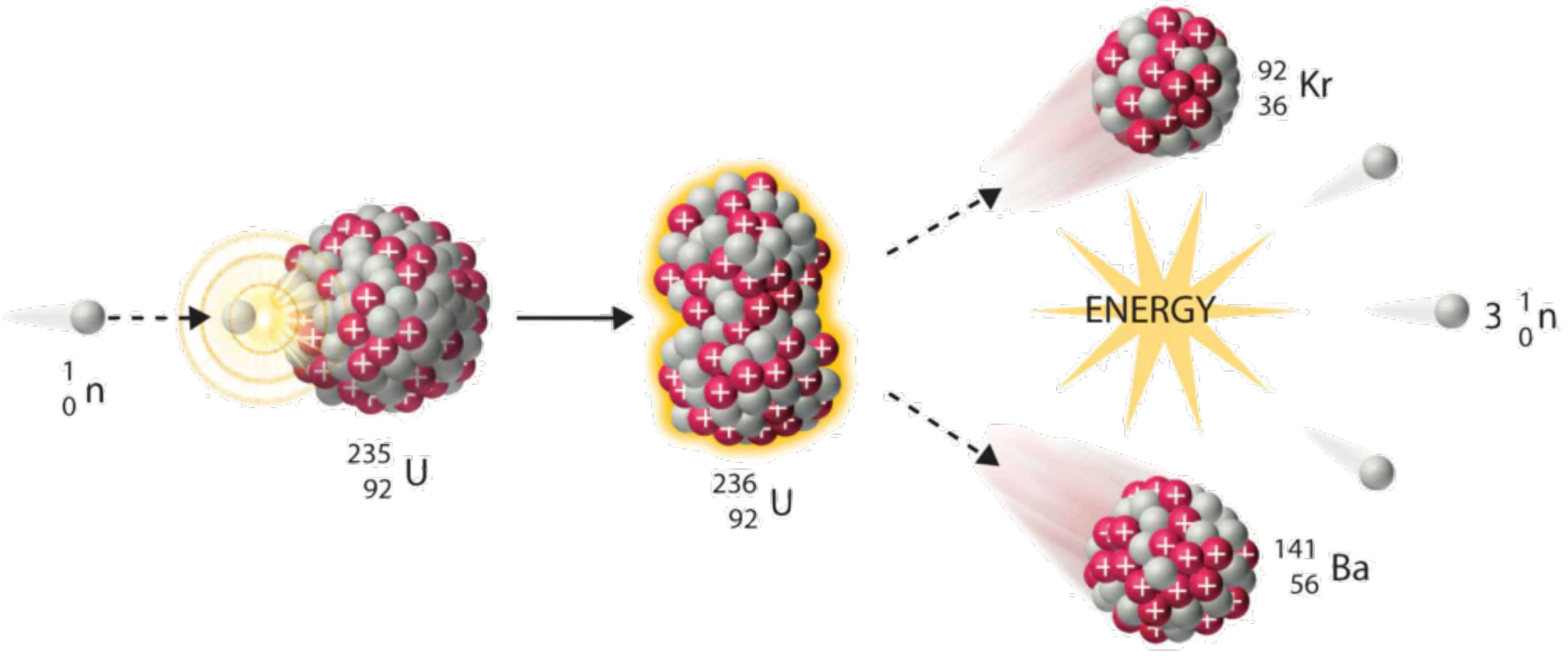
Matteo M. Defranchis
Deutsches Elektronen-Synchrotron (DESY)

The concept of mass at the test of time



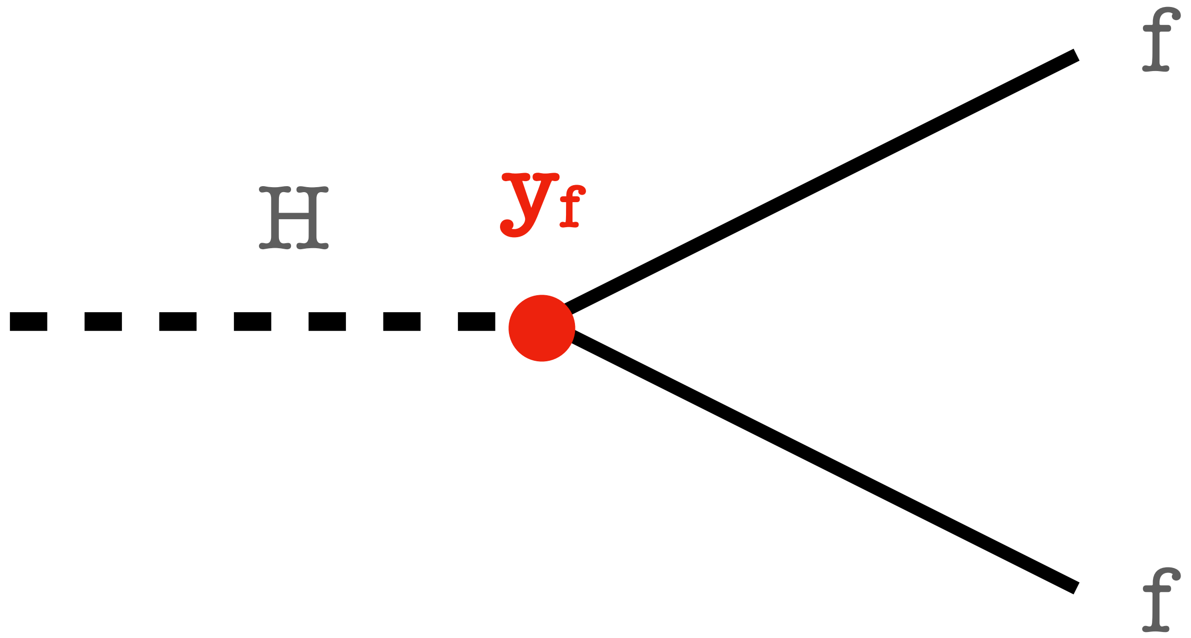
Galileo studia il moto accelerato di una pallina che rotola lungo un piano inclinato (*dis. G. Gamow*).

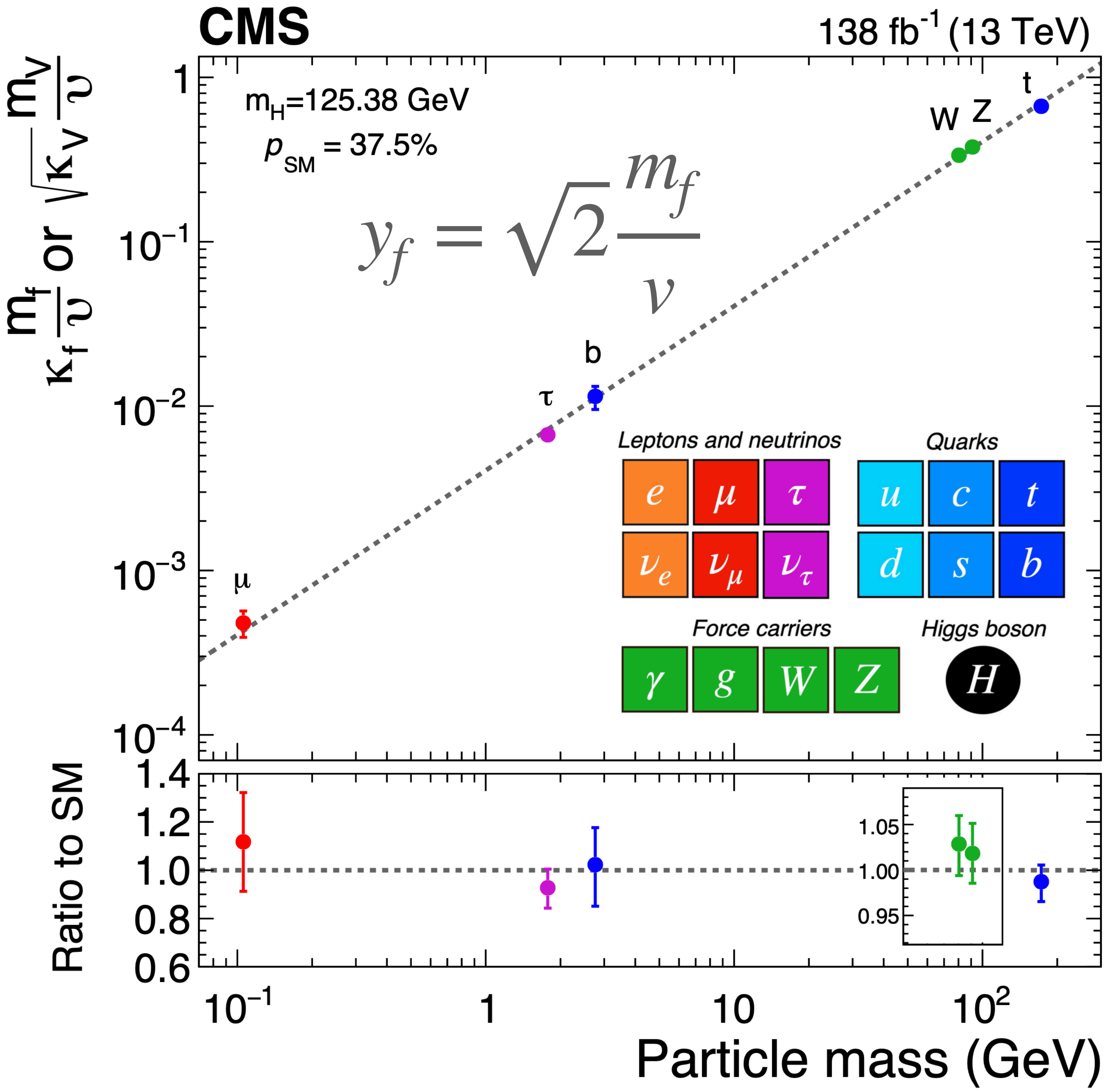
- 1687 **Newton**: inertial mass, laws of gravitation
- 1905 **Einstein**: equivalence between mass and energy



- 1964 **Brout-Englert-Higgs**: coupling to the Higgs field (in the case of elementary particles)

Is the top quark the only elementary particle with a “natural” mass?



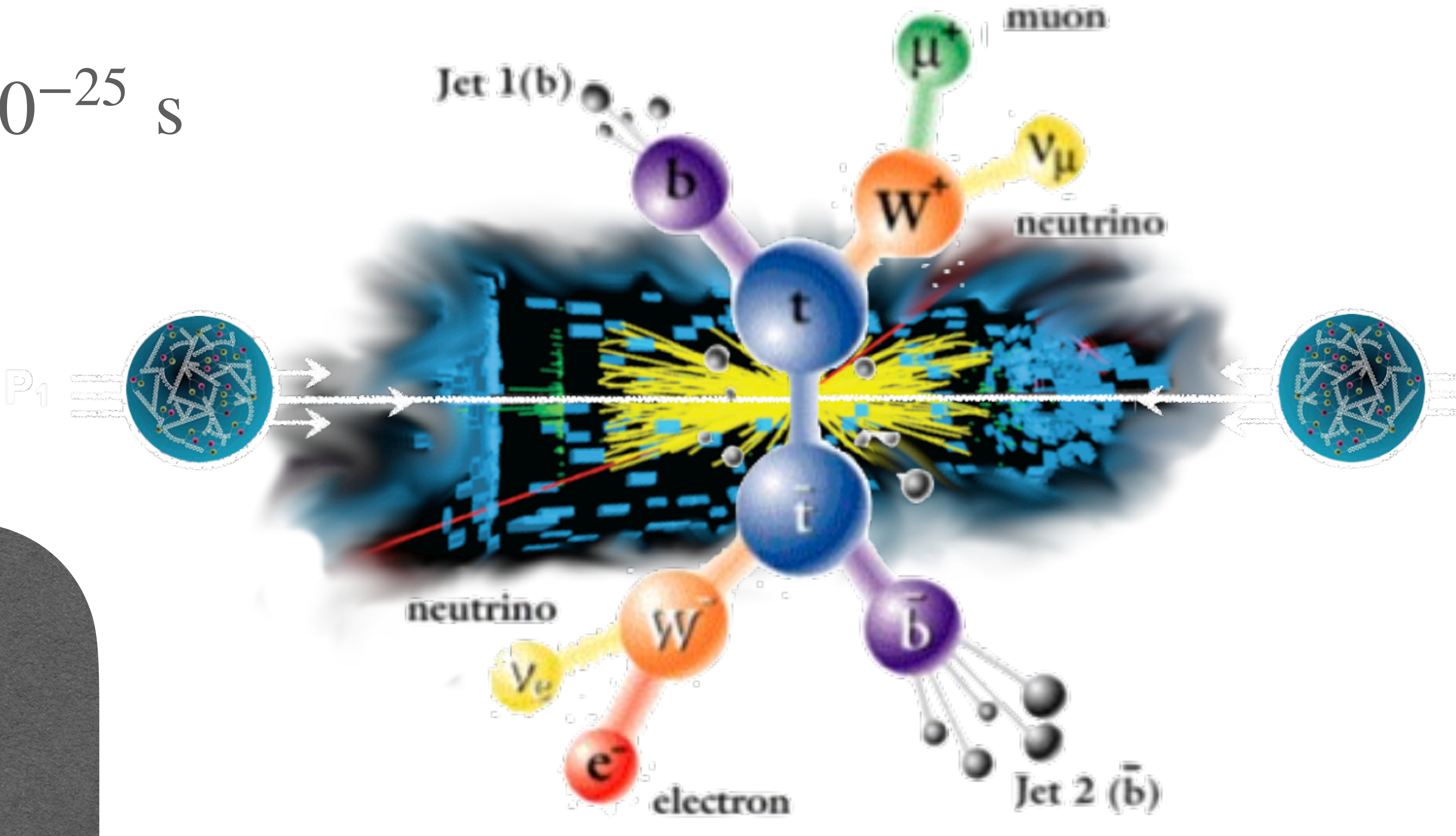


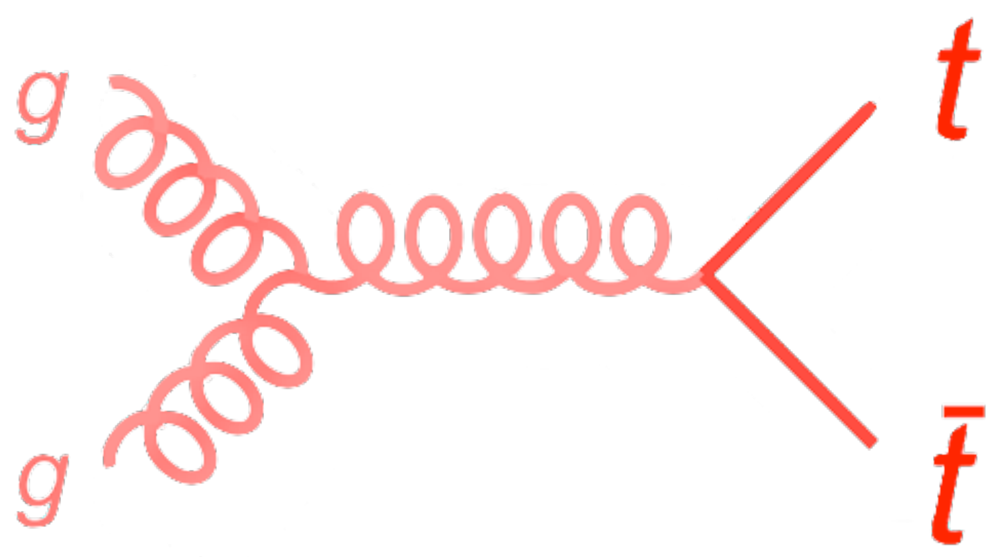
m_t makes the top quark special

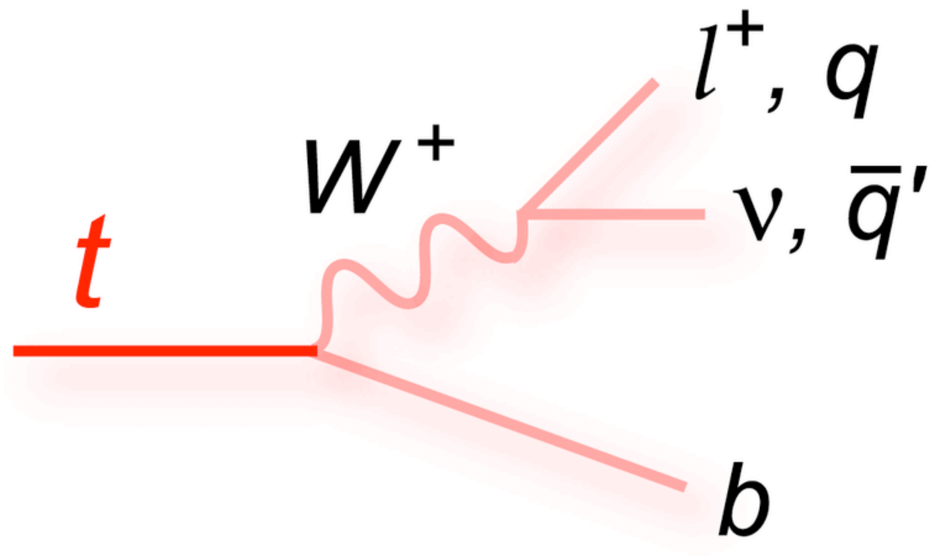
Unlike all other quarks, the top quark decays before forming bound states

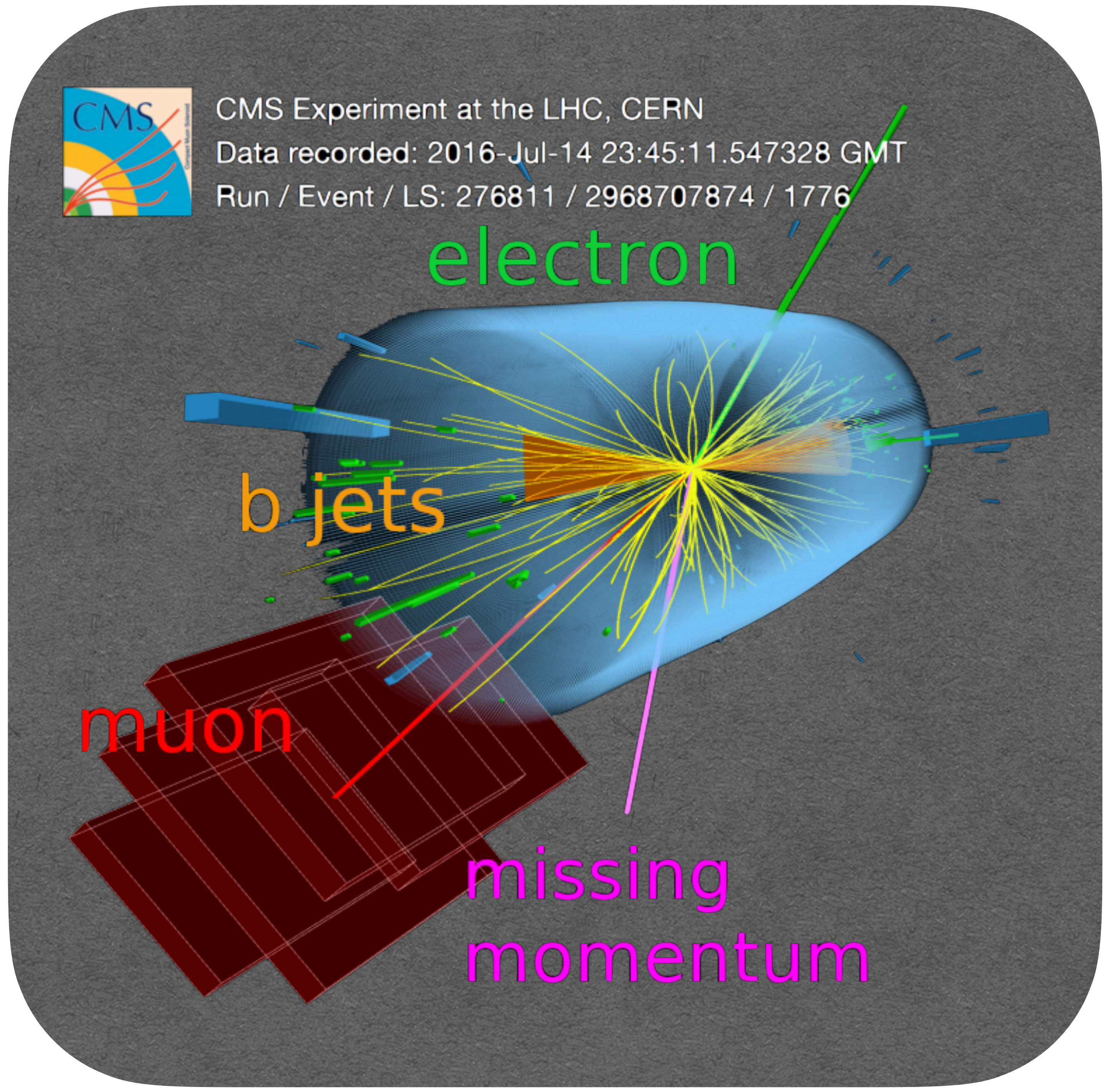
- Behaves (approximately) as a **free particle**
- Mass can be reconstructed from decay products

$$\tau_t = \frac{1}{\Gamma_t} \sim 10^{-25}\ \mathrm{s}$$







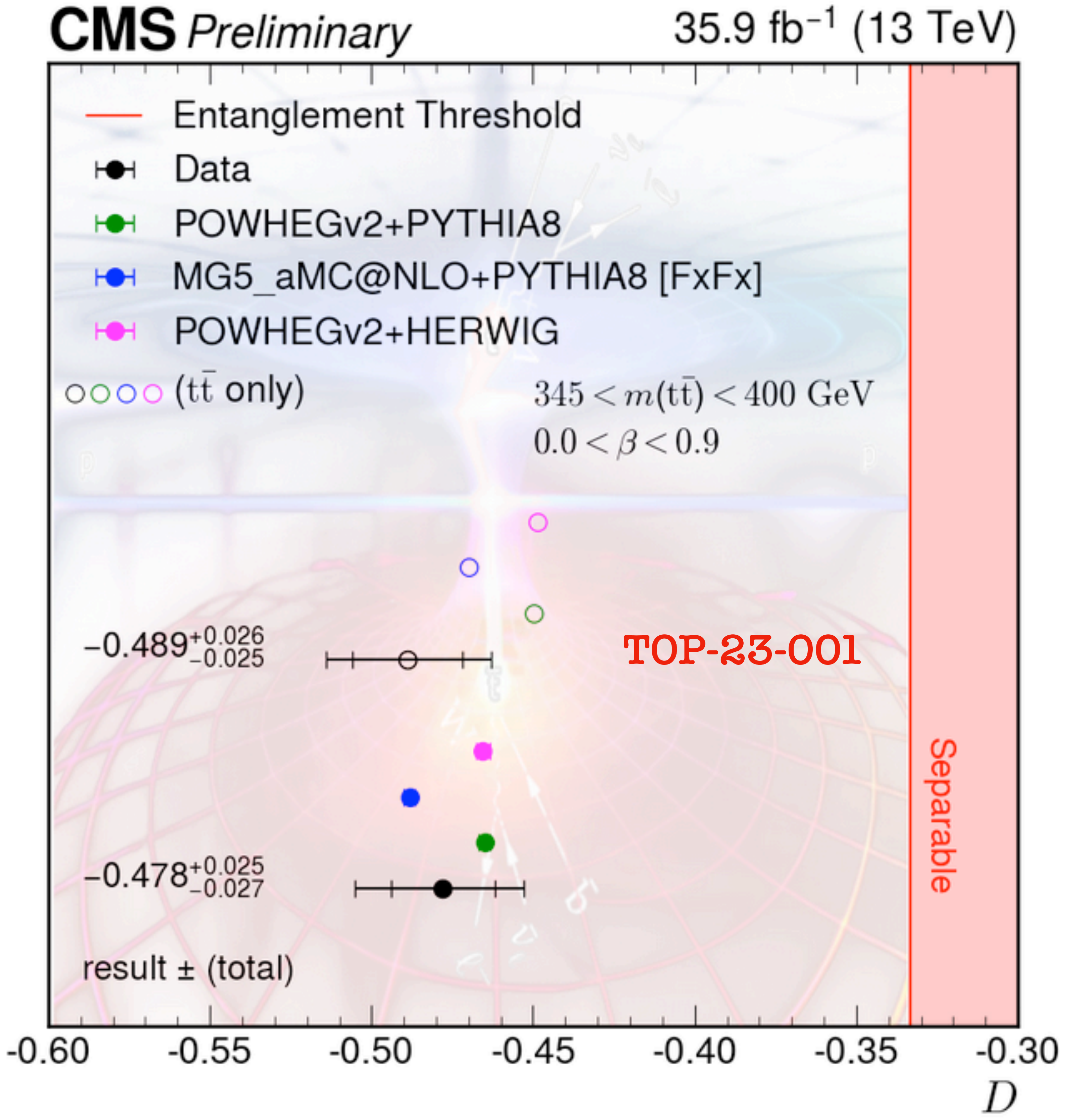

CMS Experiment at the LHC, CERN
Data recorded: 2016-Jul-14 23:45:11.547328 GMT
Run / Event / LS: 276811 / 2968707874 / 1776

Relatively narrow resonance
-> conceptually we can factorise
top quark production and decay

EW decay makes top quark “easy” to identify experimentally

m_t makes the top quark special

- **Spin correlation and quantum entanglement** information transferred to decay products



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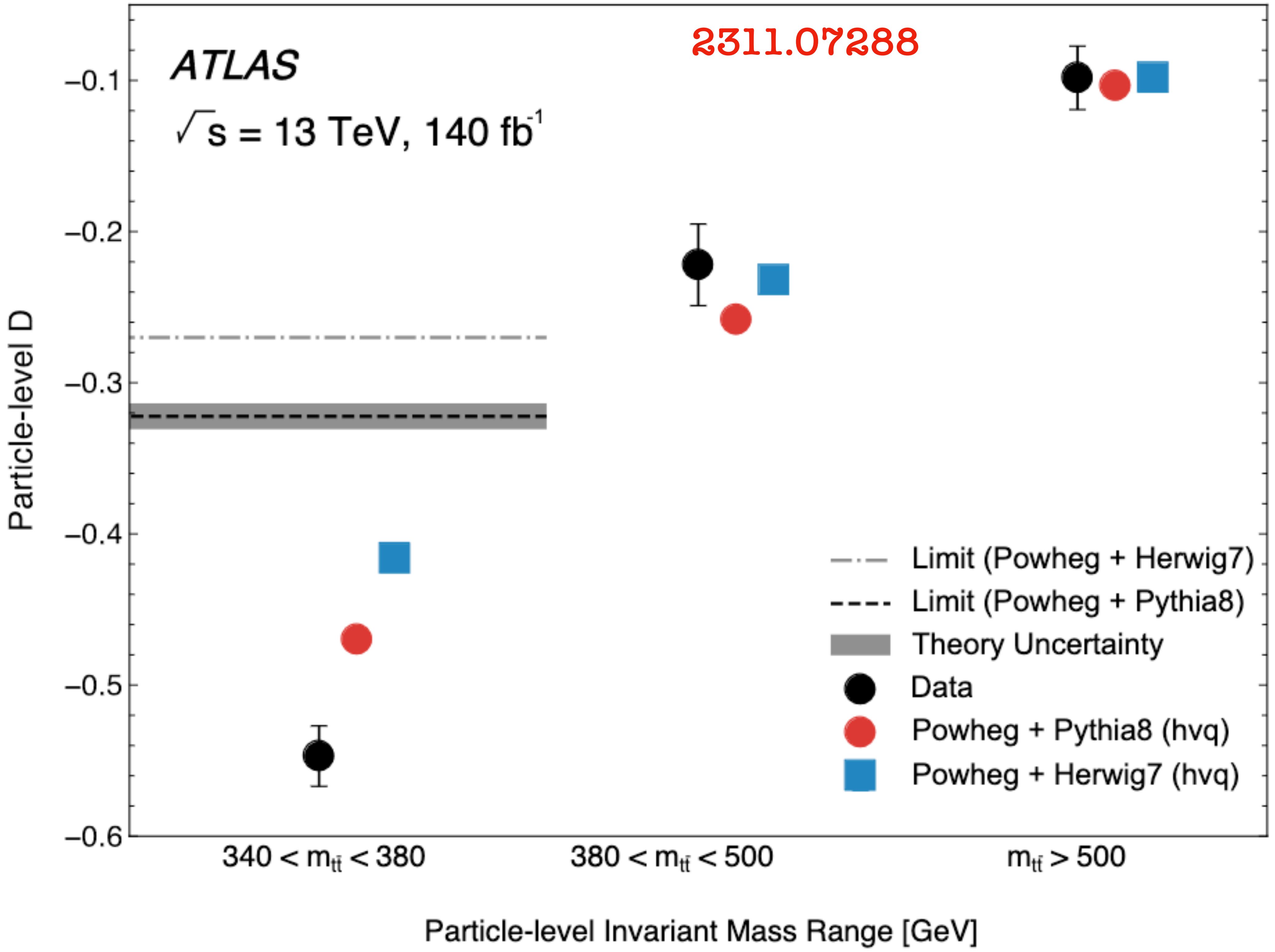


2311.07288

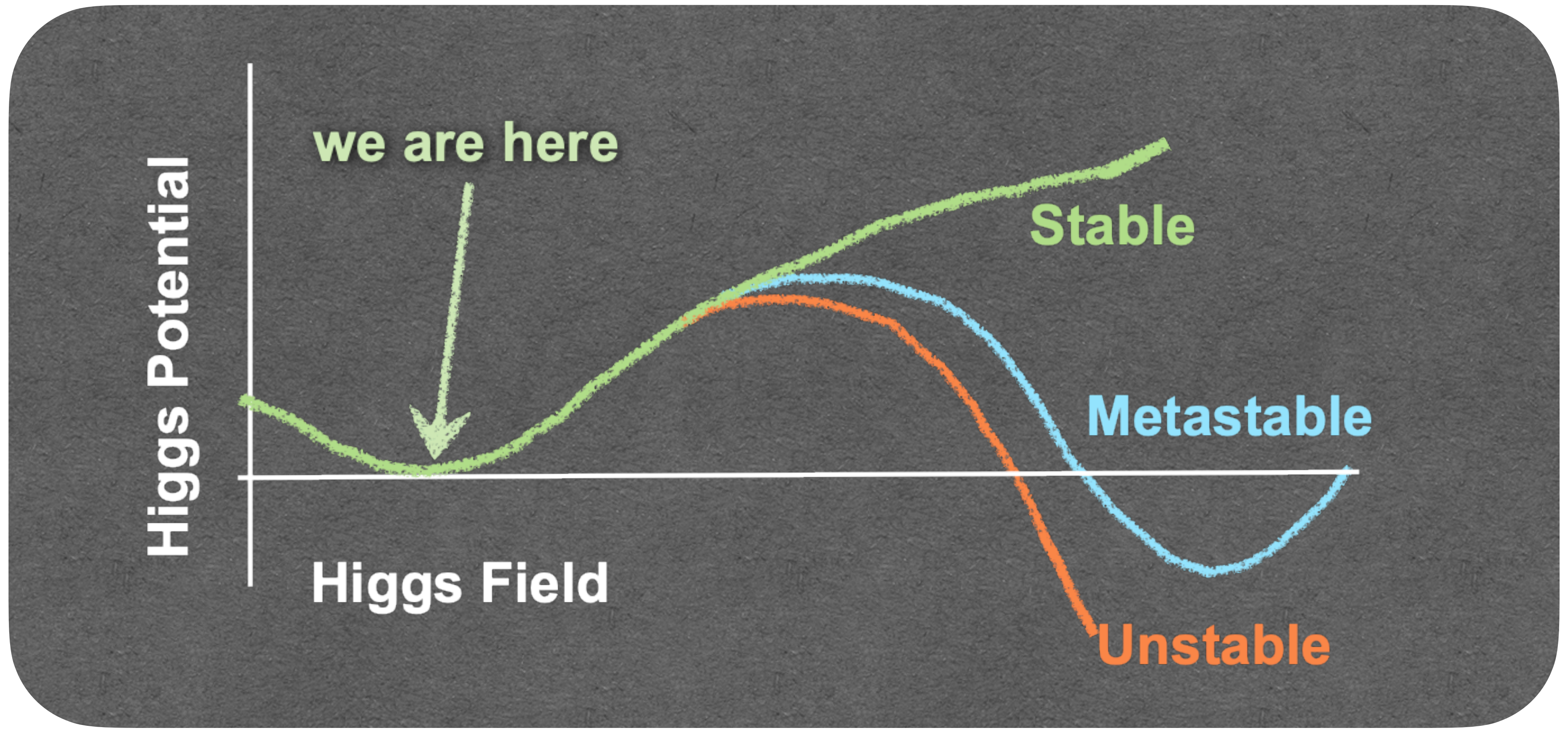
Highest energy observation (so far) of quantum entanglement between elementary particles

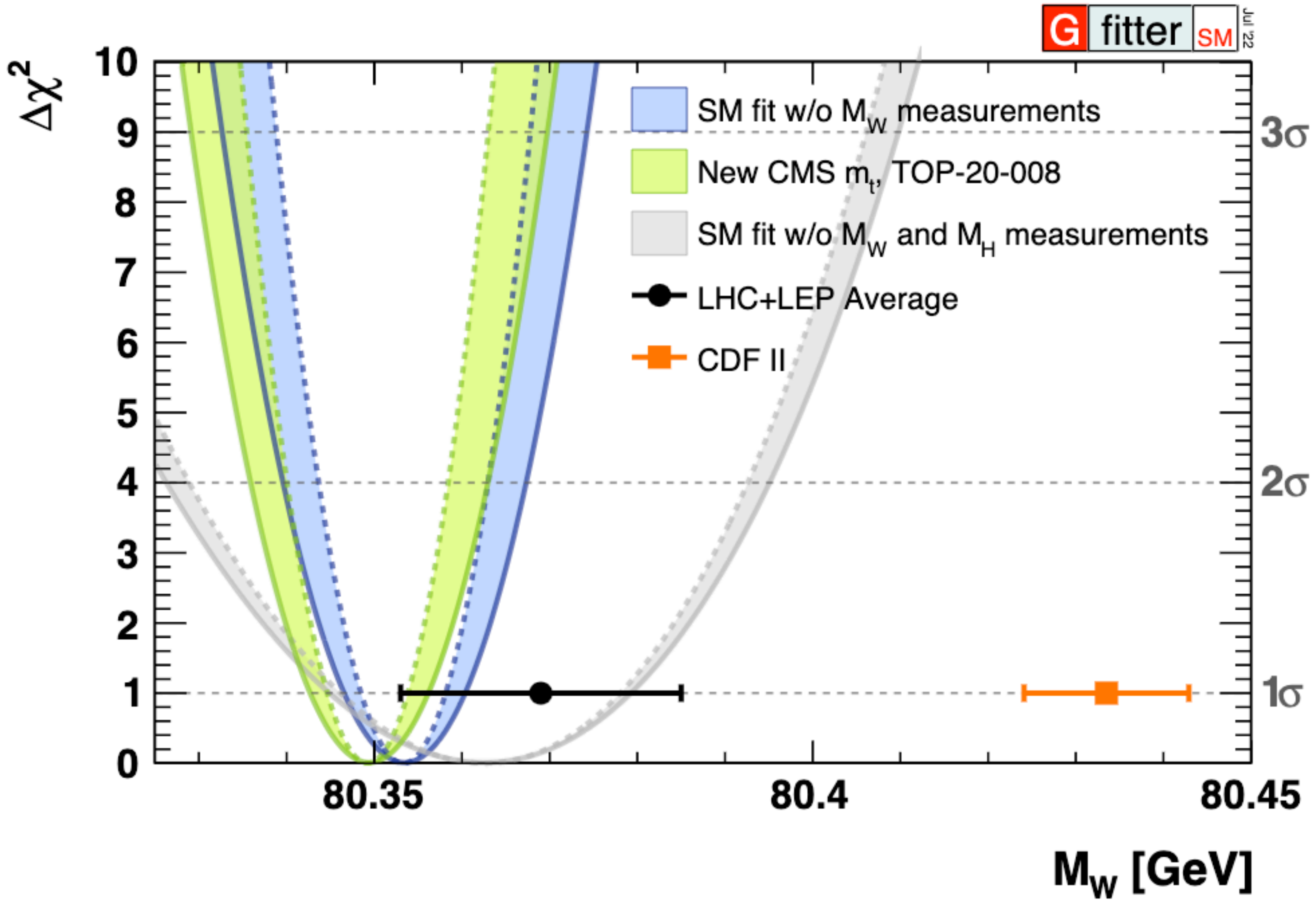
In the near future this will be used as a probe for physics beyond the SM

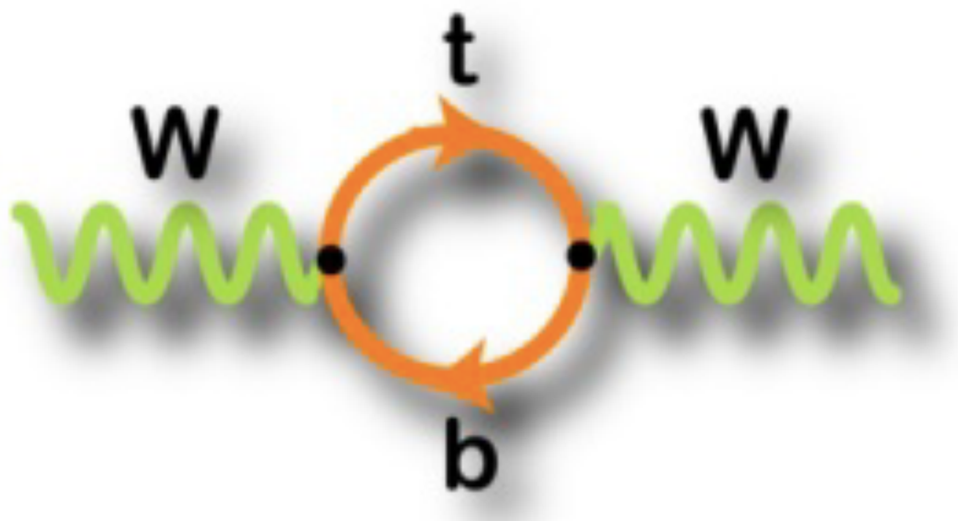
Can m_t break the standard model?

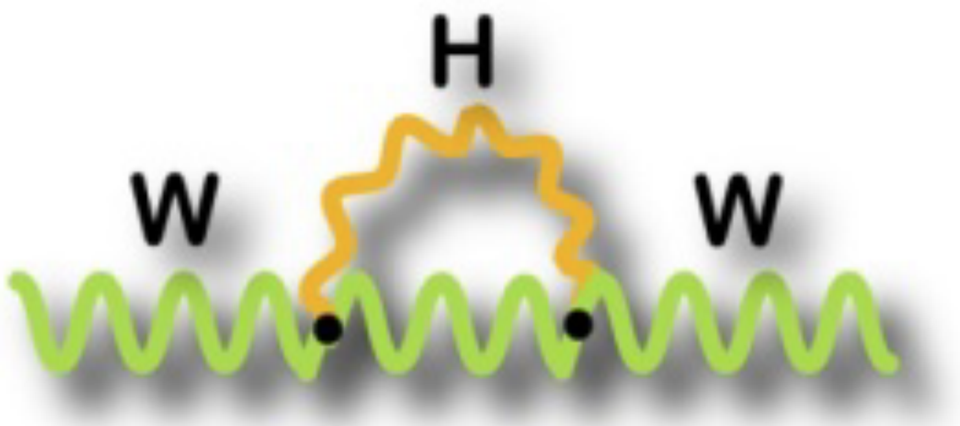
In the SM, m_t can be related to m_W and m_H thanks to loop corrections to precision EW observables
-> **internal consistency of SM**

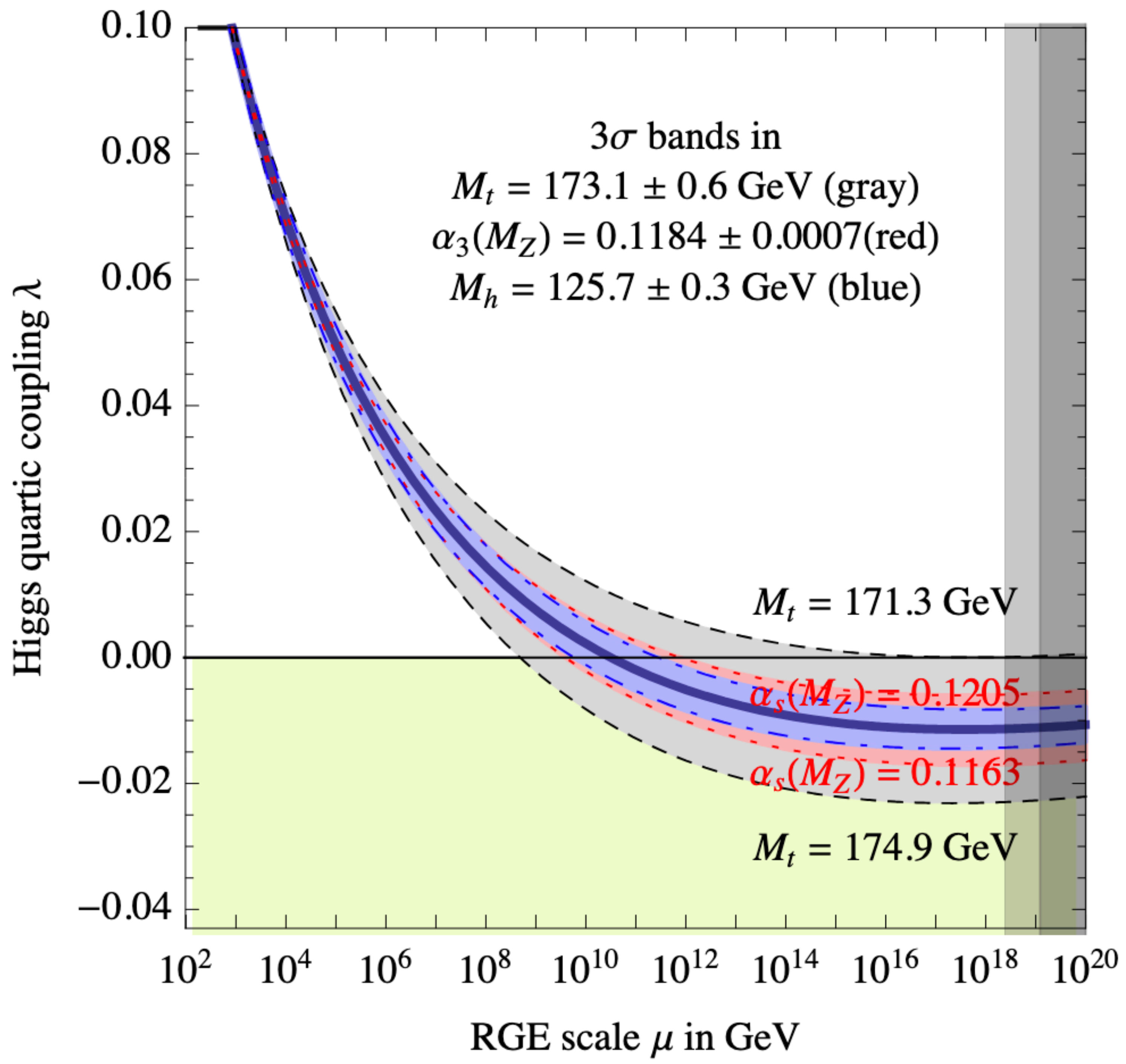
Stability of Higgs potential at the Planck scale depends on value of m_t
-> $\lambda < 0$ would be indirect evidence of BSM physics











Where do we stand?

CMS review of top quark mass measurements [arXiv:2403.01313]

A RICH COLLECTION OF TOP QUARK MASS MEASUREMENTS BY THE CMS EXPERIMENT

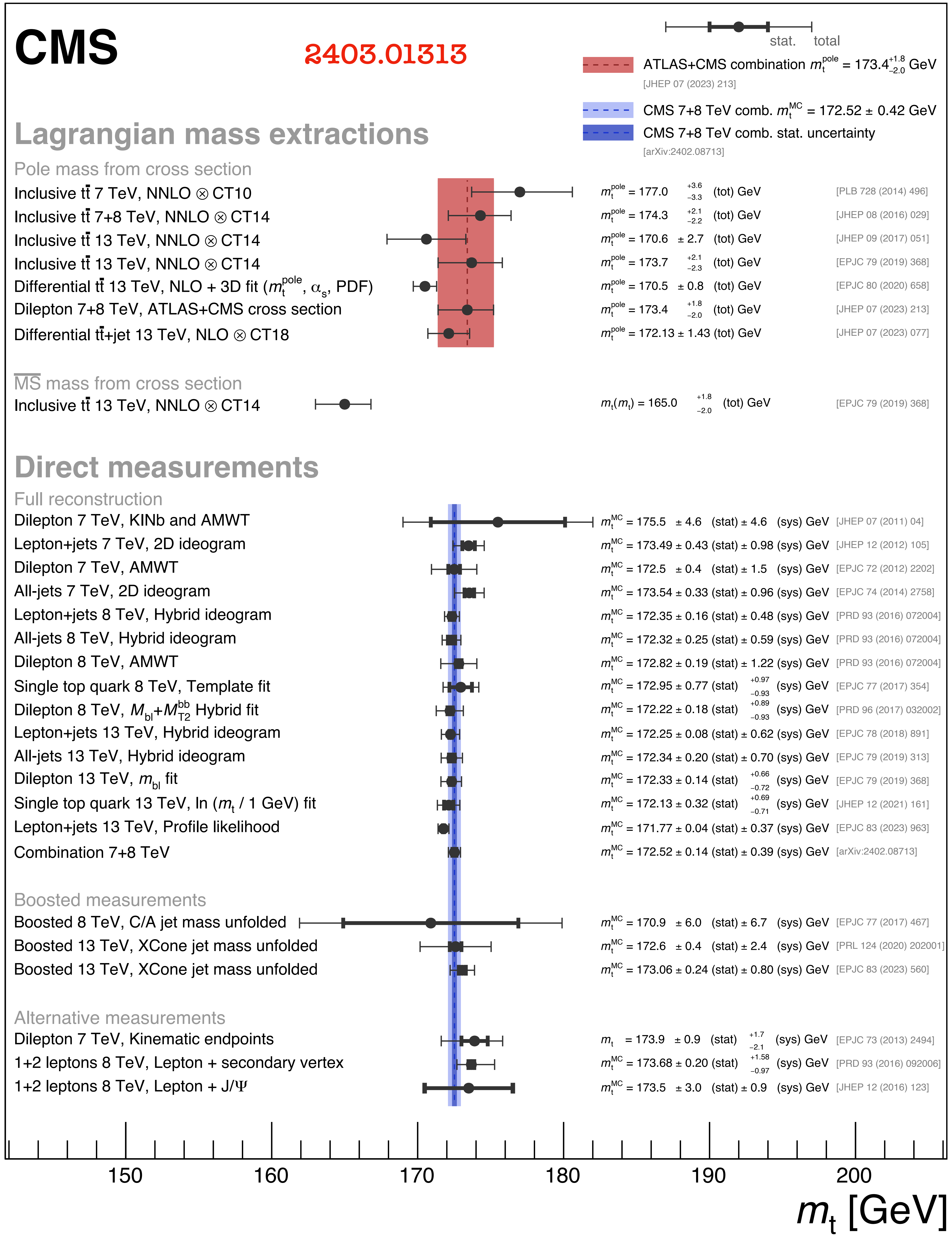
Large number of measurements performed by CMS (and ATLAS) during LHC Run1 and Run2

- Demonstrates the relevance of the topic

Measurements classified into **different classes**

- Different methods and topologies, as a cross check to each other
- But not only…

Significant improvement in precision over the years thanks to advancements in data analysis techniques



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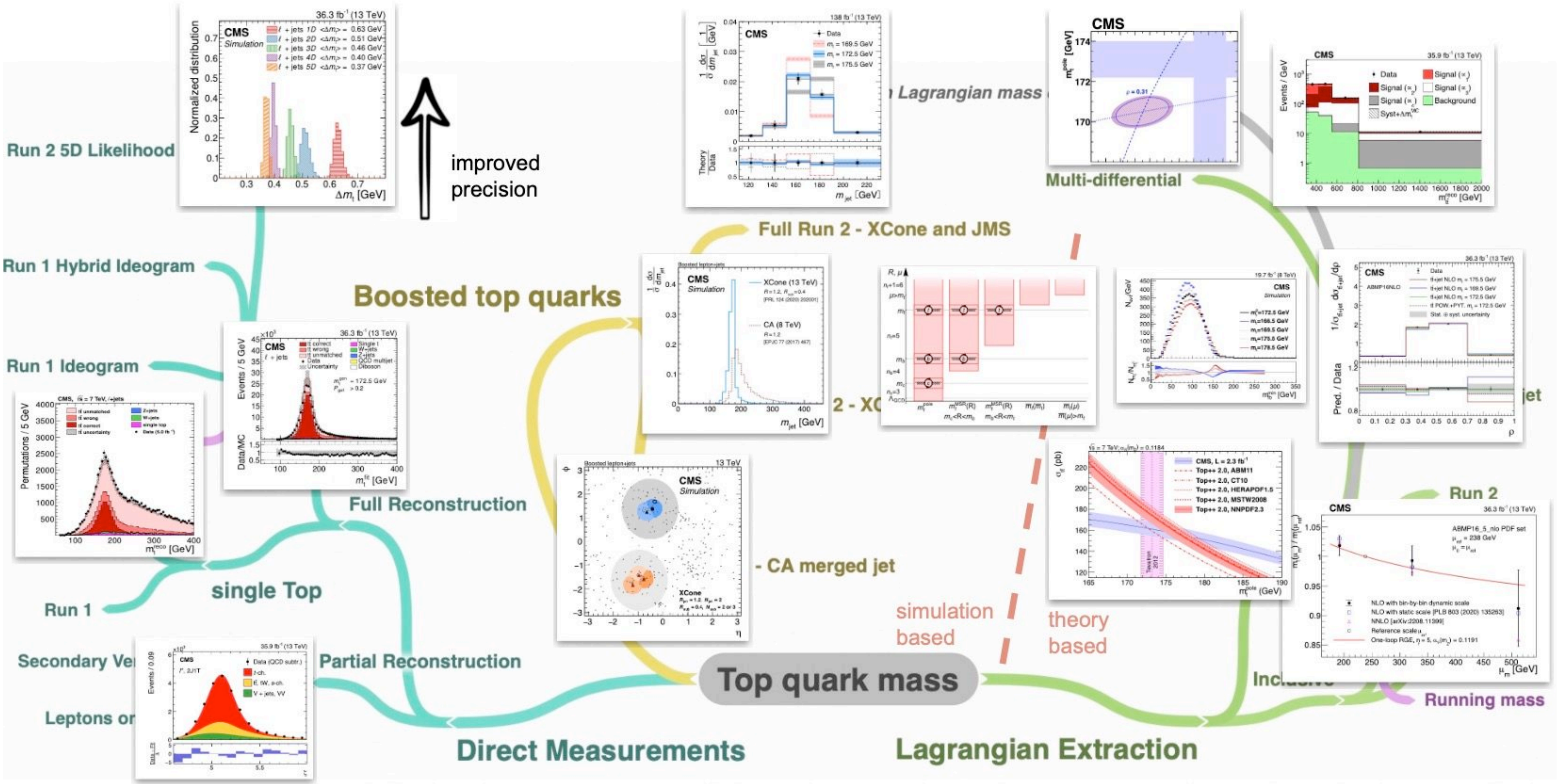

Run 2 5D Likelihood

improved precision

Run 1 Hybrid Ideogram

Run 1 Ideogram

Full Reconstruction

single Top

Run 1

Secondary Ve

Leptons or

Partial Reconstruction

Direct Measurements

Boosted top quarks

Full Run 2 - XCone and JMS

2 - XC

- CA merged jet

Top quark mass

simulation based

theory based

Lagrangian mass

Multi-differential

jet

Run 2

Inclusive

Running mass

Lagrangian Extraction

Each measurement is a piece of the puzzle…

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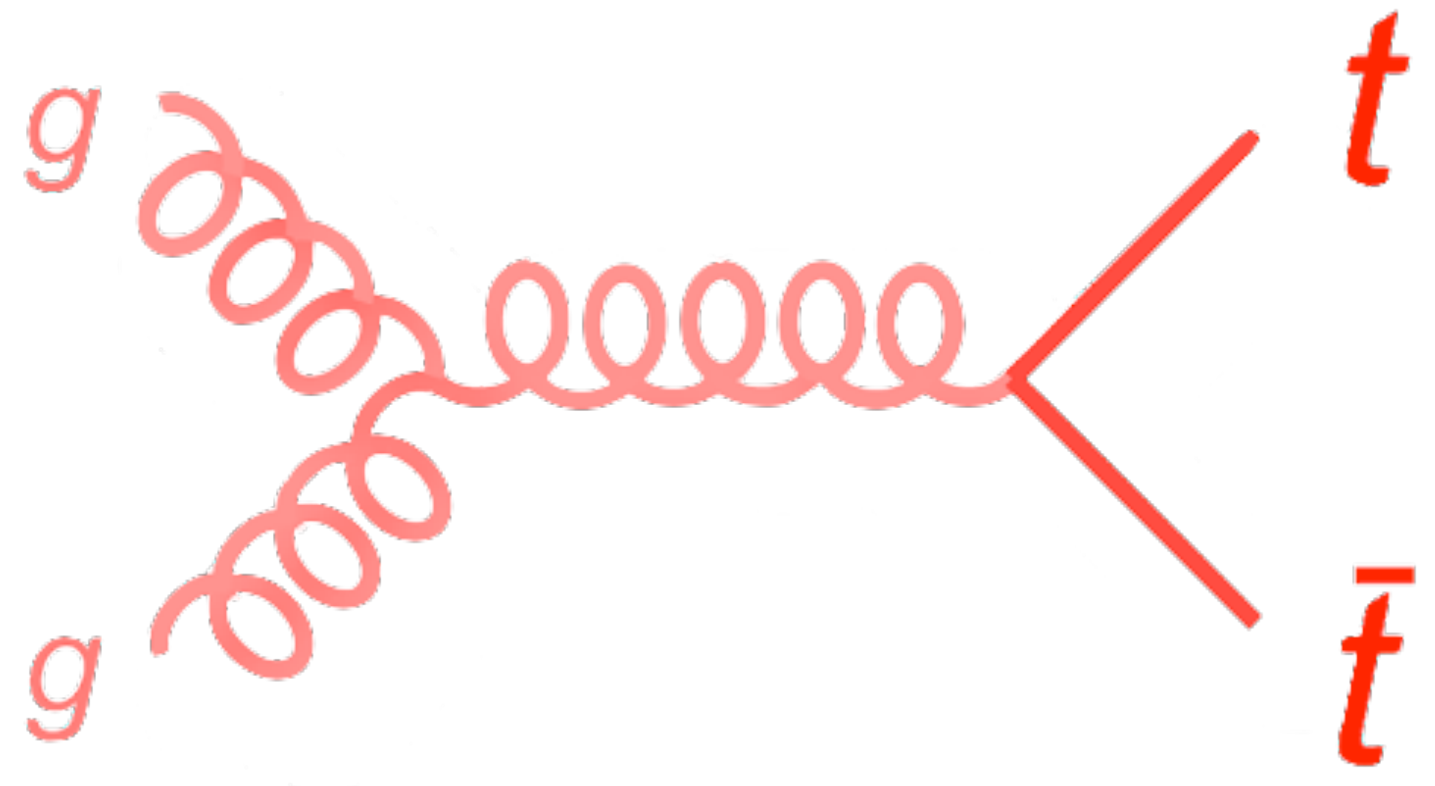
Top quark production at the LHC

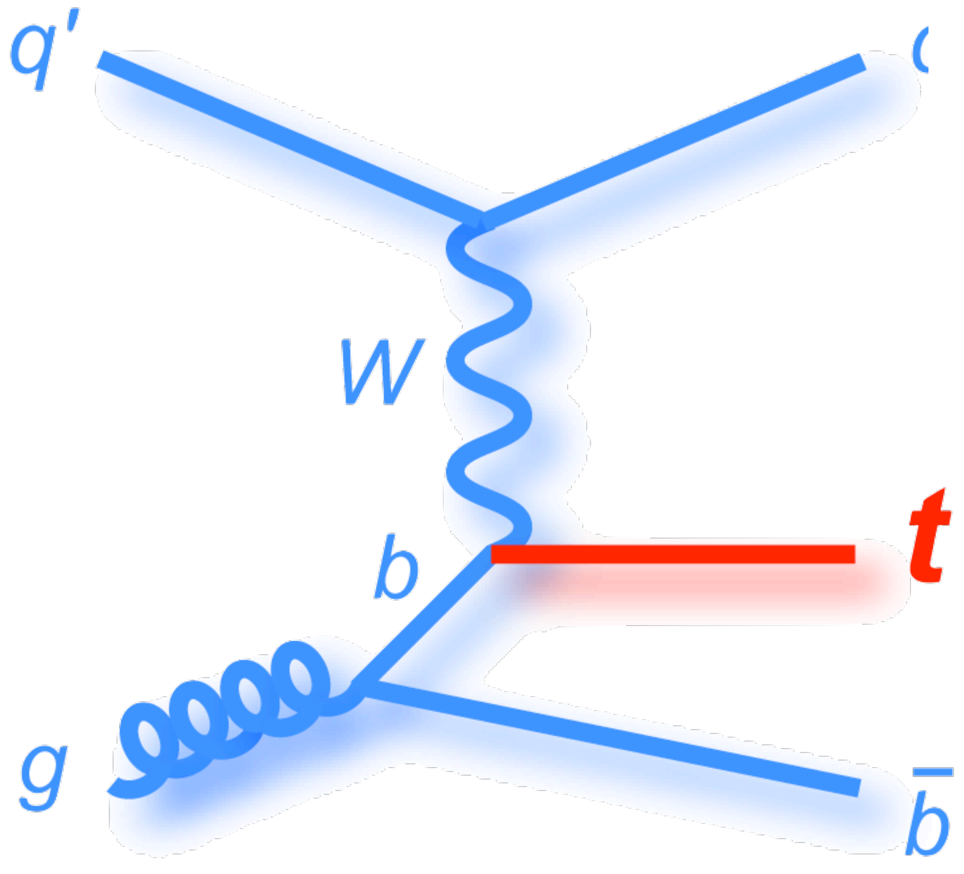
gg -> tt is by far the dominant production channel

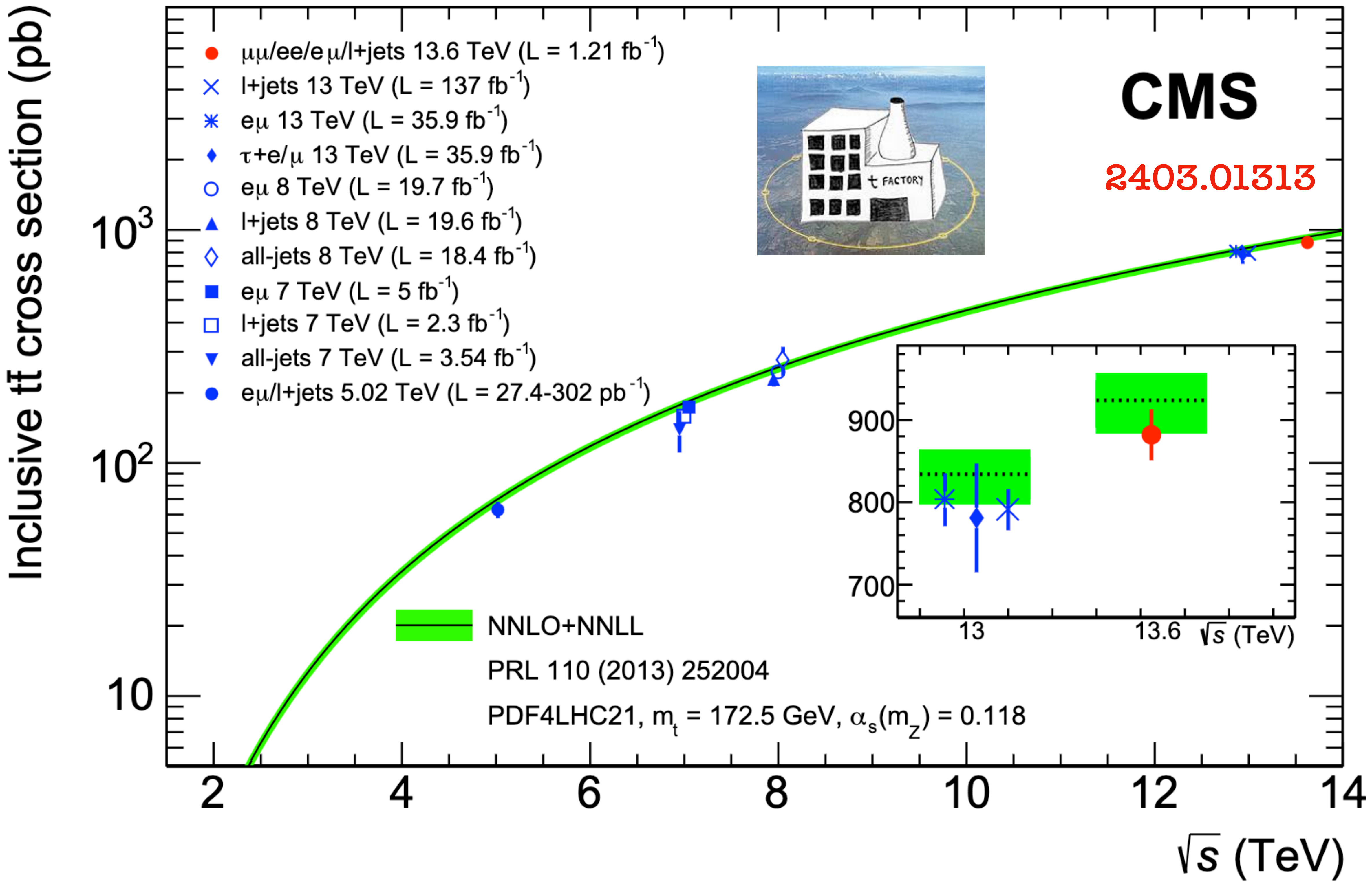
- Most relevant in the context of m_t measurements
- Other channels (e.g. t-channel) can be used, and bring some benefits in **combinations**



QCD production

EW production



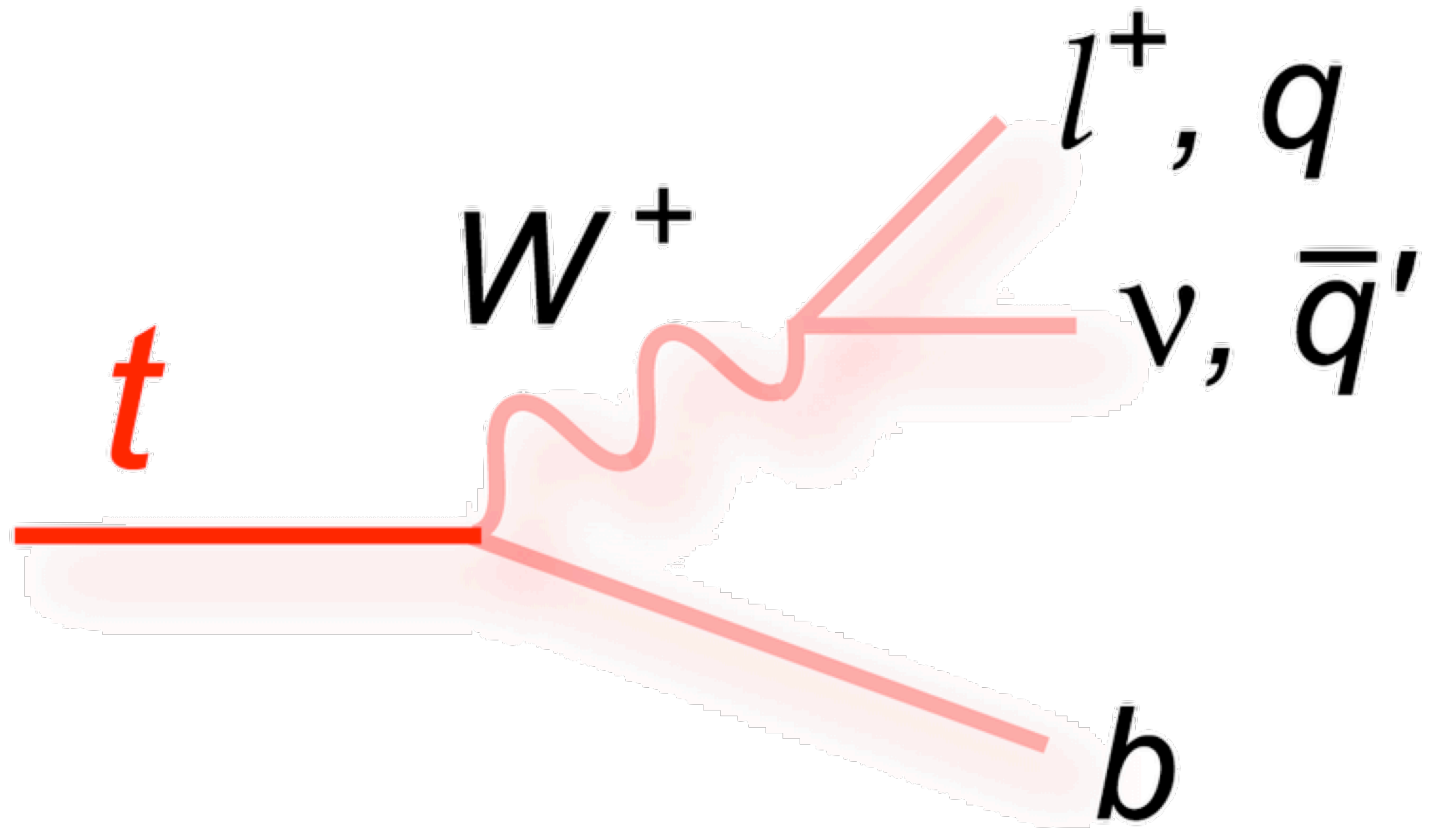


More than 10 tt pairs produced every second at the LHC

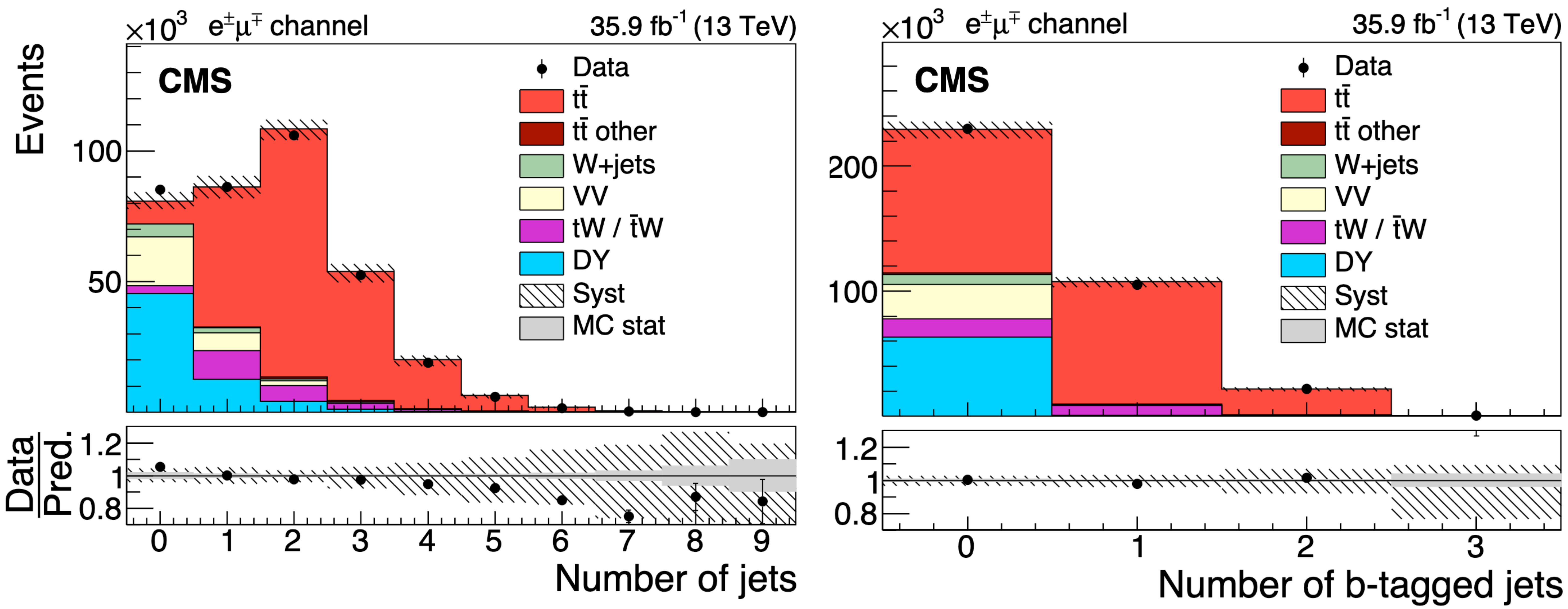
- Well understood process on a wide range of energy scales

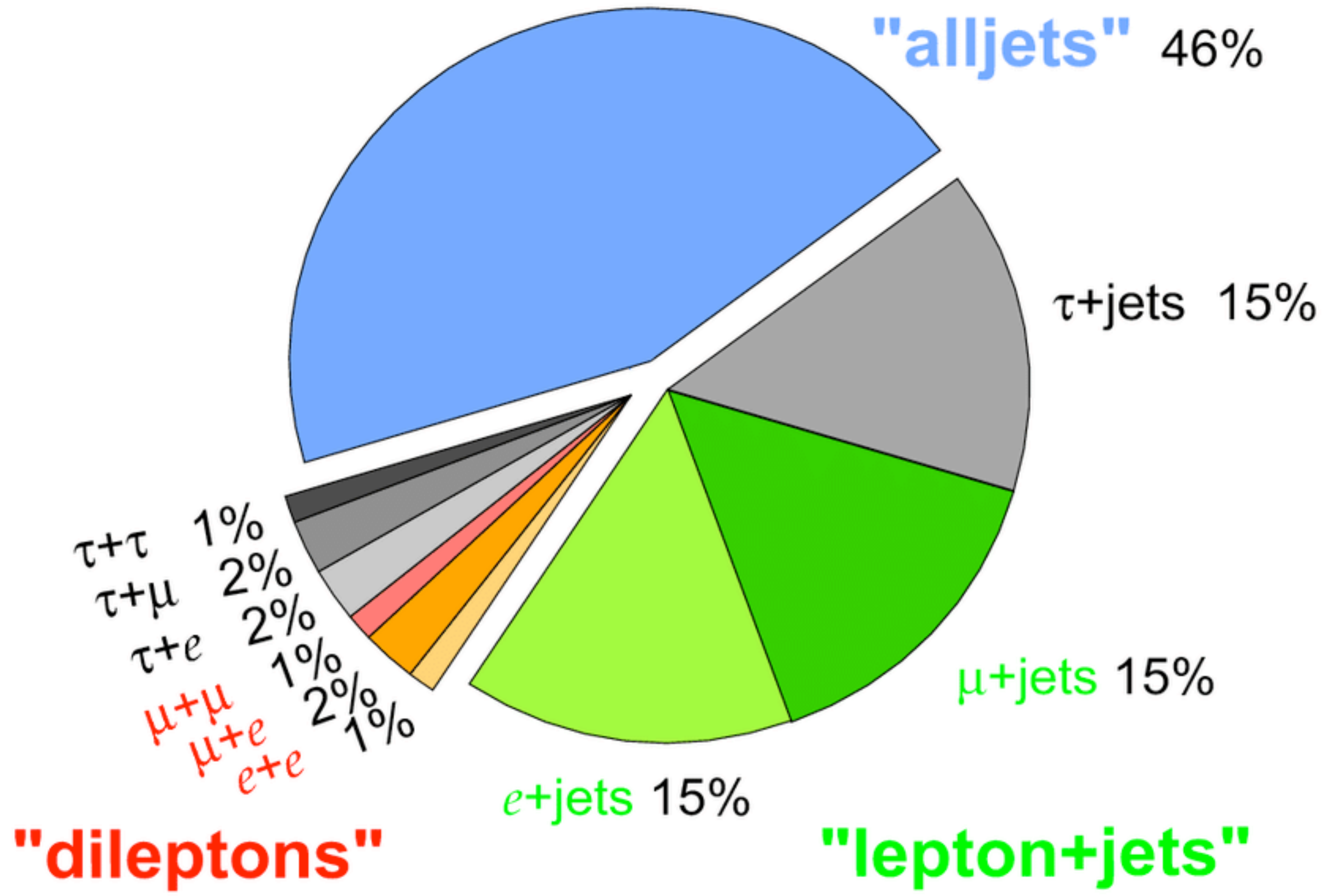
Zooming in on tt̄: the final states

- Di-lepton final state extremely clean, but only **partial reconstruction** of the top quark mass is possible. Ideal for cross section measurements
- Fully hadronic final state pays the price of large QCD background
- Lepton+jets final state allows for **full reconstruction of m_t**

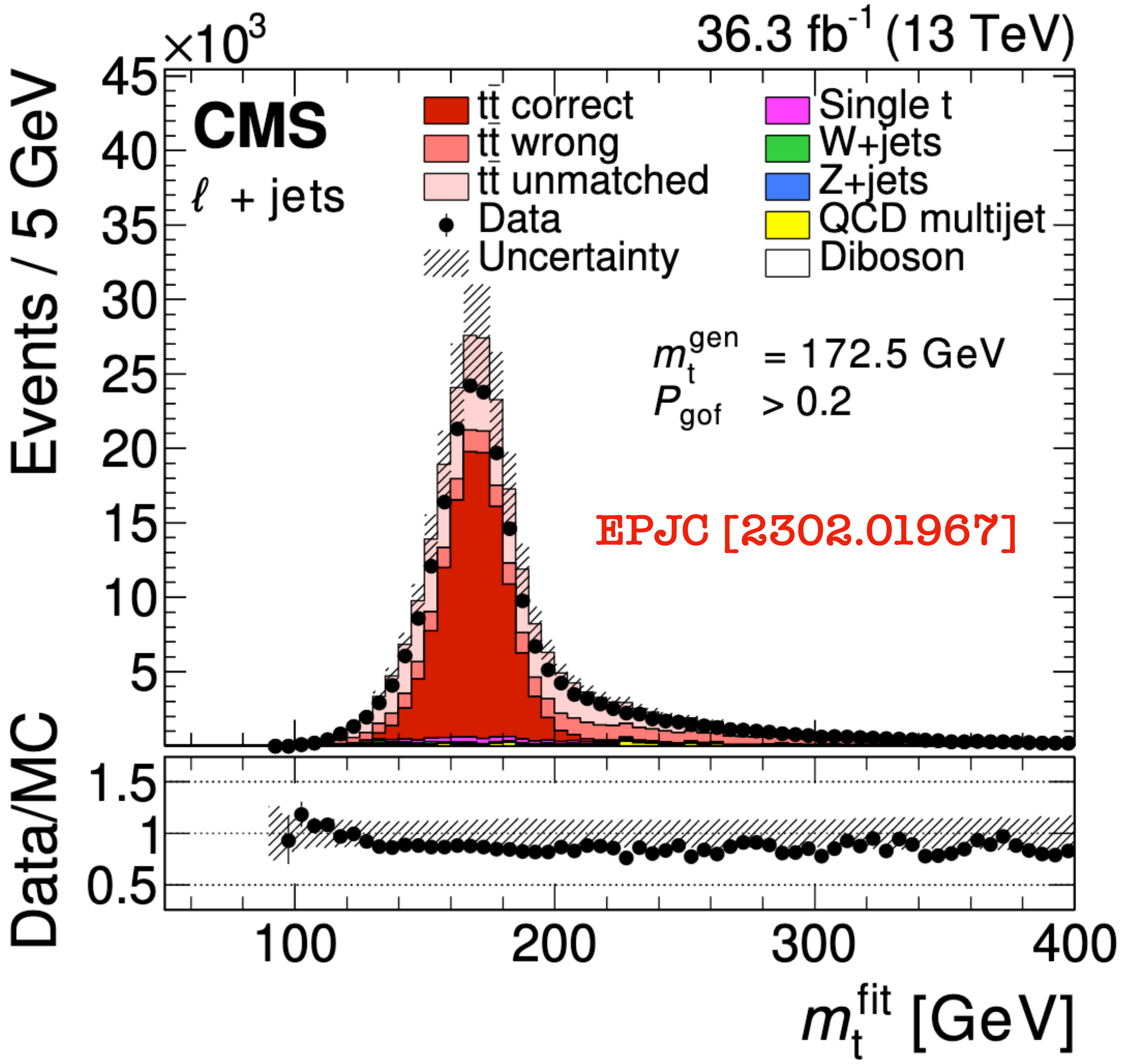


EPJC [1812.10505]





Direct measurements



EPJC [2302.01967]

$$\chi^2 \equiv (\mathbf{x} - \mathbf{x}^m)^{\mathrm{T}} G (\mathbf{x} - \mathbf{x}^m)$$

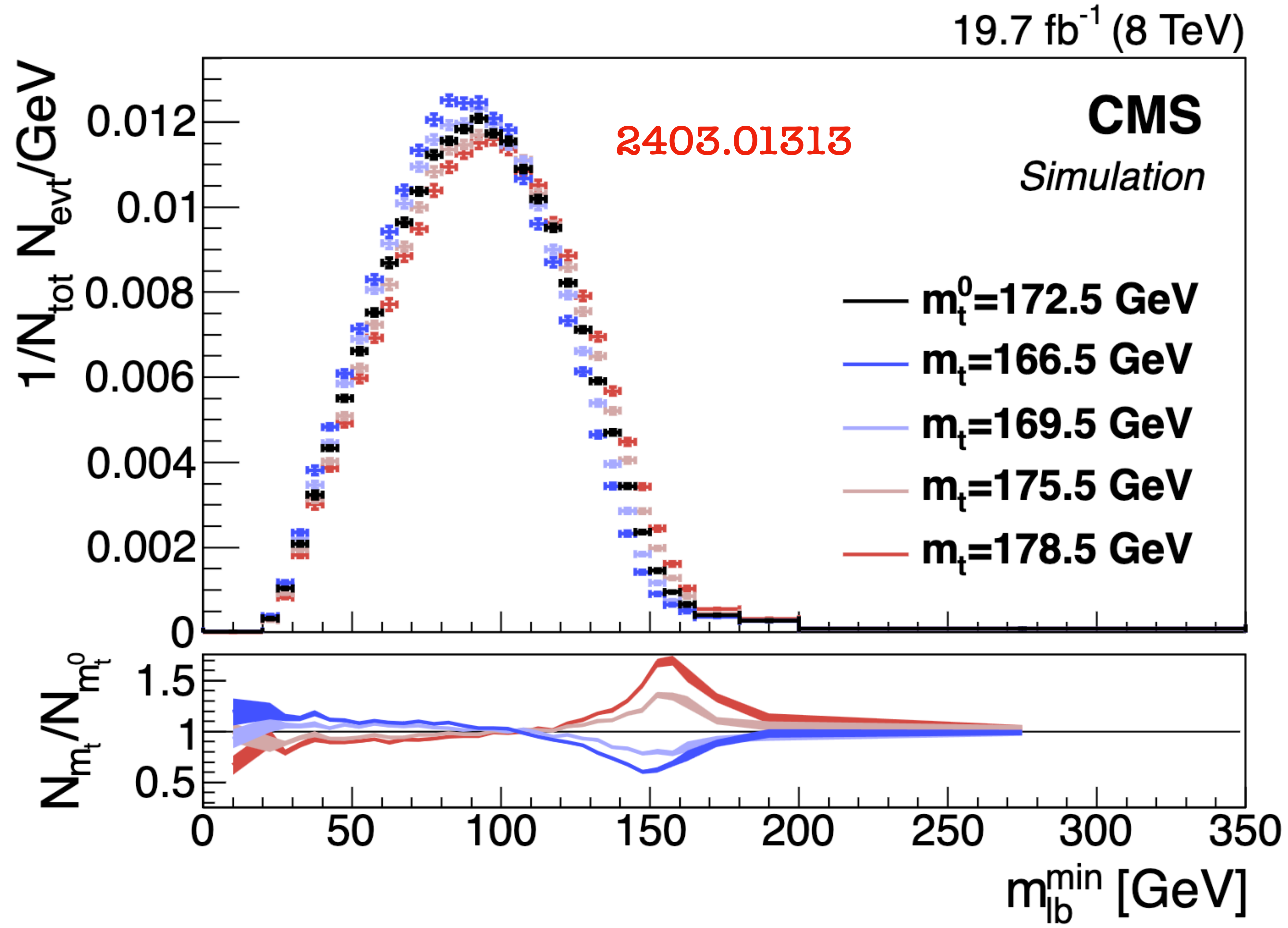
$$P_{\mathrm{gof}} = \exp(-\chi^2/2)$$

Full reconstruction

- Only possible in l+jets and fully hydronic channels
- In l+jets, longitudinal component of neutrino momentum estimated imposing a **W mass constraint**

- m_t constraint needed in di-lepton channel -> cannot be used in m_t measurement -> **partial reconstruction**

P_{gof} cut suppresses backgrounds in l+jets channel



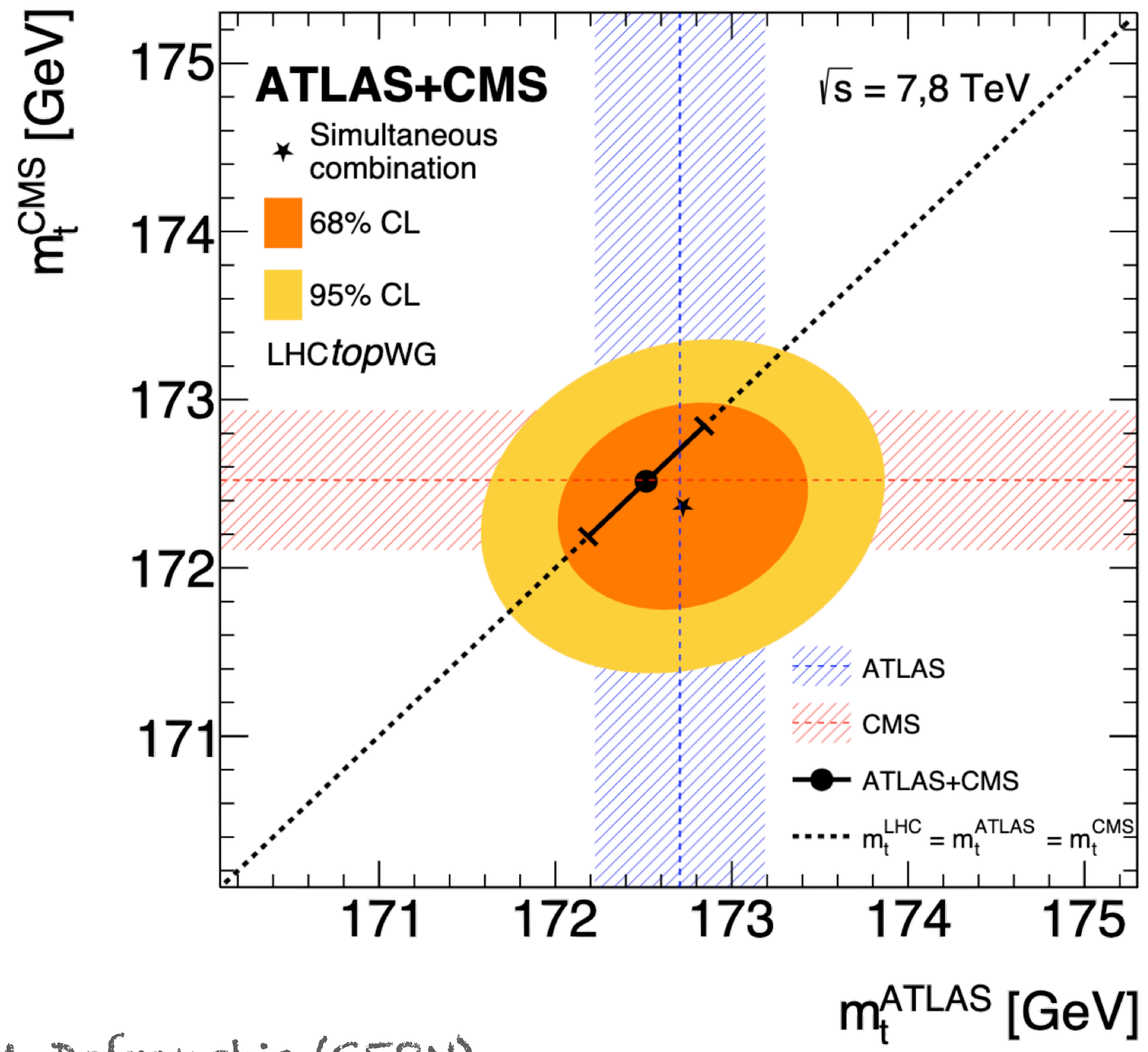
2403.01313

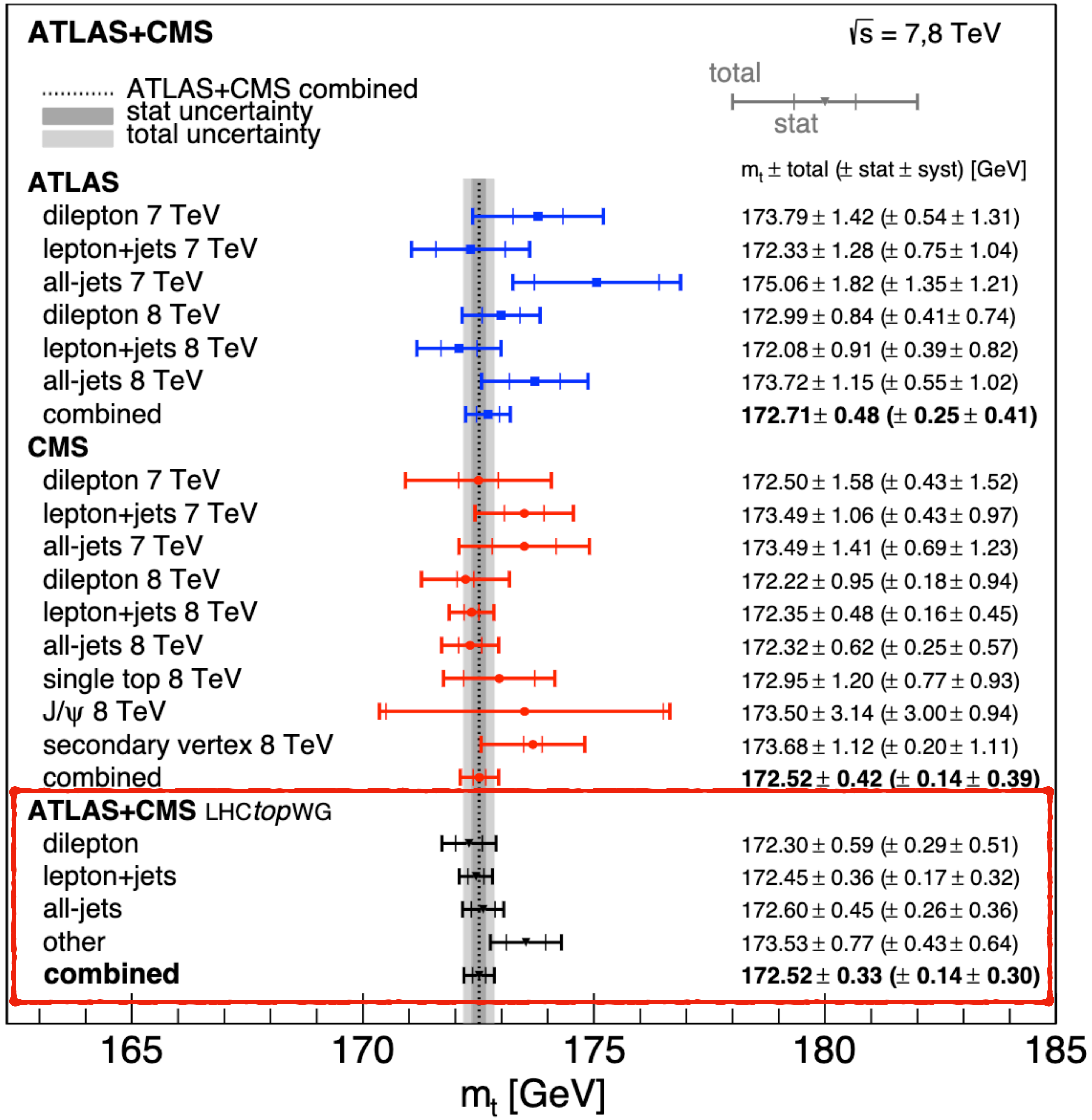
LHC Run1 combination (7 and 8 TeV)

PRL [2402.08713]

- Combination of **15 input measurements** (6 ATLAS + 9 CMS)
- Better than 0.2% precision -> **most precise result** to date
- Includes 3 CMS measurements from “alternative methods”

-> Precision limited by b-quark jet energy scale uncertainty

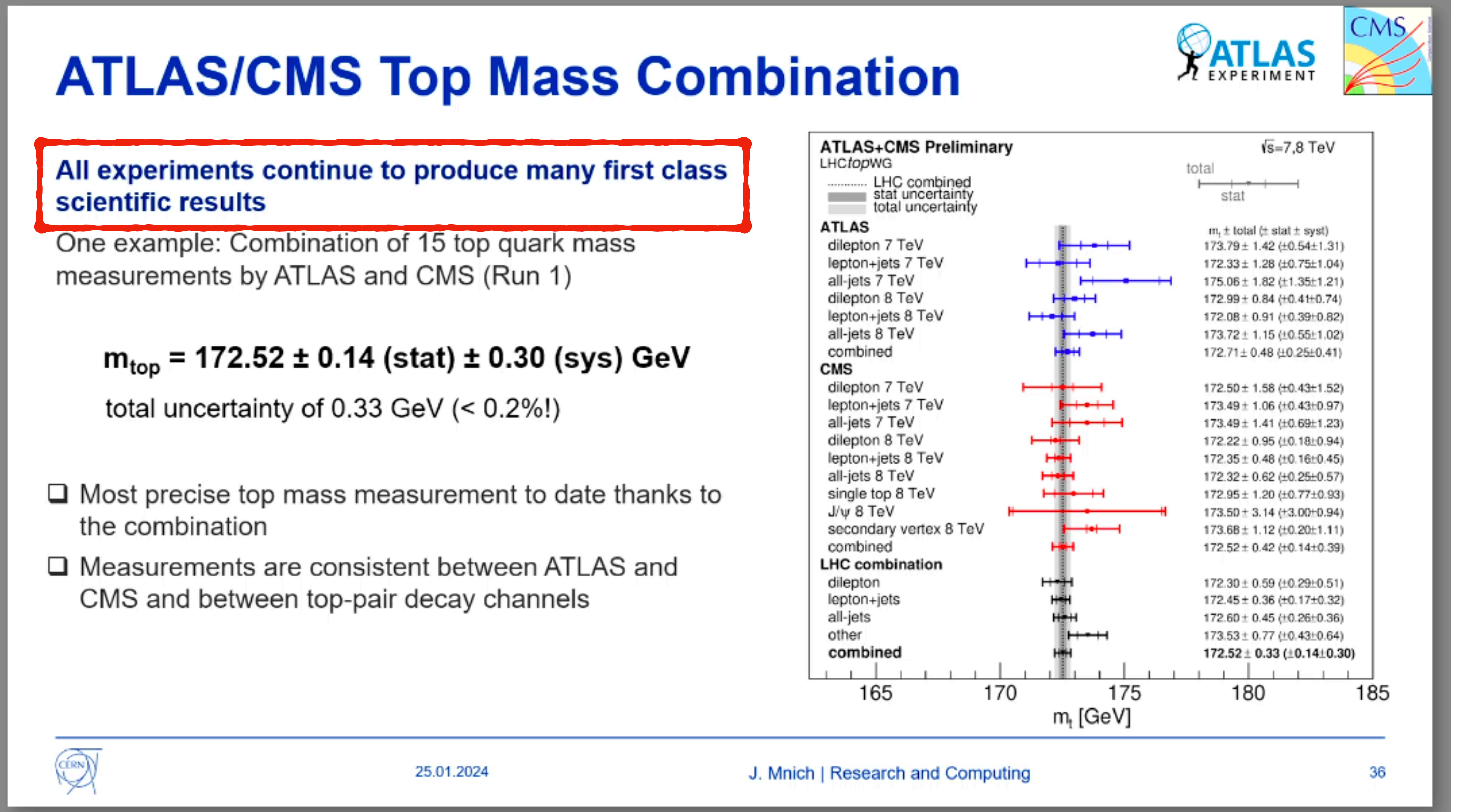




	m_t ± total (± stat ± syst) [GeV]
ATLAS	
dilepton 7 TeV	173.79 ± 1.42 (± 0.54 ± 1.31)
lepton+jets 7 TeV	172.33 ± 1.28 (± 0.75 ± 1.04)
all-jets 7 TeV	175.06 ± 1.82 (± 1.35 ± 1.21)
dilepton 8 TeV	172.99 ± 0.84 (± 0.41 ± 0.74)
lepton+jets 8 TeV	172.08 ± 0.91 (± 0.39 ± 0.82)
all-jets 8 TeV	173.72 ± 1.15 (± 0.55 ± 1.02)
combined	**172.71 ± 0.48 (± 0.25 ± 0.41)**
CMS	
dilepton 7 TeV	172.50 ± 1.58 (± 0.43 ± 1.52)
lepton+jets 7 TeV	173.49 ± 1.06 (± 0.43 ± 0.97)
all-jets 7 TeV	173.49 ± 1.41 (± 0.69 ± 1.23)
dilepton 8 TeV	172.22 ± 0.95 (± 0.18 ± 0.94)
lepton+jets 8 TeV	172.35 ± 0.48 (± 0.16 ± 0.45)
all-jets 8 TeV	172.32 ± 0.62 (± 0.25 ± 0.57)
single top 8 TeV	172.95 ± 1.20 (± 0.77 ± 0.93)
J/ψ 8 TeV	173.50 ± 3.14 (± 3.00 ± 0.94)
secondary vertex 8 TeV	173.68 ± 1.12 (± 0.20 ± 1.11)
combined	**172.52 ± 0.42 (± 0.14 ± 0.39)**
ATLAS+CMS LHC*top*WG	
dilepton	172.30 ± 0.59 (± 0.29 ± 0.51)
lepton+jets	172.45 ± 0.36 (± 0.17 ± 0.32)
all-jets	172.60 ± 0.45 (± 0.26 ± 0.36)
other	173.53 ± 0.77 (± 0.43 ± 0.64)
combined	**172.52 ± 0.33 (± 0.14 ± 0.30)**

> 30% improvement over most precise input measurement

Only LHC result highlighted at Directorate's 2024 new year presentation



ATLAS/CMS Top Mass Combination

All experiments continue to produce many first class scientific results

One example: Combination of 15 top quark mass measurements by ATLAS and CMS (Run 1)

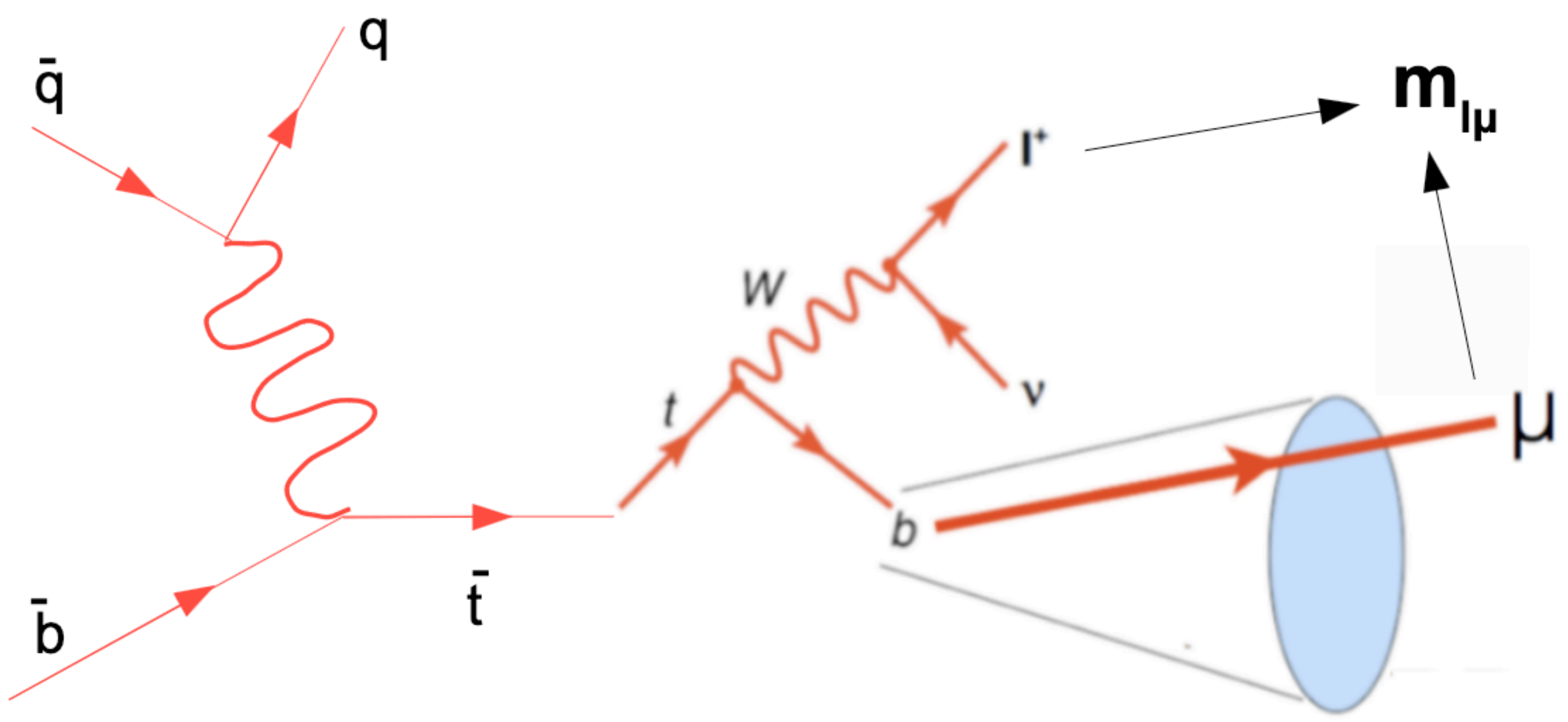
m_{top} = 172.52 ± 0.14 (stat) ± 0.30 (sys) GeV

total uncertainty of 0.33 GeV (< 0.2%!)

- ☐ Most precise top mass measurement to date thanks to the combination
- ☐ Measurements are consistent between ATLAS and CMS and between top-pair decay channels

25.01.2024 J. Mnich | Research and Computing 36

“Alternative” methods (e.g. soft muon)



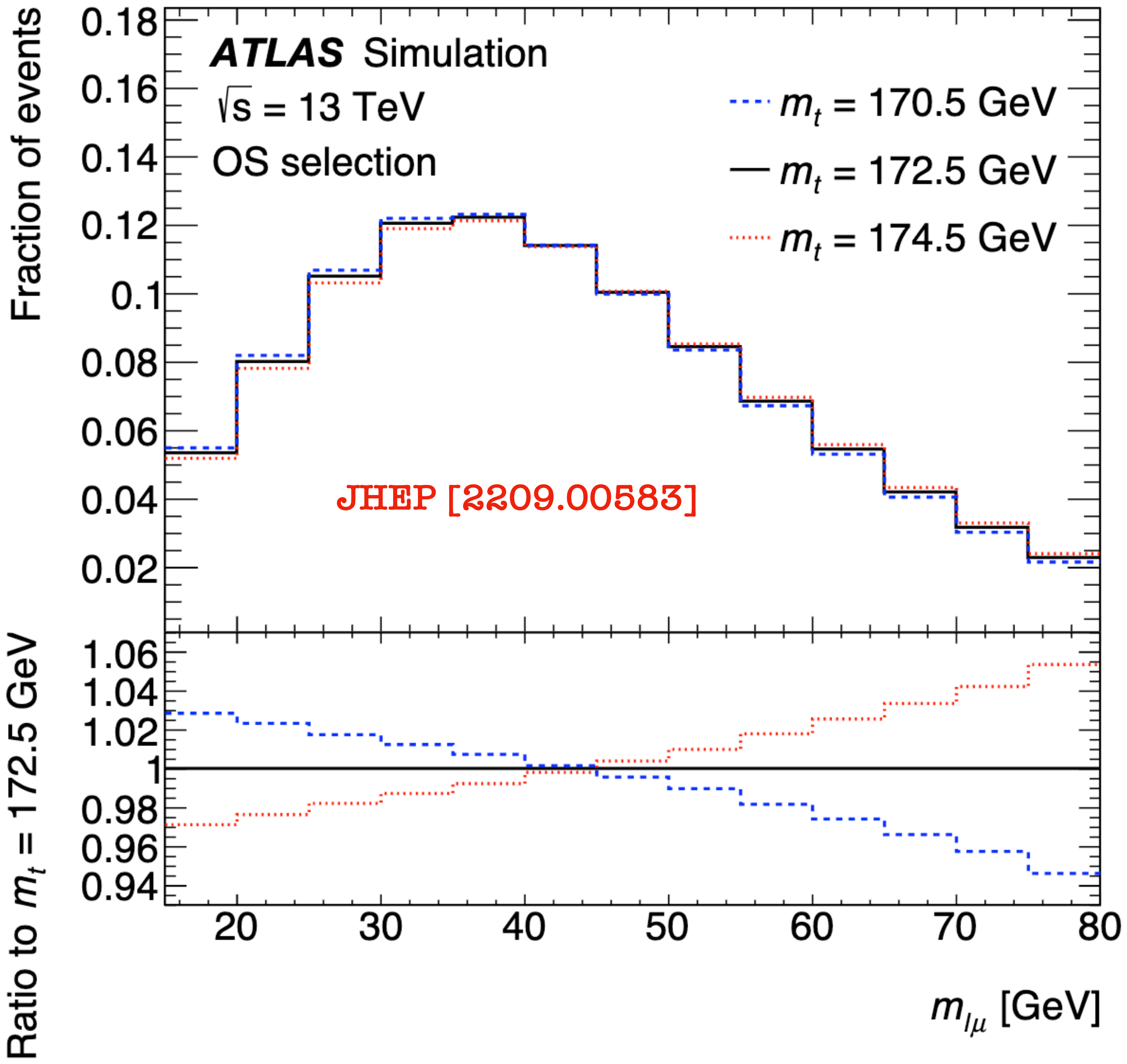
- **Partial reconstruction** of invariant mass of top decay products
- Often consider only certain constituents of b quark jets
  - Soft lepton from B decays, J/Ψ, secondary vertex

Pros: reduced sensitivity to b-quark jet energy scale
-> beneficial for combinations

Cons: sensitive to fragmentation effects,
-> overall less precise

$$m_t = 174.41 \pm 0.39\ (\text{stat.}) \pm 0.66\ (\text{syst.}) \pm 0.25\ (\text{recoil})\ \text{GeV}$$

$$f(z) = \frac{1}{z^{1+br_b m_b^2}}(1-z)^a \exp(-bm_\mathrm{T}^2/z)$$



JHEP [2209.00583]

13 TeV measurement w/ profile likelihood

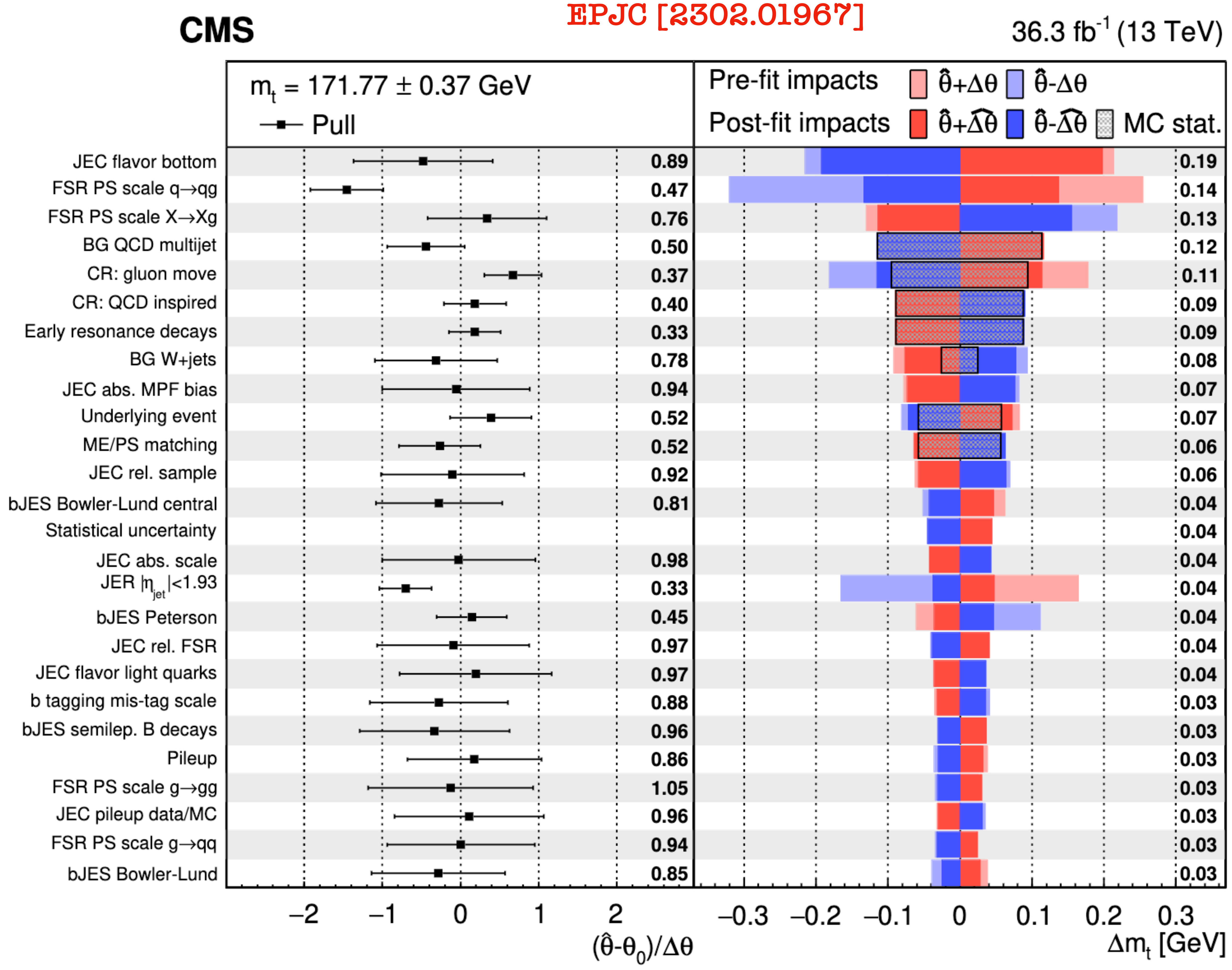
“Traditional” method: vary each parameter in the analysis and repeat extraction of parameter of interest

- Simple to implement and interpret
- **Neglects correlations between systematics**
- Does not make use of data to constrain systematics

Profile-likelihood approach: fit parameter(s) of interest and systematic uncertainties (nuisance parameters) all at once

- Takes all correlations into account
- Makes optimal use of **multiple distributions**
- Can **constrain** systematic effects from data
- Harder to diagnose, unclear interpretation of “theory” nuisance parameters

- Can **significantly mitigate bias** on parameter(s) of interest



EPJC [2302.01967]

$$\mathscr{L}(m_t\,, \lambda_1, \lambda_2 \ldots, \lambda_N)$$

13 TeV profile likelihood result

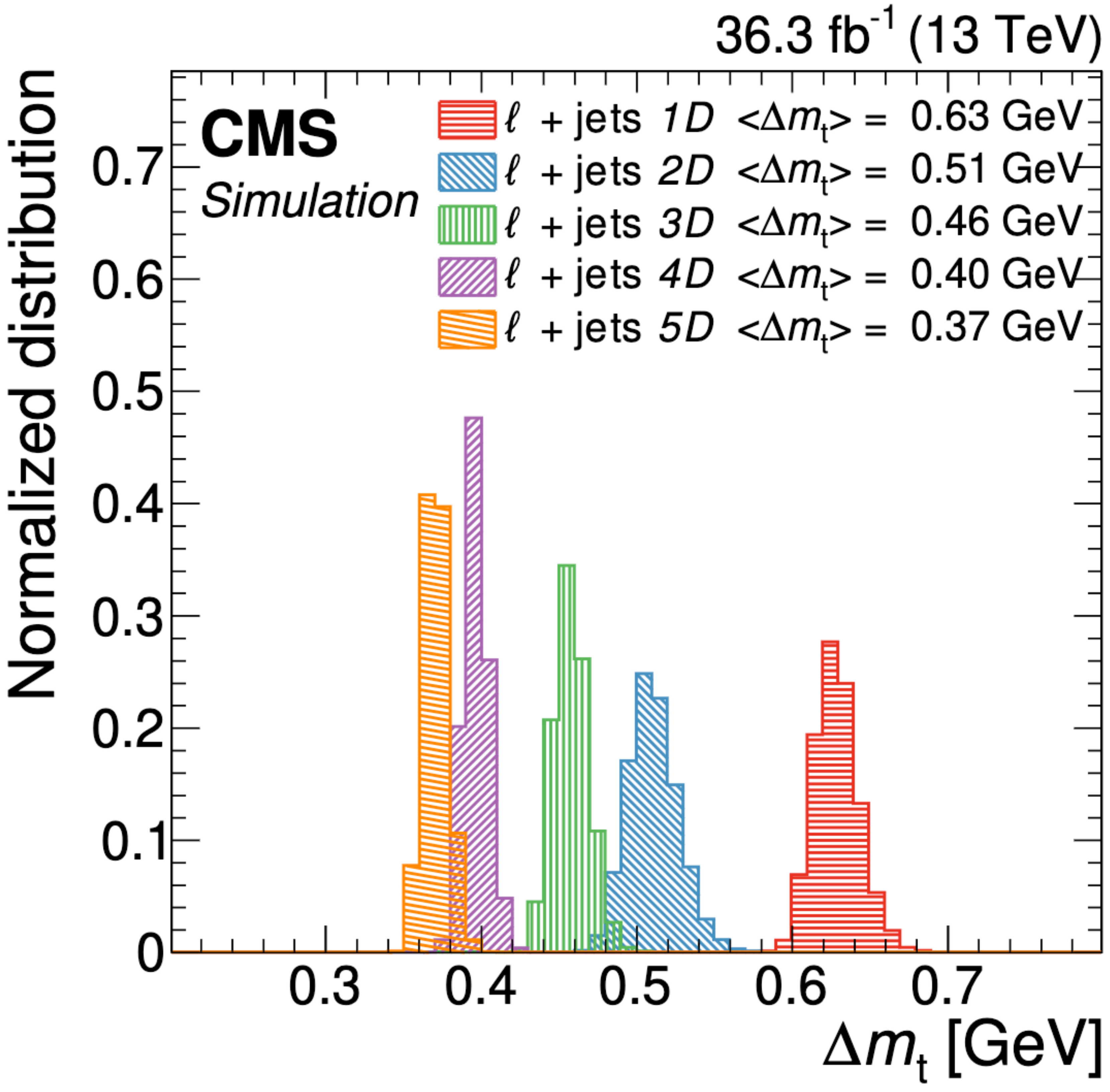
EPJC [2302.01967]

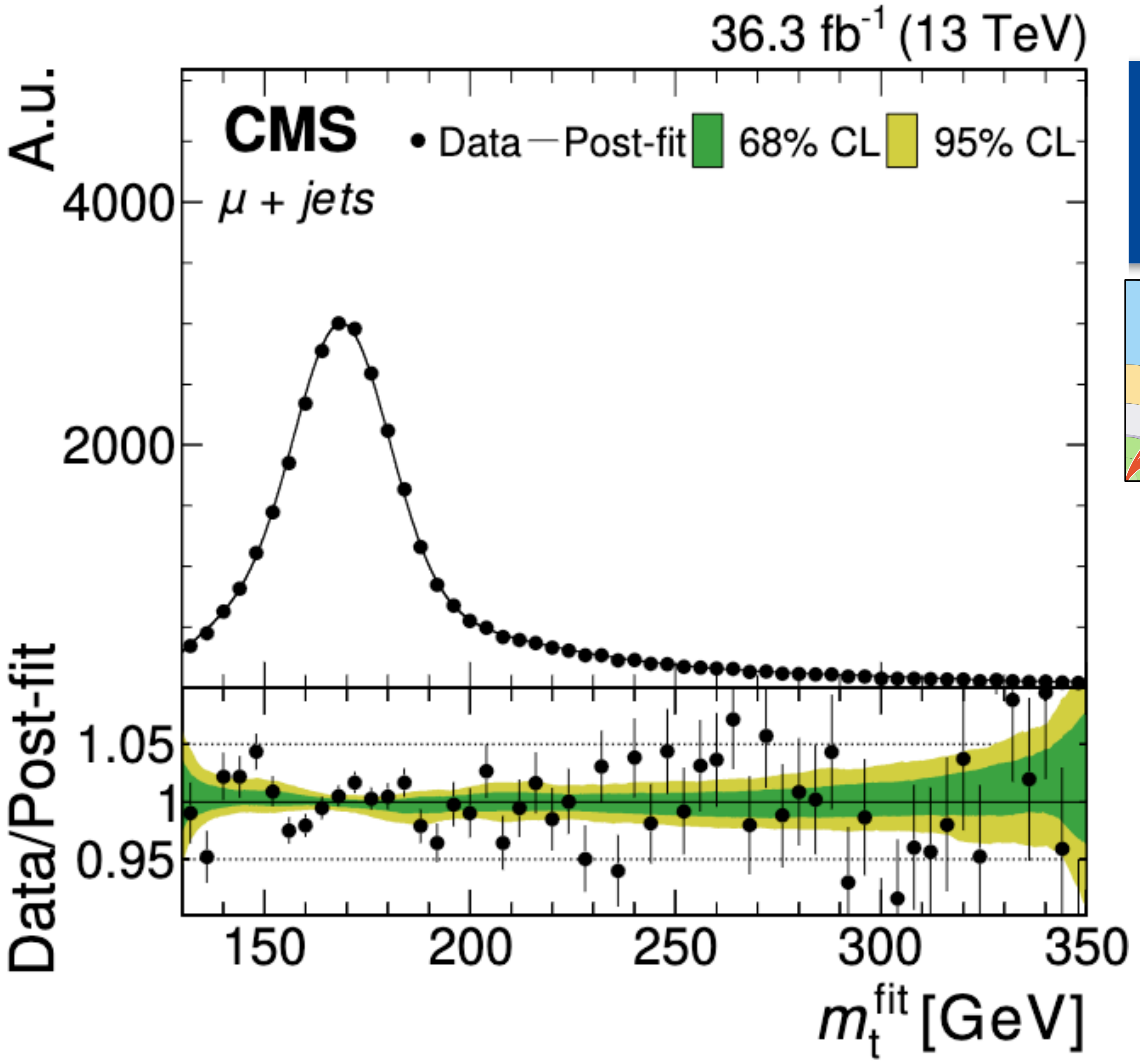
$$\begin{aligned} \text{Electron+jets:} \quad m_{\mathrm{t}}^{5D} &= 172.11 \pm 0.49\,\mathrm{GeV}, \\ \text{Muon+jets:} \quad m_{\mathrm{t}}^{5D} &= 171.98 \pm 0.42\,\mathrm{GeV}, \\ \text{Lepton+jets:} \quad m_{\mathrm{t}}^{5D} &= 171.77 \pm 0.37\,\mathrm{GeV}. \end{aligned}$$

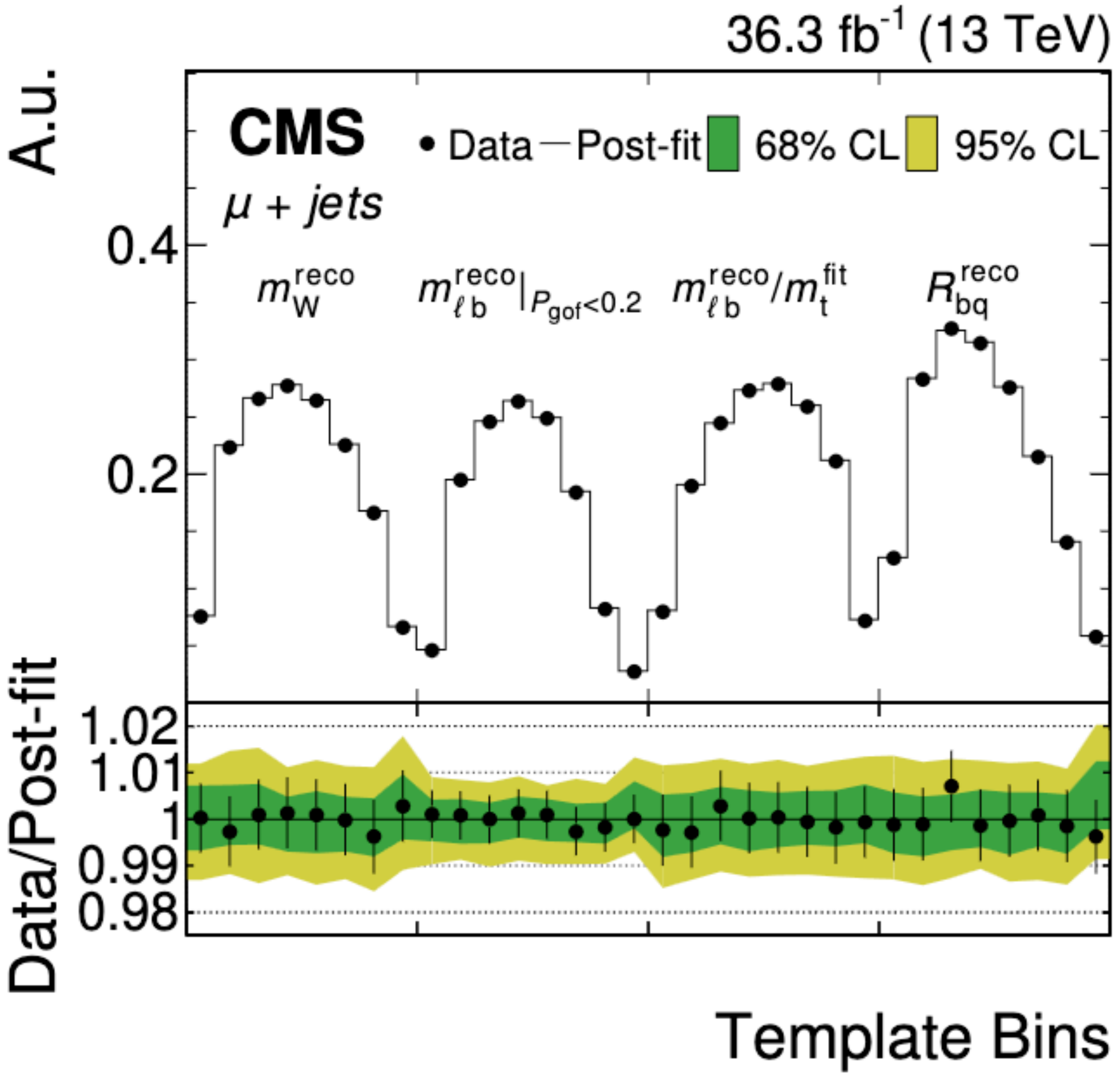
Most precise standalone result

- Combination of different decay channels and distributions significantly improves precision

- **Nearly as precise as the combination of 15 Run1 measurements!**







Uncertainty on m_t vs time

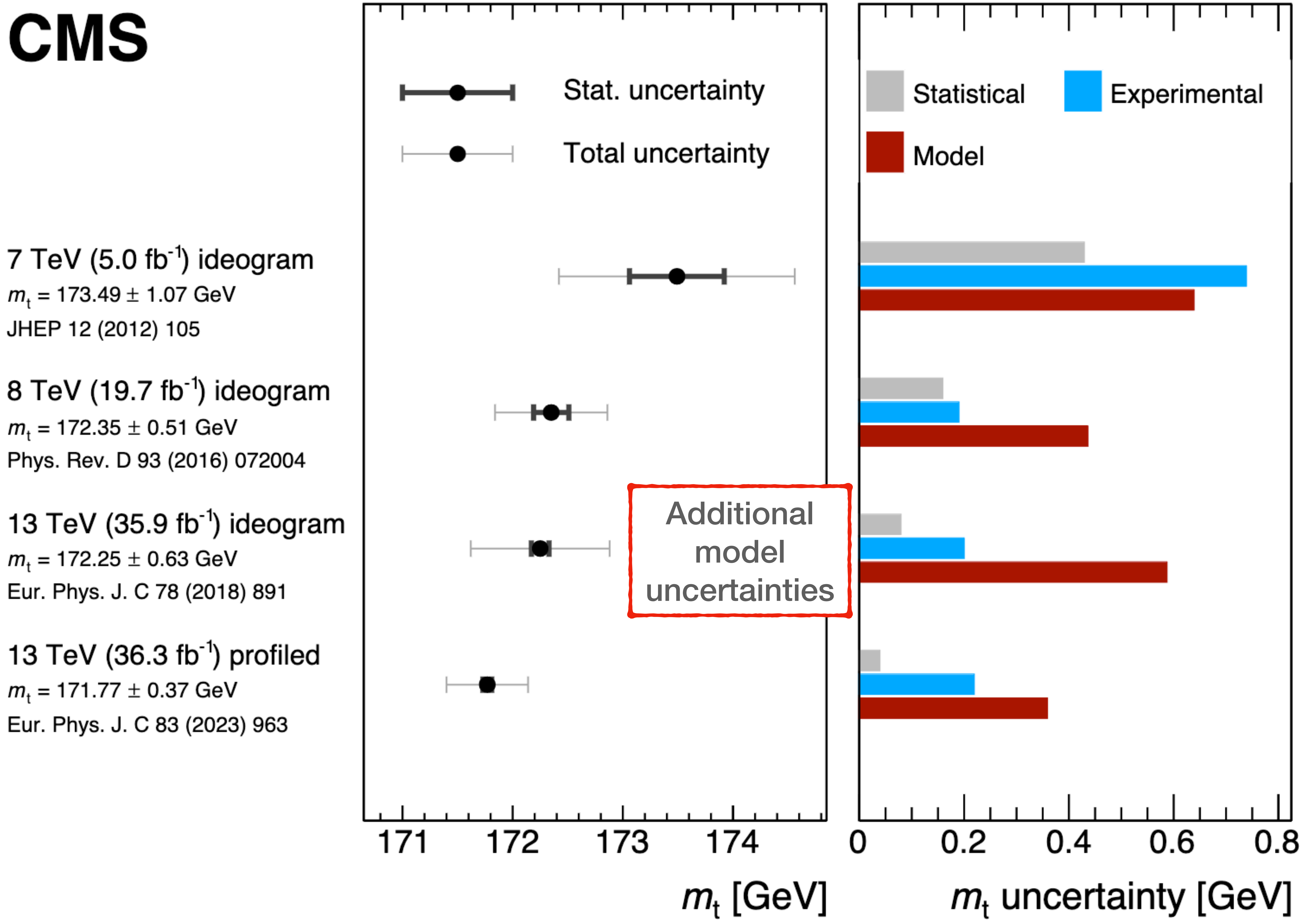
Improvement in precision well **beyond** decrease in statistical uncertainty

- Like many other class of measurements, significant benefit from **advancement in data analysis techniques**

Natural questions:

- Do we need to know m_t more precisely than this?
- How much more precisely can we measure it?

2403.01313



Additional model uncertainties

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Issues with direct measurements

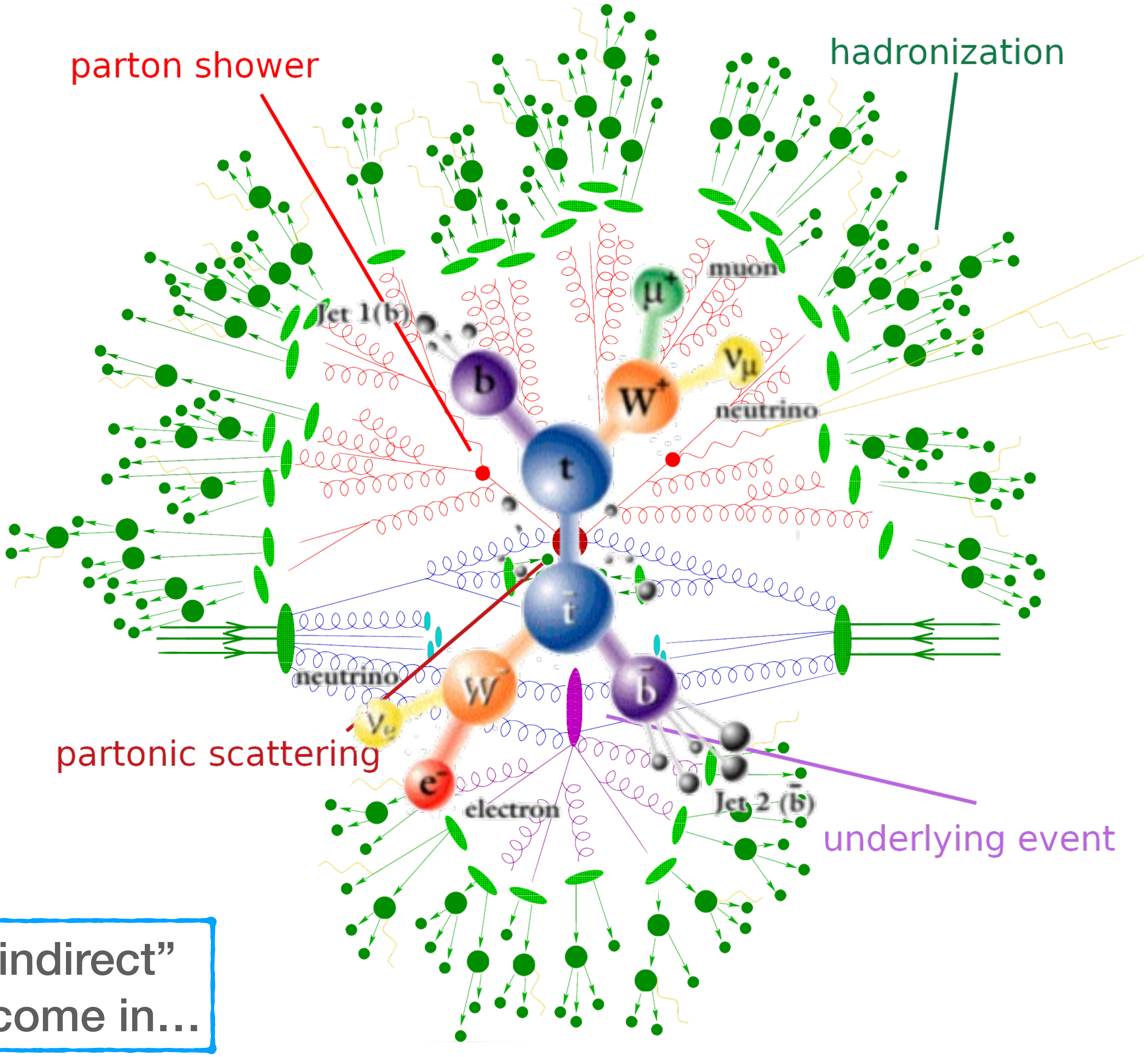
- Matrix element: calculated at fixed order in QCD (NLO)
- Parton-shower: leading-logarithmic (LL) approximation of soft and collinear emissions
- Colour reconnection, hadronisation, and underlying event: QCD-inspired heuristic models

Hard to interpret results of direct measurements in a consistent theoretical framework (Lagrangian mass)

$$m_t^{\mathscr{L}} = m_t^{\mathrm{MC}} + \Delta_{\mathrm{MC}}^{\mathrm{bias}} \pm \Delta_{\mathrm{MC}}^{\mathrm{unc}}$$

Cannot be estimated from first principles (yet)

This is where "indirect" measurements come in…



Run 2 5D Likelihood

improved precision

Run 1 Hybrid Ideogram

Boosted top quarks

Run 1 Ideogram

Full Reconstruction

single Top

Run 1

Secondary Ve

Leptons or

Partial Reconstruction

Direct Measurements

Full Run 2 - XCone and JMS

2 - XC

- CA merged jet

Top quark mass

Lagrangian mass

Multi-differential

simulation based

theory based

jet

Run 2

Incl

Running mass

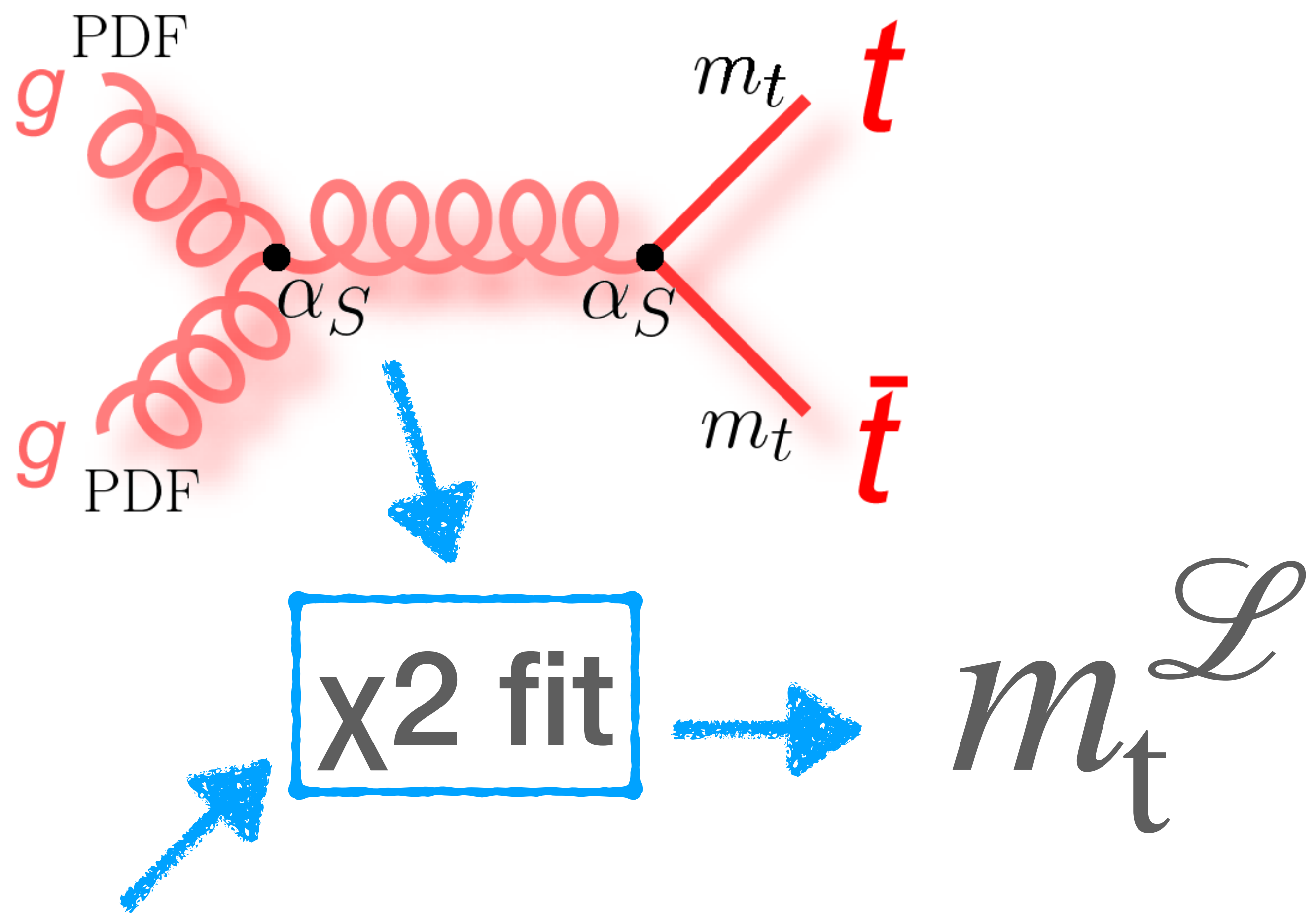
Lagrangian Extraction

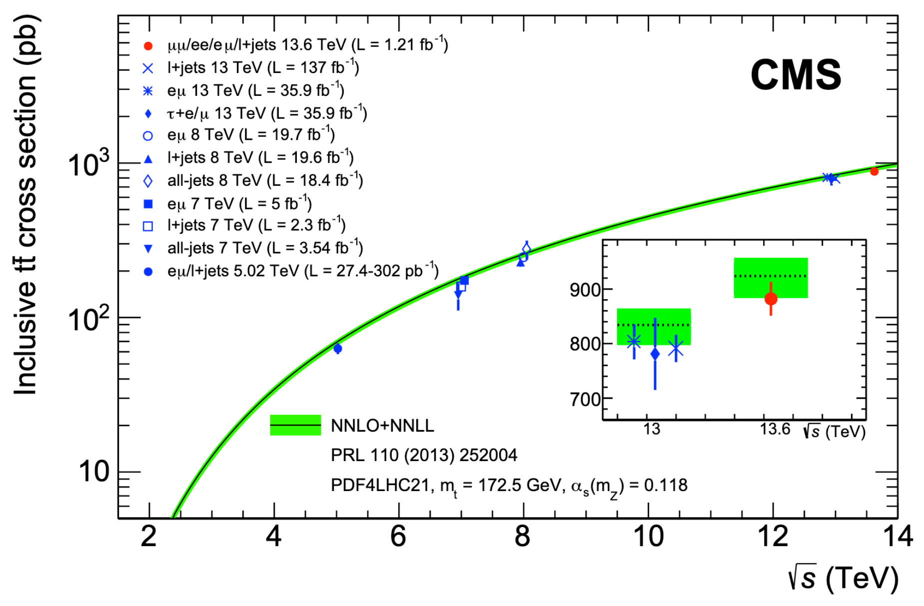
Matteo M. Defranchis (CERN)

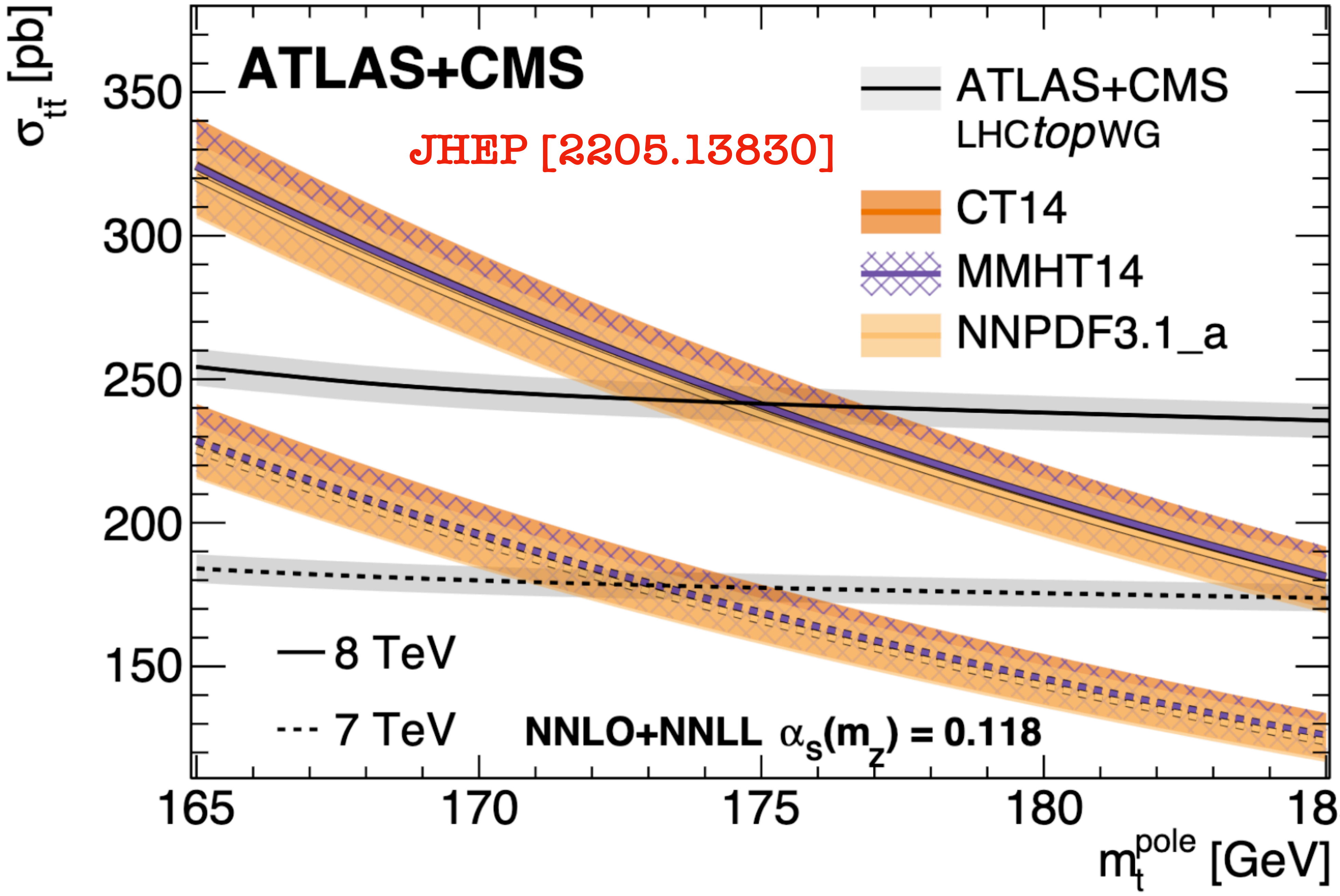
The principle of indirect measurements

First of all, “admit” that m_t is not a physics observable (as it renormalisation-scheme dependent)

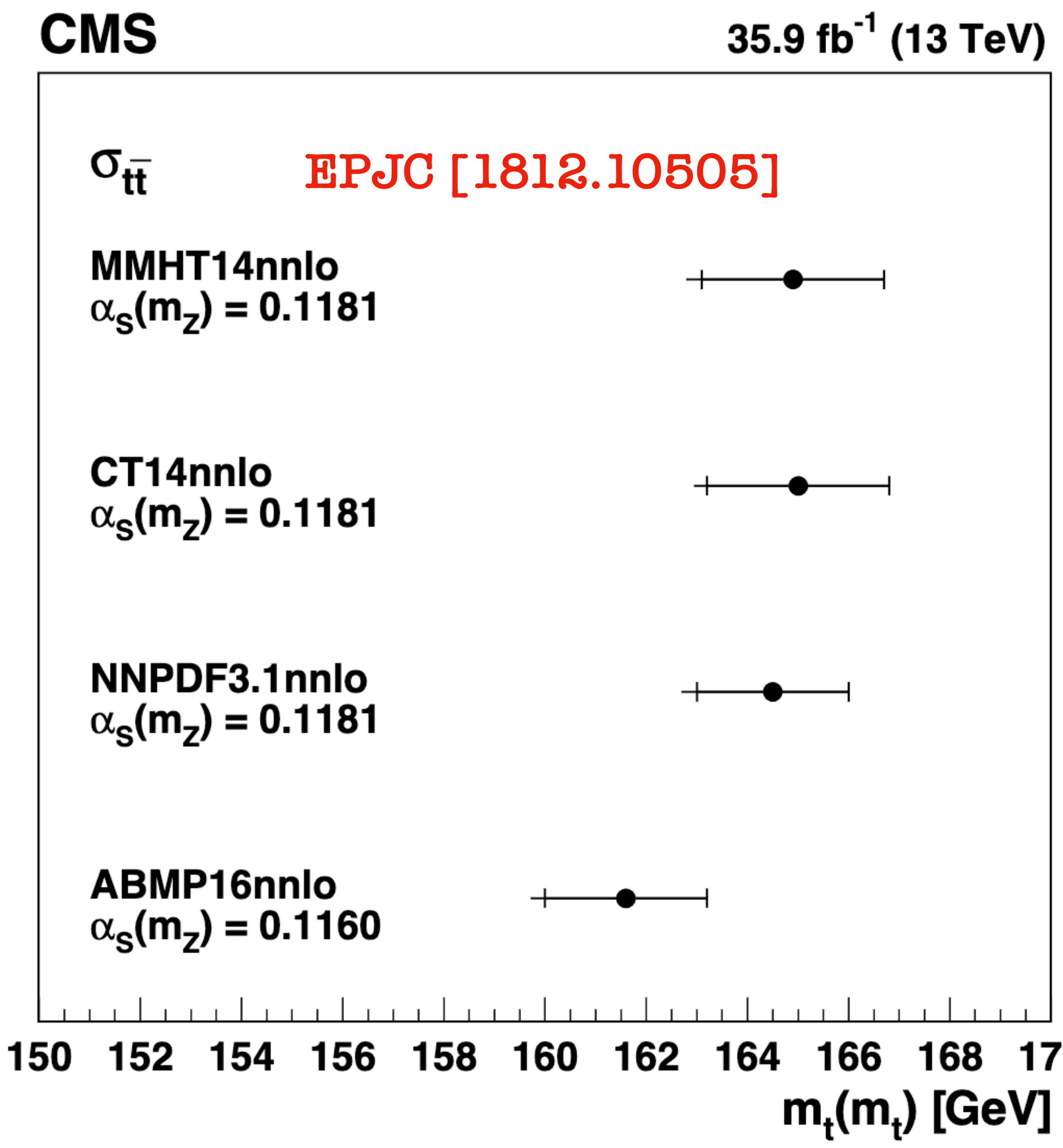
- Measure an observable (cross section) sensitive to m_t
- Use **standalone theory prediction** to extract m_t in a given renormalisation scheme







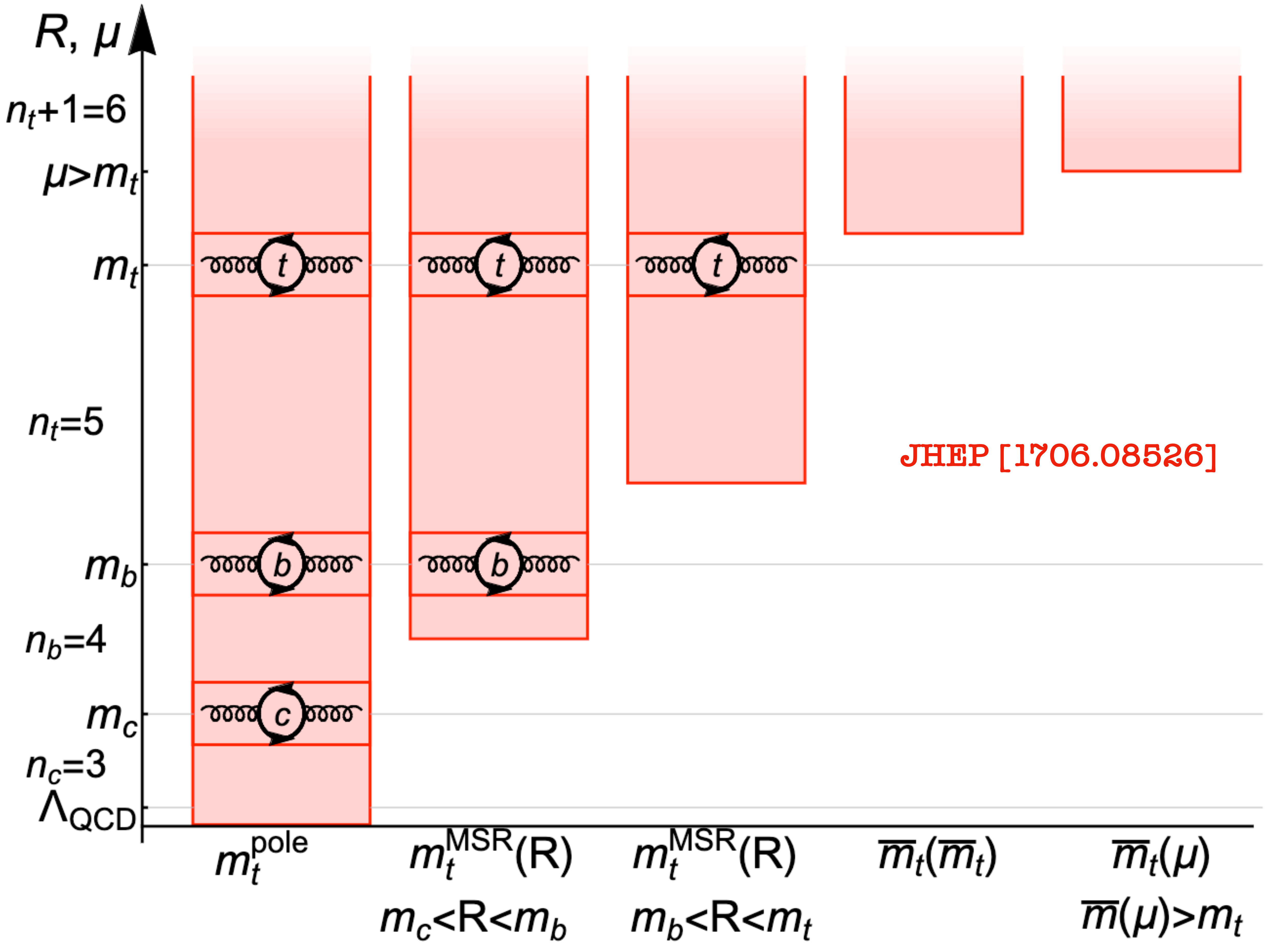
JHEP [2205.13830]



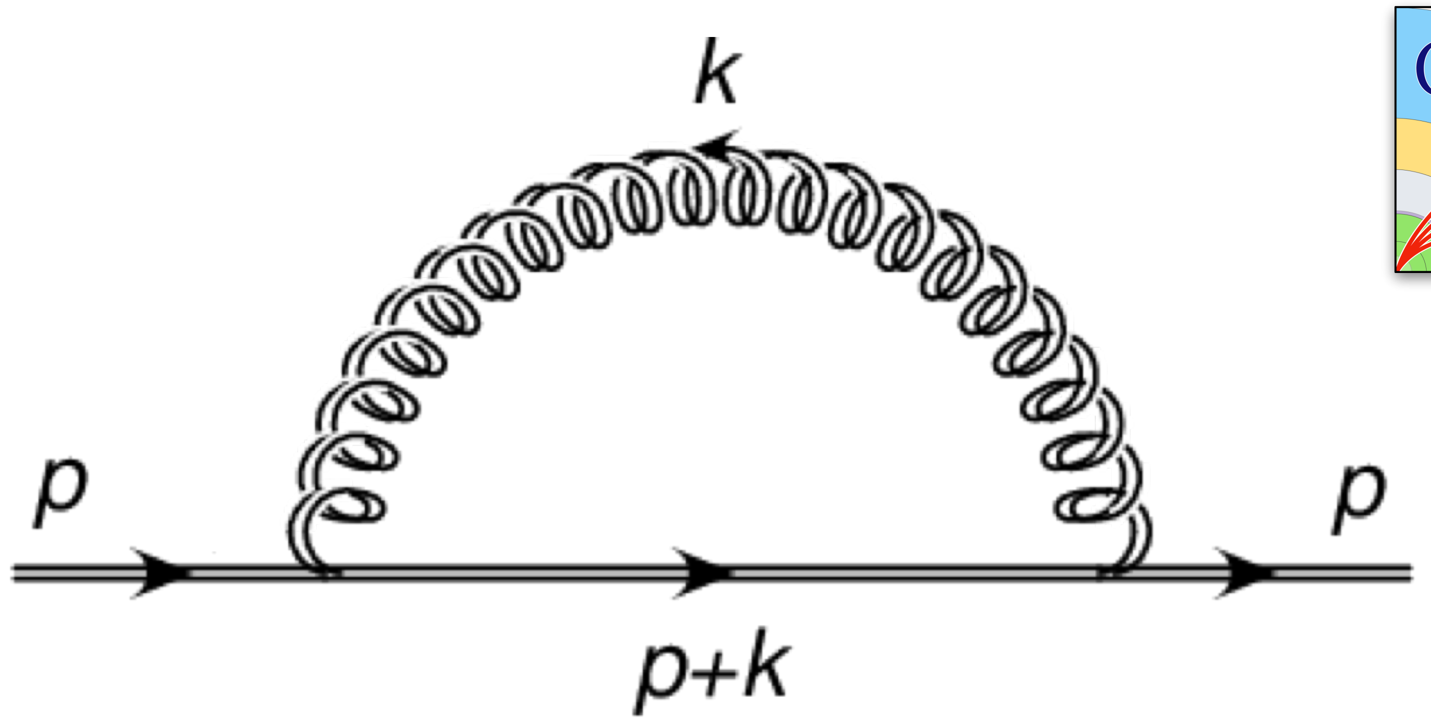
EPJC [1812.10505]

$$m_t^{\text{pole}} = 173.4^{+1.8}_{-2.0}\ \text{GeV} \neq m_t(m_t) = 164.5\,^{+1.8}_{-2.1}\ \text{GeV}$$

The Lagrangian top quark mass



JHEP [1706.08526]



m_t^{pole}: pole of the top quark propagator

- Numerically similar to m_t^{MC}
- Sensitive to IR effects that give rise to fundamental (renormalon) **ambiguity** of order $\Lambda_{QCD} \simeq 250$ MeV

MSbar: short distance mass

- Same renormalisation scheme as α_S
- Reabsorbs IR divergencies into bare mass
- Free of (linear) renormalon

-> as α_S, the top mass becomes **scale dependent**

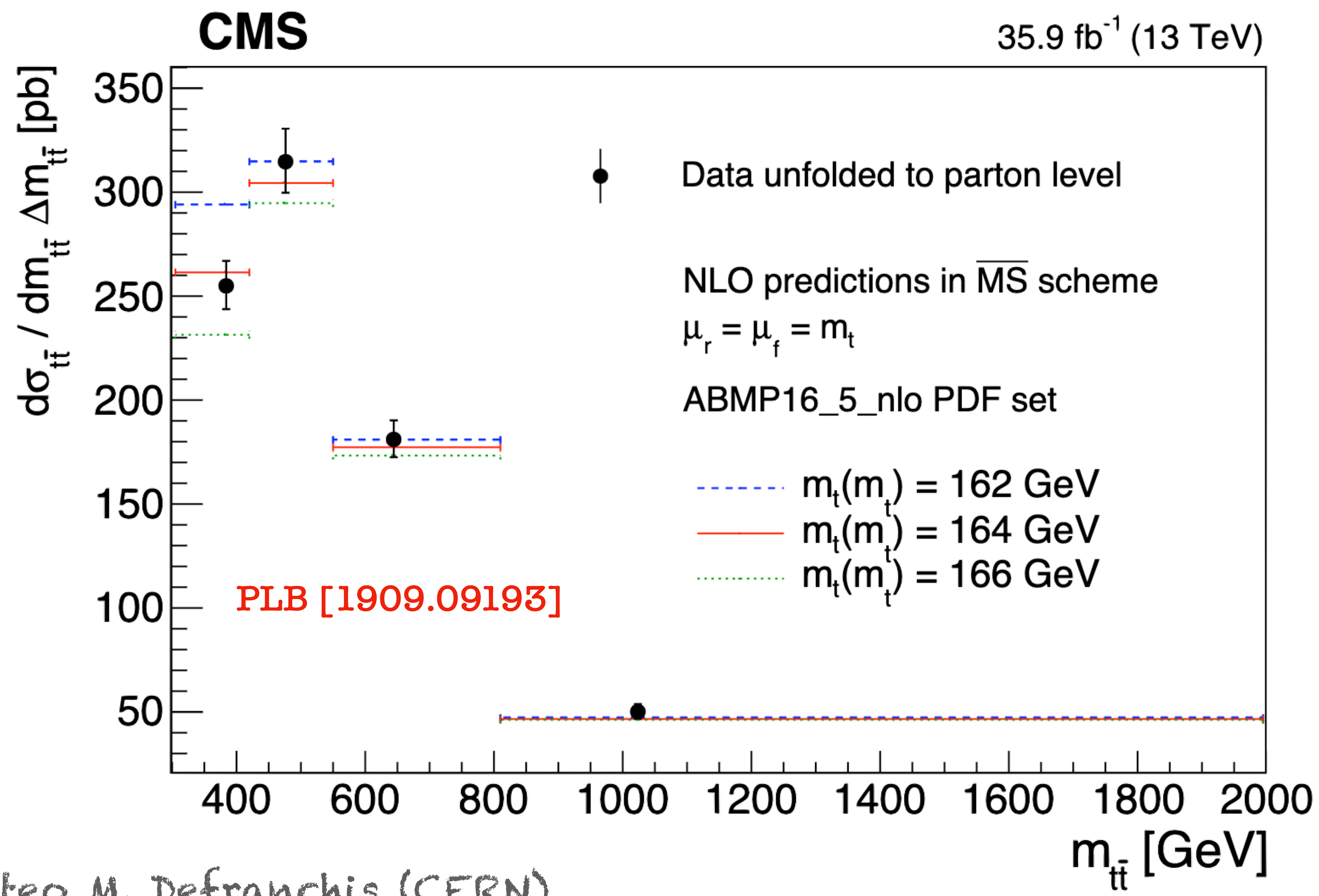
$$m_q(m_q) = m_q^{\text{pole}}\left[1-\frac{4}{3\pi}\alpha_S(m_q)+\mathcal{O}(\alpha_S^2)\right]$$

The running of m_t

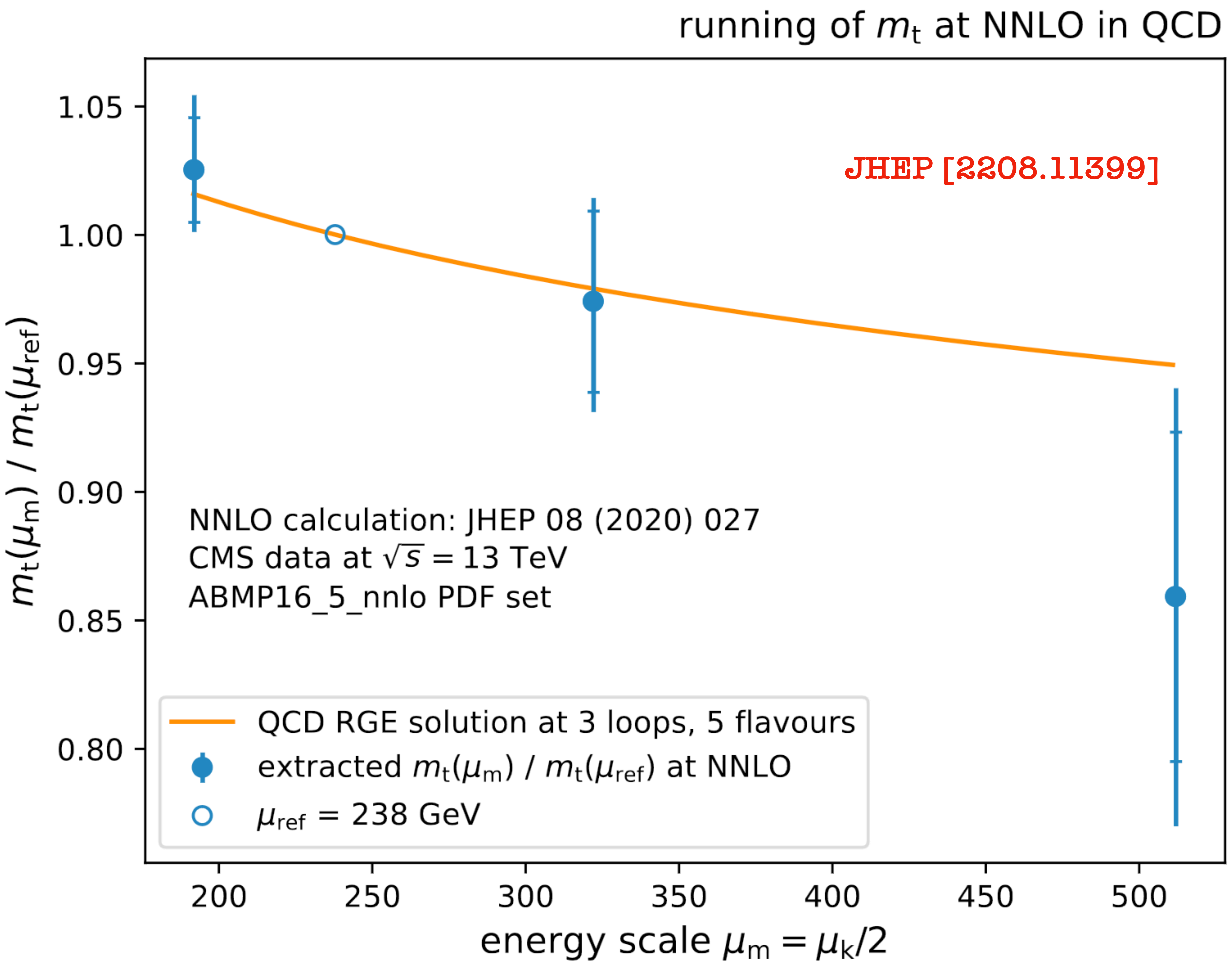
- Described by **renormalisation group equations** of QCD
- Can be extracted from a measurement of a differential cross section as a function of the energy scale of the process (similar to α_S extraction)

- Good agreement with RGE solution at 3 loops
- Can be used as a probe of **BSM physics**

$$\mu^2 \frac{\partial m}{\partial \mu^2} = -\gamma_m(\alpha_S)\, m$$



PLB [1909.09193]



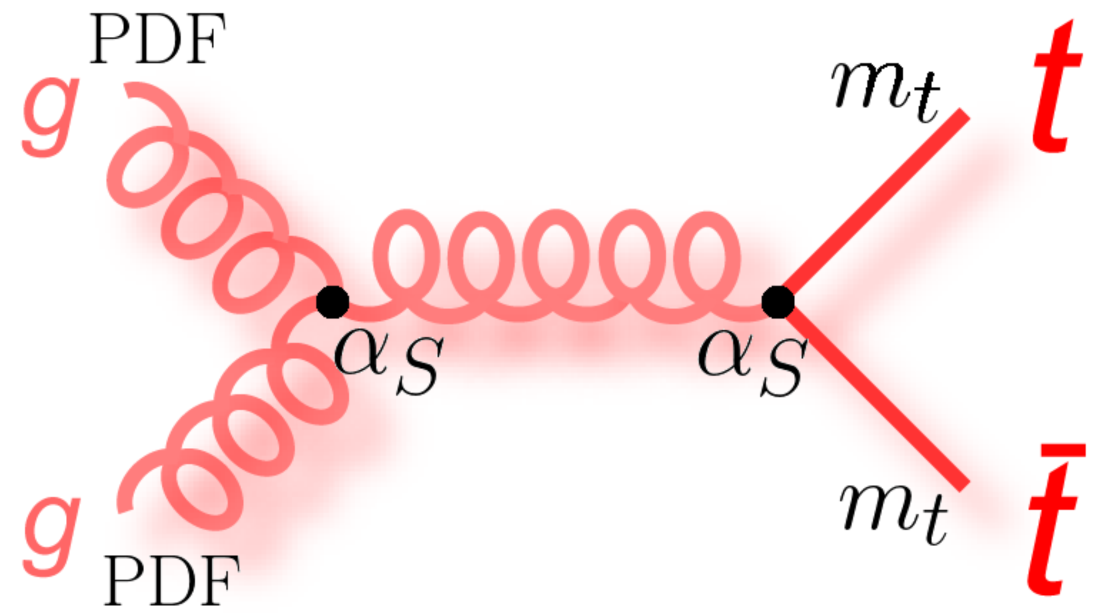
JHEP [2208.11399]

The “ultimate” pole mass measurement?

Multi-differential distributions allow to simultaneously constrain top mass, strong coupling constant, and PDF

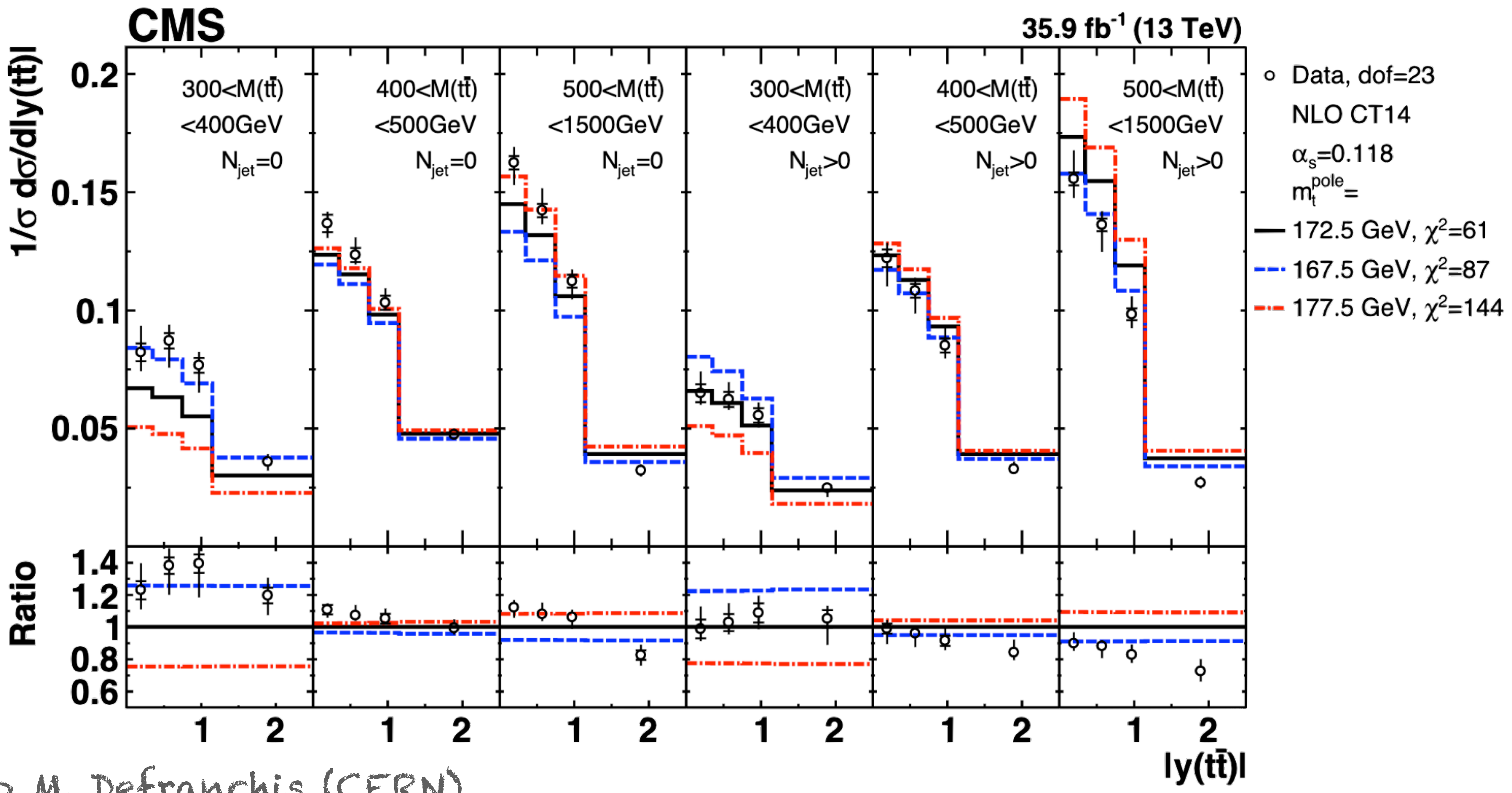
Simultaneous fit with HERA deep inelastic scattering data (NLO)

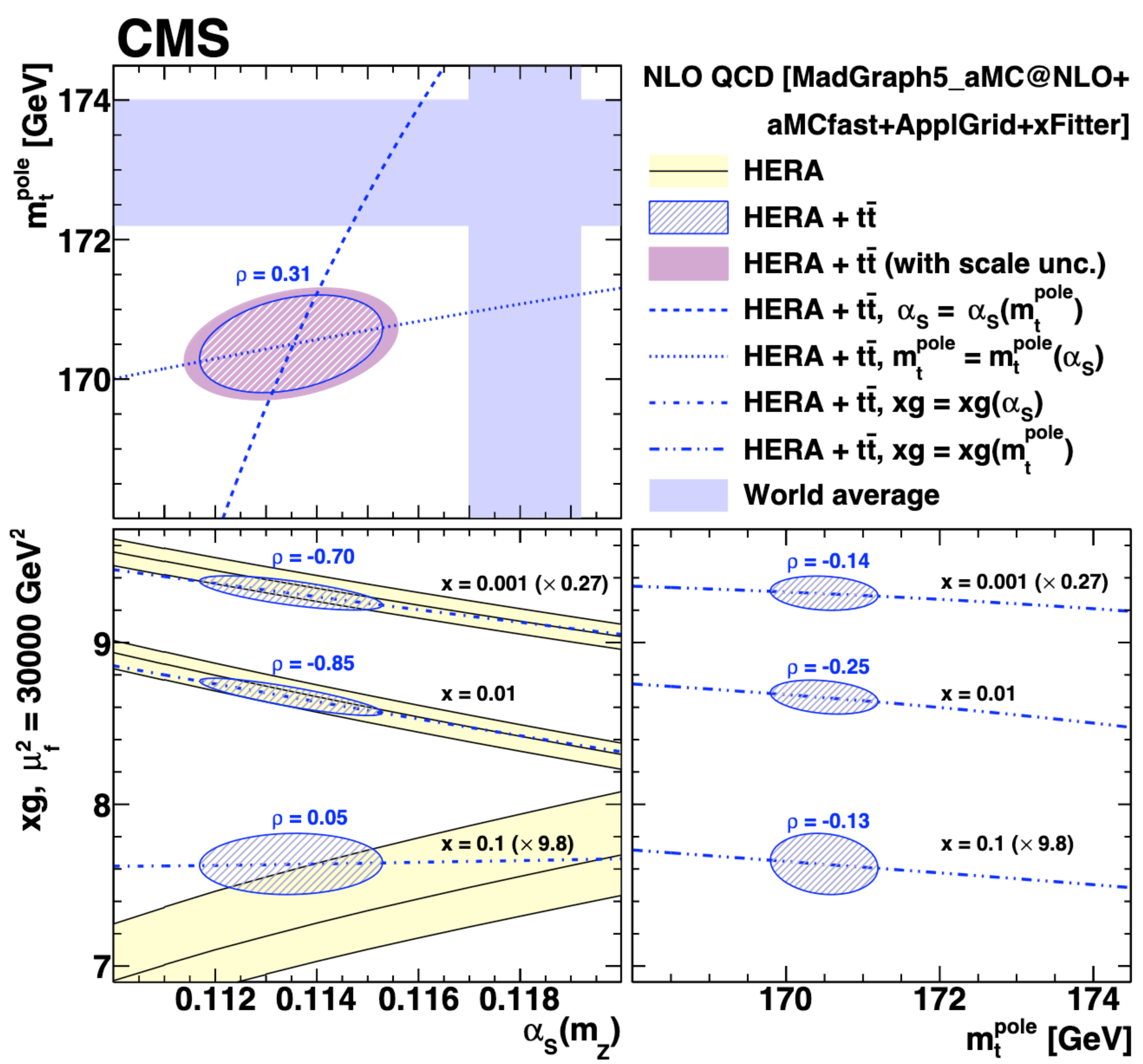
Results in remarkable precision of **0.8 GeV**, which be further improved by using the full 13 TeV dataset



EPJC [1904.05237]

$$m_{\text{t}}^{\text{pole}} = 170.5 \pm 0.8\,\text{GeV}$$



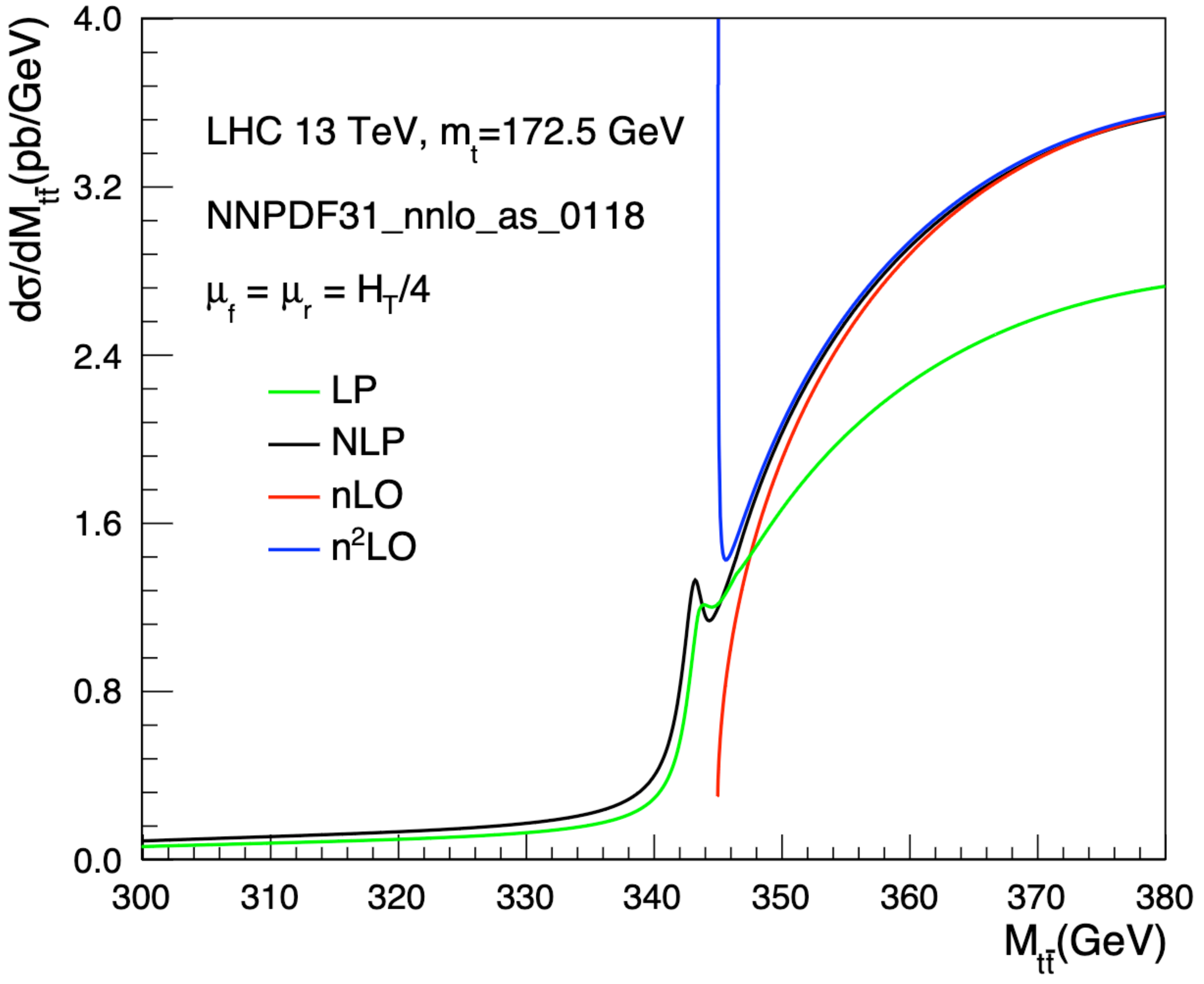


A closer look at the 3D measurement

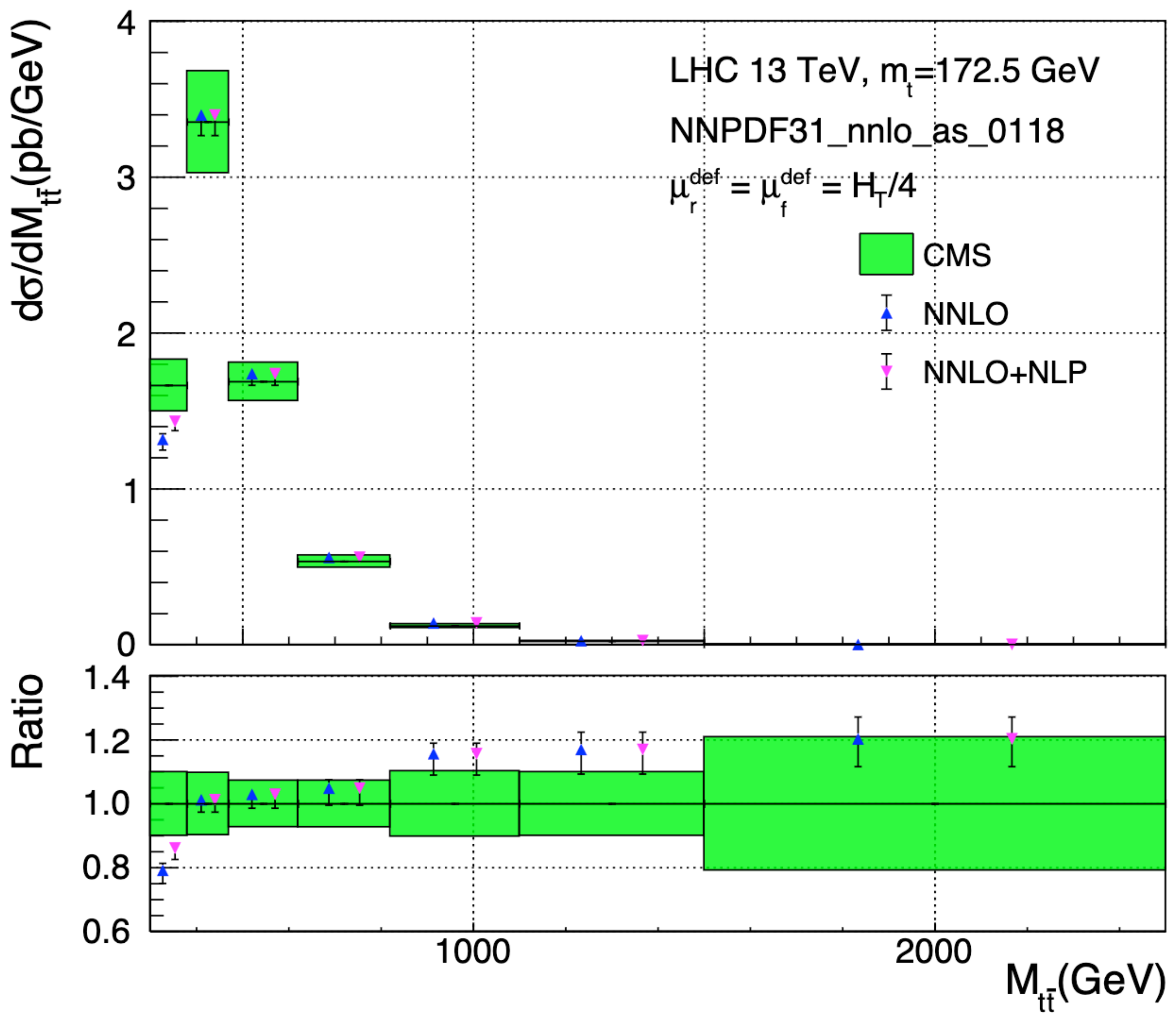
Result in **tension** with ATLAS+CMS combination of 7 and 8 TeV measurements

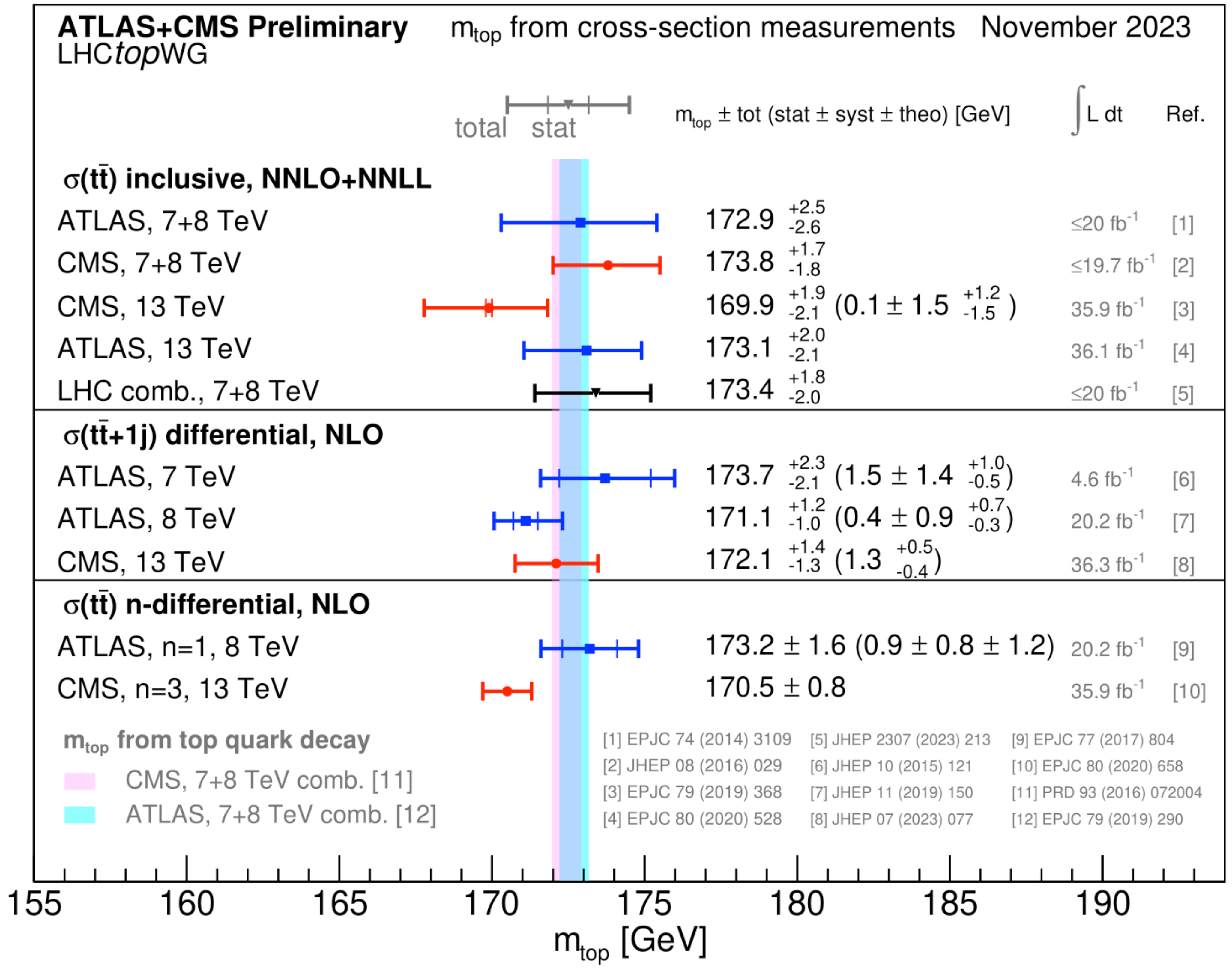
Limitations of fixed order calculations are particularly pronounced in the **threshold region**

- Soft gluon resummation (up to NNLL)
- **Toponium**-like bound state effects -> can lead to a bias of up to +1.4 GeV in first bin



JHEP [2004.03088]



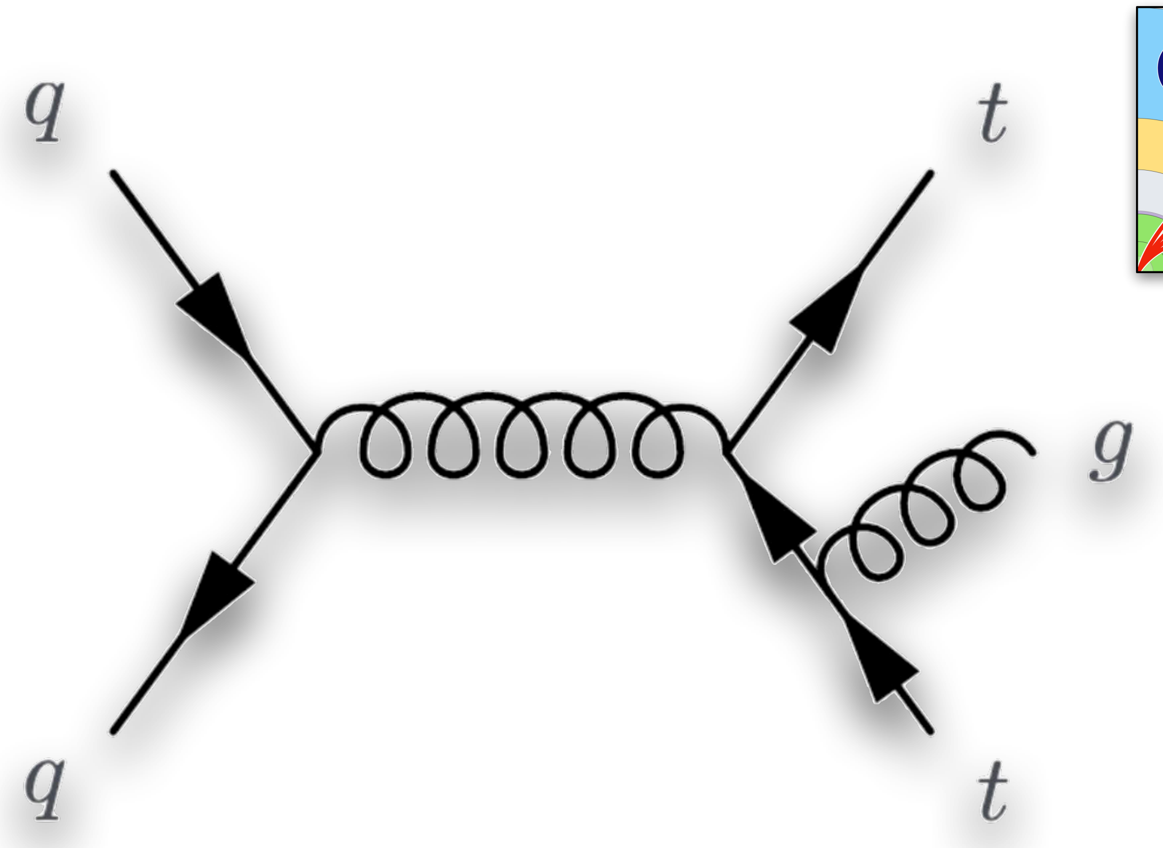


Modelling of the tt production threshold limiting factor for Lagrangian mass extraction

Are we doomed to pay this price?

$$\rho = \frac{2m_0}{m_{\mathrm{t\bar{t}+jet}}}$$

$$m_{\mathrm{t}}^{\mathrm{pole}} = 172.93 \pm 1.26\,(\mathrm{fit})\,^{+0.51}_{-0.43}\,(\mathrm{scale})\,\mathrm{GeV}$$

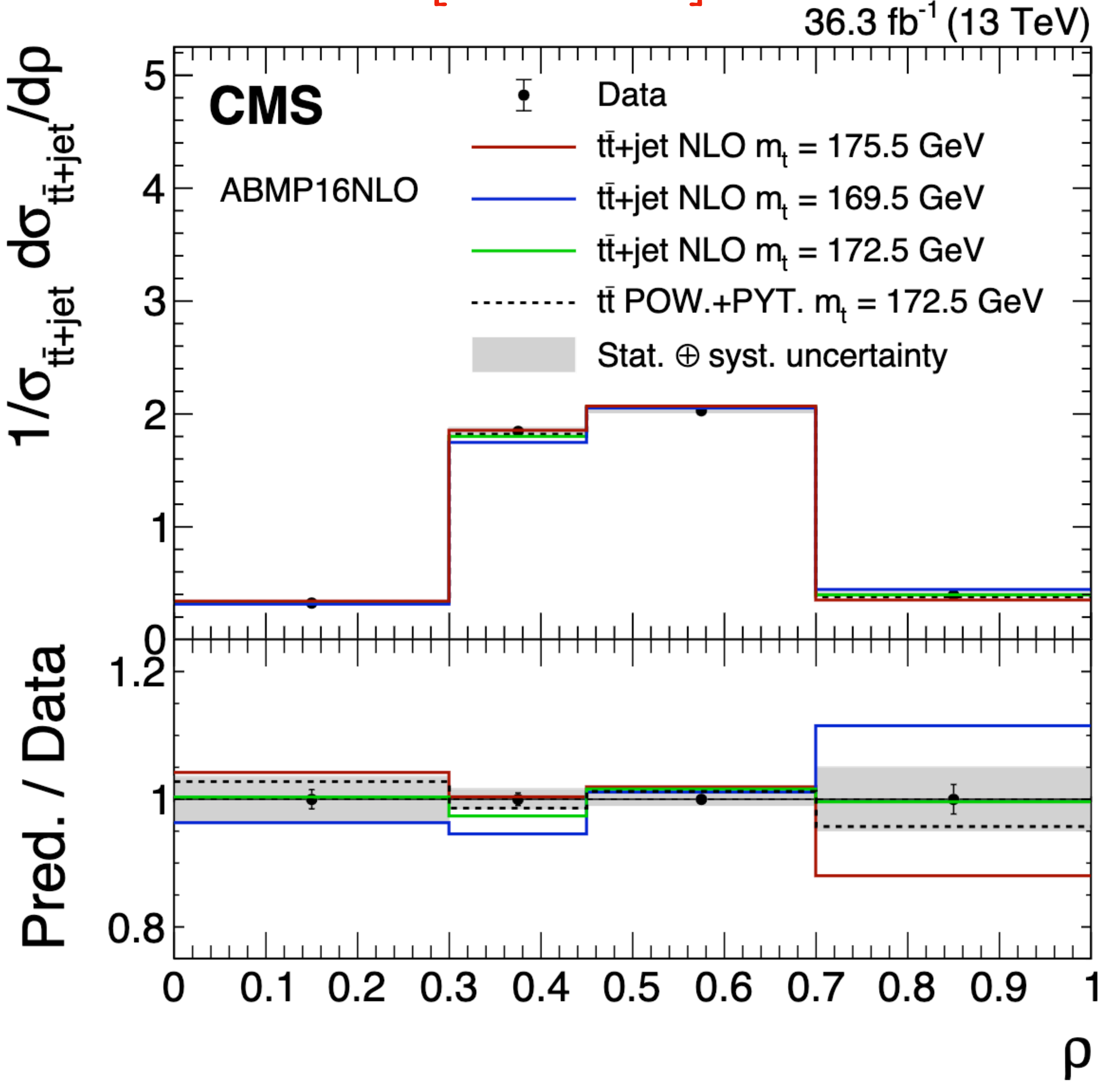


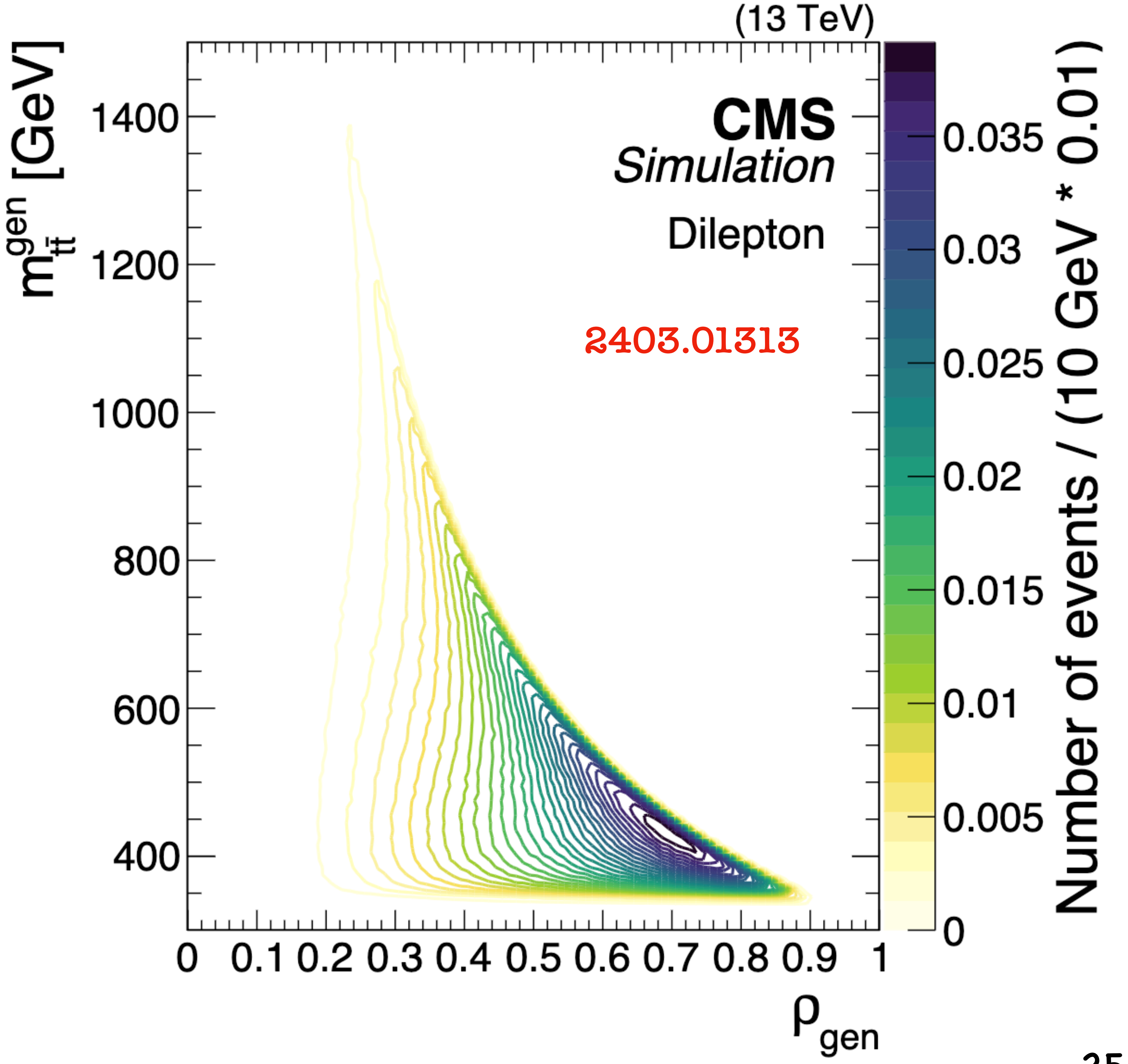
Invariant mass of **tt+jet** system significantly less sensitive to modelling of production threshold

Theoretical uncertainties under better control

Precision can be significantly improved with additional data

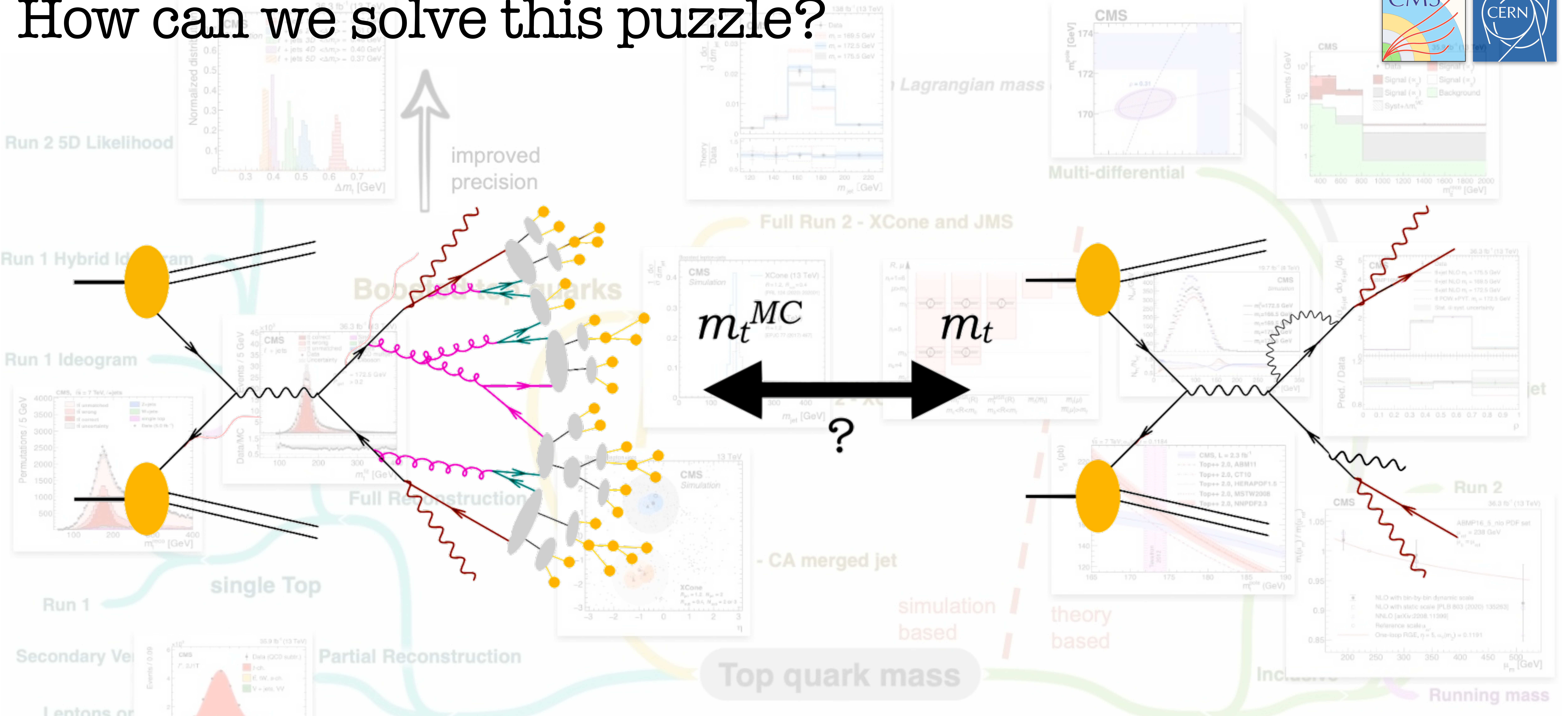
JHEP [2207.02270]





2403.01313

How can we solve this puzzle?



m_t^{MC} ⟷ m_t

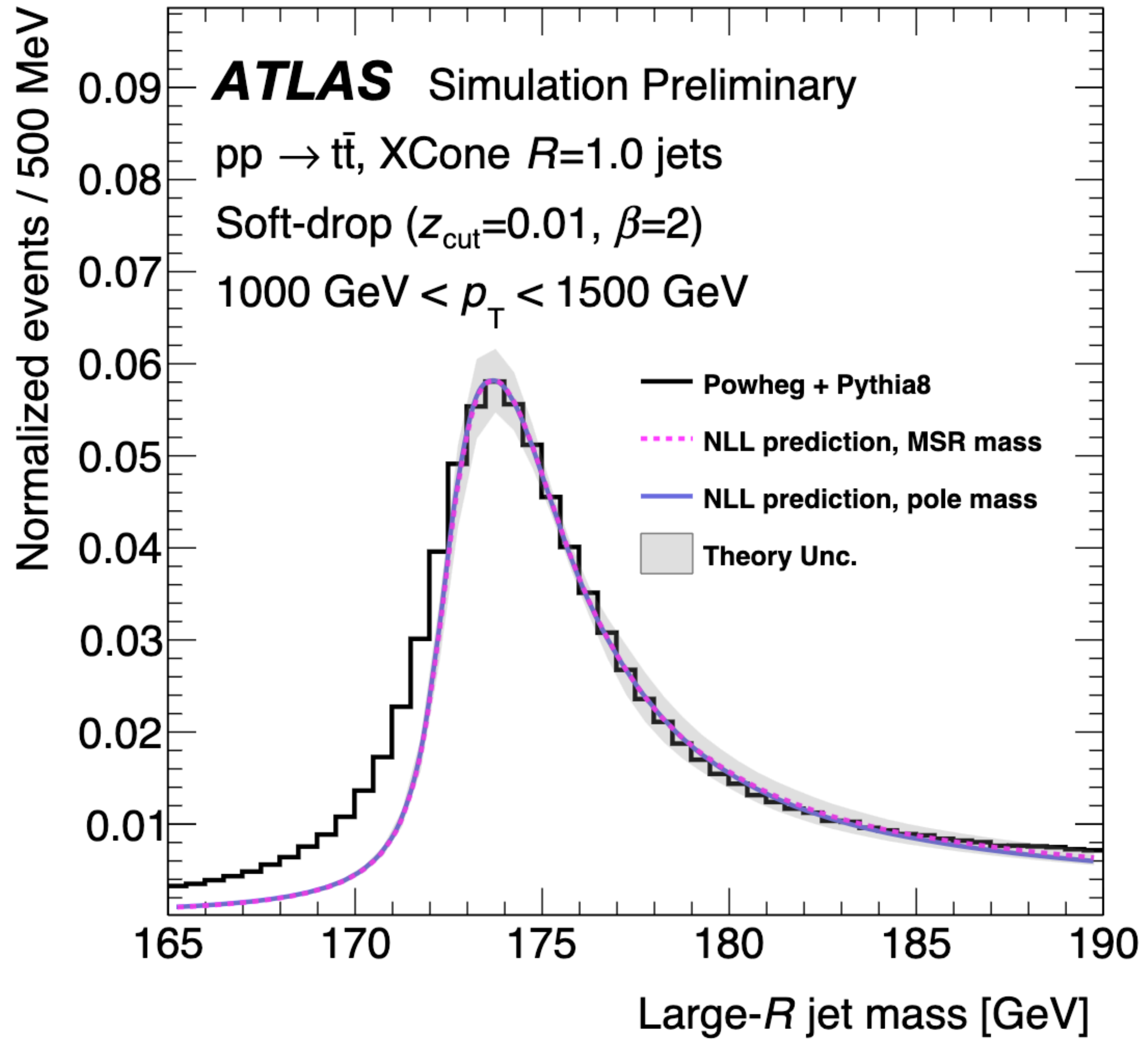
?

This is where boosted measurements come in…

Why are boosted measurements special?

Large-R jet mass from boosted top decays can be compared to both MC simulation and standalone theory prediction in a given renormalisation scheme

$$m_t^{\text{MC}} - m_t^{\text{MSR}}(R = 1\ \text{GeV}) = 80^{+350}_{-410}\ \text{MeV}$$



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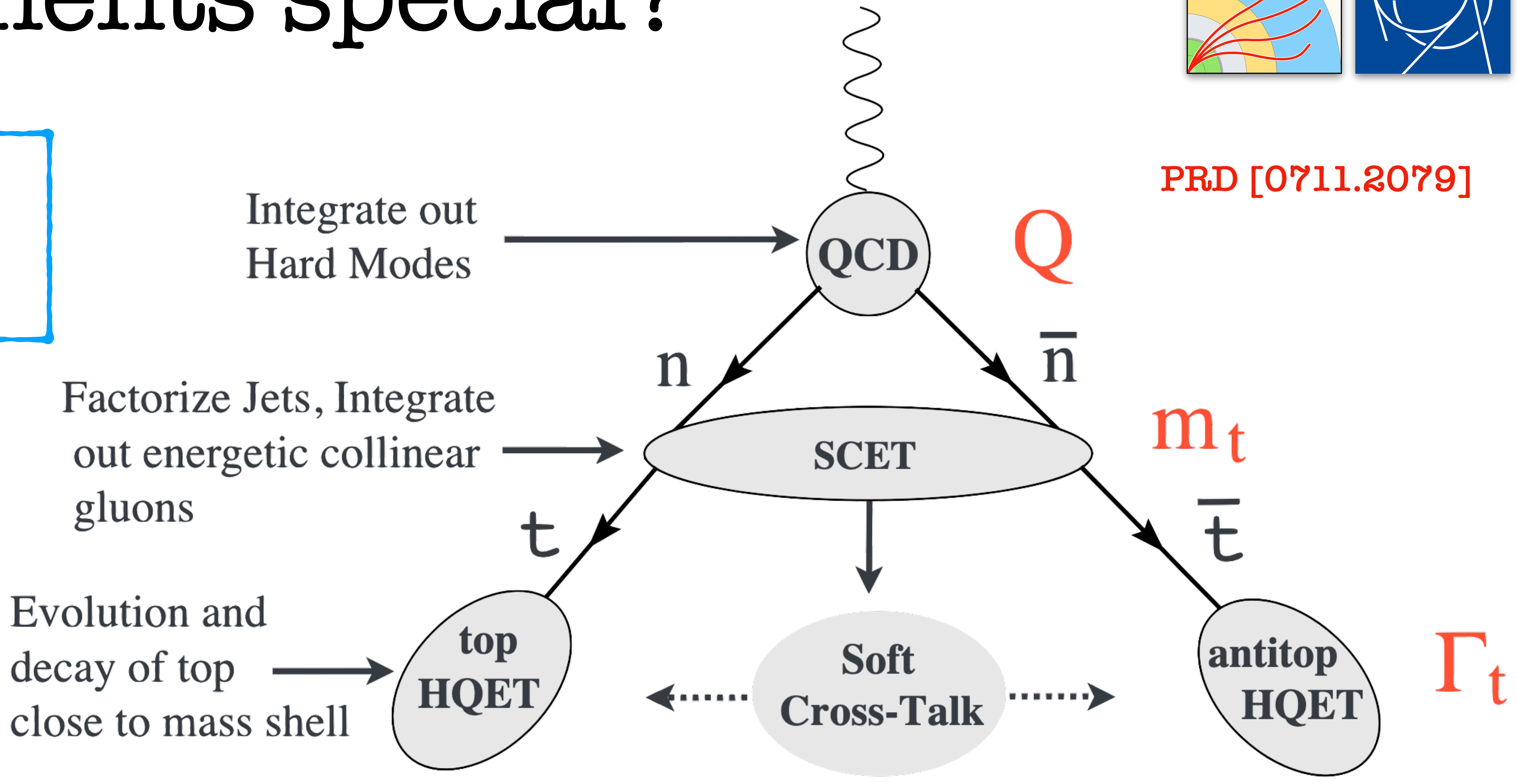
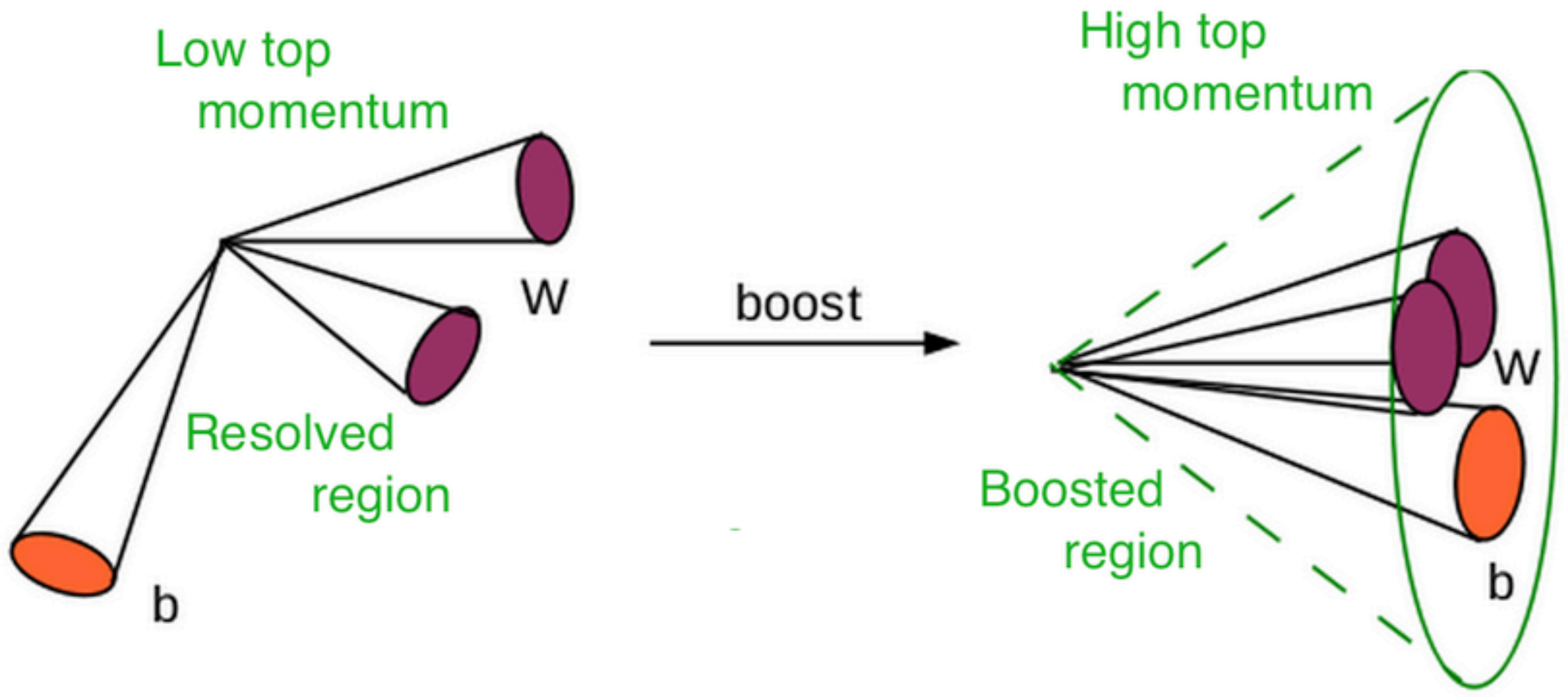
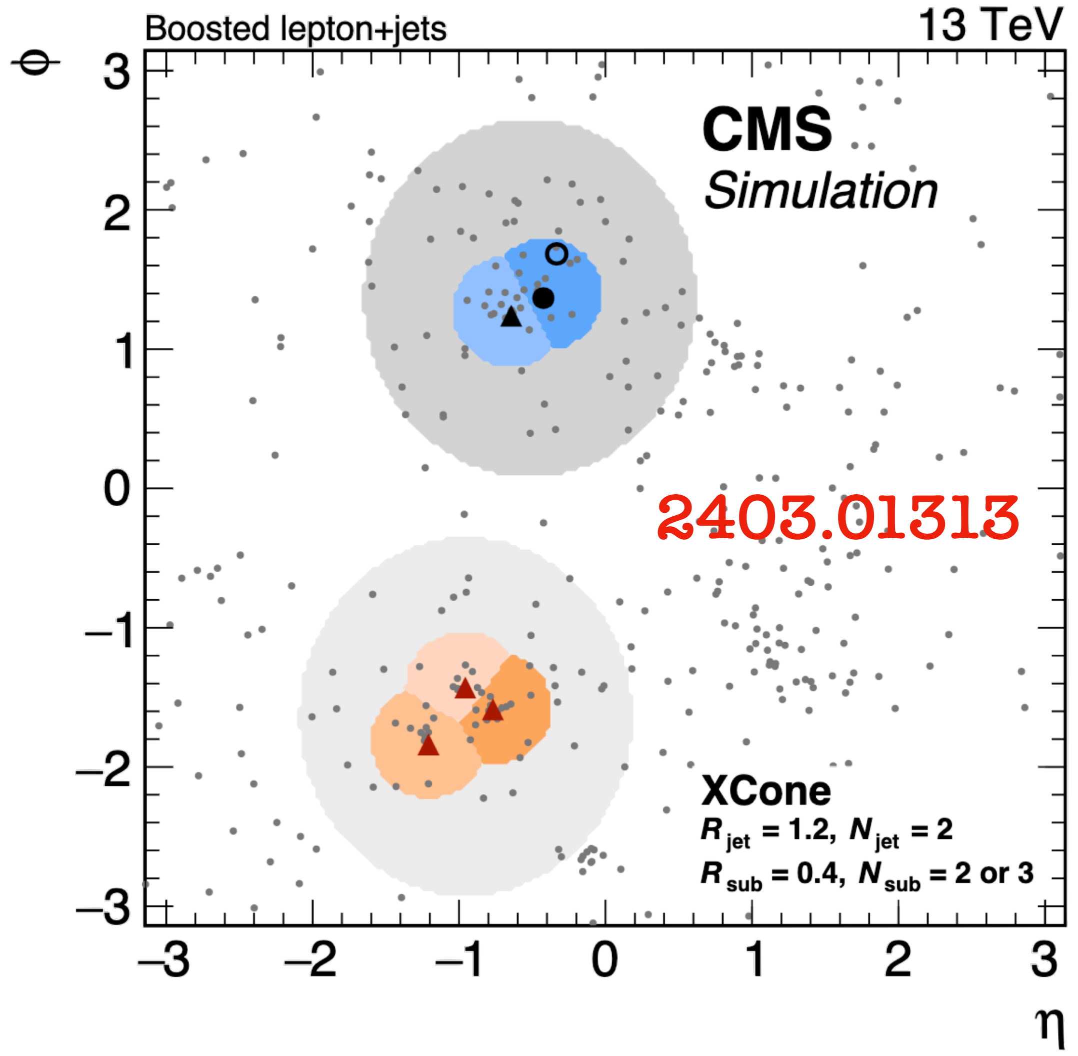


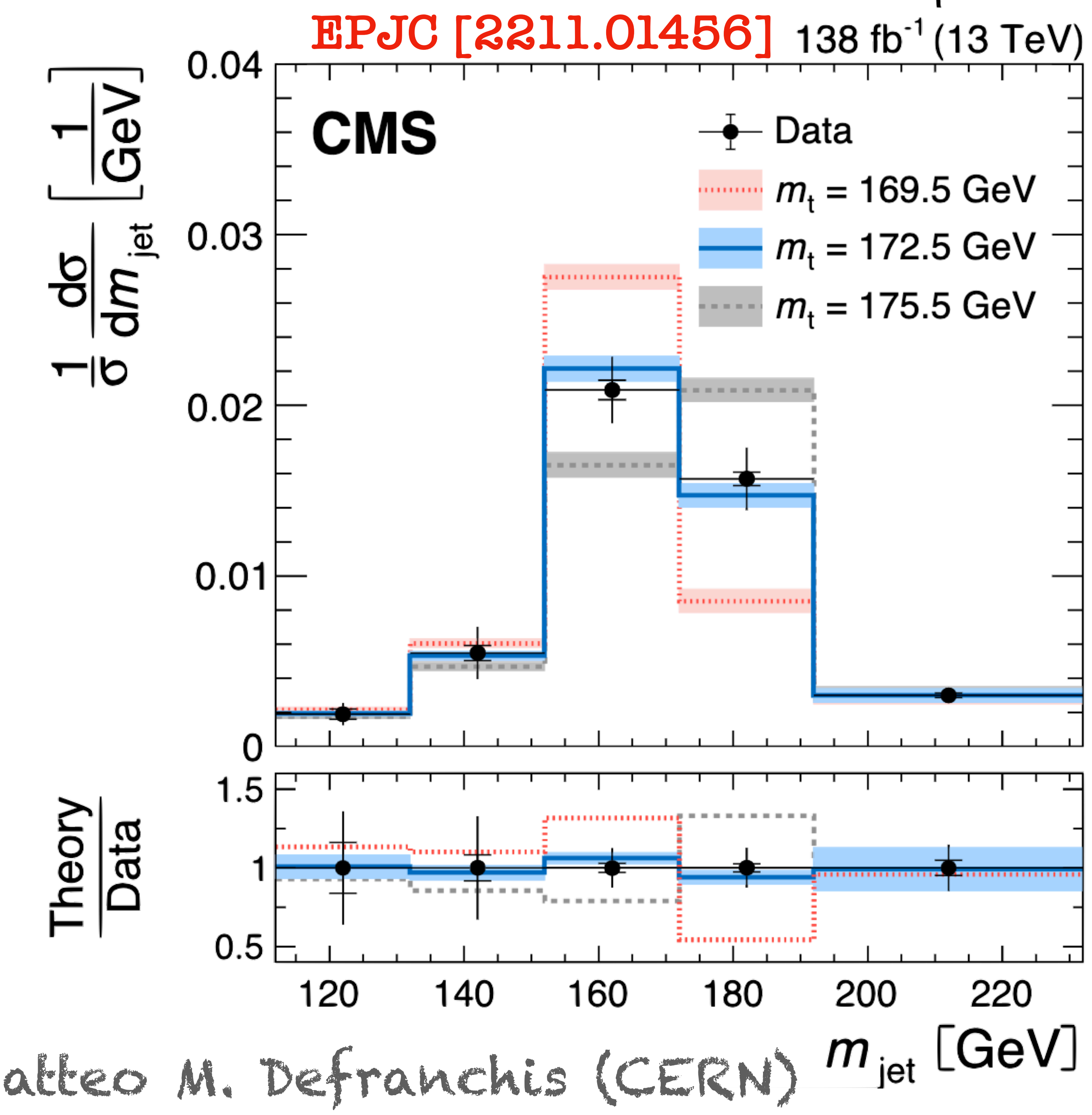
PRD [0711.2079]

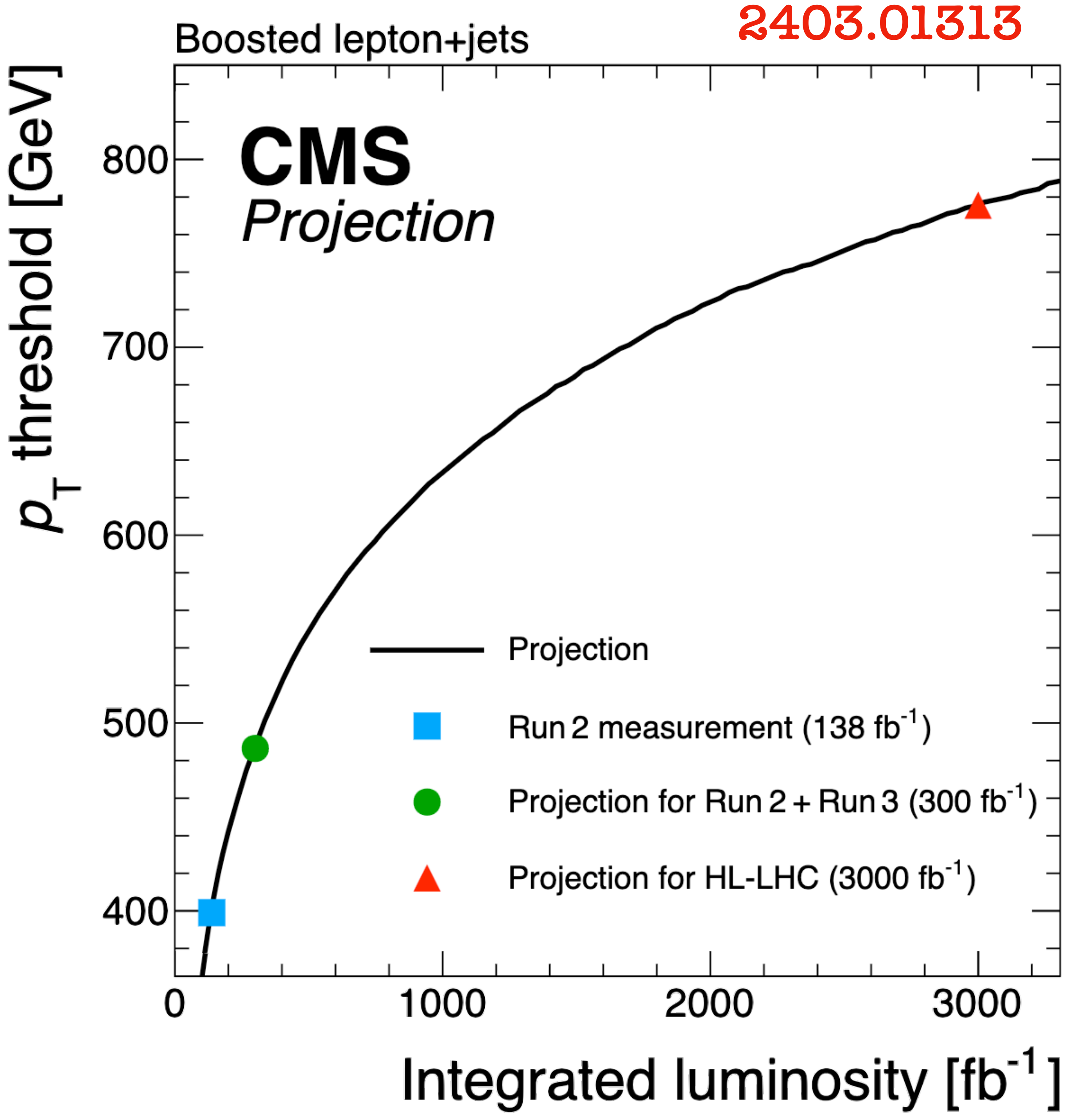
Measurements in the boosted regime can be used to relate m_t^{MC} to well-defined Lagrangian mass definition

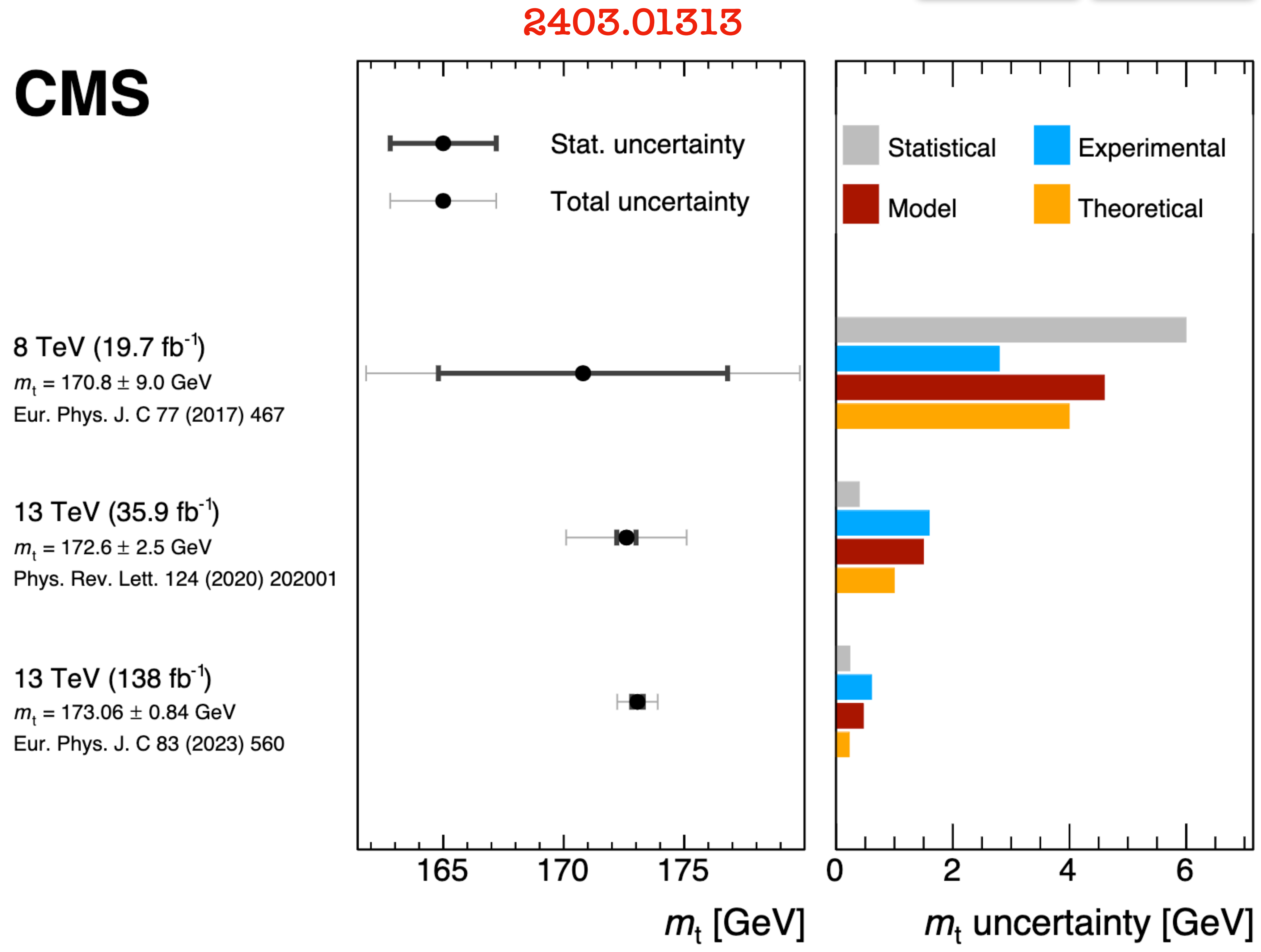


The present and future of boosted measurements









- **Improvement in precision of x10** compared to first 8 TeV result
- Will significantly benefit from **HL-LHC** dataset
- Bold (personal) prediction: the future of m_t at the LHC is boosted!

Top mass beyond the (HL-) LHC

m_t can be measured via **e^+e^- scan** e.g. at FCC-ee

- Extracted from the peak position corresponding to the **spin-1 toponium state** (J/Ψ-like)
- Requires mass definition that is not sensitive to large correction to Coulomb potential -> potential subtracted (PS) mass

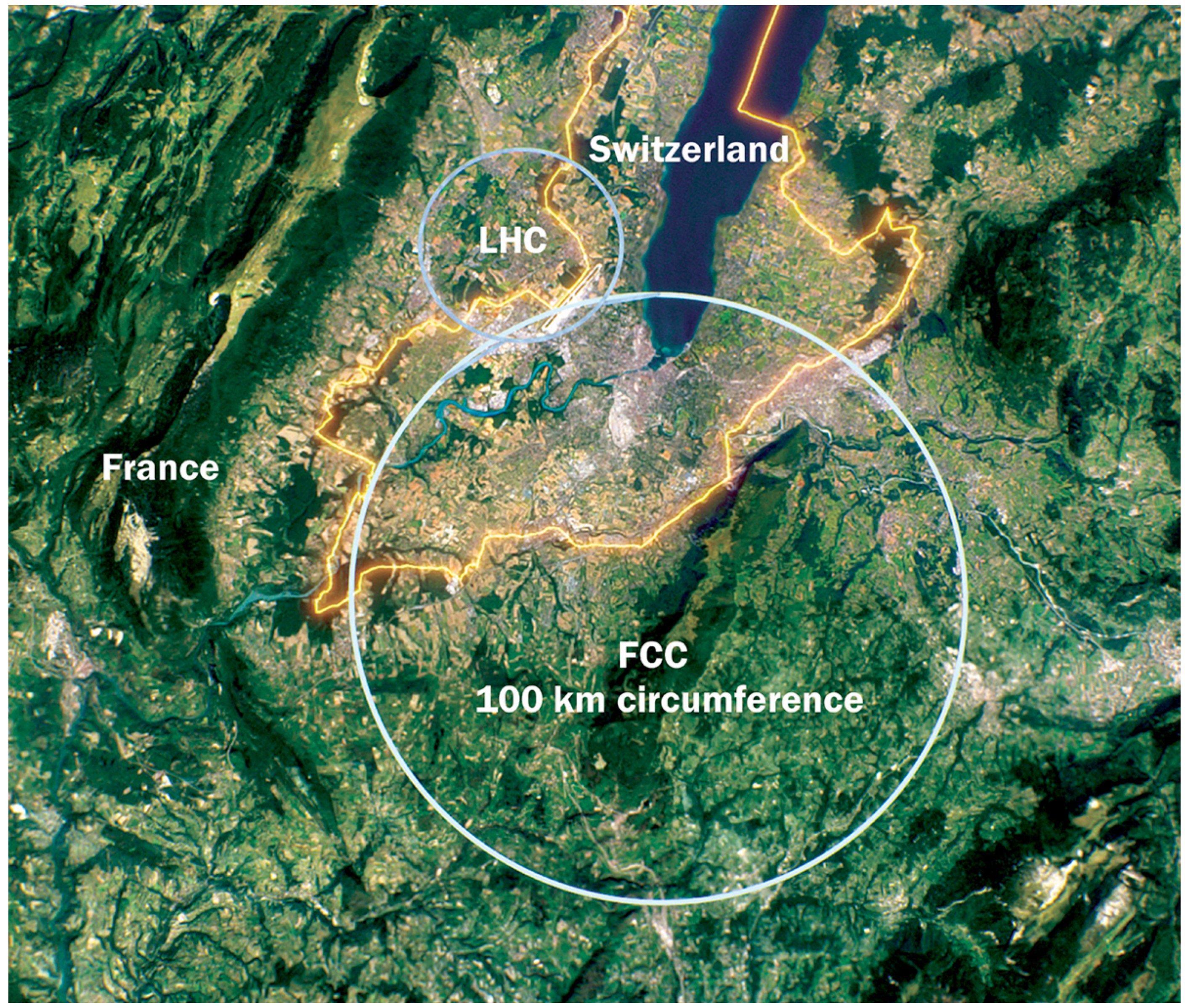
PLB [9804241]

$$m_{t,\mathrm{PS}}(\mu_f) = m_t - \delta m_t(\mu_f)$$

2203.06520

At FCC-ee mt could be measured at the level of 50 MeV or better

Estimates currently limited by theoretical uncertainties



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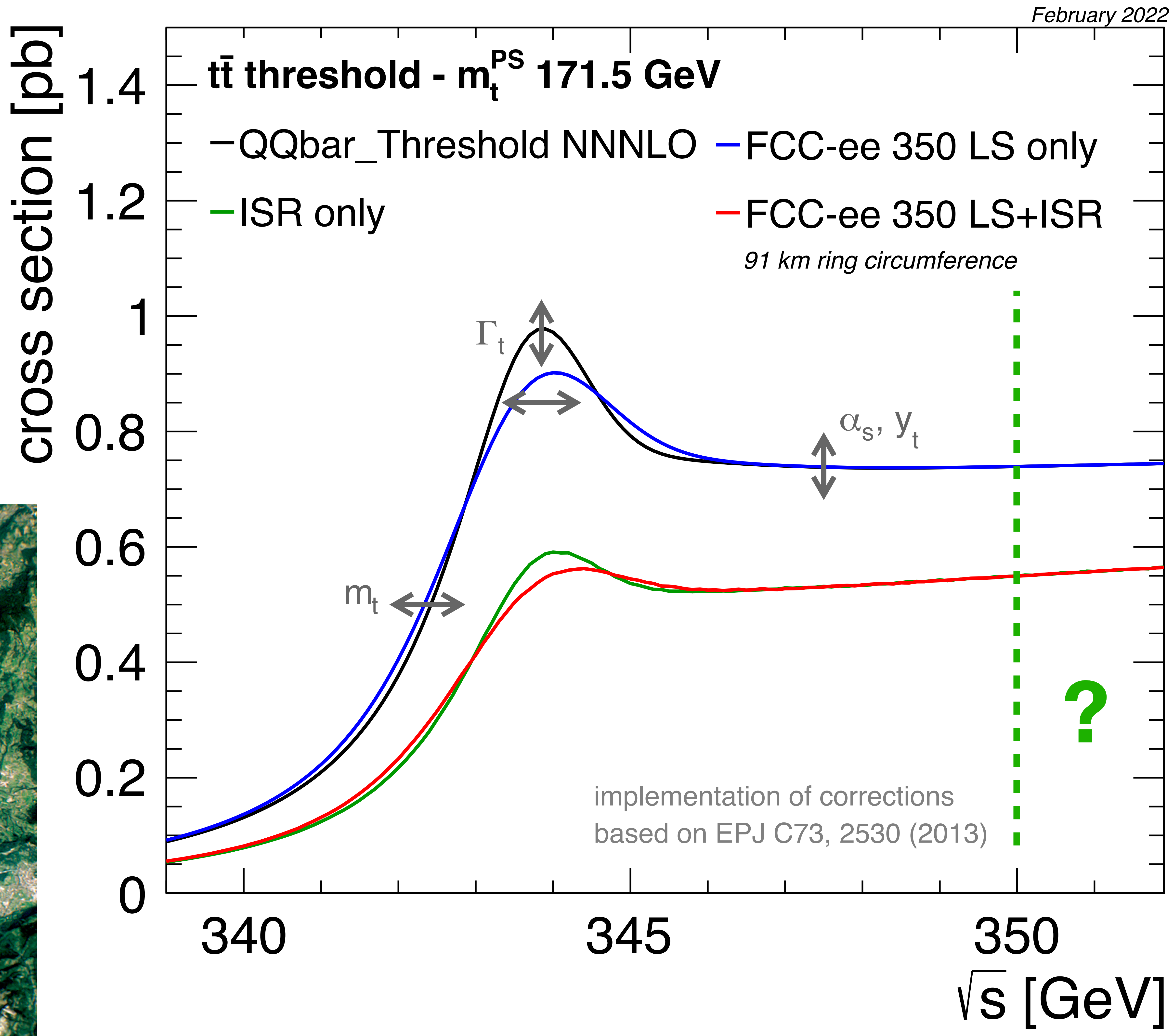


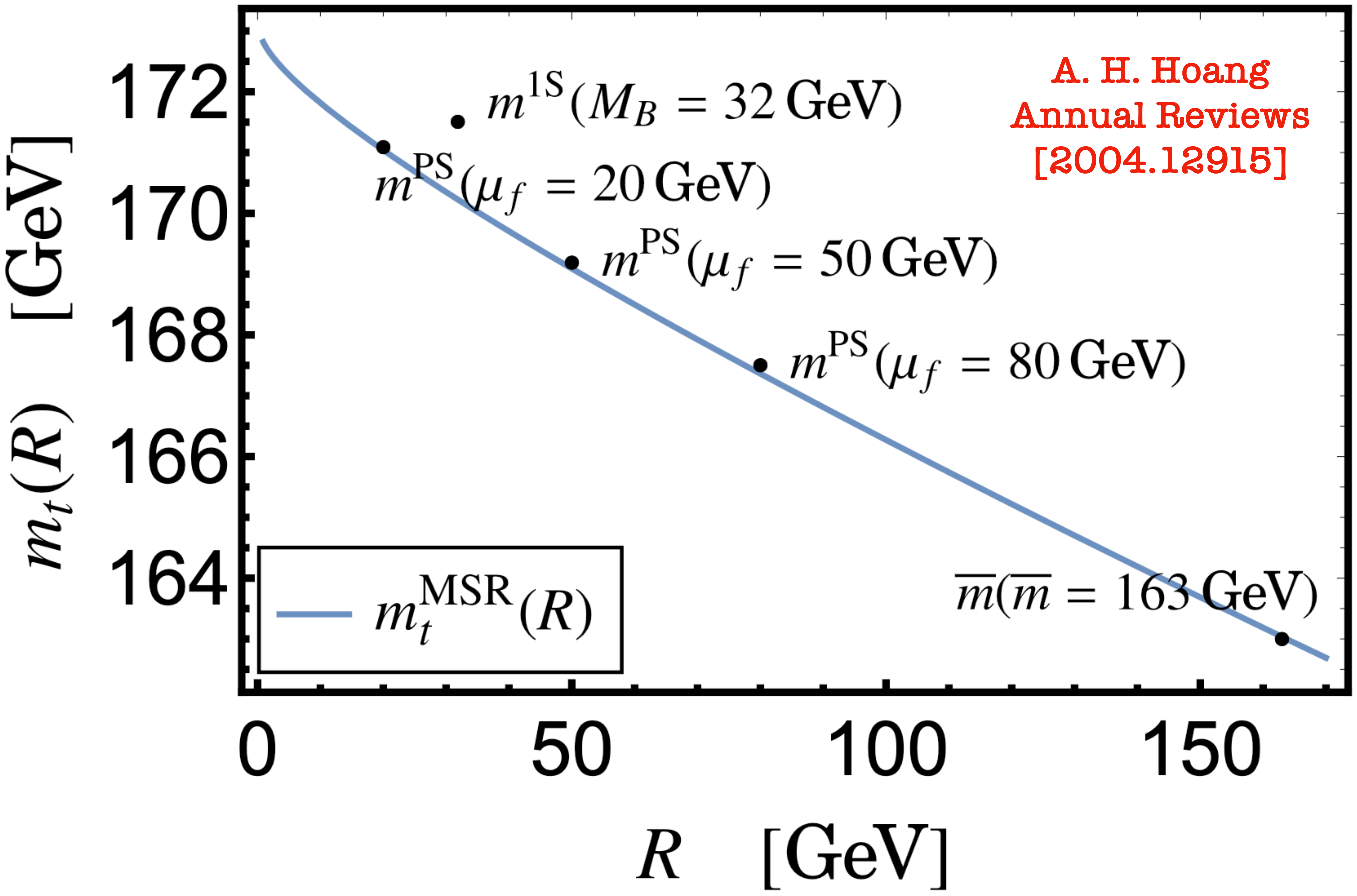
The grand plan for ultimate precision

The MSR mass can be used as a tool to provide a **unified picture**

- Smoothly interpolates between different mass definitions (pole, PS, MSbar)
- Can be **naturally related to m_t^{MC}** via boosted measurements, thanks to its intrinsic IR cut-off

Once all the experimental and theoretical ingredients are in place:

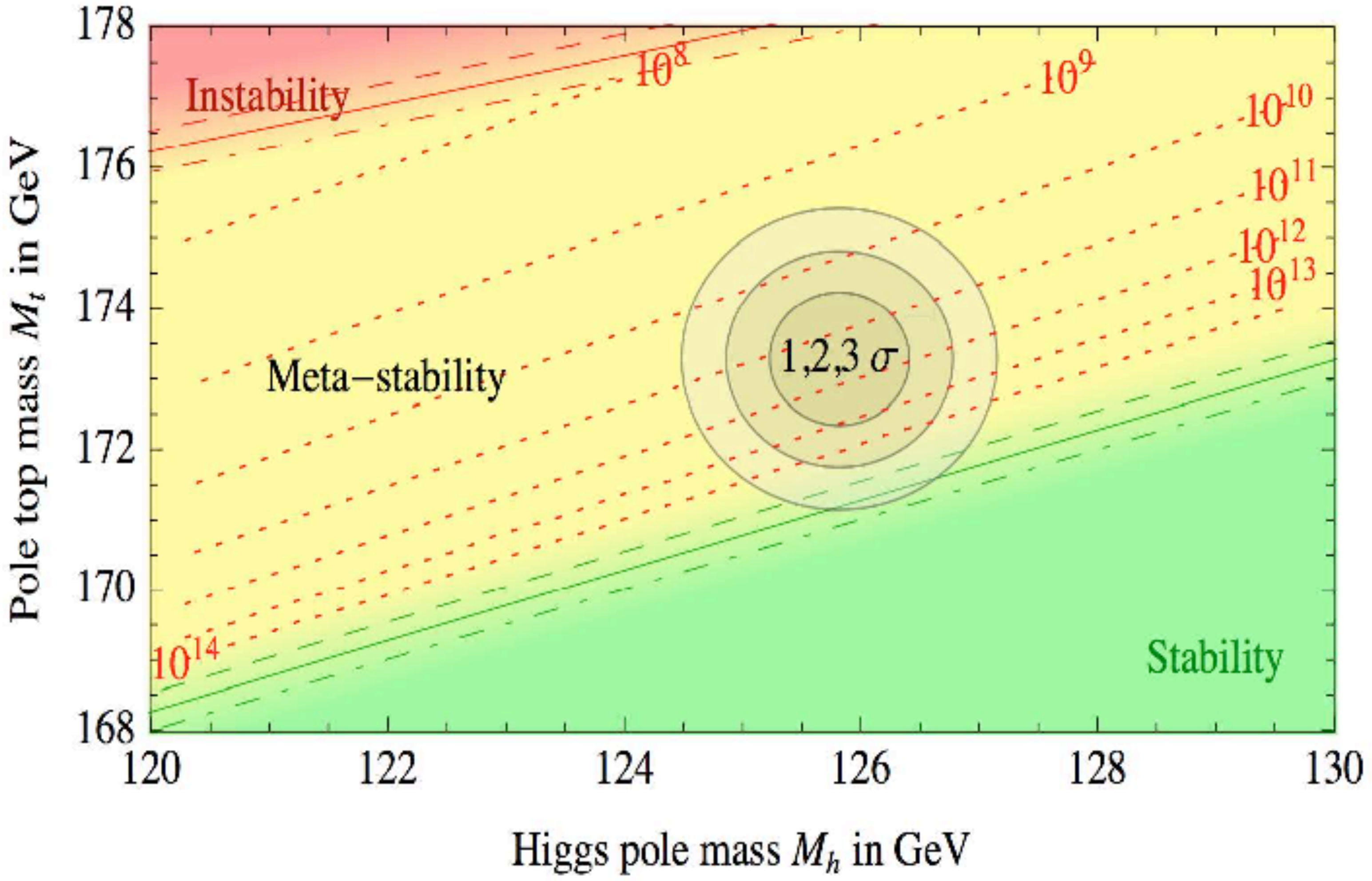
- Can check consistency between results direct and indirect measurements
- Can perform the “final” grand-combination and obtain the “ultimate” result



A. H. Hoang Annual Reviews [2004.12915]

From the mess to the mass

- It is crucial to measure m_t at the highest level of precision in order to test the **consistency** of the SM
- Despite the remarkable precision achieved so far, **fundamental theoretical issues** make the interpretation of the results somewhat unclear
- **Remarkable progress** is being made on the experimental side, which can provide vital input for theoretical developments -> watch out for HL-LHC and (hopefully) a future e^+e^- collider



See you in 2029… if the universe is still there!