

BLACK HOLE SPECTROSCOPY: GR AND BEYOND

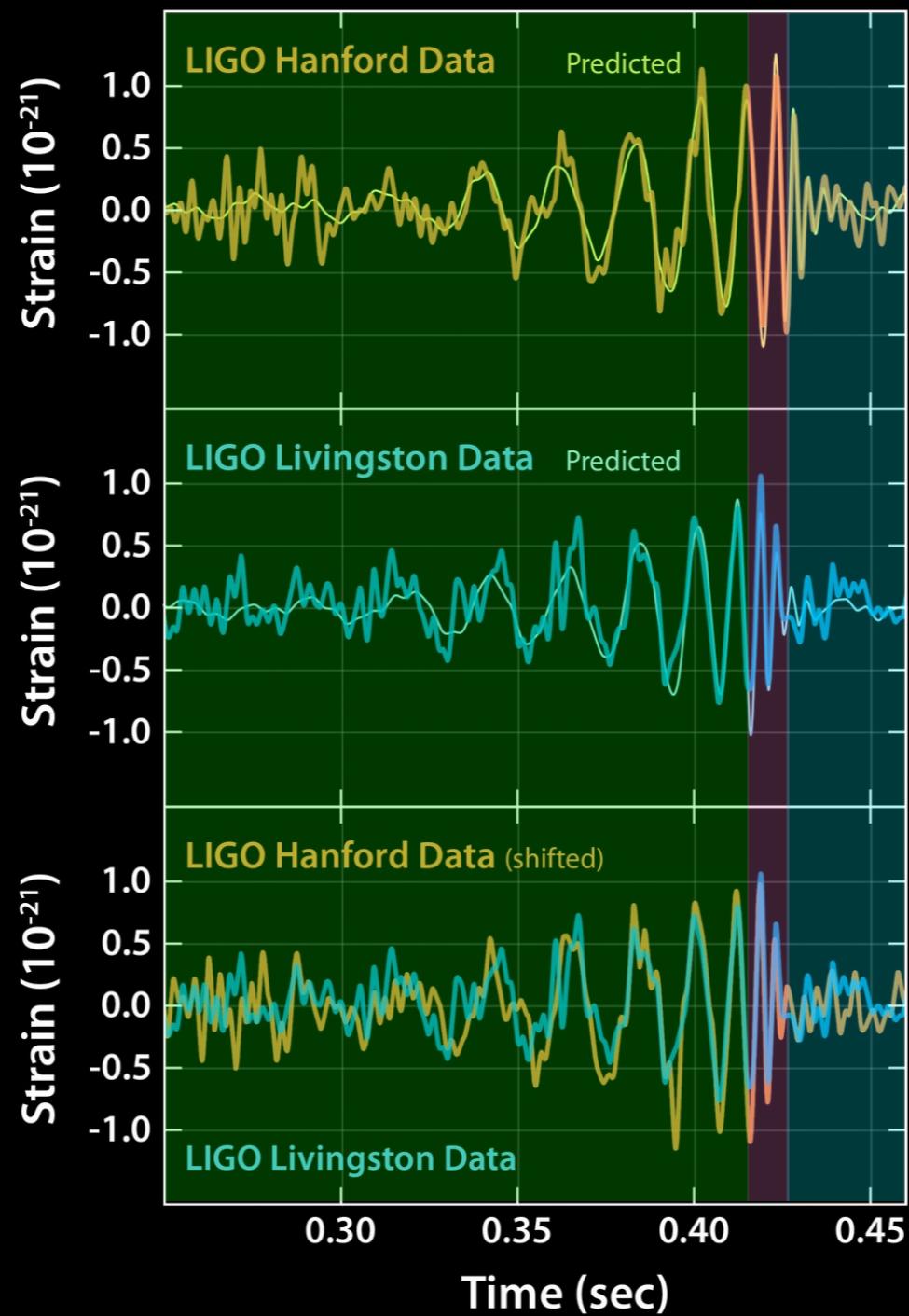
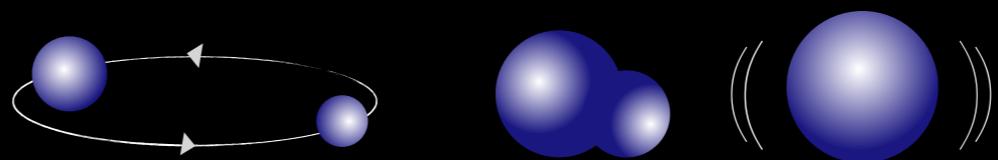
GIADA CANEVA SANTORO

IN COLLABORATION WITH ANIA LIU, NICOLA FRANCHINI, VASCO GENNARI, GREGORIO CARULLO

EPPG SEMINAR



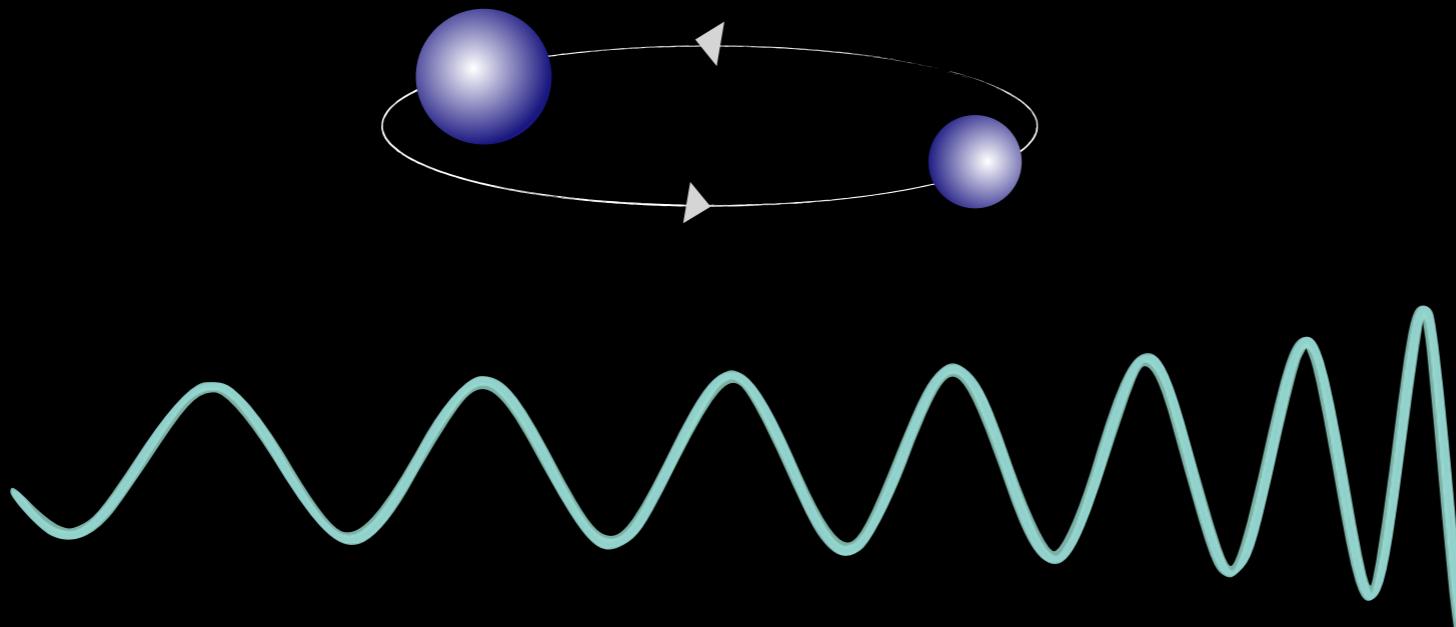
COSMIC DANCE



Inspiral

Bound orbits, shrinking and circularising due
to the emission of GWs;

Post-Newtonian approximation (perturbative
expansion in powers of v/c).



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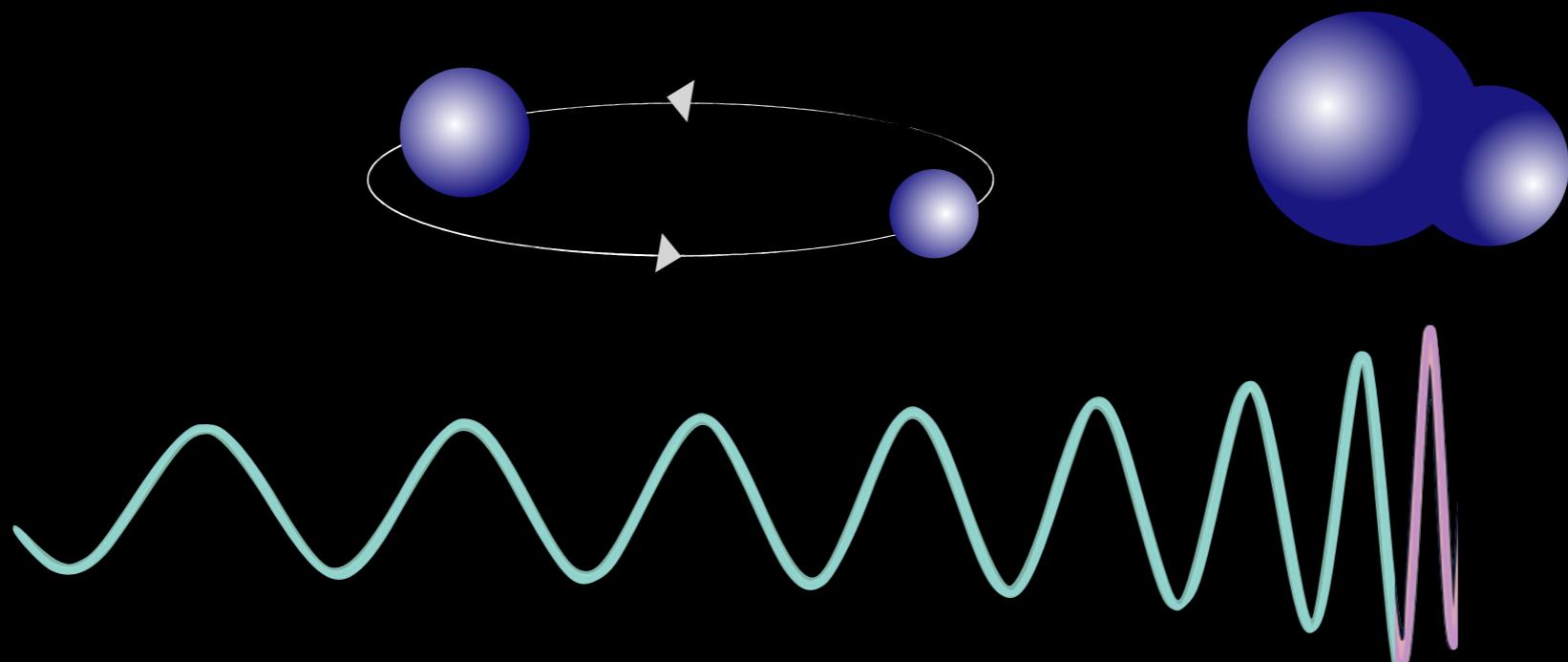
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Merger

Plunge into each other due to energy and orbital angular momentum losses;

Formation of a common horizon, single final BH is formed;

Numerical relativity simulations.



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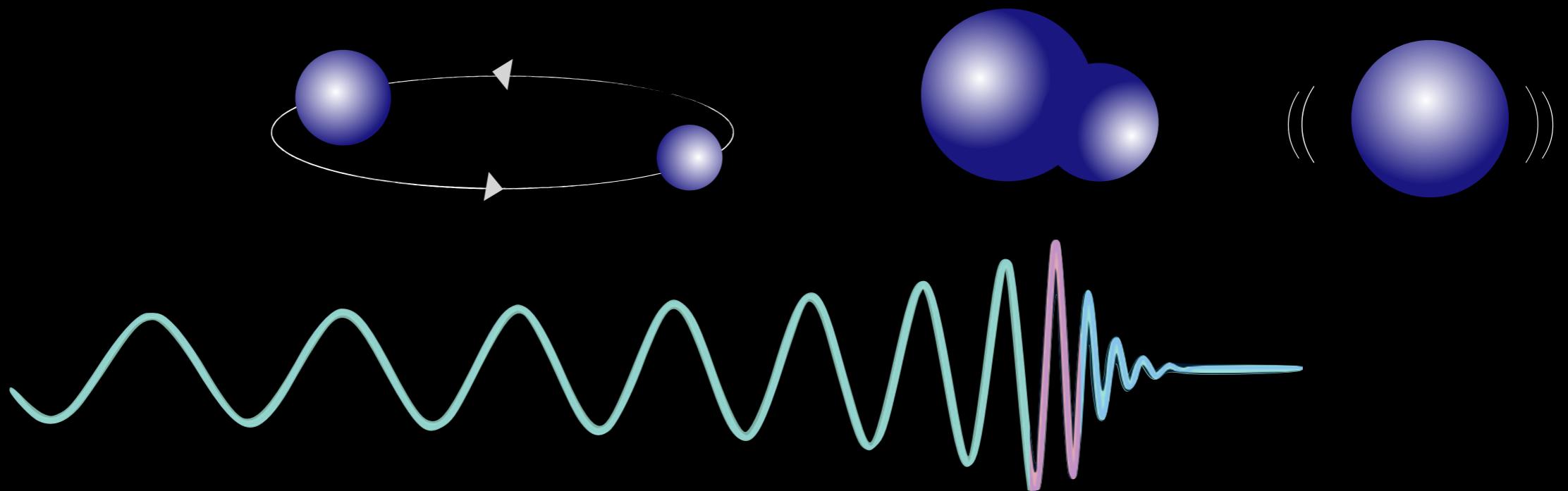
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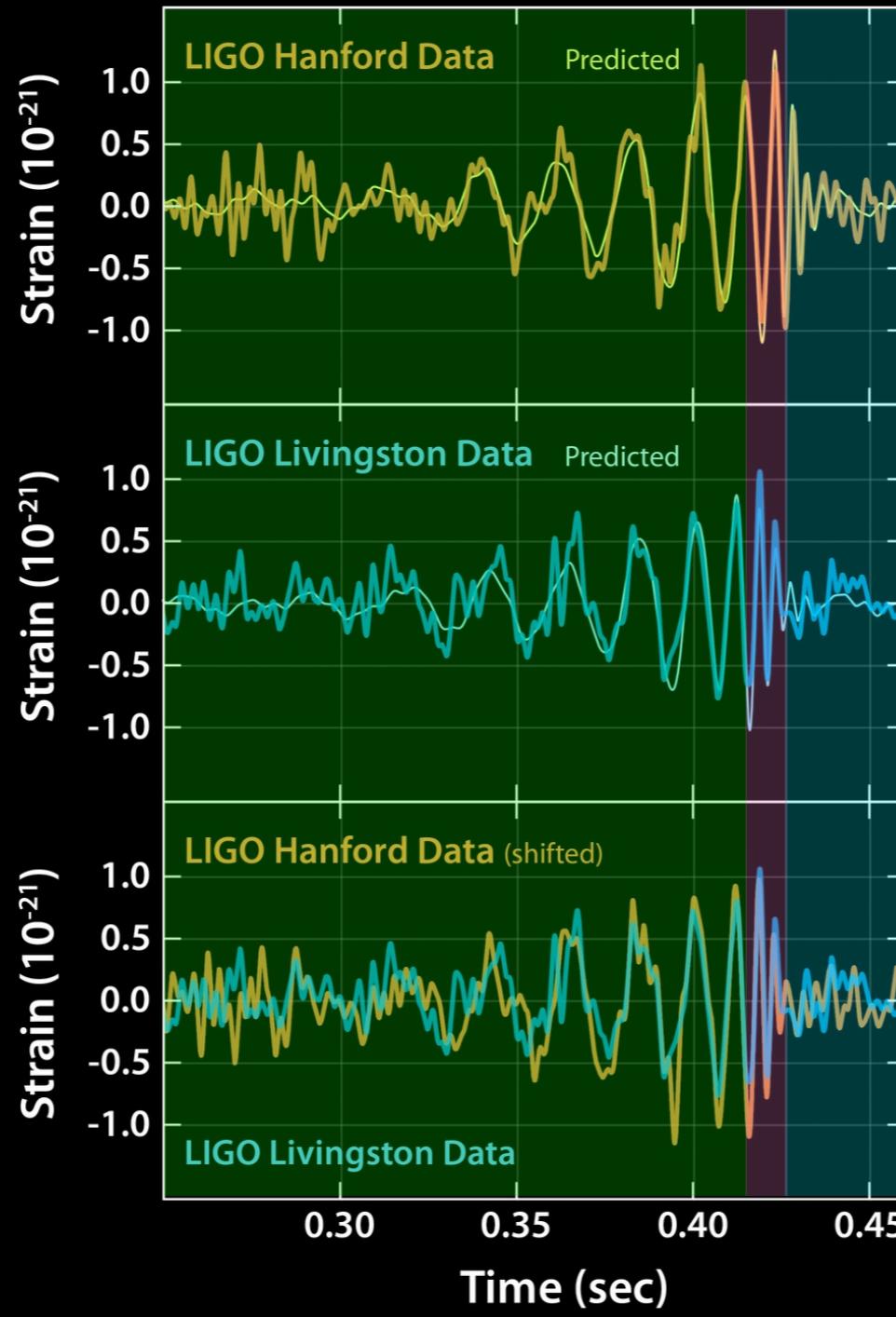
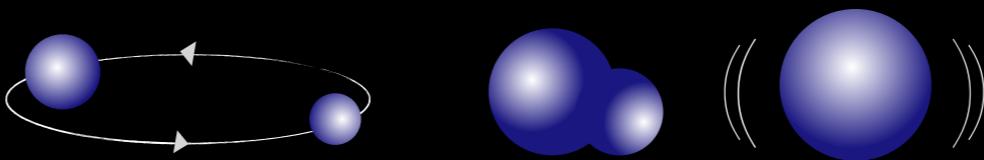


Ringdown

Highly asymmetric BH radiates away GWs to reach stable equilibrium state;

Perturbation theory.

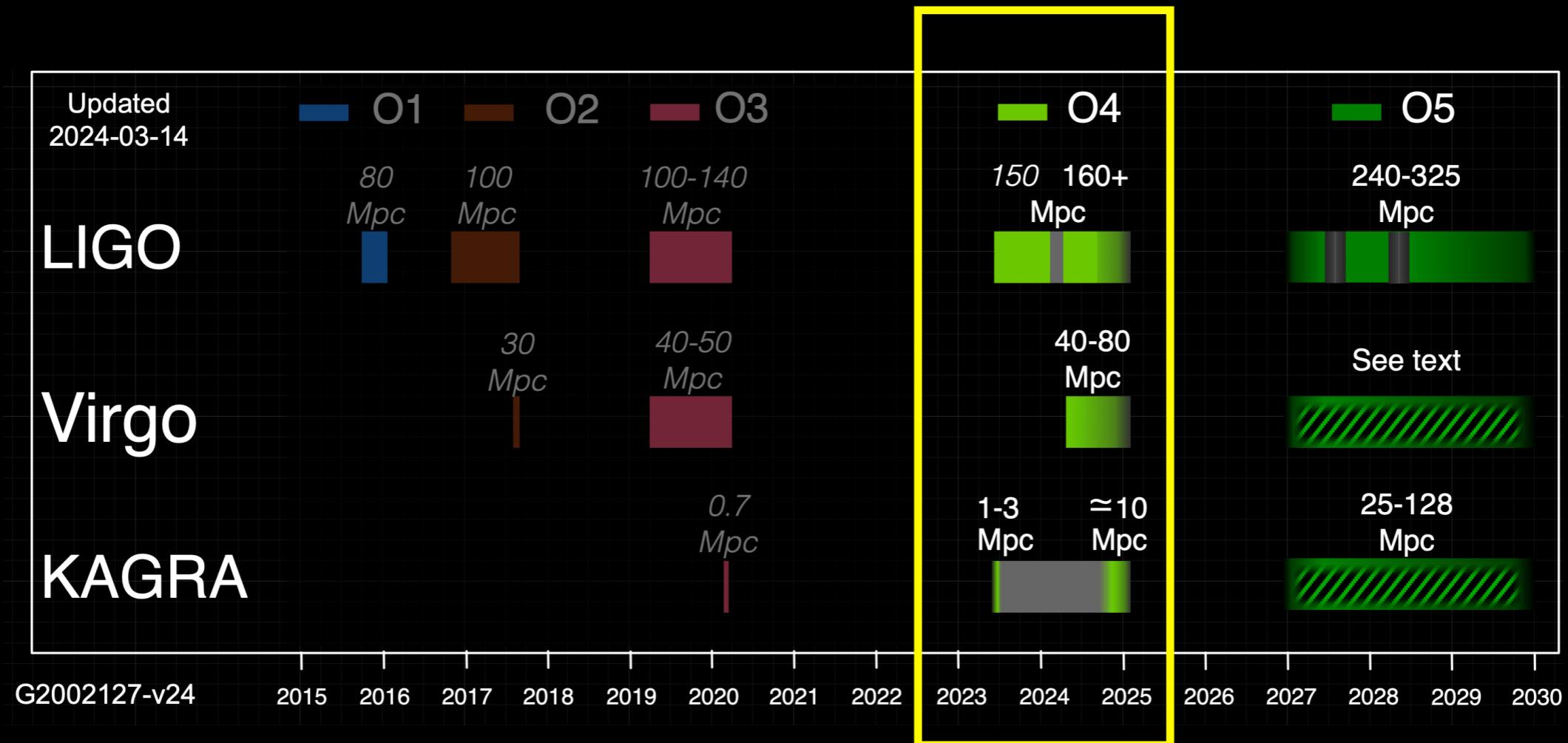
COSMIC DANCE



Courtesy of Cezary Turski

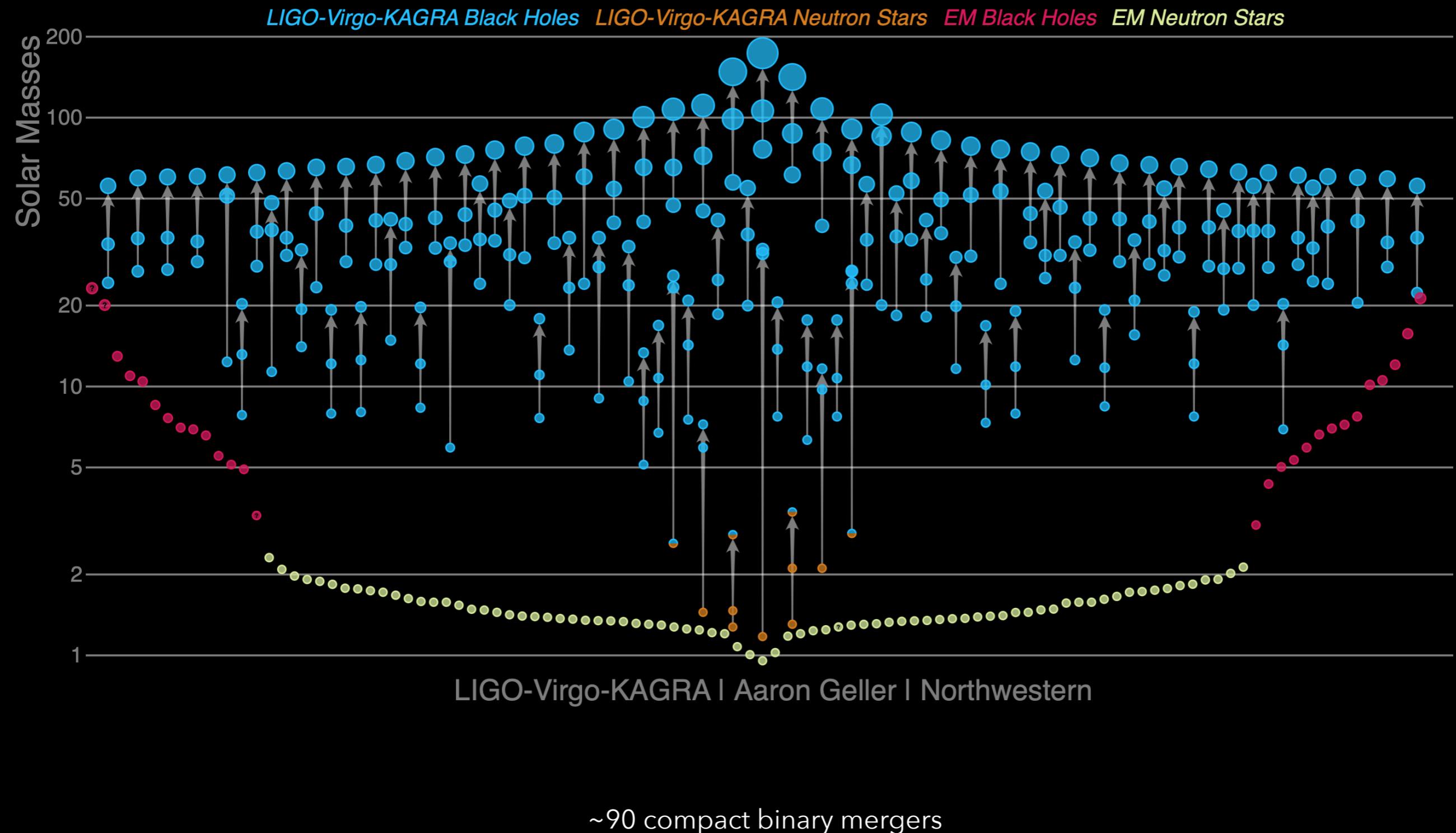
Caltech/MIT/LIGO Lab.

WHAT IS HAPPENING NOW?

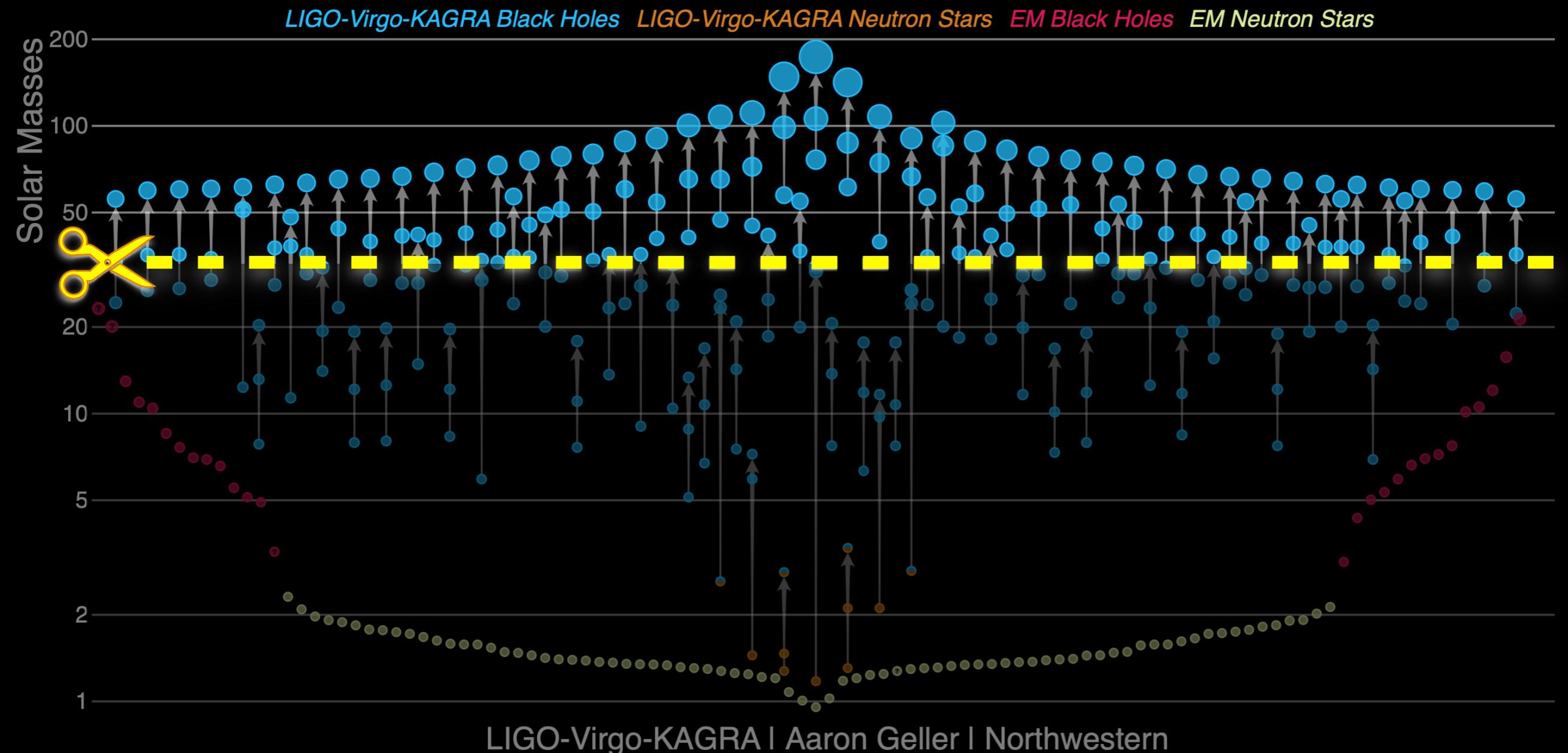


<https://observing.docs.ligo.org/plan/>

STELLAR NECROPOLIS



RINGDOWN

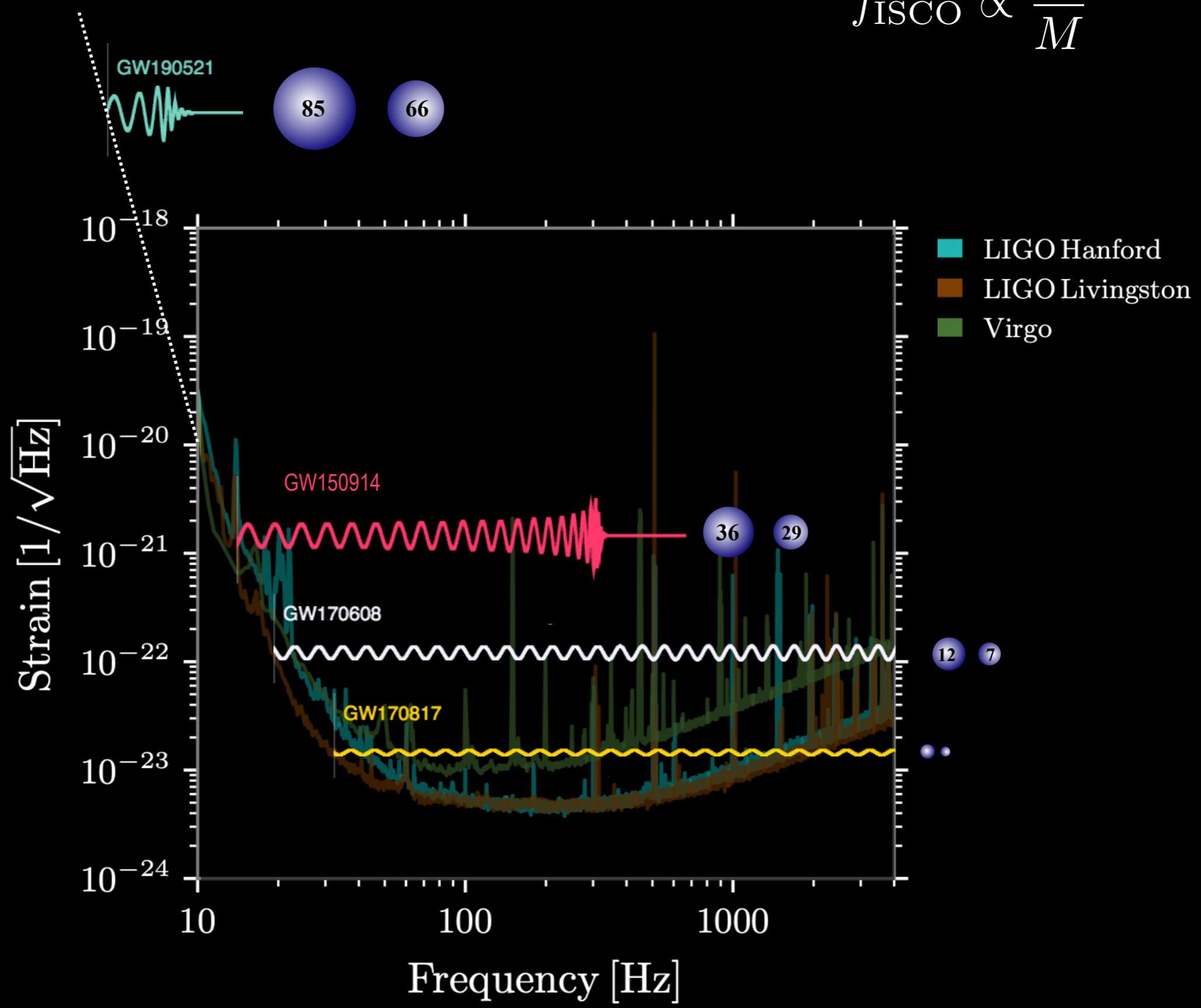


~90 compact binary mergers

~49 events confident enough to test GR

~22 event with confident ringdown signatures

$$f_{\text{ISCO}} \propto \frac{1}{M}$$



WHAT IS RINGDOWN?

Vibrates with many harmonic frequencies.

Vibrations give away energy through...

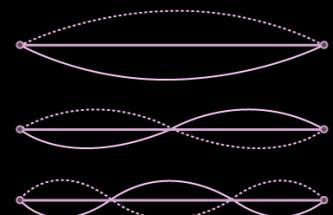
Harmonic oscillations can be described as...

Fundamental mode

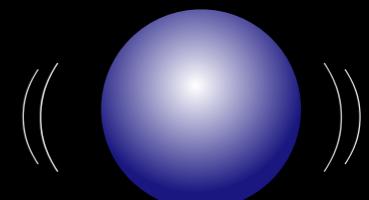
1st overtone

2nd overtone

...



..sound waves.



...gravitational waves.

...standing sine waves.



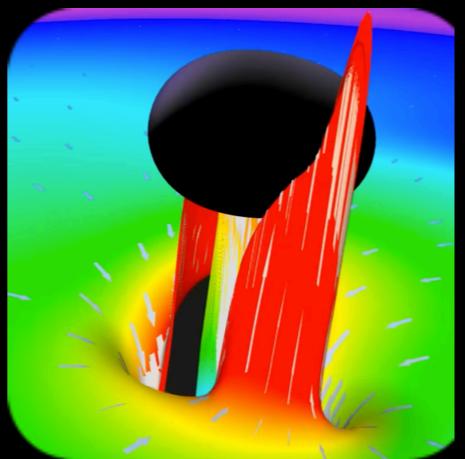
... QuasiNormal Modes
(QNMs)



DYNAMICAL VS PERTURBATIVE

In a realistic scenario, right after merger:

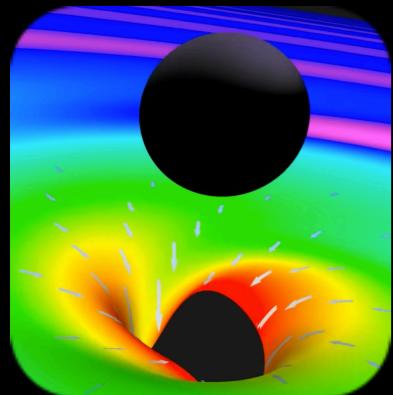
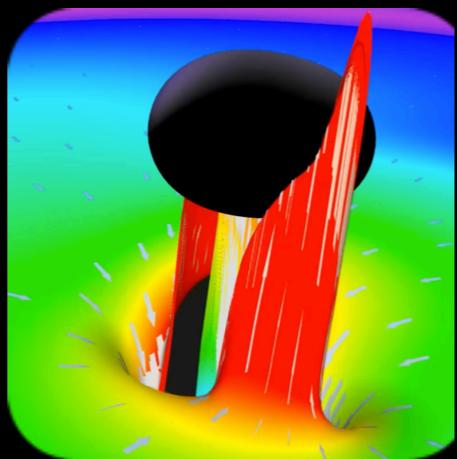
Highly non-linear regime.



DYNAMICAL VS PERTURBATIVE

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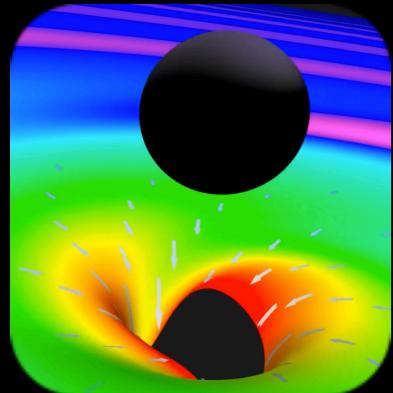
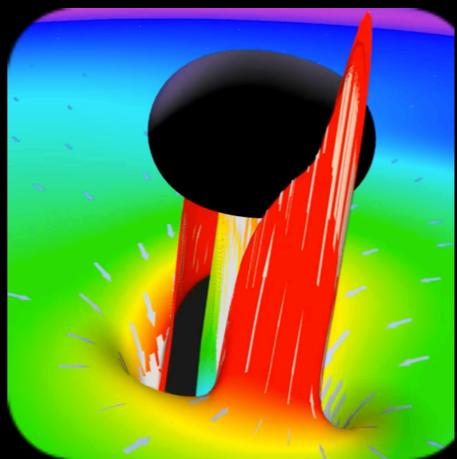


Dynamical horizon forming and...

DYNAMICAL VS PERTURBATIVE

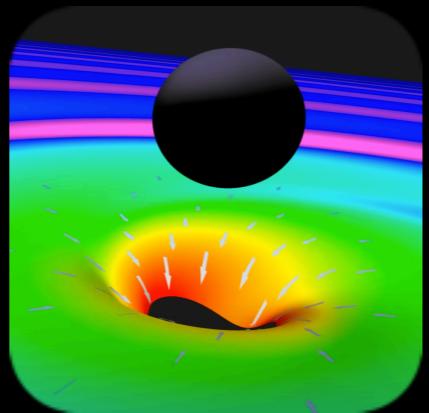
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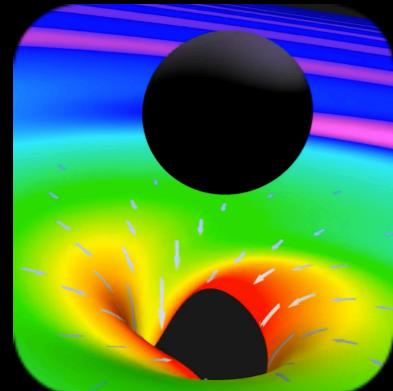
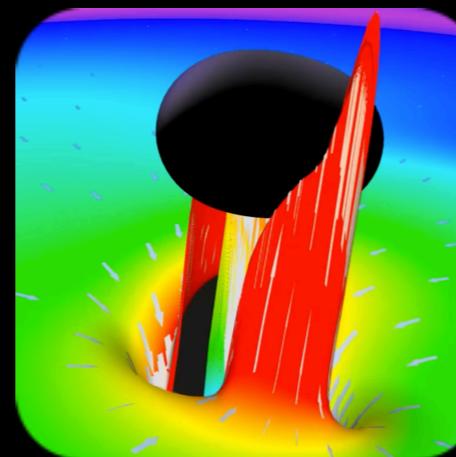
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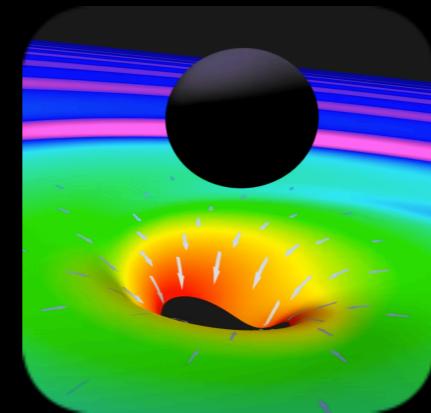
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In perturbation theory:

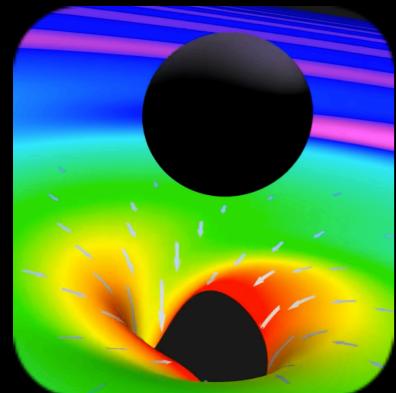
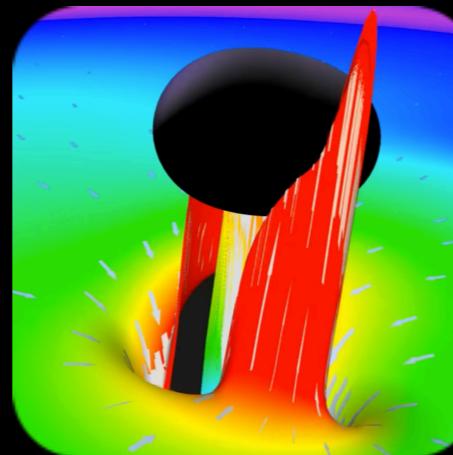
1. Linearize Einstein field equation on a BH background metric:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

DYNAMICAL VS PERTURBATIVE

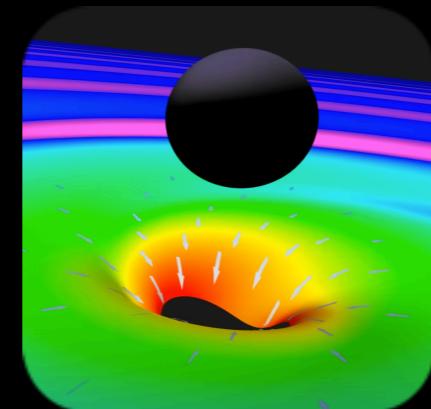
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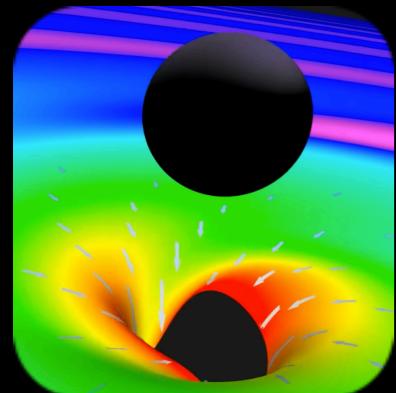
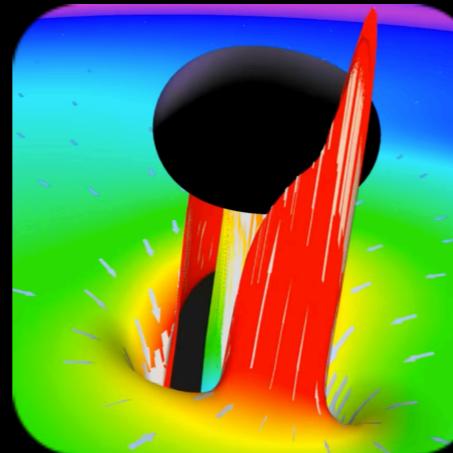
2. Wave-like equation with an effective potential:

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - V(r_*) \right) \Psi = S$$

DYNAMICAL VS PERTURBATIVE

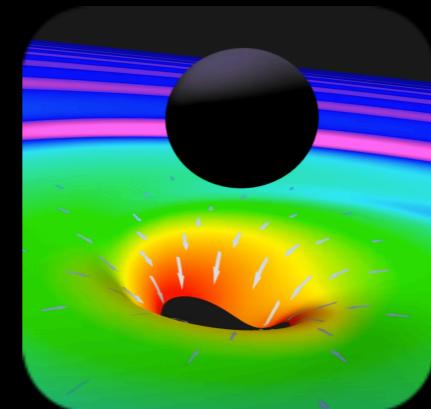
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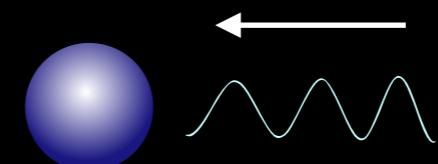
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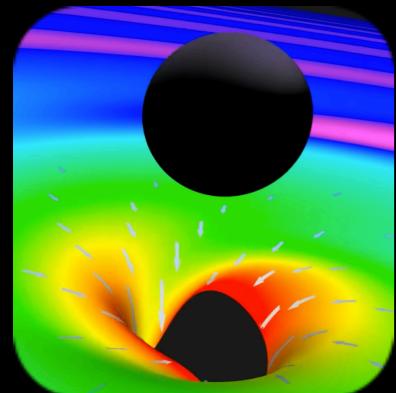
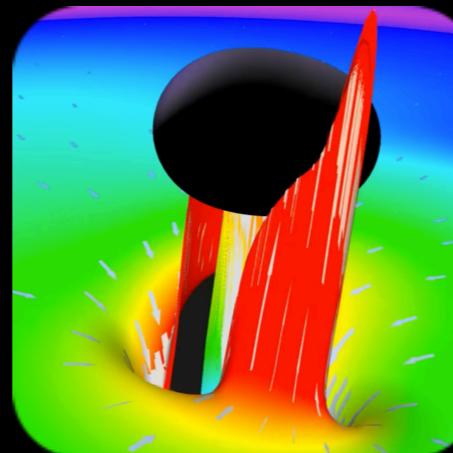
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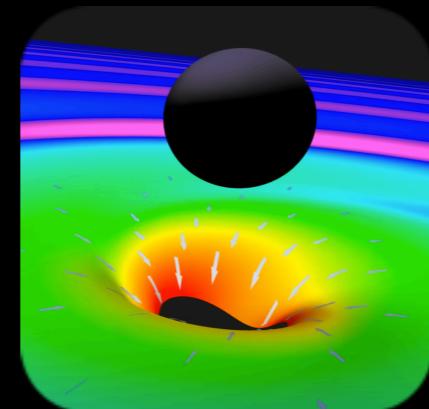
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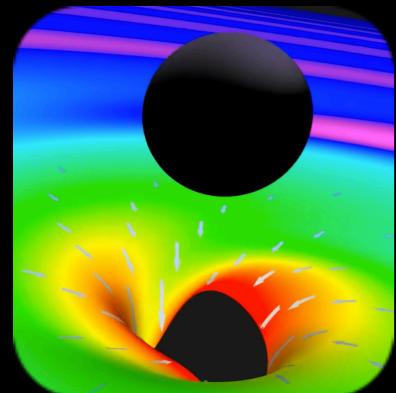
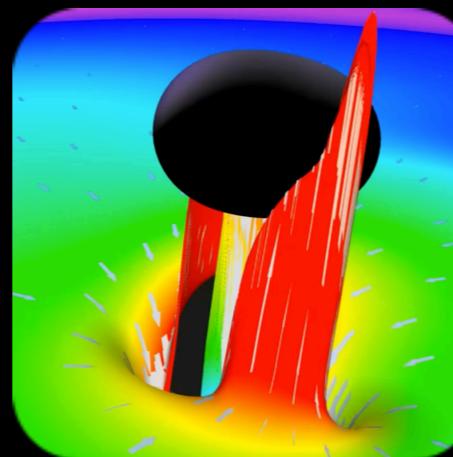
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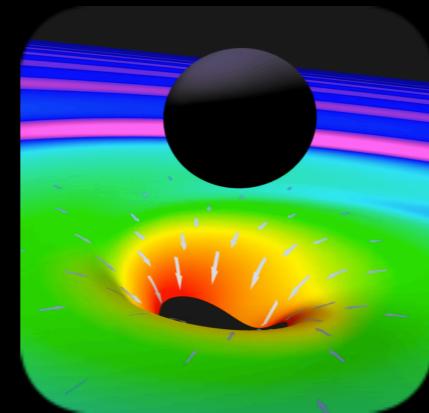
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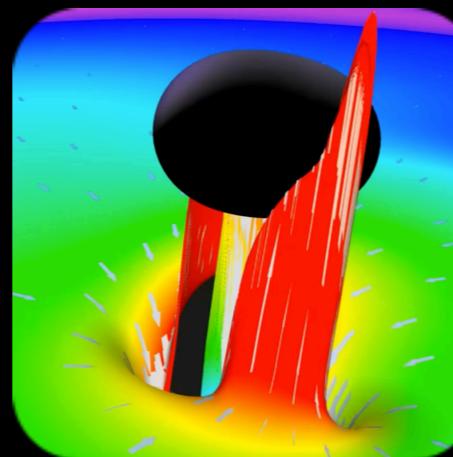


$$h_+ - i h_\times = - \frac{M}{r} \sum_{lmn} \mathcal{A}_{lmn} S(\theta, \phi) e^{i\tilde{\omega}_{lmn} t}$$

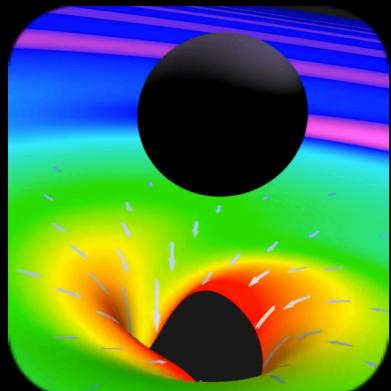
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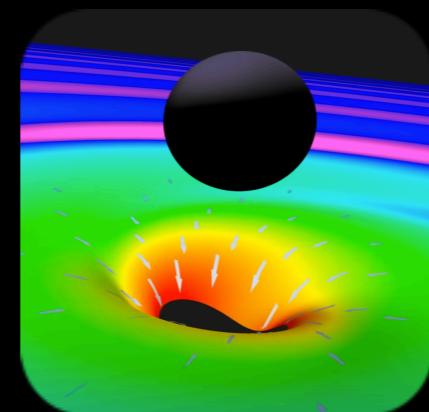
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$$\tilde{\omega}_{lmn} = \omega_{lmn} + \frac{i}{\tau_{lmn}}$$

BH vibrates

dissipative system

WHERE IS THE NON-LINEAR DESCRIPTION?

WHY ARE BHs SO COOL?

1. "Black holes have no hair" - J. Wheeler

In standard Schwarzschild coordinates:

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

fully characterized by a single parameter, the total gravitational mass M , can be generalised to include electric charge and rotation.

WHY IS RINGDOWN SO COOL?

2. The QNMs spectrum only depends on the mass and angular momentum of the Kerr BH:

$$\omega_{lmn}(M_f, a_f), \tau_{lmn}(M_f, a_f)$$

This allow to test the Kerr nature of a BH:

- measure the dominant mode $\omega_{220}(M_f, a_f), \tau_{220}(M_f, a_f)$;
- Infer the M_f and a_f ;
- Measure the $\omega_{330}(M_f, a_f)$ and check it is consistent with the one predicted by perturbation theory.

Black hole spectroscopy: observational study of the QNMs spectrum.

BLACK HOLE SPECTROSCOPY

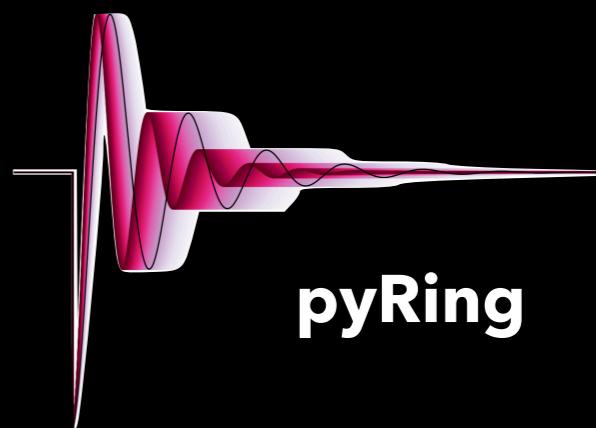
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In reality...

- Analyse only post-merger signal;
- Time domain analysis avoids contamination of pre-merger frequencies when cutting the signal;
- Different models used to estimate remnant parameters and perform model selection.

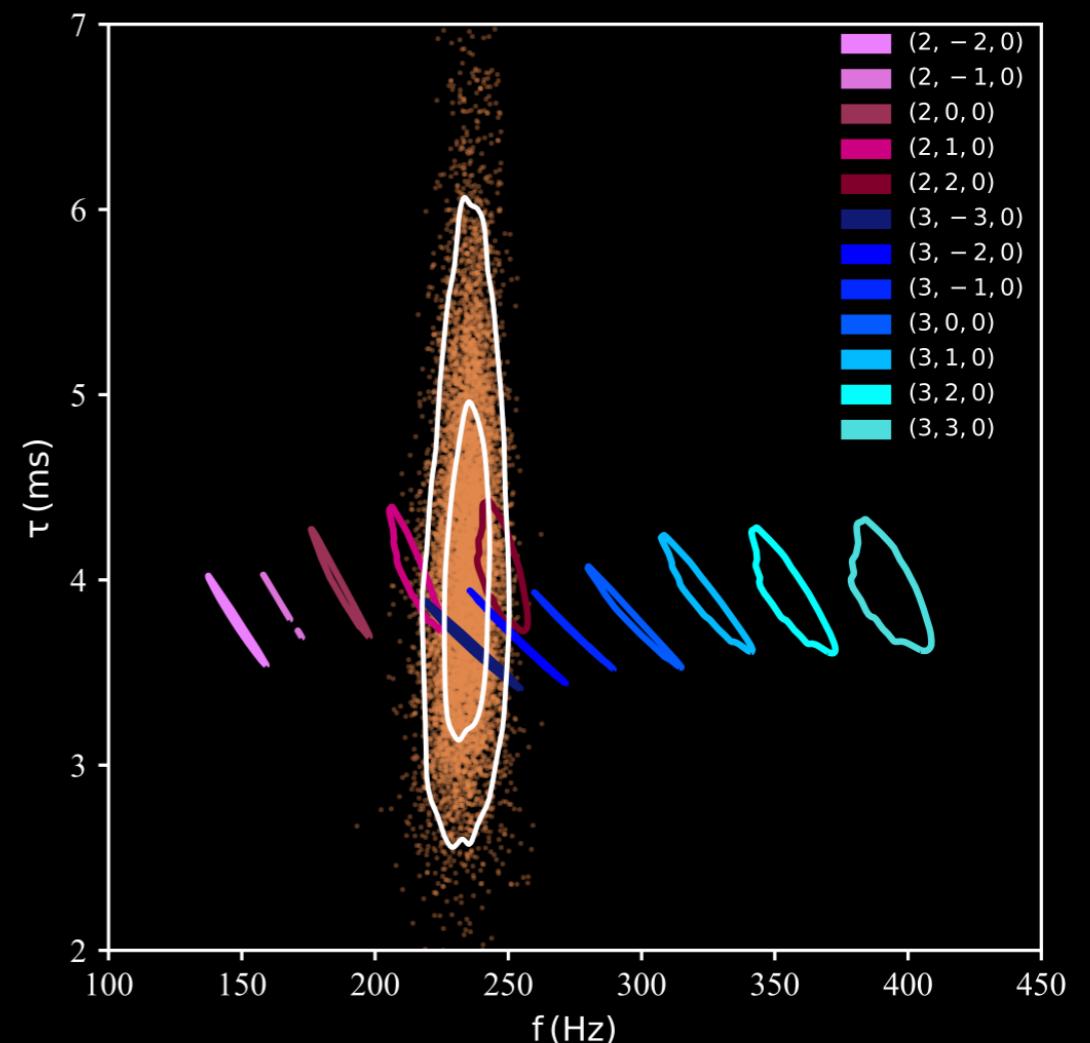


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In reality...



Carullo et al, PRD (2019)

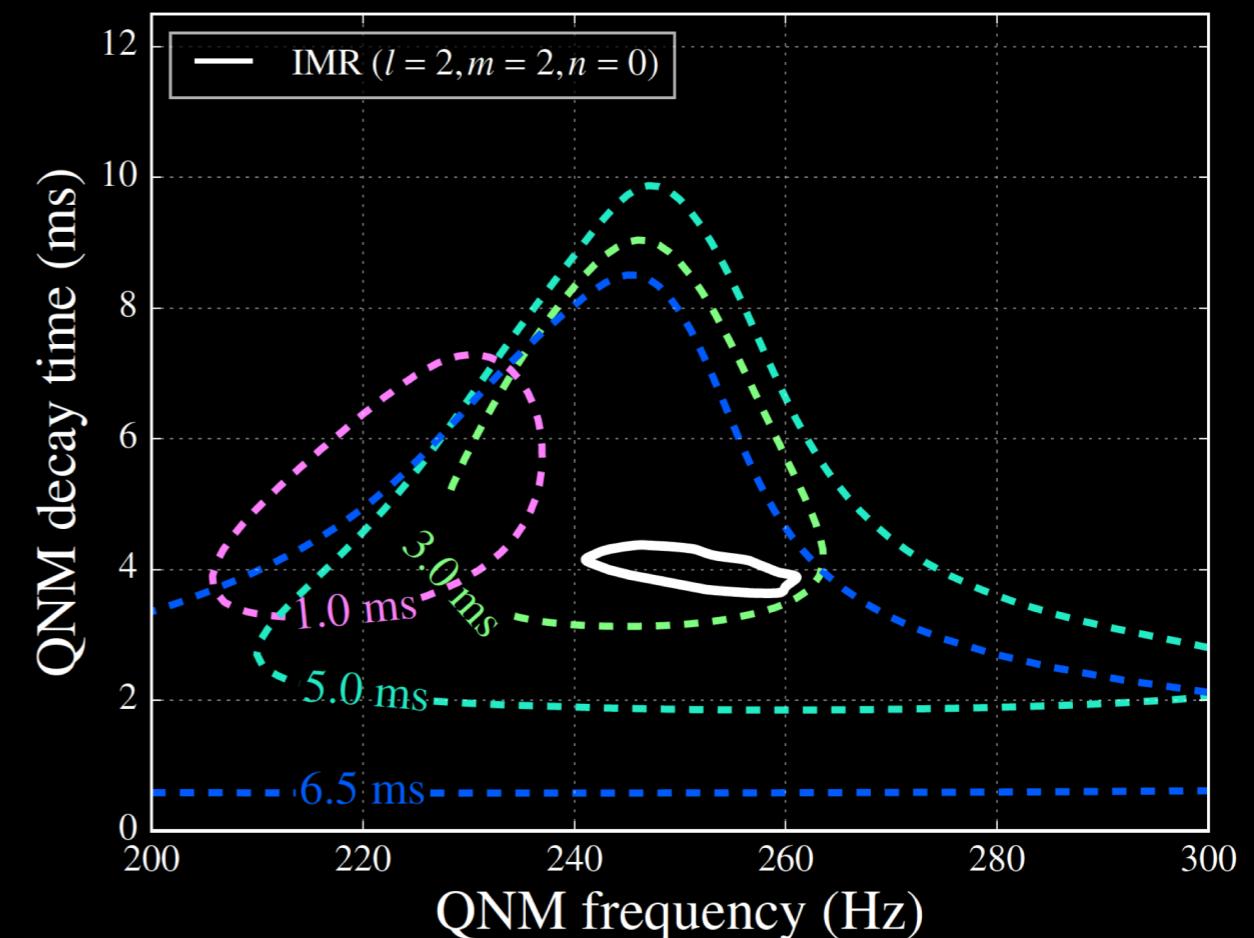
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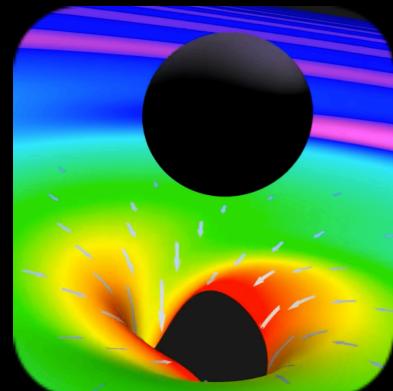
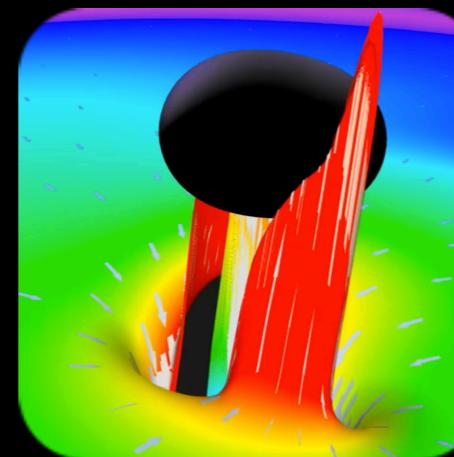


LVC, PRL 116, 221101 (2016)

DYNAMICAL VS PERTURBATIVE

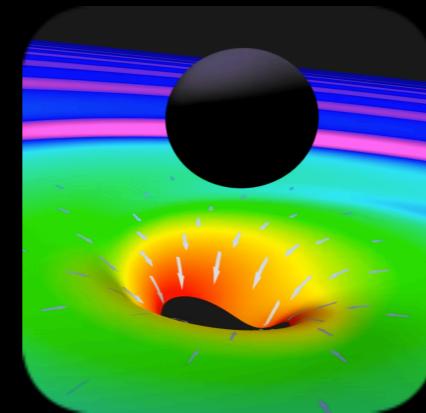
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Highly non-linear regime.

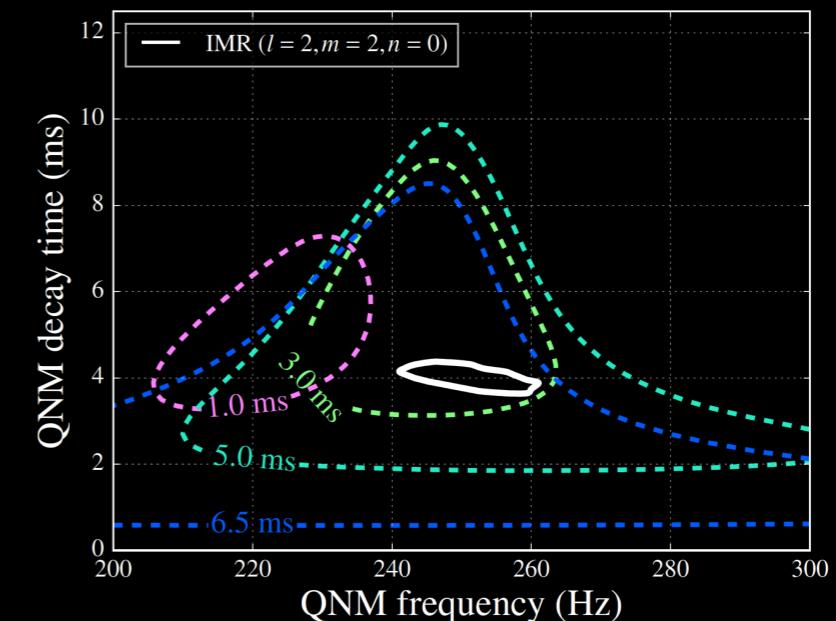


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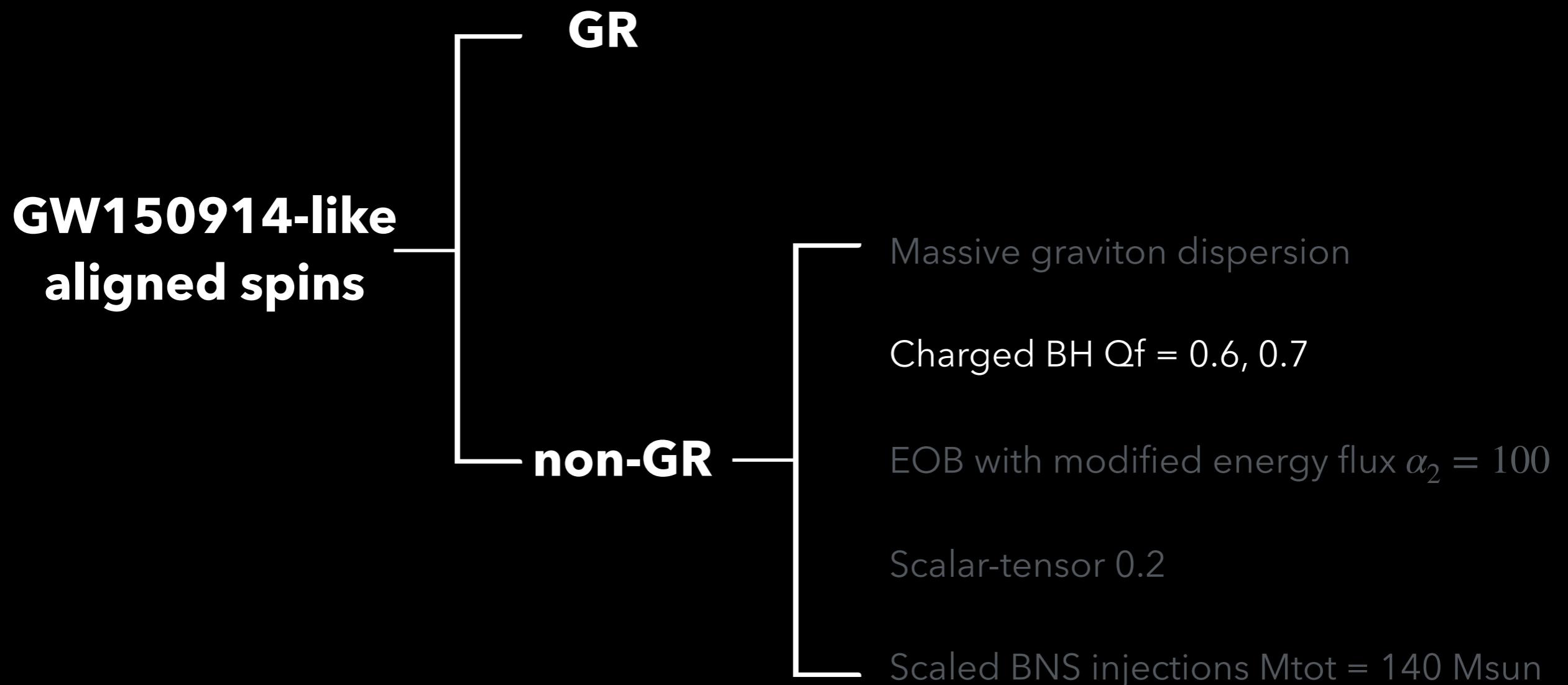
WHERE IS THE NON-LINEAR DESCRIPTION?



At *early times* non-linearities are non-negligible.

At *late times* QNMs description works better but SNR goes down.

THE DATA



THE MODELS

AGNOSTIC
INFORMED

$$t_0 = [8-15]M_f$$

$$t_0 = [13-20]M_f$$

$$t_0 = [-6,6]M_f$$

GR

GR + deviation

Damped Sinusoids

- 1 mode
- 2 modes

Kerr (non precessing)

- 220
 - 220+210
 - 220+221
 - 220+330
- Free Amplitudes
Ordered Amplitudes

MMRDNP [LONDON, 2020]

- 220
- HM (up to $\ell=4$)

TEOBPM [DAMOUR & NAGAR, 2014]

- 220
- 220+210
- 220+330
- 220+210+330

$+ \delta\omega$

$+ \delta\tau$

$+ \delta\omega\delta\tau$

GR SYSTEMATICS

SPIN ALIGNED

Agnostic superposition of damped sinusoids:

$$h_s = \sum_j A_j^s \cos(\omega_j^s t + \phi_j^s) e^{-(t-t_j^{0,s})/\tau_j^s}$$

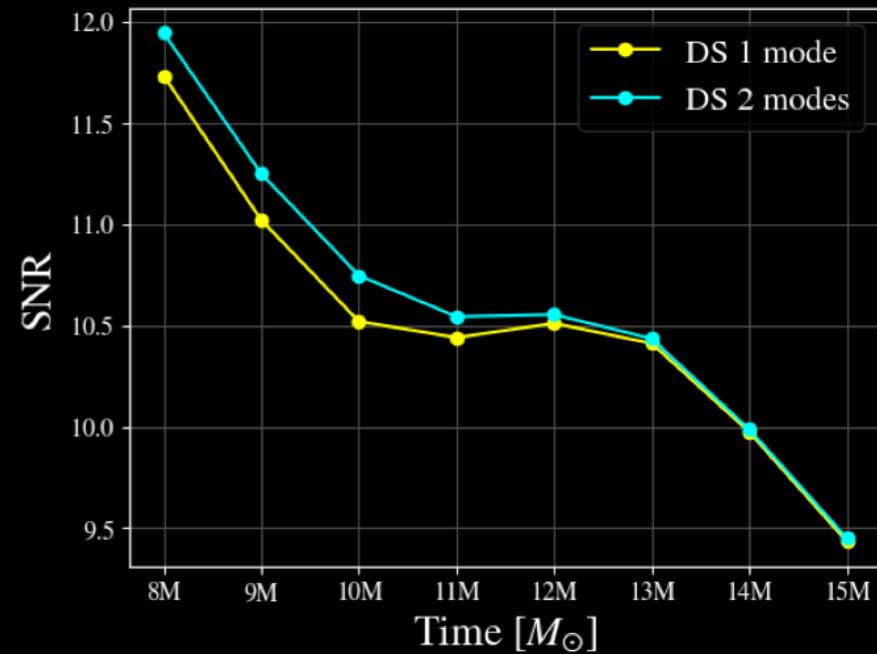
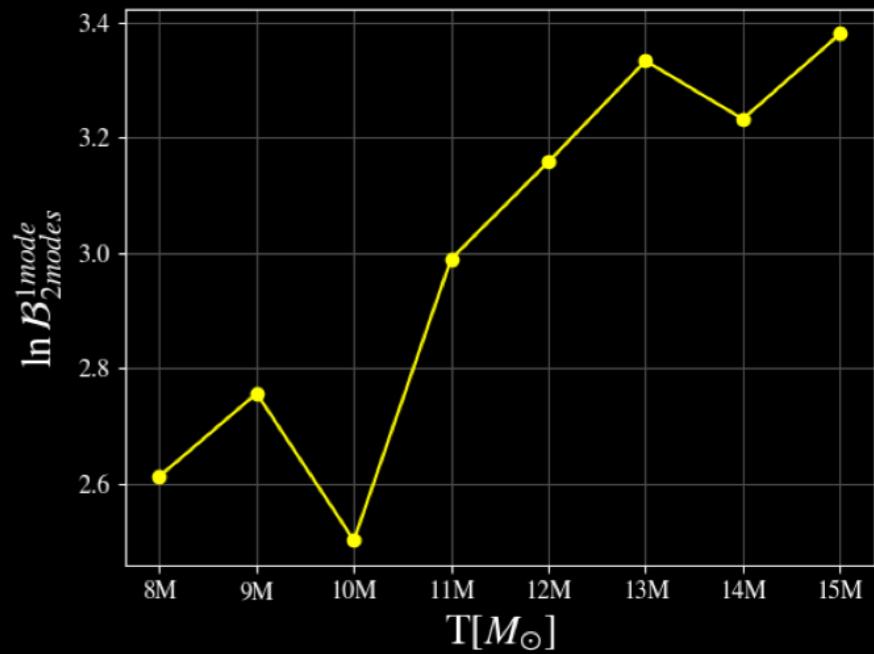
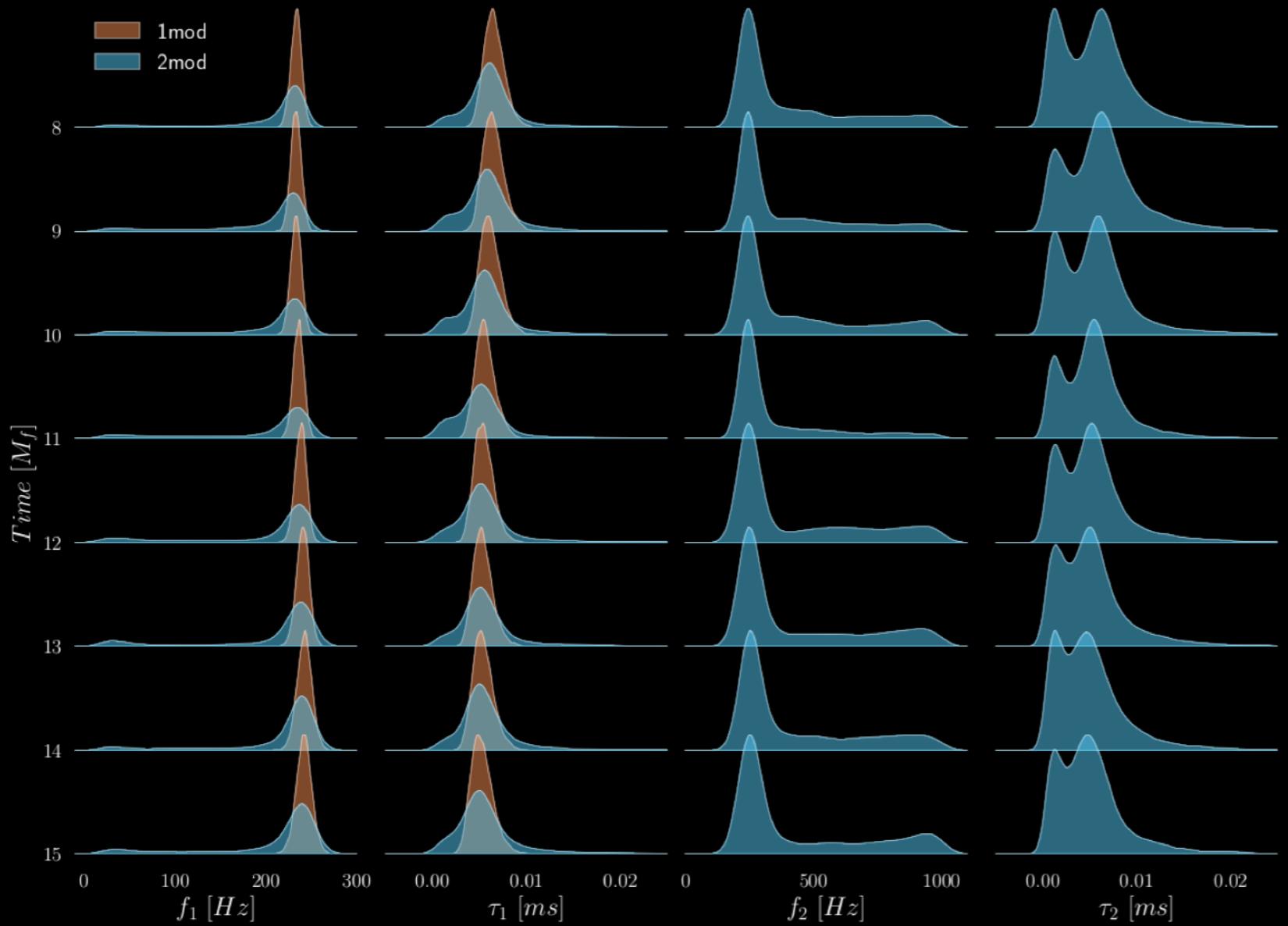
Each mode (fixed j, s) has $\{A, \phi, f, \tau\}$ are free to vary (assuming t^0 known).

All pyRing models available for generic (scalar, vector, tensor) polarisations: s -index.

SYSTEMATICS

DS - GR recovery

Only one mode can be resolved.



All plots are made using [namib](#).

SYSTEMATICS

Kerr

Model with Kerr spectrum predictions:

non precessing
reflection symmetry implies
 $\tilde{A}^- = (-1)^l \cdot \tilde{A}^+ *$
 where:
 $\tilde{A}^\pm = \mathcal{A}_{lmn}^\pm e^{i\phi_{lmn}^\pm}$

$$h_+ + i h_\times = \frac{M_f}{D_L} \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{+\ell} \sum_{n=0}^{\infty} (h_{\ell mn}^+ + h_{\ell mn}^-)$$

$$\begin{aligned} h_{\ell mn}^+ &= \mathcal{A}_{\ell mn}^+ S_{\ell mn}(\iota, \varphi) e^{i[(t-t_{\ell mn})\tilde{\omega}_{\ell mn} + \phi_{\ell mn}^+]} \\ h_{\ell mn}^- &= \mathcal{A}_{\ell mn}^- S_{\ell mn}^*(\pi - \iota, \varphi) e^{-i[(t-t_{\ell mn})\tilde{\omega}_{\ell mn}^* - \phi_{\ell mn}^-]} \end{aligned}$$

Ordered Amplitudes

Add more information, impose semi-agnostic assumptions, e.g.

$$A_{330}/A_{220} < 0.4$$

$$A_{210}/A_{220} < 0.7$$

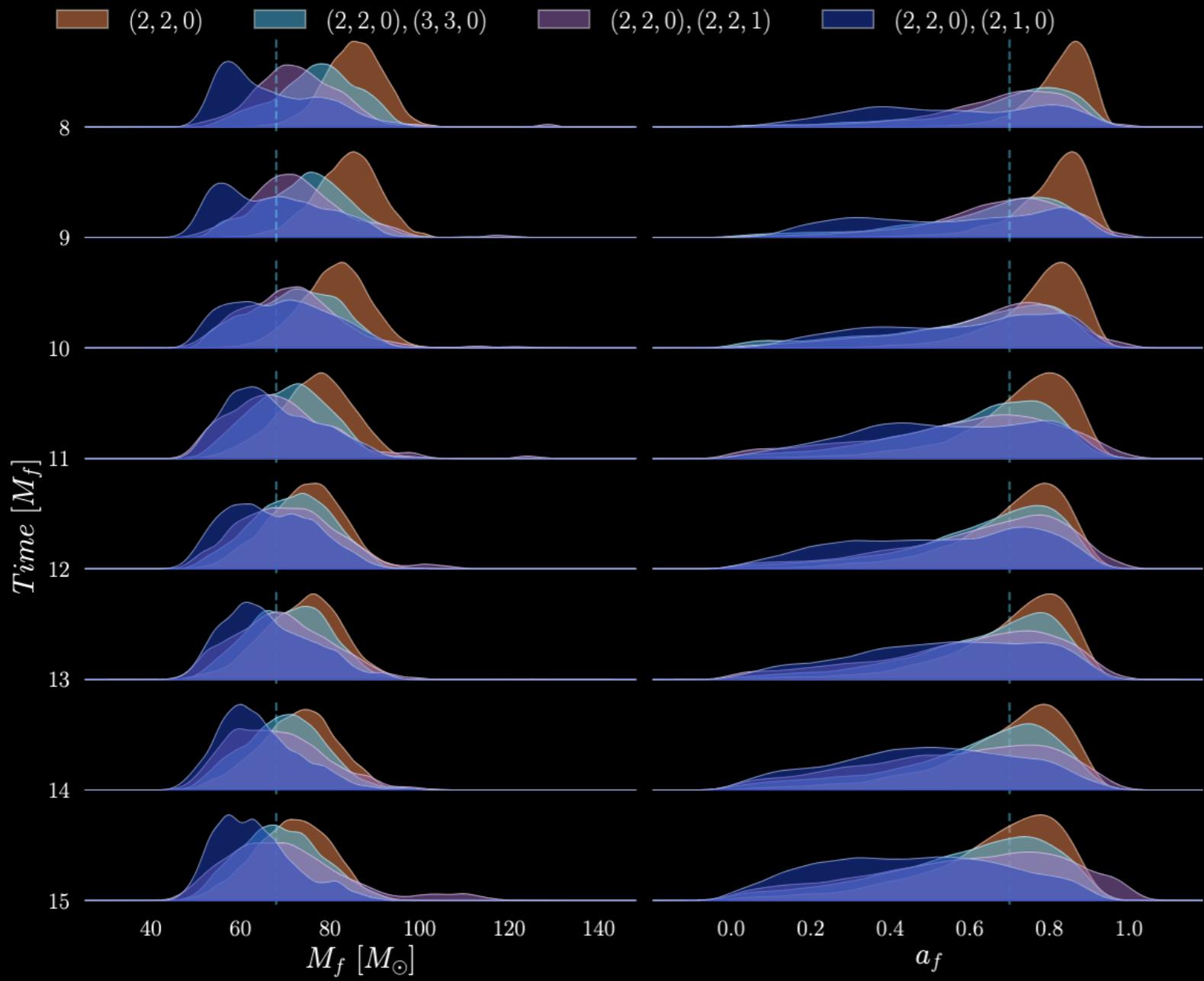
taken from NR

(e.g. Forteza+, 2205.14910
 Capano+, 2105.05238)

Each mode has $\{A_{lmn}, \phi_{lmn}\}$ as free parameters, $\{f_{lmn}(M_f, a_f), \tau_{lmn}(M_f, a_f)\}$ GR predicted.

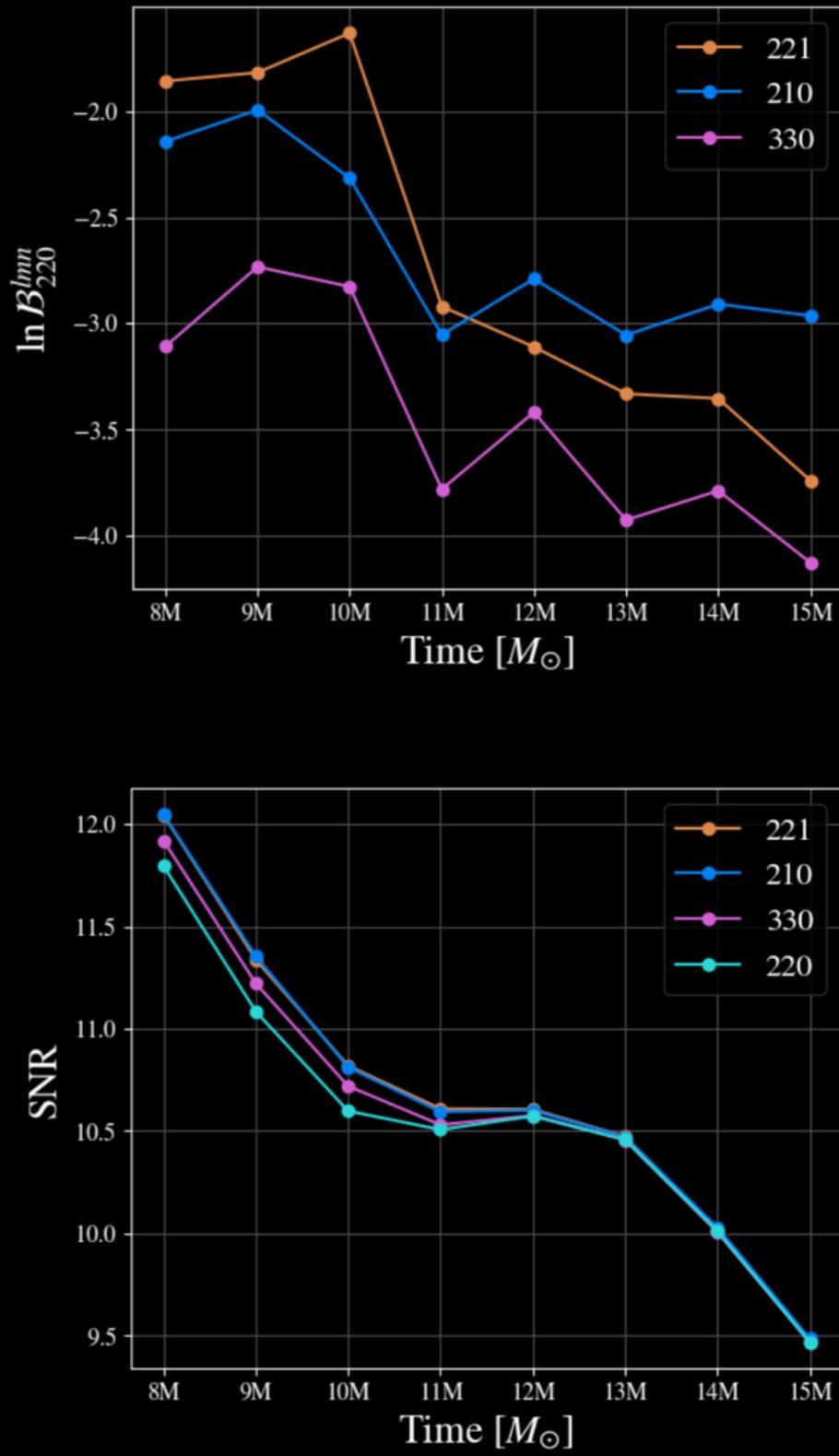
SYSTEMATICS

330 and 221 give the best recovery for M_f and a_f
 However, BF strongly favours 220 across all cases.



Presence of additional modes beyond 220 but not high enough SNR for given q prevents resolution of additional modes.

Kerr (Ordered amplitudes) - GR recovery



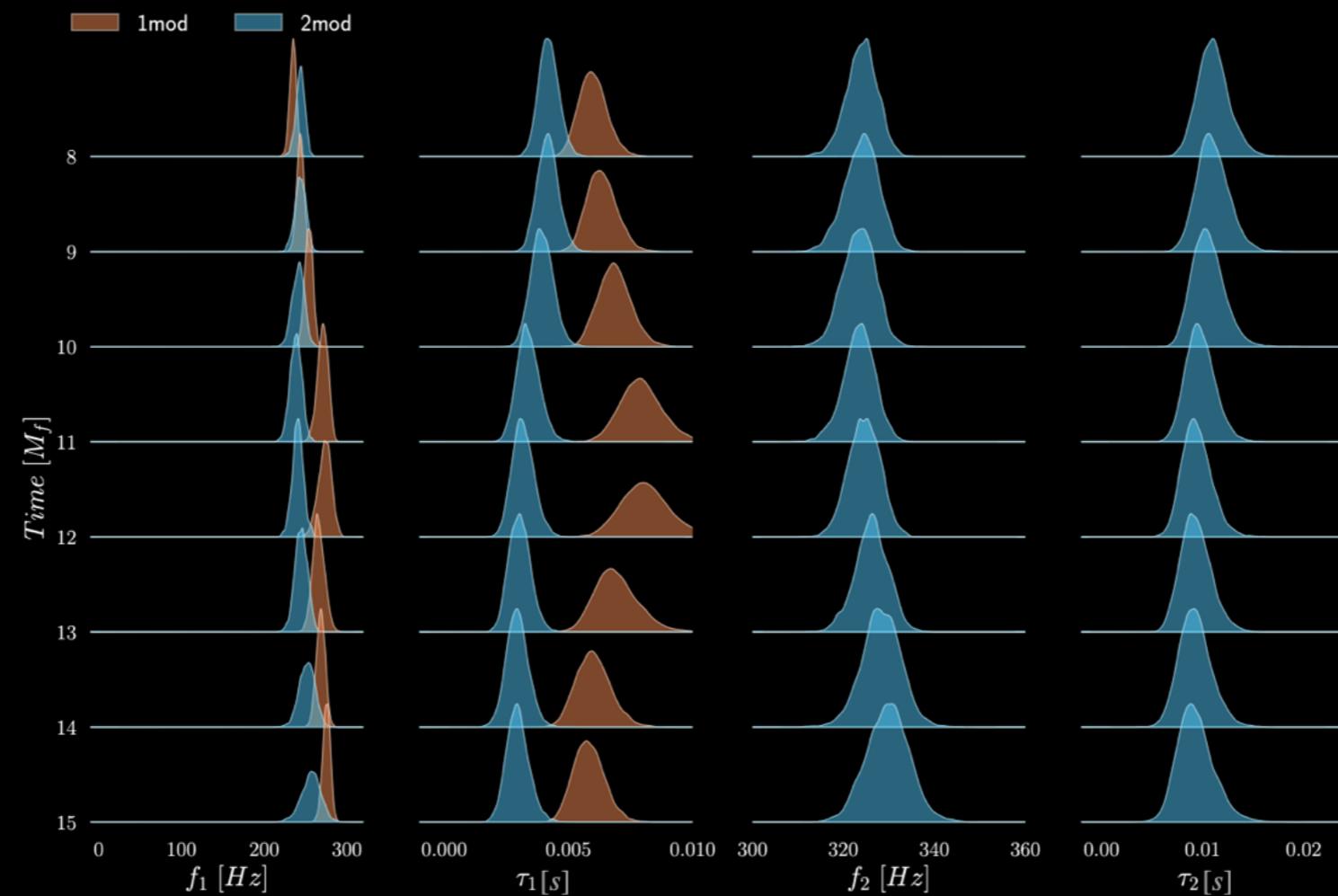


NONGR

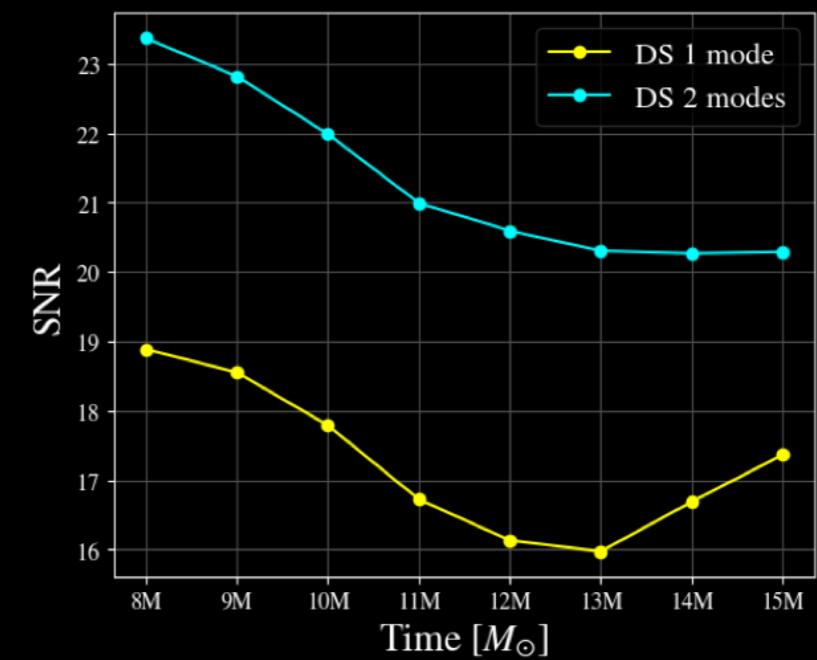
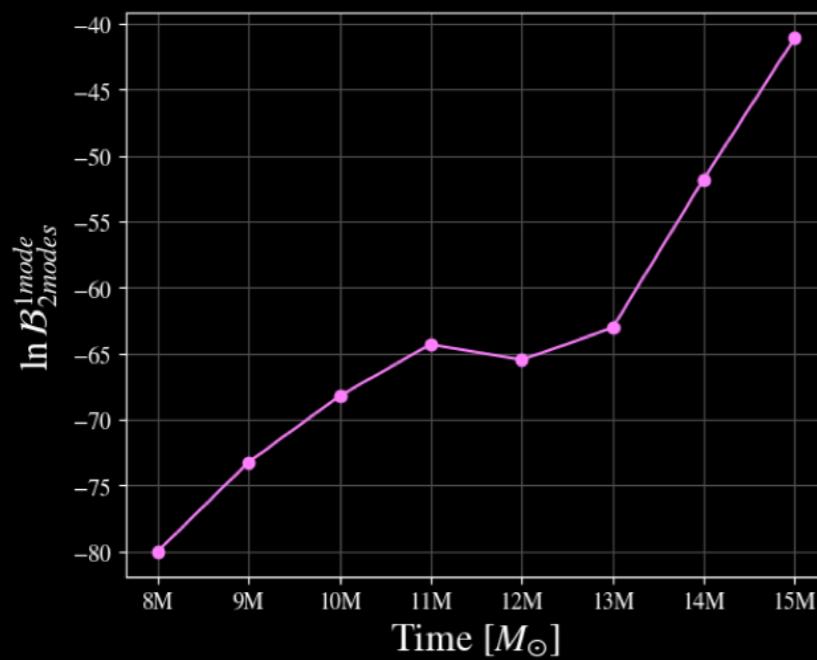
MODIFIED QNM SPECTRUM $Q_F=0.7$

DS - GR recovery

Presence of 2 modes strongly favoured*.

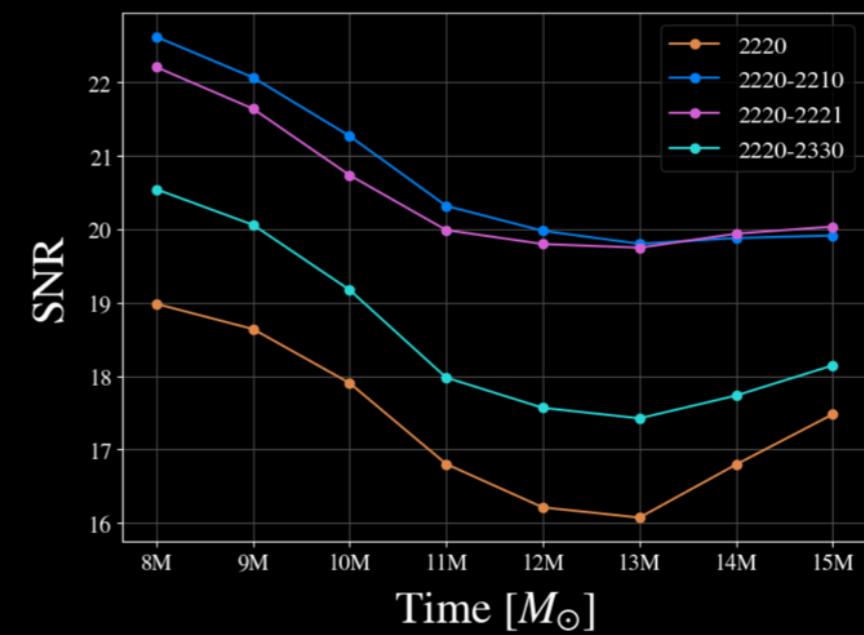
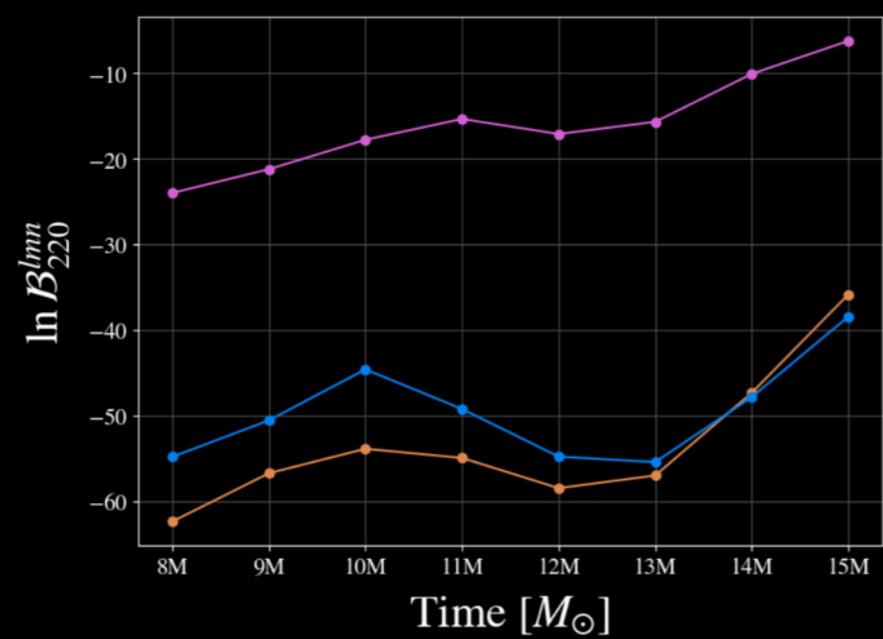
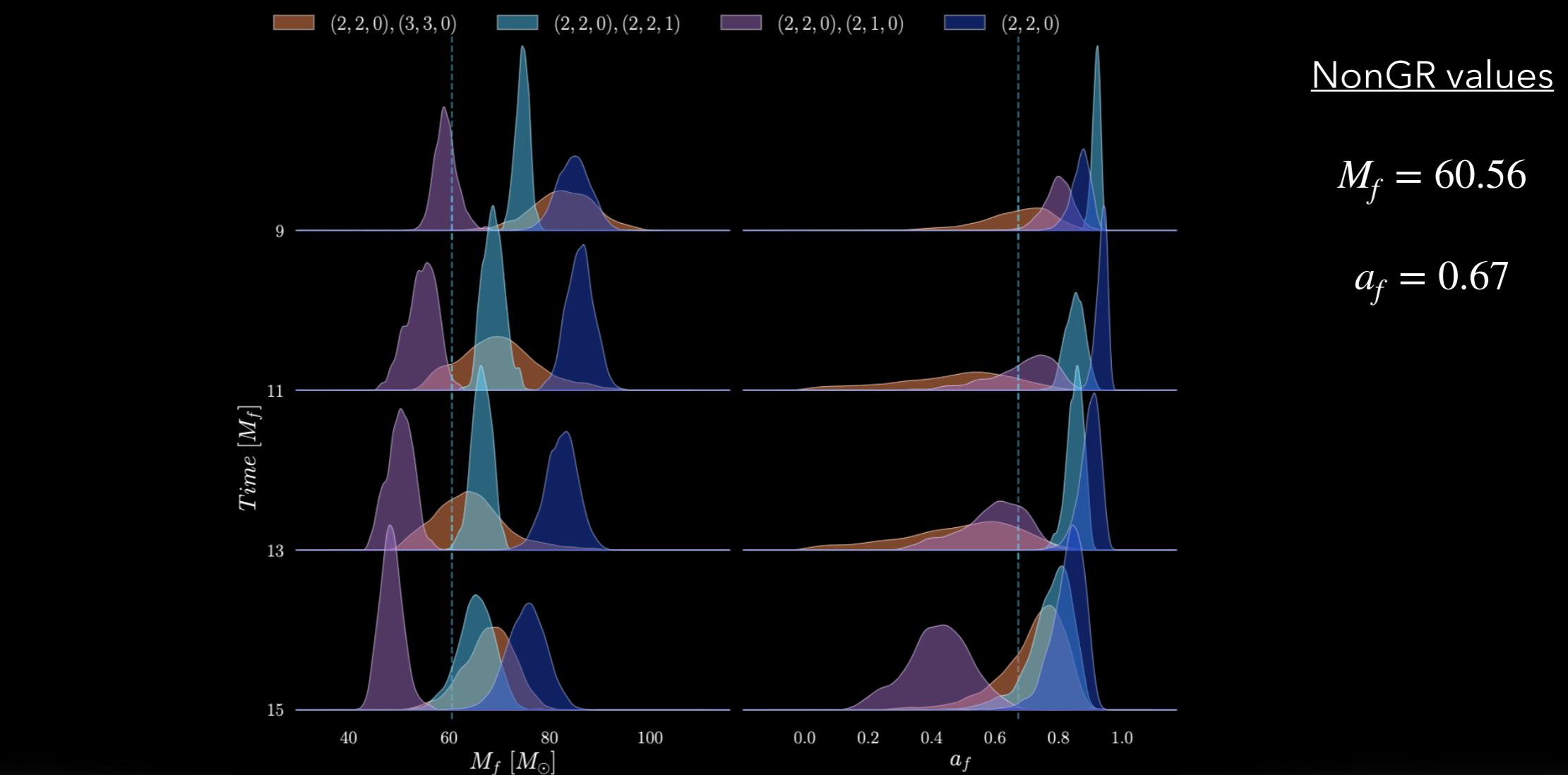


*3-modes analysis shows no evidence of additional mode.



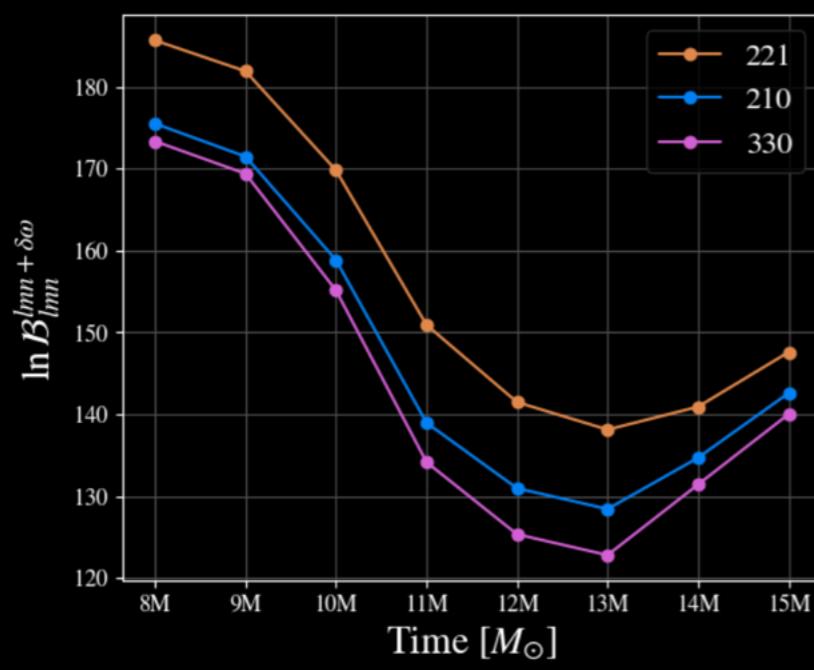
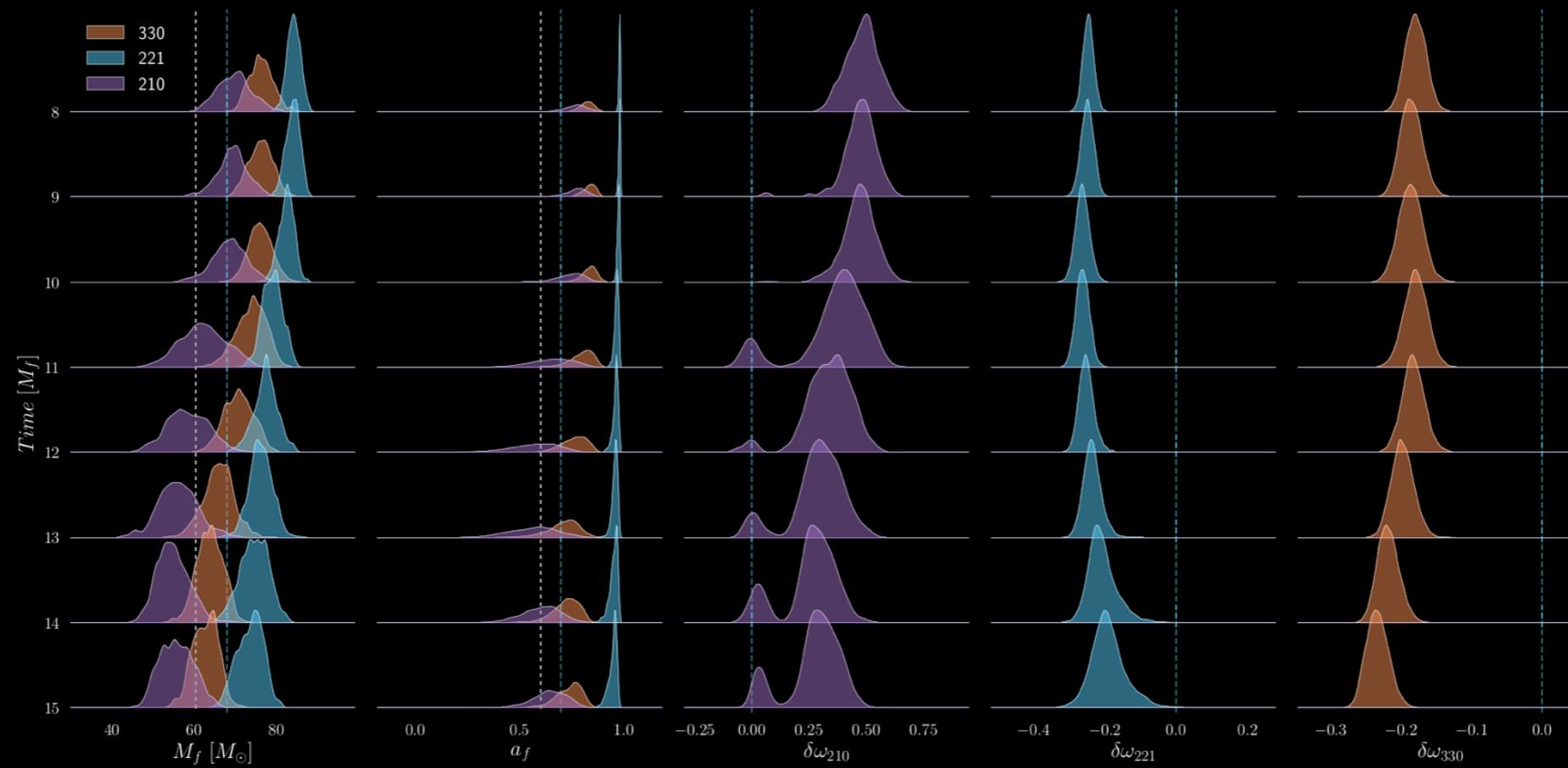
MODIFIED QNM SPECTRUM QF=0.7

Kerr (OA) - GR recovery

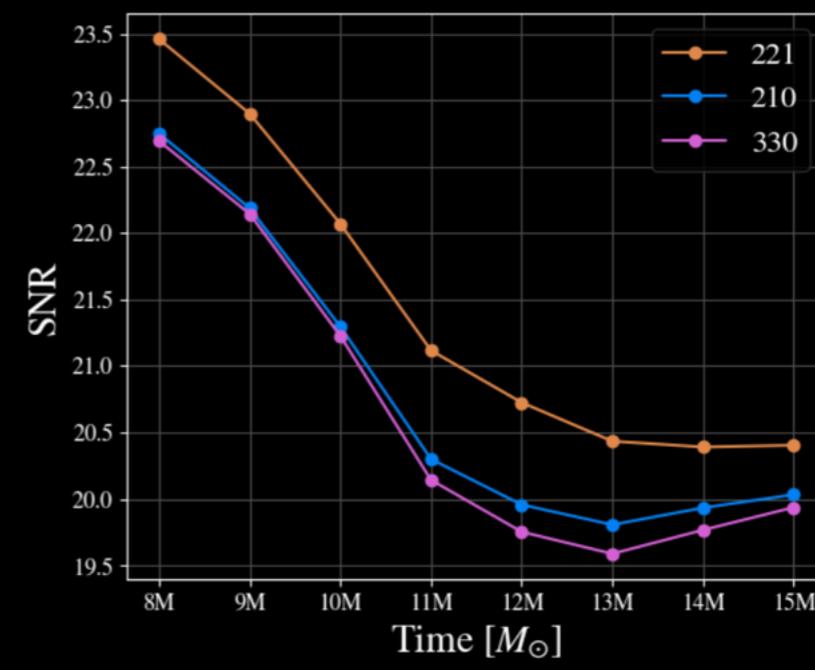


MODIFIED QNM SPECTRUM $QF=0.7$

Kerr - GR + DEV recovery (OA)



nonGR
GR



CONCLUSIONS

Systematics

- No evidence of HMs in the data;
- PE is biased at early times for 220 due to non-linearities being important;
- SNR not high enough to resolve additional modes.

NonGR

- Deviation from GR recovered only in the case of the modified QNM spectrum (both for charge of 0.6 and 0.7);
- Modification of GR might extend non-linearities regime.